

# VLTI Tutorial

## Introduction to Stellar Interferometry

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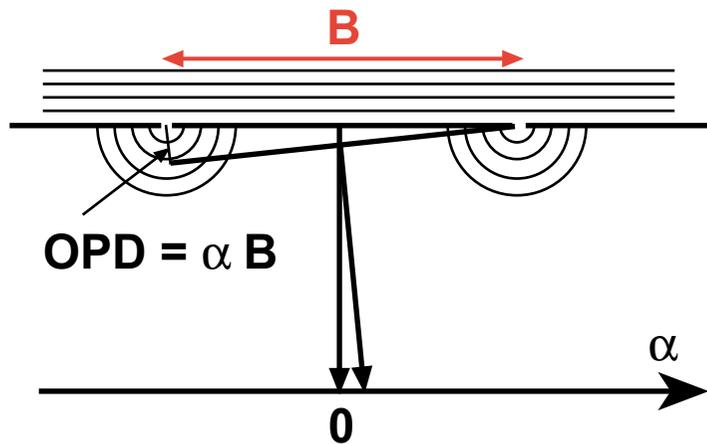


# Project 'VLT Tutorial'

## Milestones:

- Principle of Interferometry: concept of coherence length and spatial coherence, van Cittert-Zernike Theorem
- Types of Interferometers: Michelson and Fizeau, Homothetic mapping, measurement of the visibility
- Atmospheric turbulence: What does AO do for interferometry, why do we need a fringe tracker and PRIMA?
- Interpretation of VINCI data

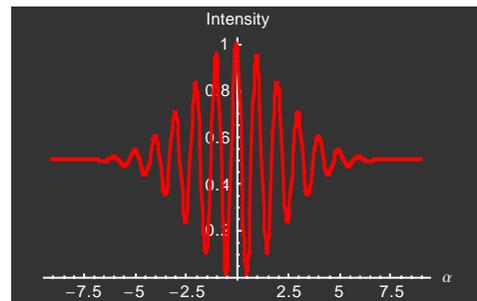
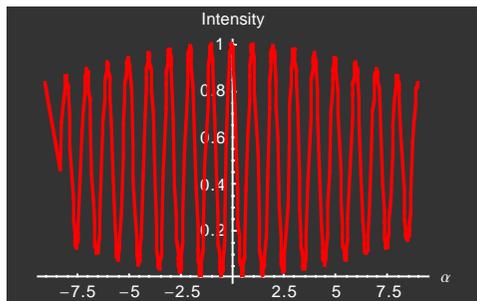
# Young's Two-Pinhole Experiment



**1st Min:  $OPD = \lambda/2 \Rightarrow \alpha_{\min} = \lambda/(2B)$**

- Light source at infinity at  $\alpha = 0$
- Intensity pattern  $\sim 1 + \cos$  as a function of  $\alpha$ , period length:  $\lambda/B$
- $OPD >$  coherence length  $\Rightarrow$  fringes disappear

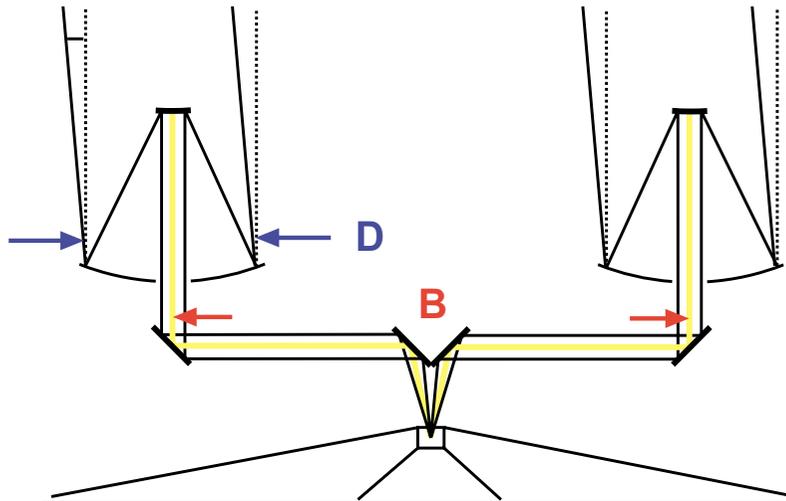
Light source at angle  $\alpha_0$   
 $\Rightarrow$  fringe pattern shifts accordingly



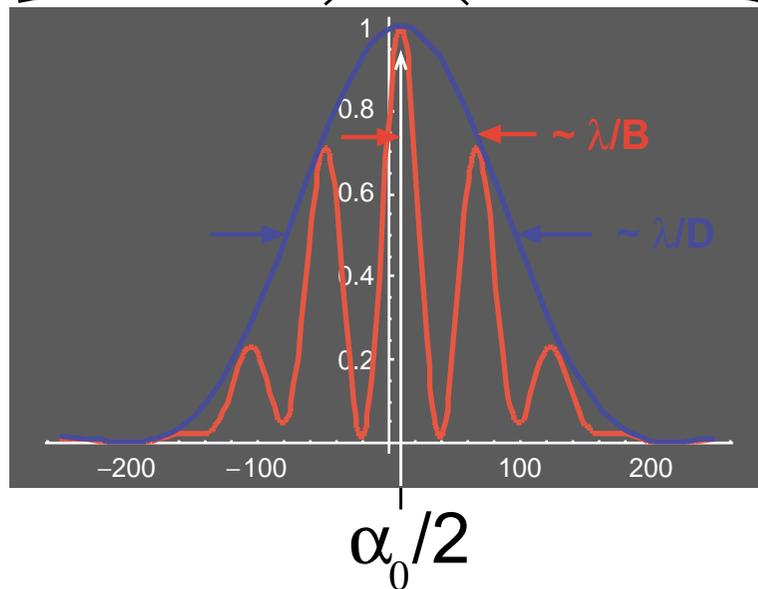
(First and last picture of a movie)

# Michelson stellar interferometer

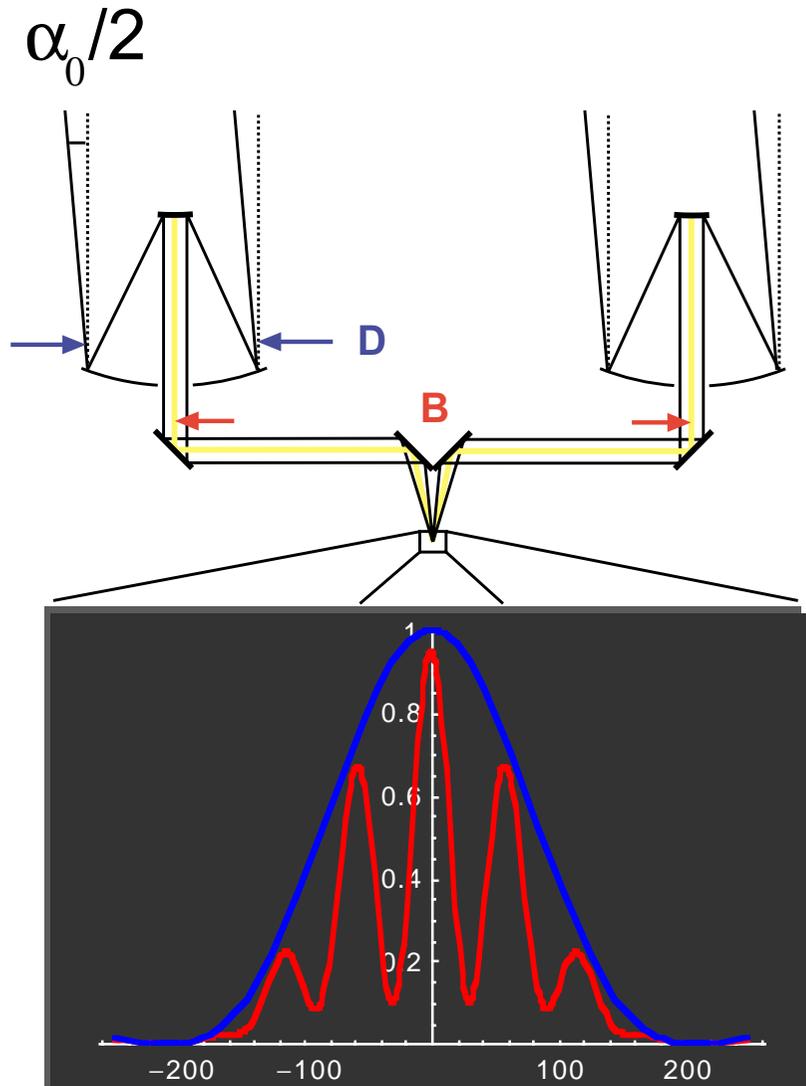
$\alpha_0/2$



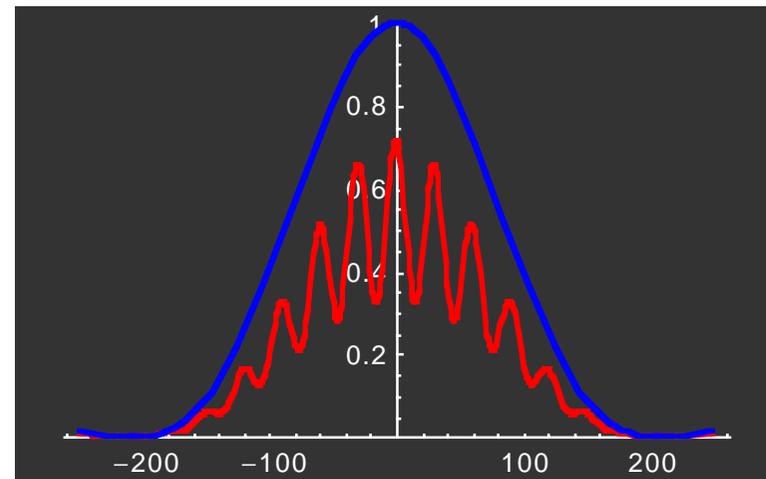
- Stellar source with angular size  $\alpha_0$
- Add fringe patterns (i.e. intensities) between  $\pm\alpha_0/2$



