



# THE CCD RIDDLE REVISTED: SIGNAL VERSUS TIME – LINEAR SIGNAL VERSUS VARIANCE – NON-LINEAR

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## Abstract

The photon transfer curve is one of the most valuable tools for calibrating, characterizing, and optimizing the performance of CCDs. Its primary purpose is to determine the conversion gain of the CCD system from which many of the other performance parameters such as read noise, dark current, QE, full well etc. are determined. Non linearity in the photon transfer curve has been reported by the author in previous papers and confirmed by others. Previous studies isolated the source of the non linearity to the CCD image area. Spatial autocorrelation analysis showed that the mechanism behind the non linearity was due to a "sharing of charge" between pixels in the image area which increases with signal level. This paper reports on further investigations carried out to explain the mechanism behind the non-linearity.

## Introduction

The photon transfer curve (PTC) is one of the most widely used techniques to determine the end-to-end conversion gain (e-/ADU) of a camera system. As it is simple to use (only requires the taking of progressive increasing time series of two flats at constant illumination) and does not require complicated or specialized equipment, it is widely used at the telescope to check the health of Charge Coupled Devices (CCDs) and Complementary Metal-Oxide-Semiconductor (CMOS) imagers and their camera system. Most other parameters such as read noise, dark current, quantum efficiency (QE), and full well are determined using this conversion gain.

The method relies on photon events being detected in a statistically independent way by the imager such that the characteristic shot noise of the light source is maintained and thus for a linear (conversion of photons to ADU) system, the mean signal (S) versus variance is related by a constant system conversion gain as follows:

$$K(e^- / ADU) = \frac{S(ADU)}{Var(ADU)^2}$$

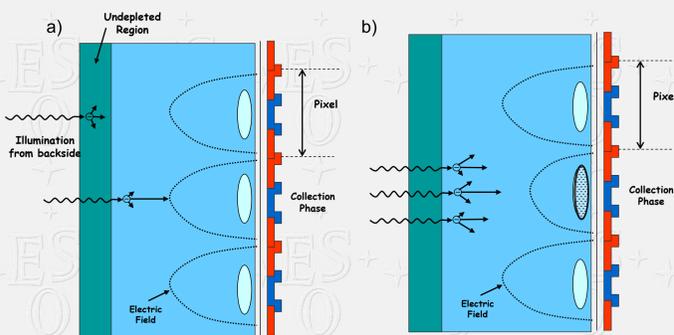
Previous studies by the author have shown this is not the case in reality and non-linearity in excess of 20% have been observed on backside illuminated CCDs even though having excellent signal linearity. In addition, the non-linearity is greater for thicker device made from higher resistivity silicon. An investigation to locate the cause of the non-linearity concluded that it was due to the amount of charge collected within a pixel and not due to lateral diffusion of charge in the undepleted region at the back of the imager, the clocking or transport of charge in the image area or serial register to the read out, the output amplifier, or the detector electronics.

Spatial autocorrelation analysis showed that the mechanism behind the non-linearity was due to correlation, a sharing of charge, between pixels and this increases linearly with signal level.

This paper continues the investigation by providing evidence to support the theory that the effect is due to a change of "sharing of charge" amongst neighboring pixels from the time an electron first experiences the electric fields of the pixels and until it is captured in the potential well of a pixel.

## "CHARGE SHARING" MECHANISM

The apparent non-linearity seen in the PTC is not due to just the "sharing of charge" between pixels, but to the change in this sharing.

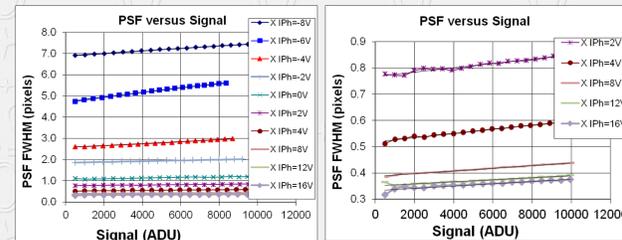


Cross-section of backside illuminated CCD illustrating the mechanism of change in "charge sharing".