# Michelson stellar interferometer

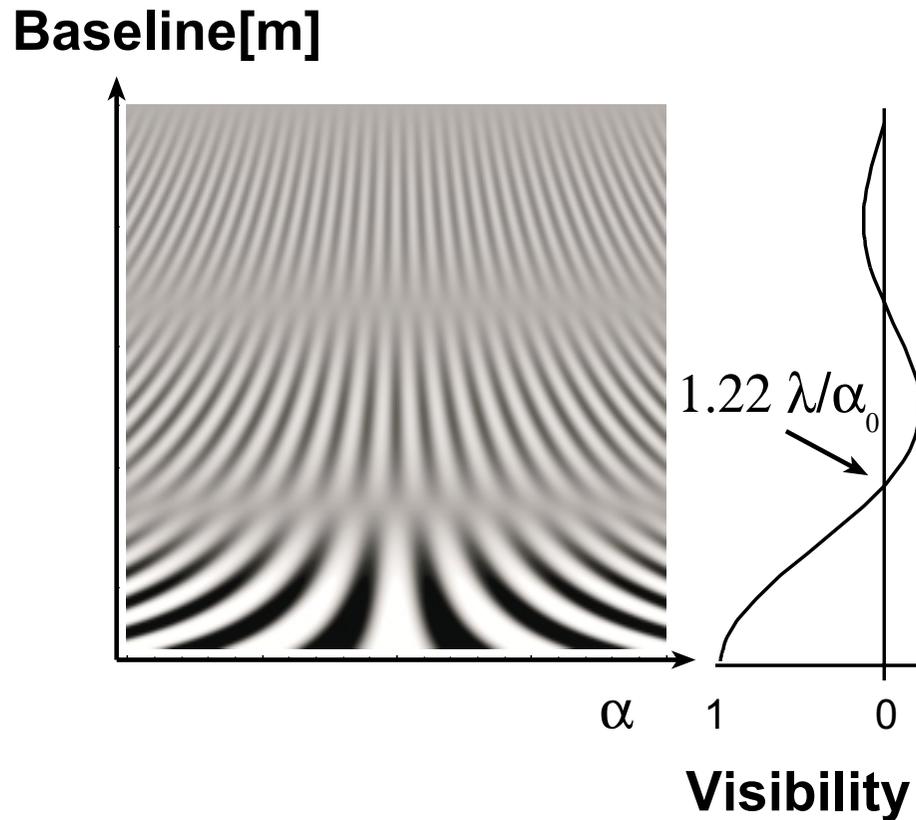


- Stellar source with angular size  $\alpha_0$
- Add fringe patterns (i.e. intensities) between  $\pm\alpha_0/2$
- Resulting fringe pattern shows reduced contrast
- Reduced contrast depends on B – and on  $\alpha_0$



$B_1$  and  $\alpha_0$  (Movies used for illustration)  $B_2 > B_1$  and  $\alpha_0$

# Visibility Function



- Analysing the resulting fringe pattern as a function of  $B$  and  $\alpha_0$  one finds that  $\text{Visibility}(B) = \mathcal{F}(I(\alpha))$
- If  $I(\alpha) = \text{Circ}(\alpha/\alpha_0)$   
 $\text{Vis}(B) = \text{Besinc}(\pi\alpha_0 B/\lambda)$

# Van-Cittert-Zernike Theorem

Definition of the coherence function:  $\Gamma(\alpha_1, \alpha_2, \tau) = \langle \Psi(\alpha_1, t+\tau) \Psi^*(\alpha_2, t) \rangle$

Split spatial and temporal terms:  $\Gamma(\alpha_1, \alpha_2, \tau) = \Gamma(\alpha_1, \alpha_2) \mathcal{F}^{-1}(G(v))$

Propagation in space:  $\Gamma(u_1, u_2) = \int \Gamma(\alpha_1, \alpha_2) \exp(ik(r_1 - r_2)) d\alpha_1 d\alpha_2$

Incoherent source (Star):  $\Gamma(\alpha_1, \alpha_2) = I(\alpha_1) \delta(\alpha_1 - \alpha_2)$

Degree of coherence:  $\gamma(u_1, u_2, \tau) = \int I(\alpha) \exp(ik(u_1 - u_2) \cdot \alpha) d\alpha \mathcal{F}^{-1}(G(v))$

This is the measured quantity in a stellar interferometer.

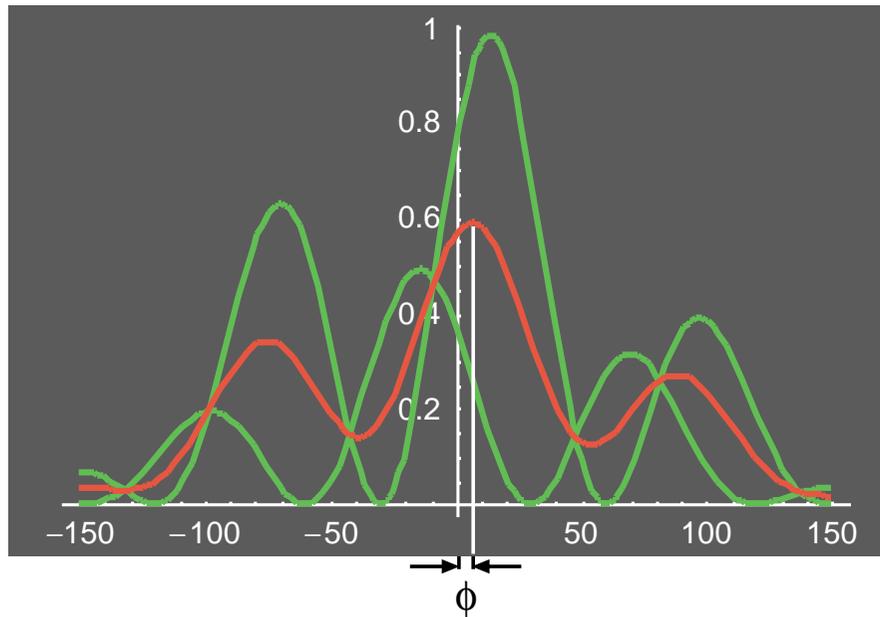
- $\gamma$  is the degree of spatial and temporal coherence, also called the visibility
- $\gamma$  is a function of  $u_1 - u_2$ , the difference vector
- for a point source the spatial coherence is 1, and the contrast is determined by the temporal coherence

# Visibility function

The visibility function  $\gamma$  is complex, with:

Modulus  $V$ : contrast of the fringe pattern

Phase  $\phi$ : Position of white light fringe with respect to  $OPD = 0$



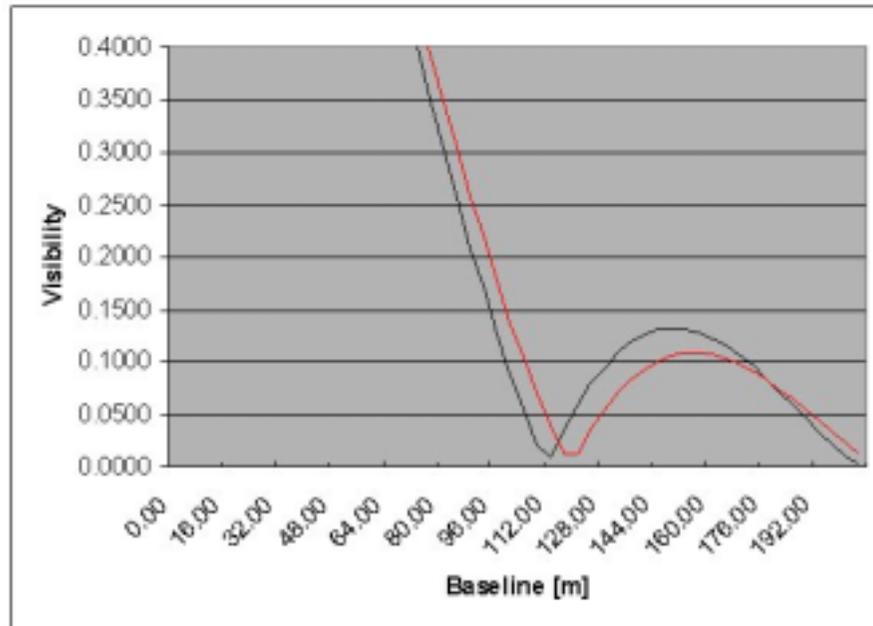
With  $\gamma = \mathcal{F}(I(\alpha))$  it is

$\phi \neq 0$  if  $I(\alpha)$  is non-symmetric  
eg spots on the stellar  
surface, dense clusters etc.

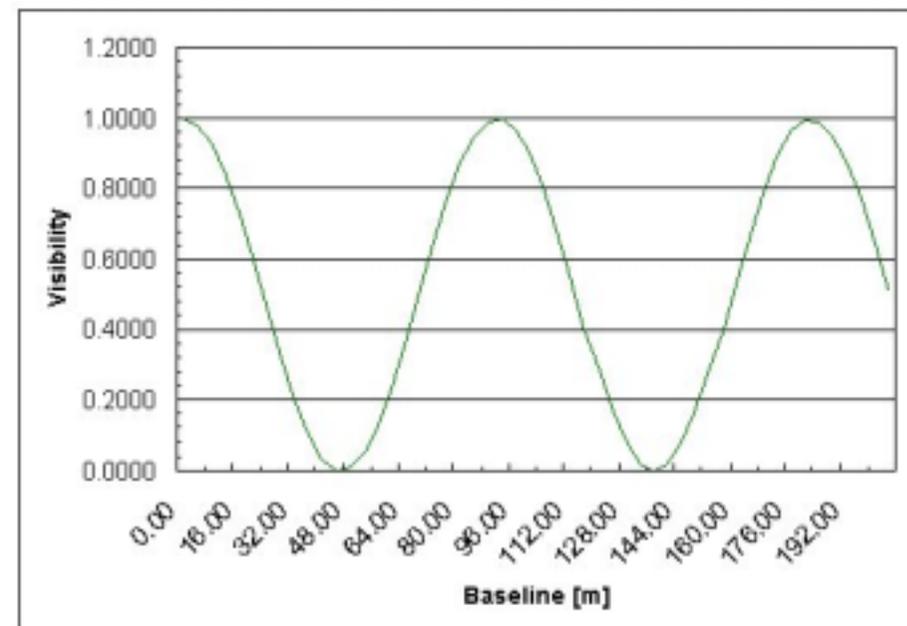
The phase is vital for imaging!

# Examples of visibility functions

Uniform disk + limb darkening:  
(UD diameter 5 milliarcsec)



Binary star:  
(separation 5 milliarcsec)



Note: Do not confuse a binary's visibility function ( $\sim \cos$ ) with the fringe pattern ( $\sim \cos$ ) of an interferometer

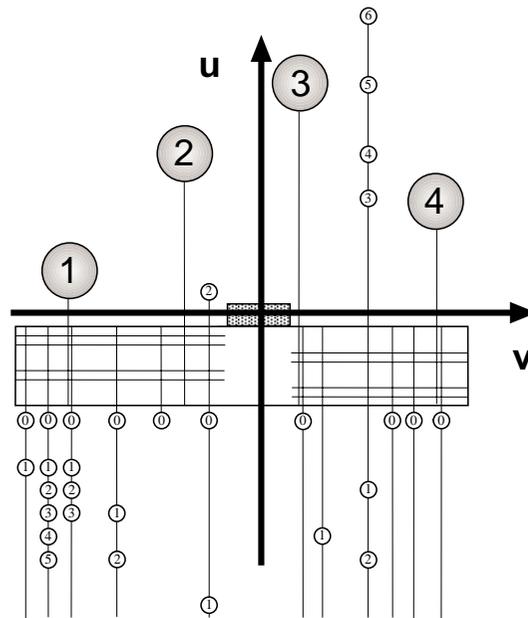
# Interferometric imaging

Image intensity  $I_{im}(\alpha) = \mathcal{F}(\Gamma(u_1 - u_2))$ , with  $u_1 - u_2 =$  Baseline vector

Fill factor in the uv plane determines 'smoothness' of the image

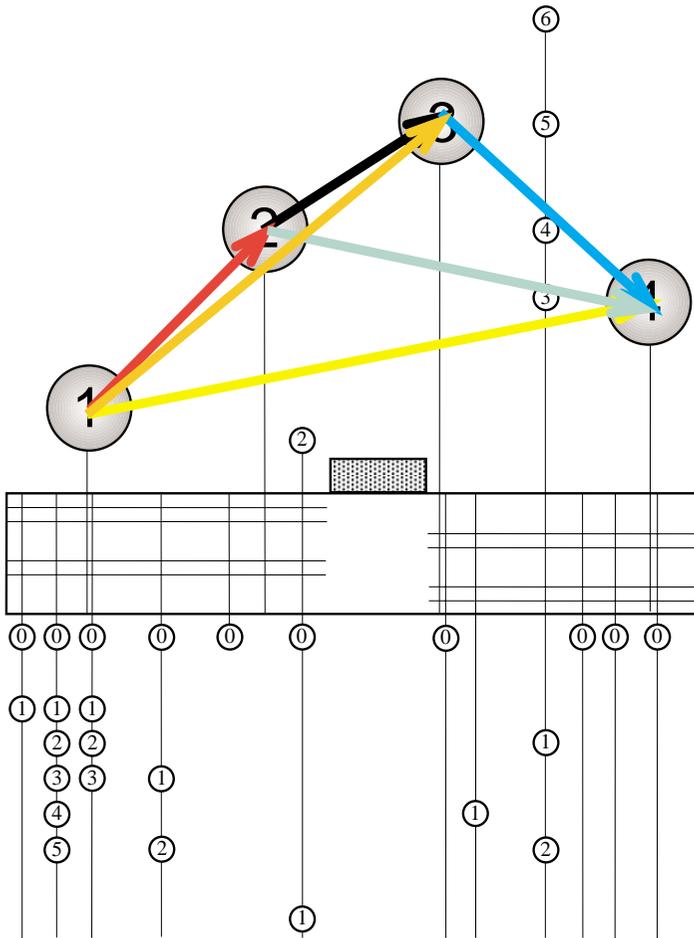
Measure Visibility and Phase for many baselines in the uv plane.

Note: This

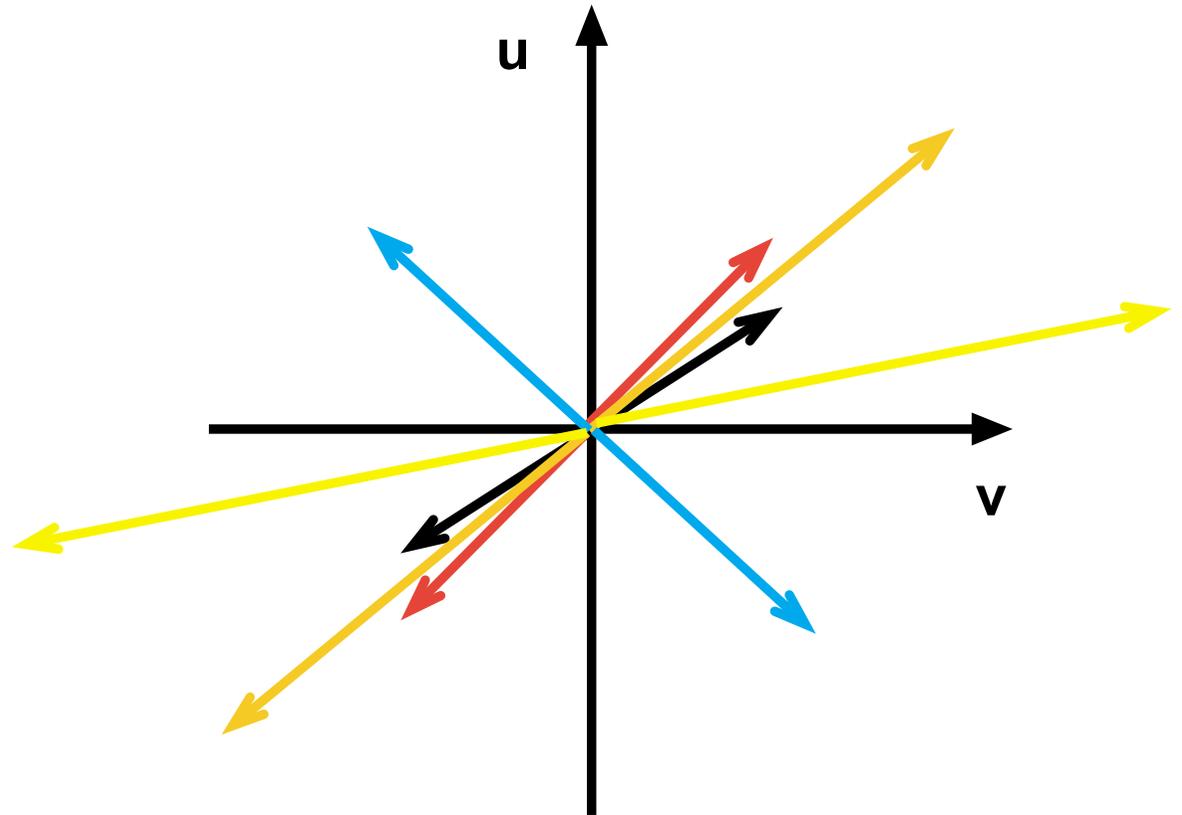


is not the uv plane!

# The uv-plane



This is the uv-plane:

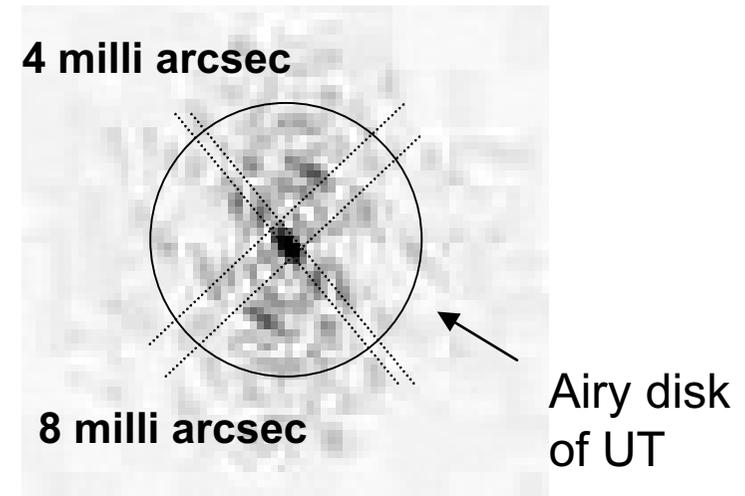


Note: This is the uv-plane for an object at zenith. In general, the projected baselines have to be used.