In Figure a) above, the pixels have not yet collected any charge and the electric field from each pixel extends evenly towards the backside to collect charge. When an electron is generated, it will drift towards the potential wells under the influence of the electric fields and be collected. In Figure b), the middle pixel has collected much more charge than its neighbors and its electric field does not extend as far as the others and thus an electron will have less probability to be collected by the middle pixel and more probability to be collected by one of its neighbors. The direction arrows attached to the electrons can be considered as probability vectors for illustration purposes. From past studies we know that the effect is not dependent on the thickness of the undepleted region at the backside of the CCD. This can be easily explained by this model. There is no electric field within the undepleted region thus it cannot change.

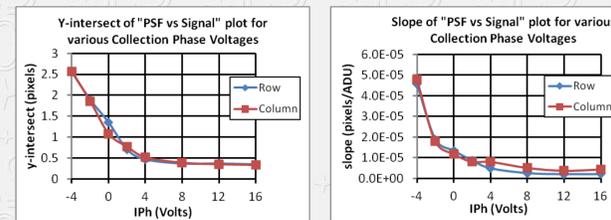
## POINT SPREAD FUNCTION (PSF)

If charge is shared between pixels and the sharing process increases linearly with signal then one would expect also the PSF of the device to vary with signal level.



PSF changing with signal level indicates that the thickness of the depleted region is reduced as electrons build up in the pixel.

The results of measurements on the Deep Depletion CCD220 clearly shows the PSF increasing linearly with signal level and this change with signal becoming less as the collection phase voltage is increased. Increasing the collection phase voltage increases the strength of the electric field in the depleted region and the drift velocity of the electrons. With a higher drift velocity, electrons have less time to 'wander' and end up in neighboring pixels.

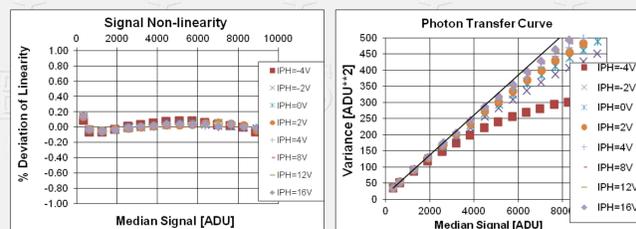


Both vary as  $\sqrt{\text{collection phase voltage}}$  as per  $PSF \approx 2.2 \sqrt{\frac{2kT}{q} \frac{x_{THICK}}{V_{IP}}}$

The plots of the slope and y-intercept of the linear fit of the PSF curves show similar square root relationship in agreement with the equation that describe the PSF, where  $x_{THICK}$  is the thickness of the depleted region and  $V_{IP}$  is voltage across the device. The linear change of PSF with signal indicate that the thickness of the depleted region is reduced as electrons build up in the pixel further validating the model.

## PHOTON TRANSFER CURVE (PTC)

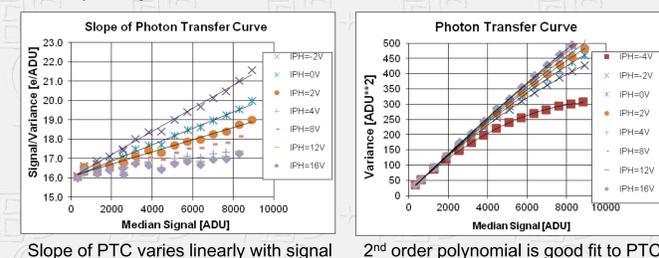
To compare results, the PTC was determined under the same conditions as the PSF



Excellent signal linearity

PTC – black trend line added to highlight non-linearity.