## The uv-plane with the UTs

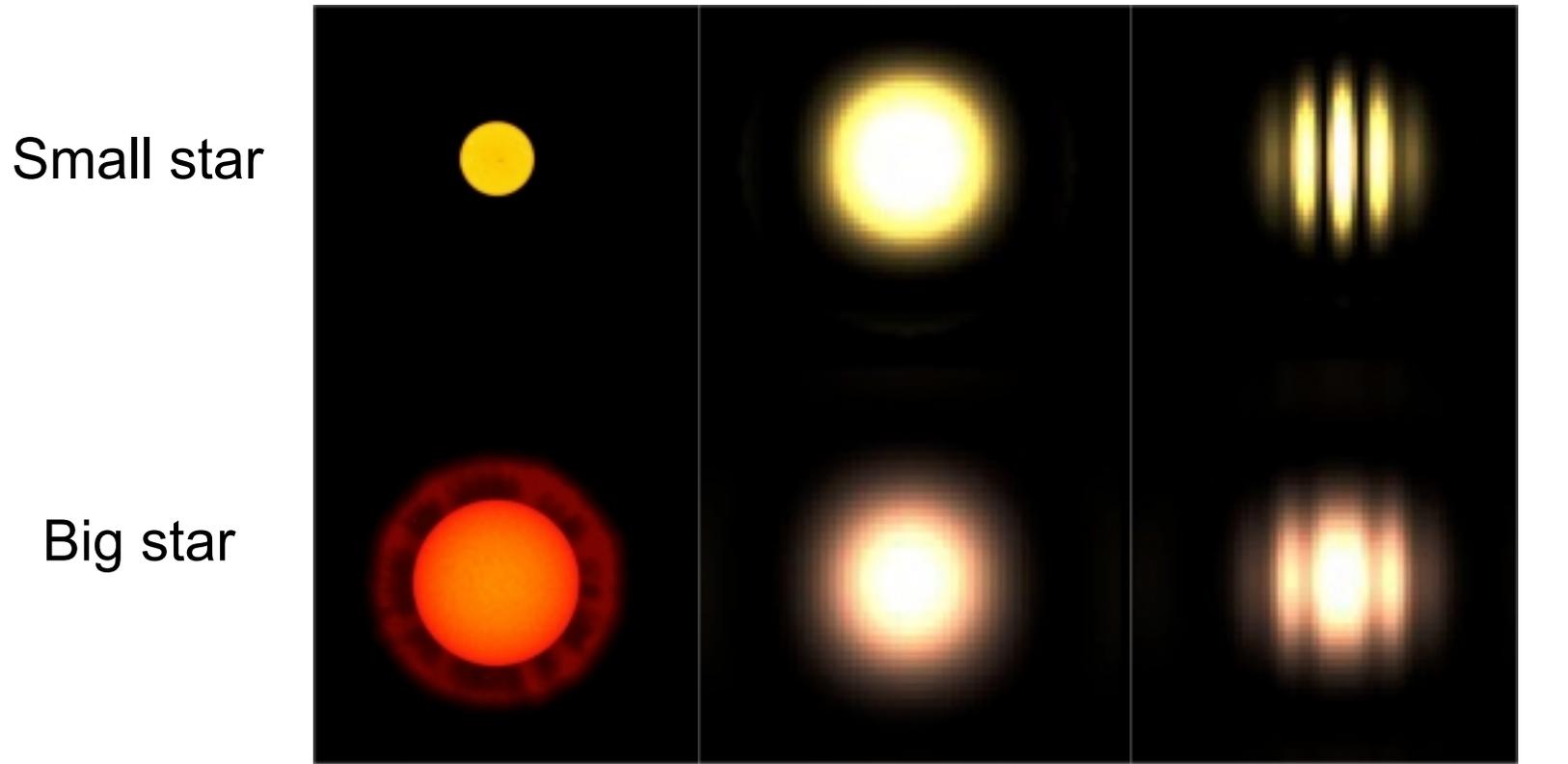


uv coverage for  
object at  $-15^\circ$   
8 hour observation  
with all UTs



Resulting PSF is the Fourier  
transform of the visibilities  
 $\lambda = 2.2\mu\text{m}$  (K-band)

# Why is interferometric imaging better?



Objects

Single Telescope

Interf. Fringes

$$I_{\text{im}}(\alpha) \sim \mathcal{F}(D)$$

$$I_{\text{im}}(\alpha) \sim \mathcal{F}(B)$$

Angular Resolution:  $\sim \lambda/D$

$\sim \lambda/B$

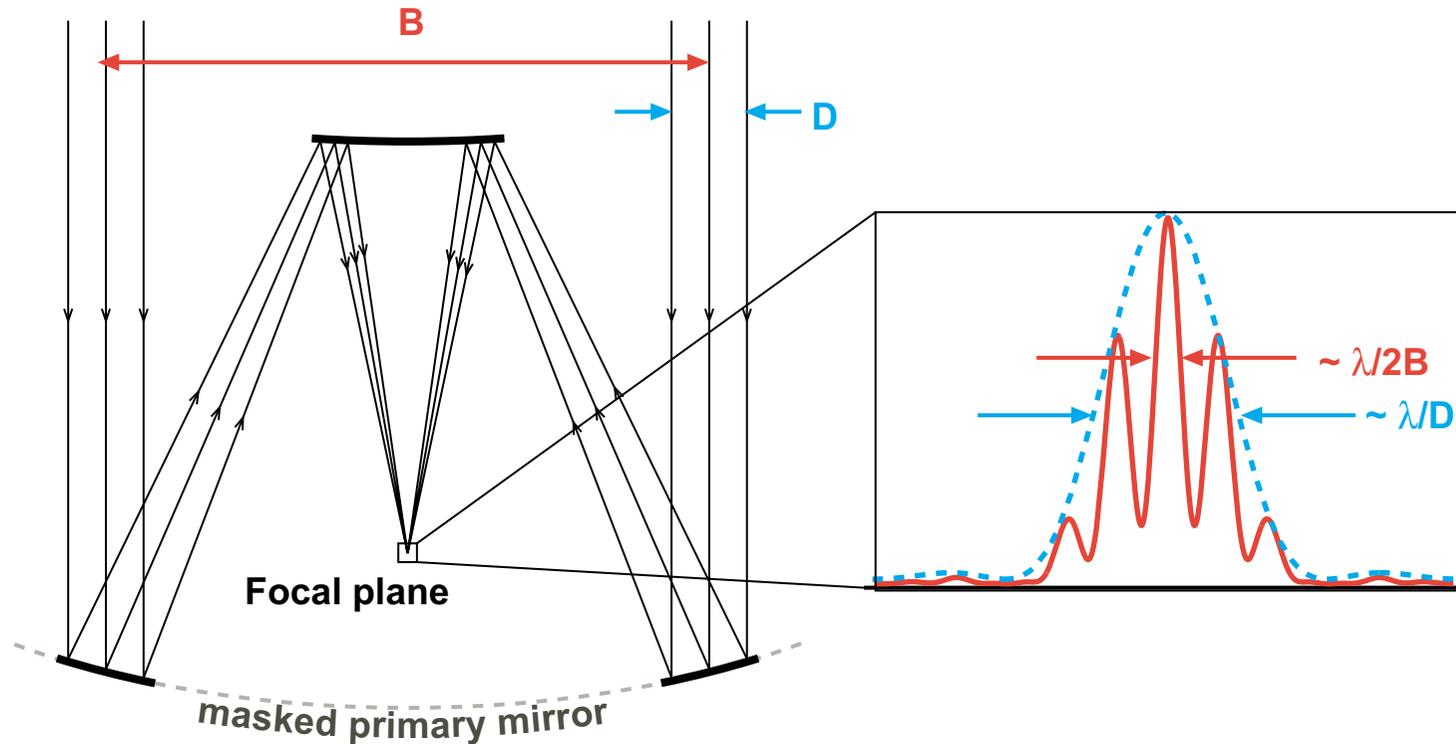
## Summary of concept

- The **Visibility** is the Fourier Transform of  $I(\alpha)$ , a complex function, the **modulus** is the contrast of the fringe pattern, the **phase** is the position of the white light fringe (wrt OPD = 0).
- A stellar interferometer measures the Visibility function at individual points in the uv plane, i.e. (projected) baseline vectors.
- **Smooth** reconstruction of the intensity distribution  $I(\alpha)$  (i.e. imaging) requires visibility measurements with **many baselines**.
- **Angular resolution** determined by the **longest baseline** and not by the diameter of the individual telescopes.
- Conclusion: Concept is good, CDR passed

## Second Part: Types of interferometers, the measurement of the Visibility function

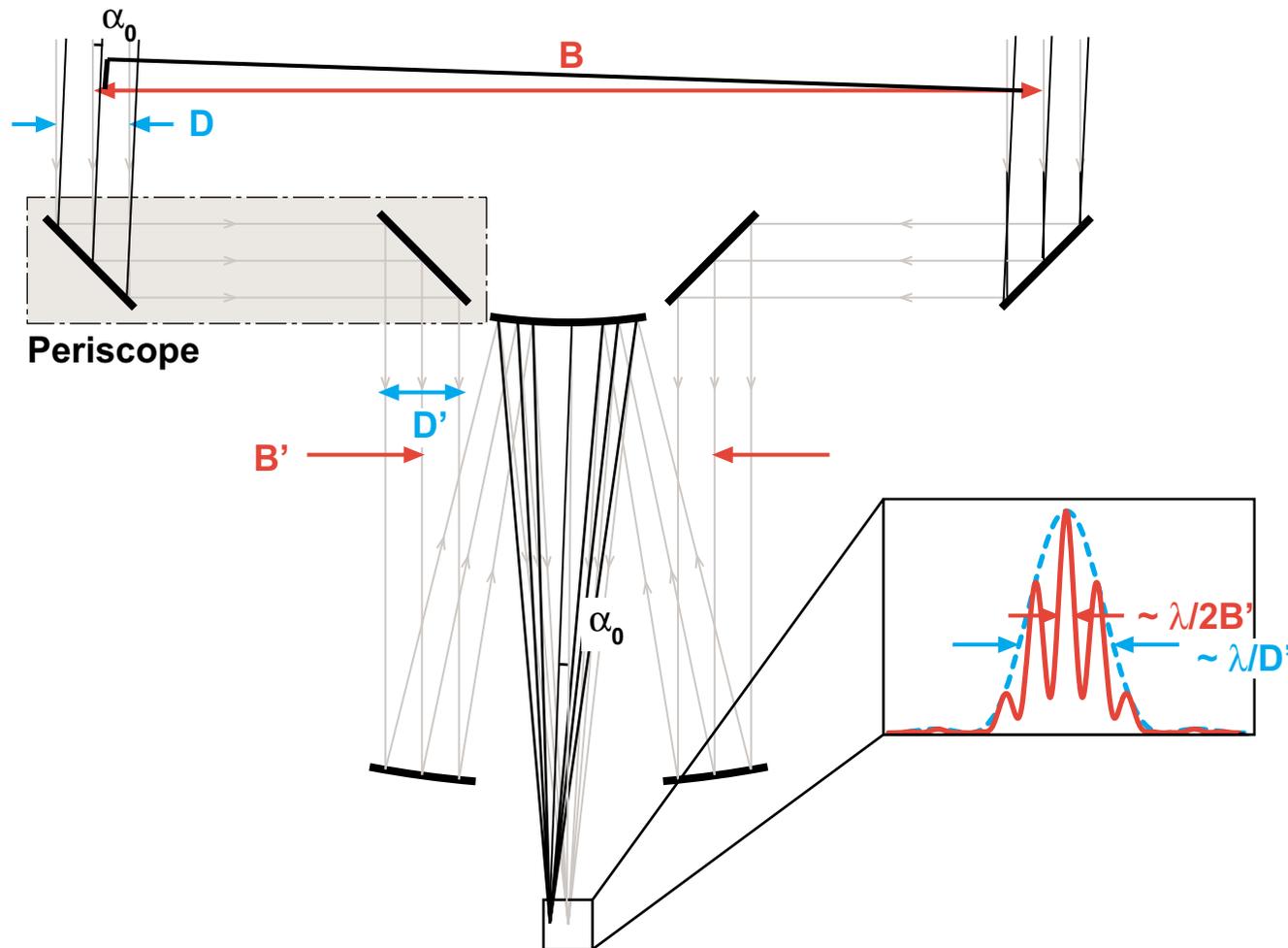
- Two major types: Michelson and Fizeau
- Main difference between the two: size of field of view  
⇒ homothetic mapping
- Three way beam combination
- Measurement principle types
- Multi-axial vs co-axial beam combination

# Masking a telescope



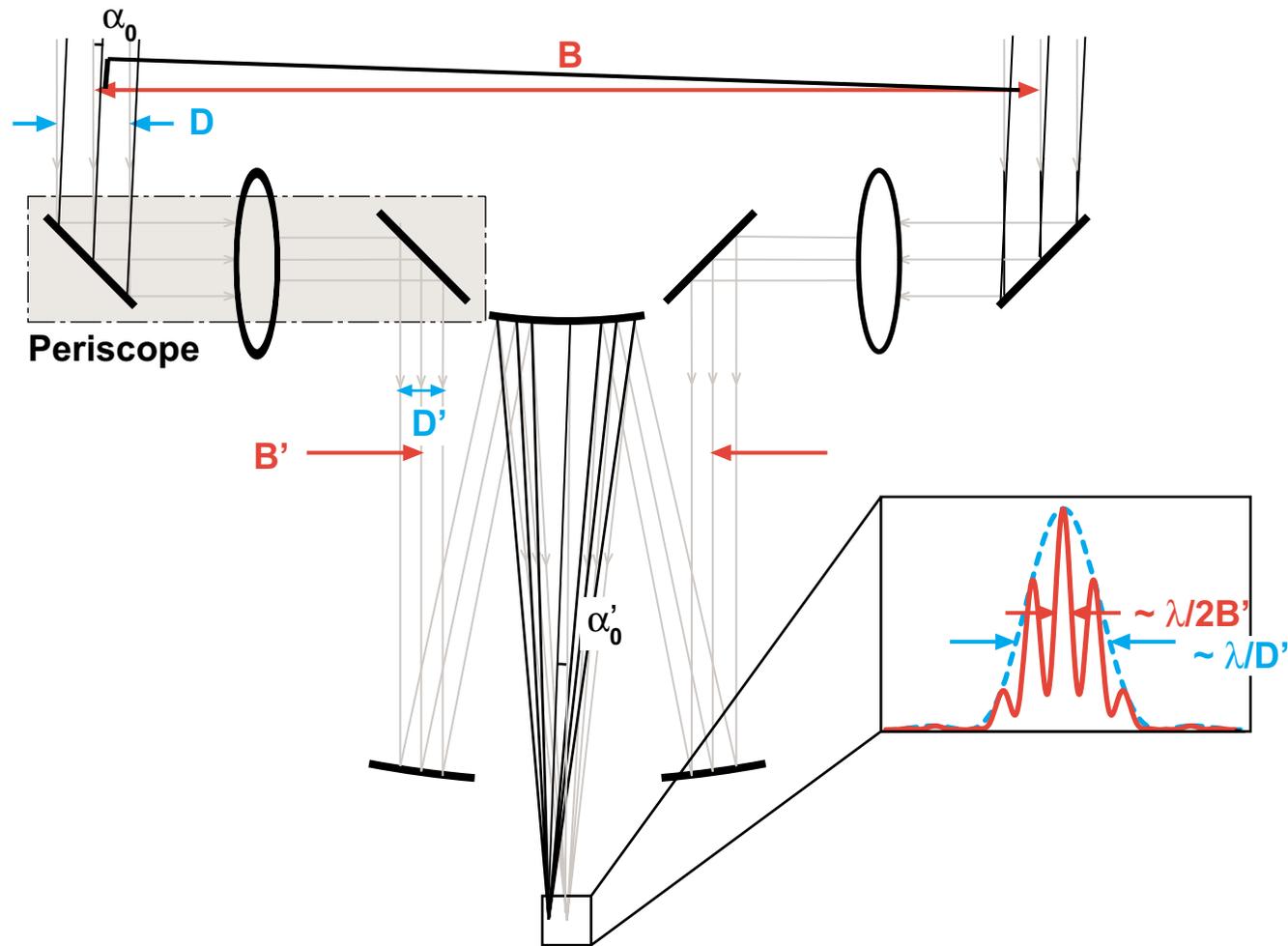
- The imaging process in a telescope is the superposition of fringe pattern from all combinations of baselines in the telescope pupil
- Masking the pupil, one can select one particular baseline
- Every star in the field of view has fringes

# Michelson interferometer



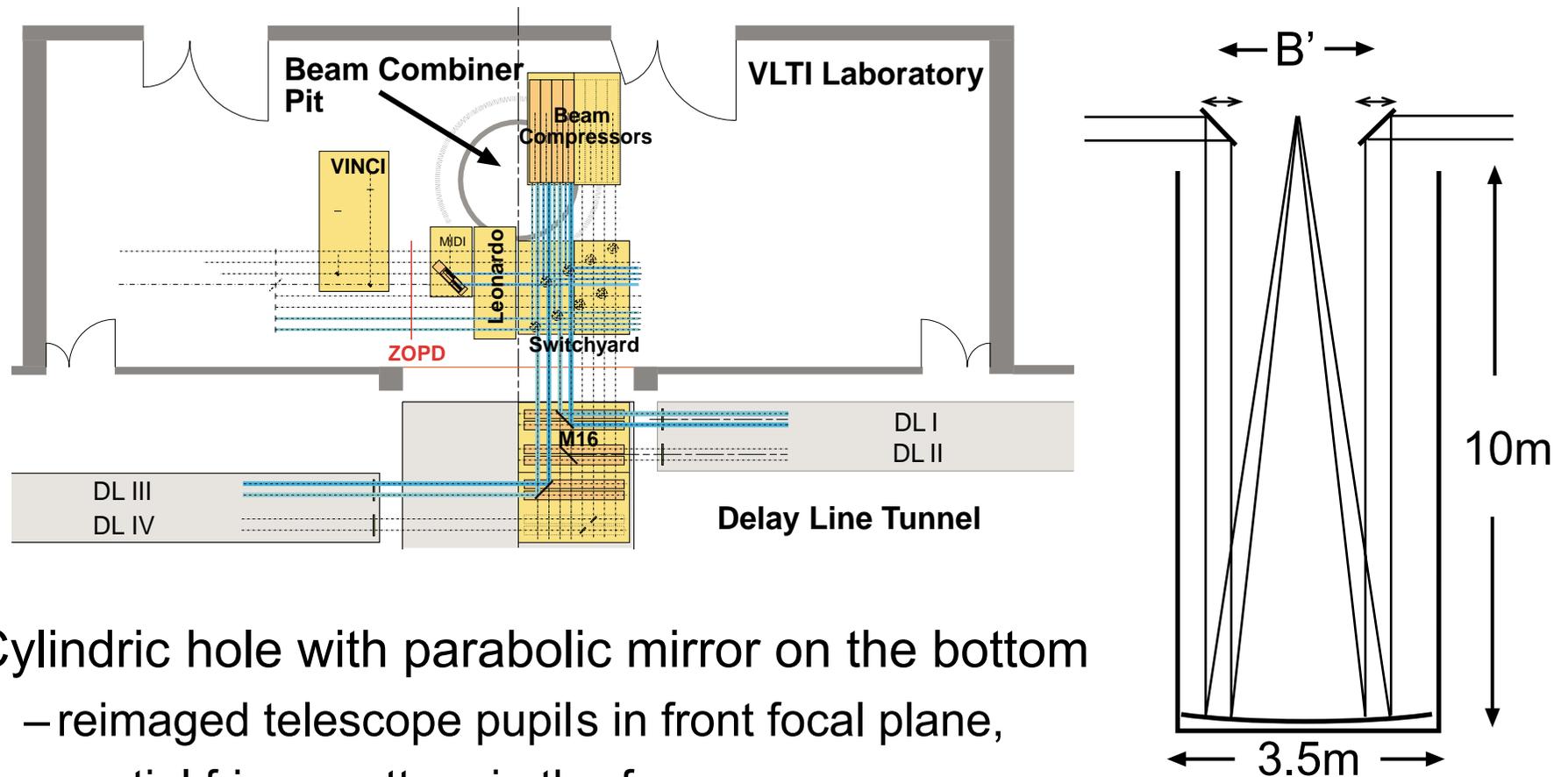
- Image at position  $\alpha_0$  (if  $D' = D$ )
- Left beam with delay  $\alpha_0 B$  , right beam with delay  $\alpha_0 B' \Rightarrow$   
OPD =  $\alpha_0 B - \alpha_0 B' \neq 0$  at image position  $\alpha_0$