The plot of signal linearity show the device to be very linear at all collection phase voltages. However, as expected, the PTC is non-linear and the non-linearity decreases as the collection phase voltage is increased. The black line shows the expected results if the PTC was completely linear.



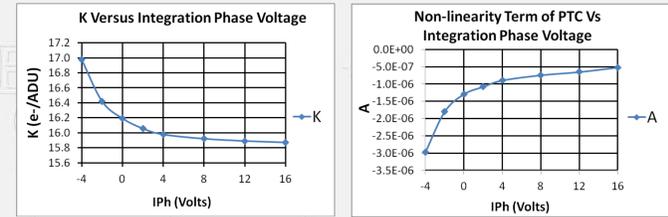
Slope of PTC varies linearly with signal

2<sup>nd</sup> order polynomial is good fit to PTC

The slope of the PTC varies linearly with signal. Thus a 2<sup>nd</sup> order polynomial,

$$Var = A \times S^2 + B \times S + C$$

must be a good fit to the PTC as shown above. From a previous study, it was suggested that the y-intercept of the slope of the PTC was a better estimate of the system gain, K e-/ADU, thus the B term of the polynomial fit can be considered a gain term (1/K) and the A term a measure of non-linearity.



K = 1/B term of the polynomial fit of PTC.

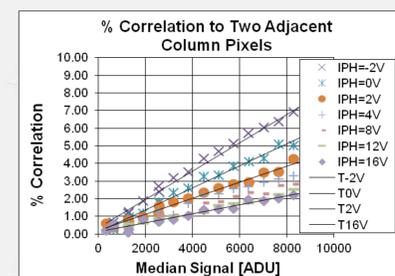
A "the non-linearity" term of the polynomial fit of PTC

Both vary as  $\sqrt{\text{collection phase voltage}}$

The plots of the PTC polynomial fit terms versus collection phase voltage show a square root relationship similar to that of the PSF results. This provides strong evidence that the same mechanism, increase in charge sharing with signal, is at play. Knowing the relationship between the non-linearity in the PTC and the change in the PSF with signal, one could then use this to apply corrections to the PSF of image data when high accuracy is required such as for ultra stable spectrometry and high resolution imagery without the need to have measured the PSF.

## AUTOCORRELATION ANALYSIS

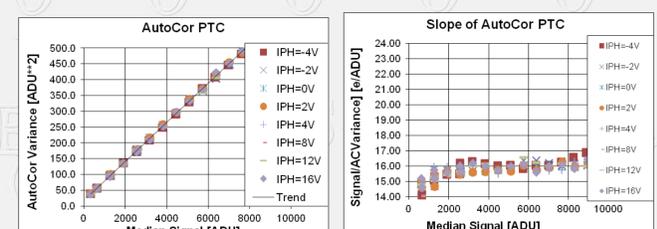
Autocorrelation analysis was performed on the same set of data as used for the PTC.



Correlation between pixels increase with signal level.

Results shows correlation between pixels as expected and that this correlation increases linearly with signal. Comparing this figure to the slope of the PTC one notes the similarity. Note the correlation results above must be multiplied by 4 to include all adjacent pixels.

Conclusive evidence that the correlation between pixels is the major factor in the non-linearity of the PTC is obtained when one uses the autocorrelation variance in determining the PTC instead of the normal variance. The autocorrelation variance takes into account the cross terms between pixels and thus the correlation; whereas the normal variance assumes no correlation and is a simple sum of the difference of pixel values squared.



PTC is now linear.

Slope of PTC is flat

PTC is now linear and gain can be accurately calculated

The plots of the PTC using the autocorrelation variance is very linear and is very almost identical for all collection phase voltages. The slope is flat and the gain can be easily calculated; in this case 16e-/ADU.

## CONCLUSION

A mechanism has been presented to describe the non-linearity seen in the PTC. Results of measurements of PSF, the PTC, and autocorrelation versus signal and collection phase voltage have validated this model.