# Fizeau interferometer



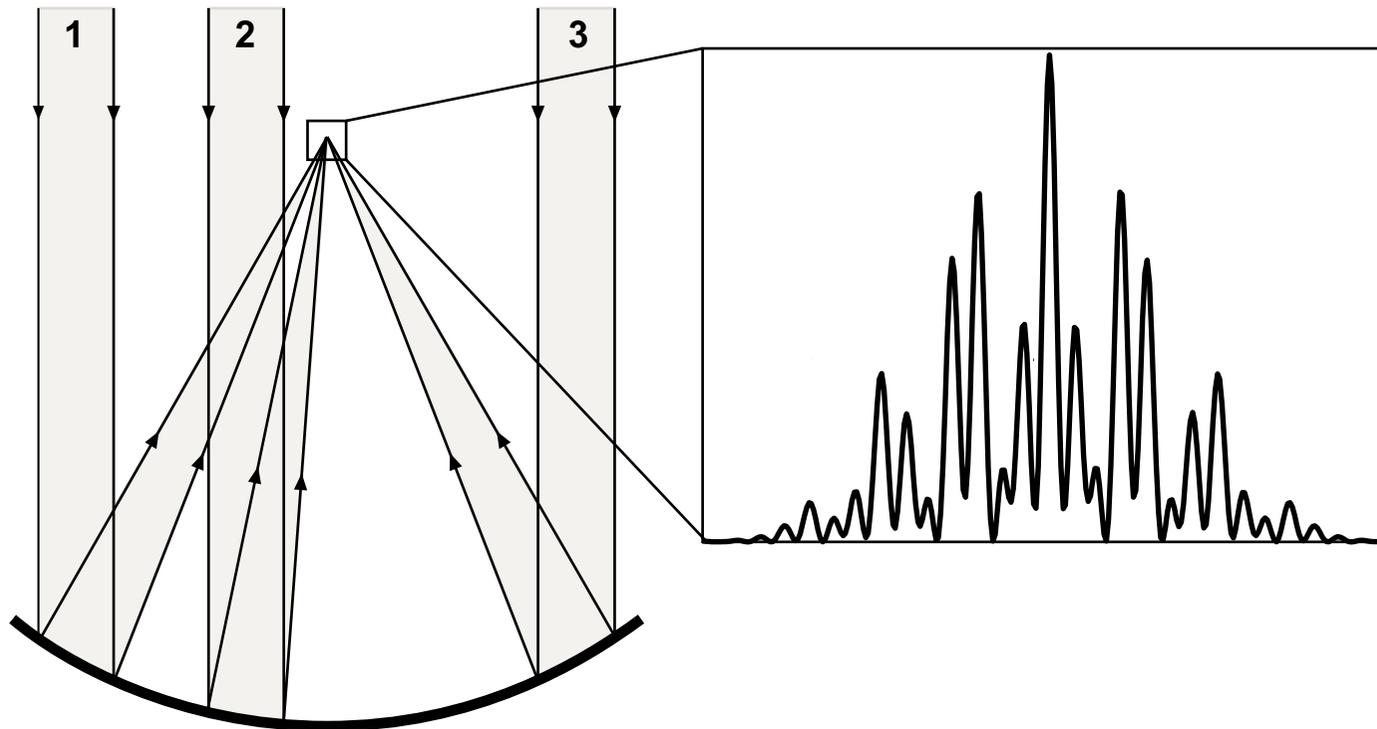
- If  $D' \neq D$  the image position is  $\alpha'_0 = \alpha_0 D/D'$
- If  $D/D' = B/B'$  one finds  $OPD = \alpha'_0 B' = \alpha_0 D/D' B' = \alpha_0 B/B' B' = \alpha_0 B$
- This kind of reimaging of the telescope pupils is called homothetic mapping

## Option: Homothetic mapping with the VLTI



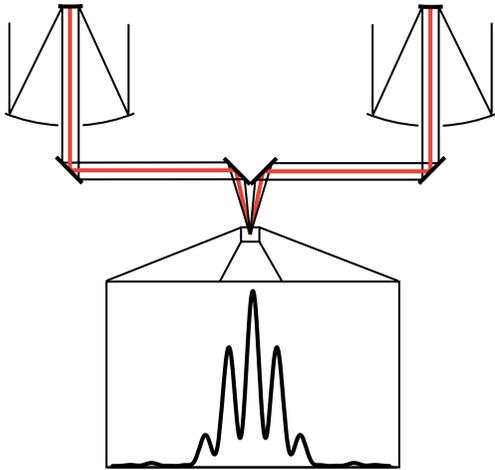
- Cylindric hole with parabolic mirror on the bottom
  - reimaged telescope pupils in front focal plane,
  - spatial fringe pattern in the focus
- Dynamic adjustment of pupil mirrors with  $\mu\text{m}$  precision required

## Three beam fringe pattern

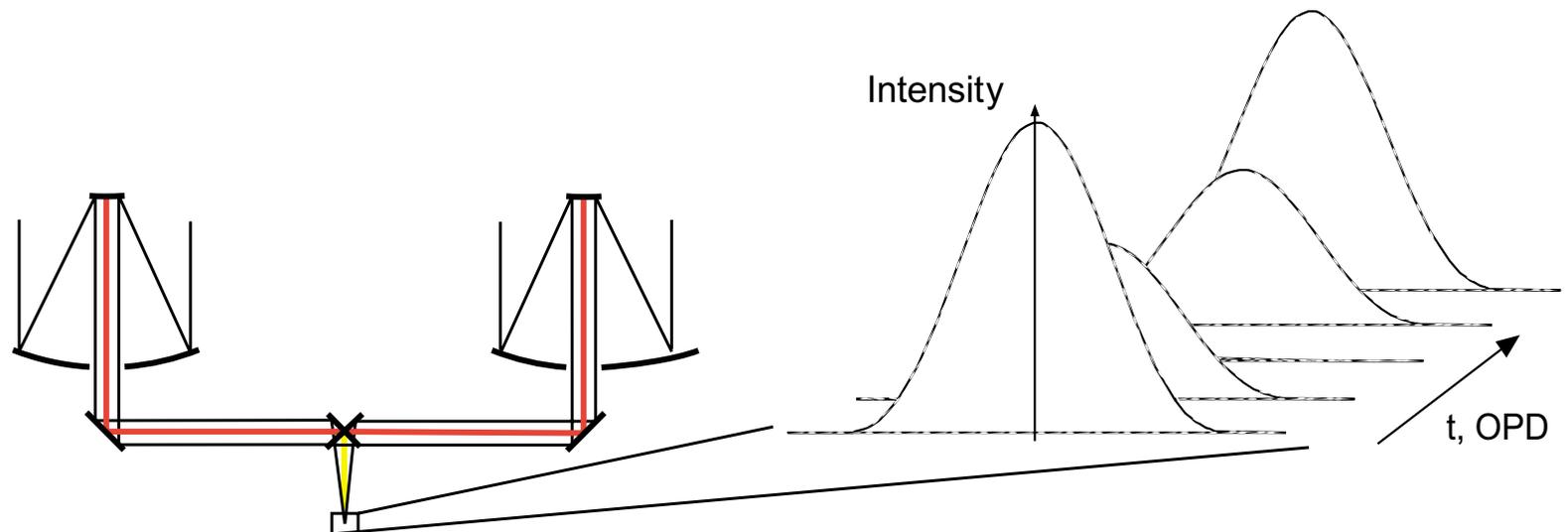


- Three baselines ( $B'_{12}$ ,  $B'_{23}$ ,  $B'_{13}$ ) with relation 1:2:3 produce three different spatial frequencies in the fringe pattern
- Disentangle the individual visibilities in Fourier space
- Planned in AMBER

# Spatial vs temporal fringe patterns

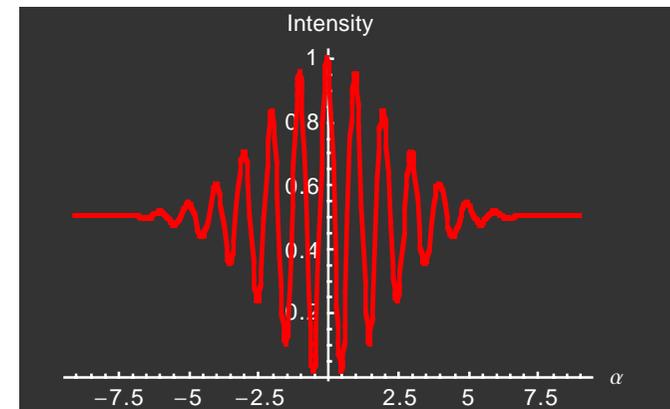
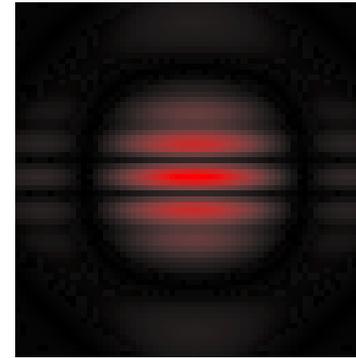


- Michelson and Fizeau interferometers discussed so far produce spatial fringe pattern (multi-axial beam combination)
- Co-axial beam combination produces Airy disk without fringes.
- Temporal OPD modulation produces  $I(t)$  (Compare to Michelson Fourier Spectrometer)

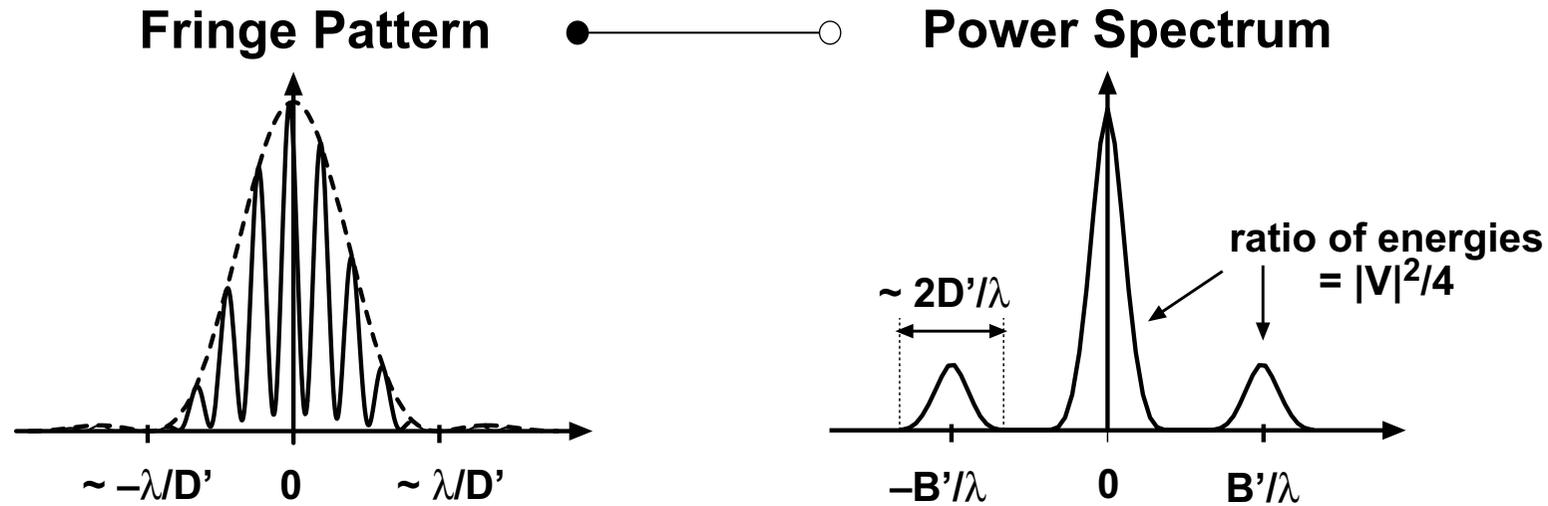


# Spatial vs temporal fringe patterns

- A spatial fringe pattern is a 2D signal as a function of the image coordinates with fringes along one coordinate enveloped by an Airy disk
- A temporal fringe pattern is a 1D signal as a function of OPD enveloped by the Fourier transform of the spectrum  $G(\nu)$

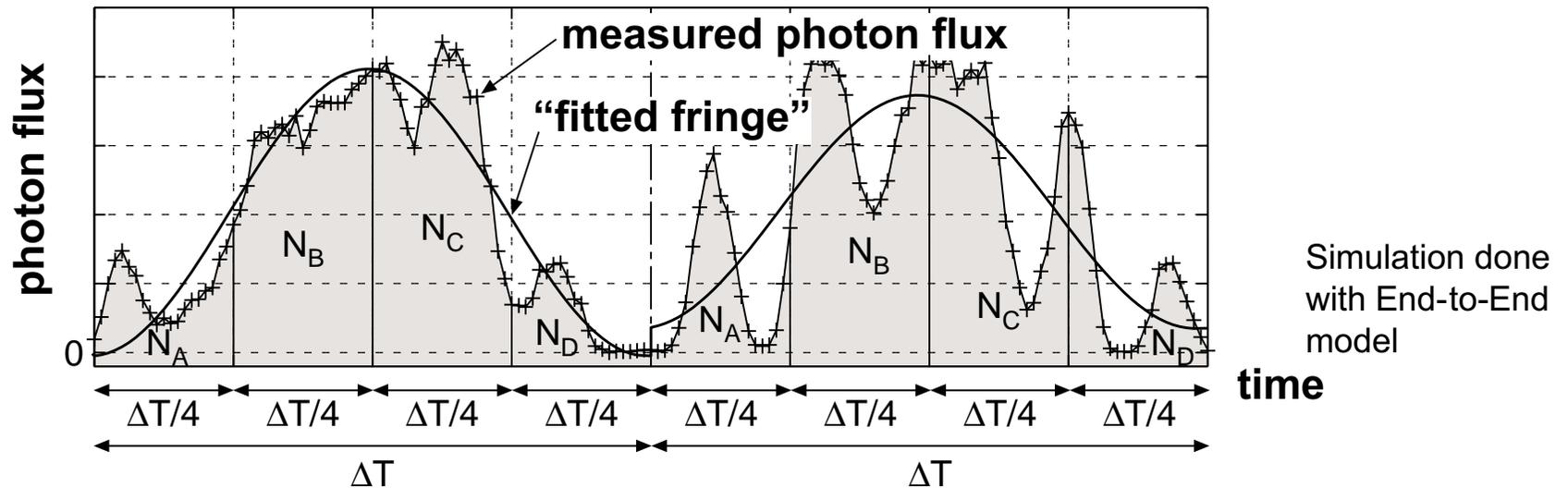


# Visibility measurement in the power spectrum



- Both spatial and temporal pattern can be analysed in the power spectrum
  - Spatial power spectrum is shown above
  - Temporal power spectrum shows intensity spectrum  $G(\nu)$  multiplied by the modulus  $V^2$  of the spatial coherence (VINCI)
- Three beam combination produces sidelobes at each of the three frequencies.
- Weak or noisy signal eventually shows up in the averaged power spectrum.

# Visibility measurement - ABCD method

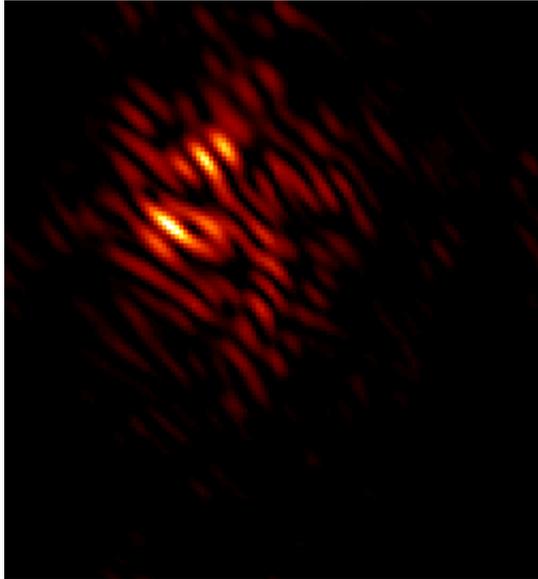


- Principle: Measure intensity at four points spaced by  $\lambda/4$ 
  - Temporal pattern shown above
  - Spatial pattern awkward, first integrate along the fringes in the Airy disk, then normalise with Airy disk intensity. Error prone.
- Fitting  $(1+|V| \cos(\alpha + \phi))$  determines modulus and phase
- Averaging is less efficient than using the power spectrum

# Summary of designs

- The position of the reimaged telescope pupils determine the interferometer type:
  - Exit pupils form scaled down model of interferometric array  
⇒ homothetic mapping, Fizeau type, ‘unlimited’ field of view
  - Exit pupils are placed to have convenient fringe spacing (e.g. to match detector pixels)  
⇒ Michelson type, very limited field of view
  - Exit pupils imaged on top of each other  
⇒ Co-axial beam combination producing temporal fringe pattern
- Visibility measurements in space/time or in Fourier space
  - ABCD method determines directly modulus  $V$  and phase  $\phi$
  - Fourier spectrum determines modulus and phase, averaged power spectrum can be used to measure weak signals
- Conclusion: preliminary design is ok, PDR passed

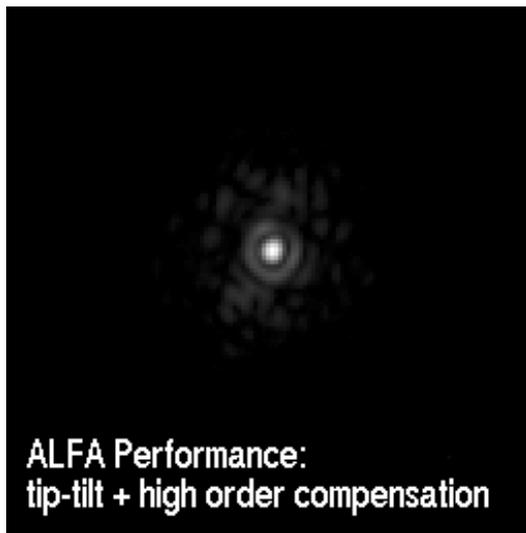
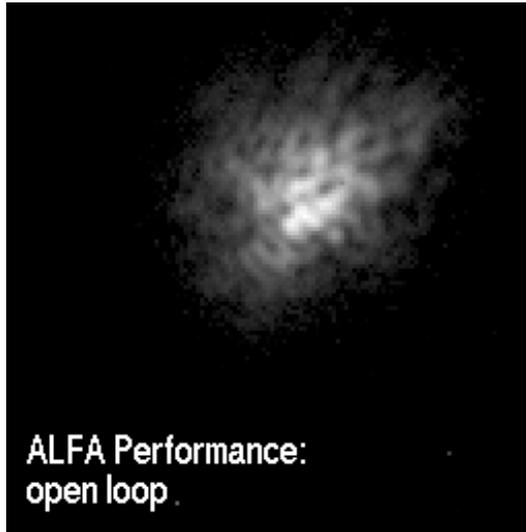
# Excursion: Atmospheric turbulence and PRIMA



Spatial fringe pattern  
of 2 UTs at  $2.2 \mu\text{m}$   
in  $0.5''$  seeing

- **Goal:** Measurement of fringe contrast and fringe position
- **Requirement:**  
Small pixel size (1- 10 milli arcsec)  
+ Short exposure time ( $\sim 10$  msec)
- **Note:**  
The angular resolution depends only on the length of the baseline.  
Any improvement of the 'image quality' only affects the sensitivity.

# Adaptive optics and interferometry



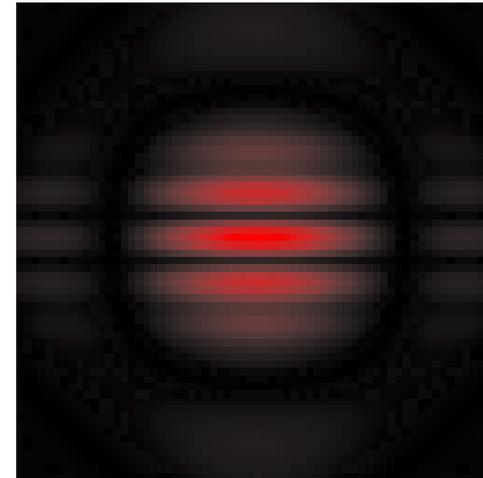
- Turbulences cause speckle pattern in individual telescope
- ‘Fishing’ for intensity with monomode fibers or using an area detector both loses sensitivity
- Perfect Airy disk has all aberrations removed, except for piston

Calar Alto 3.5m telescope  
MPIA Heidelberg

(Movie with real data)

# Fringe tracking

- Remaining atmospheric piston causes fringe wobble  
⇒ exposure time limited to some 10msec depending on  $\lambda$
- Solution:  
Bright guide star for fringe tracking  
Integrate fringes on science object
- Concept similar to Adaptive Optics
- Note: Individual Telescopes can observe faint stars without AO,  
**Interferometers cannot go faint without a fringe tracker!**

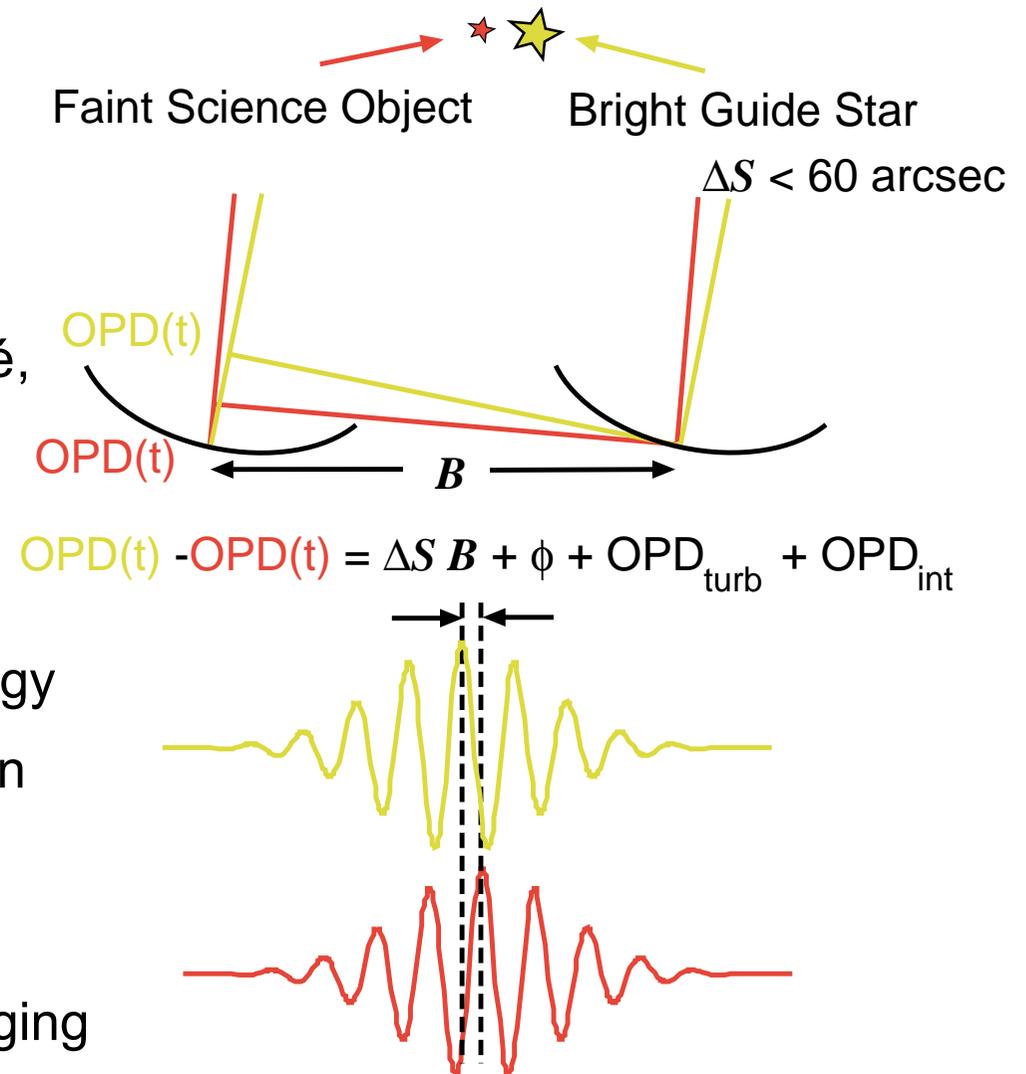


(Movie showing fringe wobble)

Spatial fringe pattern  
(multi-axial beam combination)

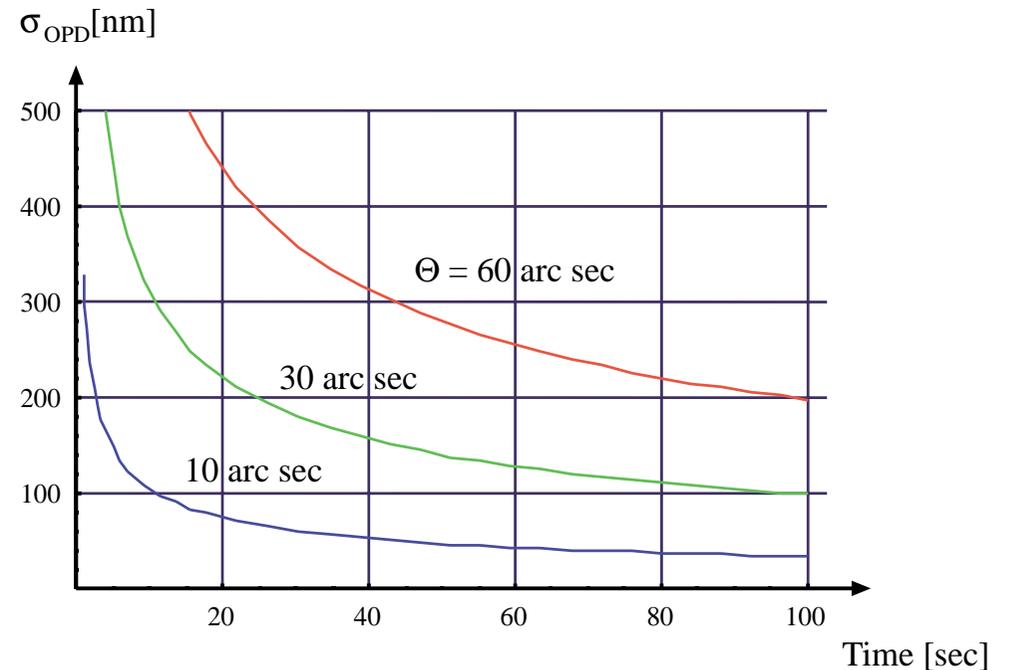
# PRIMA – the VLTI dual feed facility

- VLTI field of view 2 arcsec
- Coudé focus (in UTs and ATs)  
field of view 2 arcmin
- PRIMA picks two stars in the Coudé,  
feeds it into the Delay Lines and  
produces the fringes in the VLTI  
Laboratory
- $OPD_{int}$  measured with laser metrology
- $OPD_{turb}$  averaged by long integration
- $\Delta S B + \phi$  determined by  
interferometric instruments
- $\Delta S$  gives the astrometry,  $\phi$  the imaging



# Isoplanatic angle

- The rms OPD error is reduced to  $<200\text{nm}$  even if the separation is 1 arcmin
- After 30min the error is small enough to allow for  $10\ \mu\text{arcsec}$  astrometry
- Even individual exposures of  $\sim 10\text{sec}$  have  $\sim 100$  more signal and increase the limiting magnitude by 5



# Closure phase

- In the sum of the three phases the random fluctuation is eliminated:

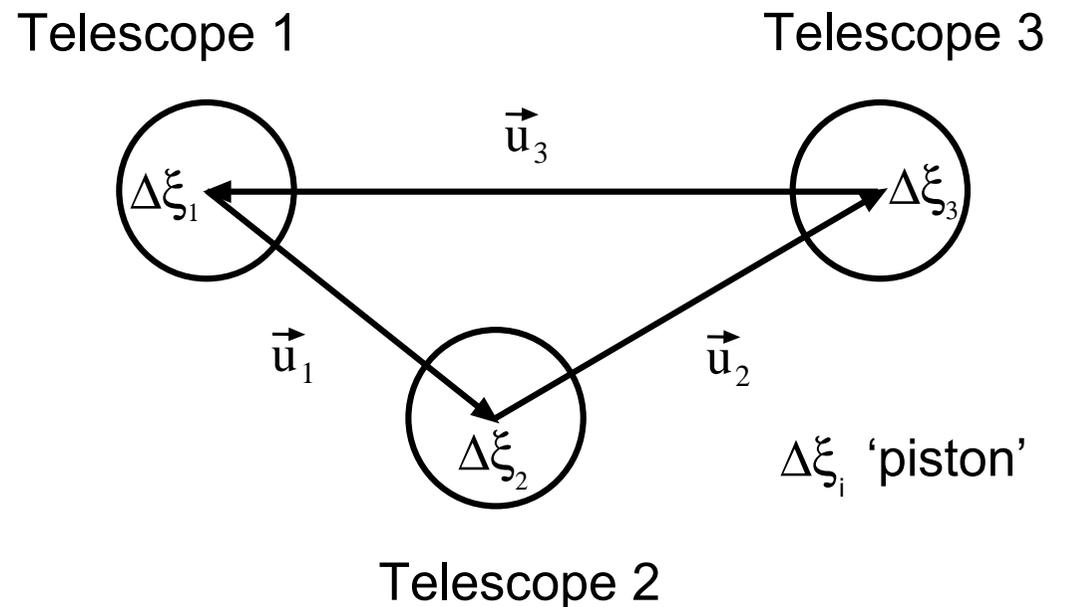
$$\psi_1(u_1) = \phi_1(u_1) + \Delta\xi_1 - \Delta\xi_2$$

$$\psi_2(u_2) = \phi_2(u_2) + \Delta\xi_2 - \Delta\xi_3$$

$$\psi_3(u_3) = \phi_3(u_3) + \Delta\xi_3 - \Delta\xi_1$$

$$\psi_1 + \psi_2 + \psi_3 = \phi_1 + \phi_2 + \phi_3$$

- Many baselines required to determine individual phases.
- The exposure time is limited, again by the individual fringe motion..

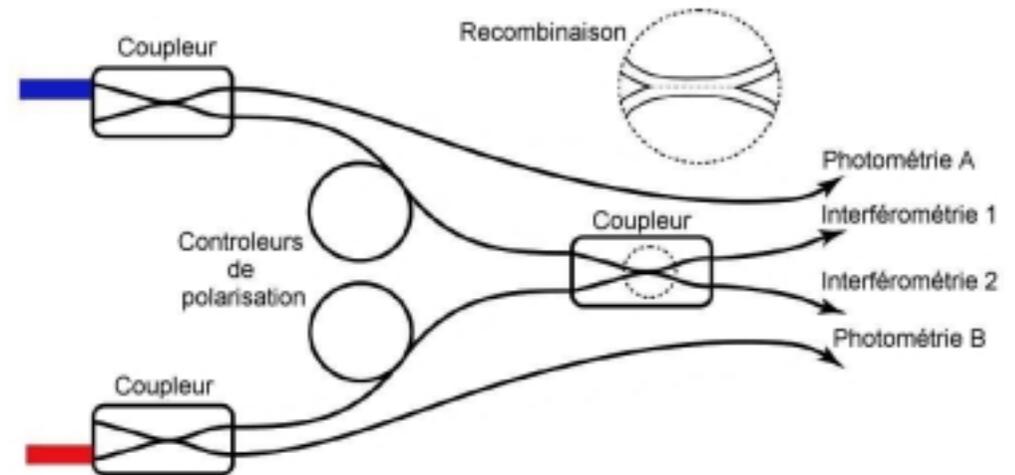


## Summary atmospheric turbulence

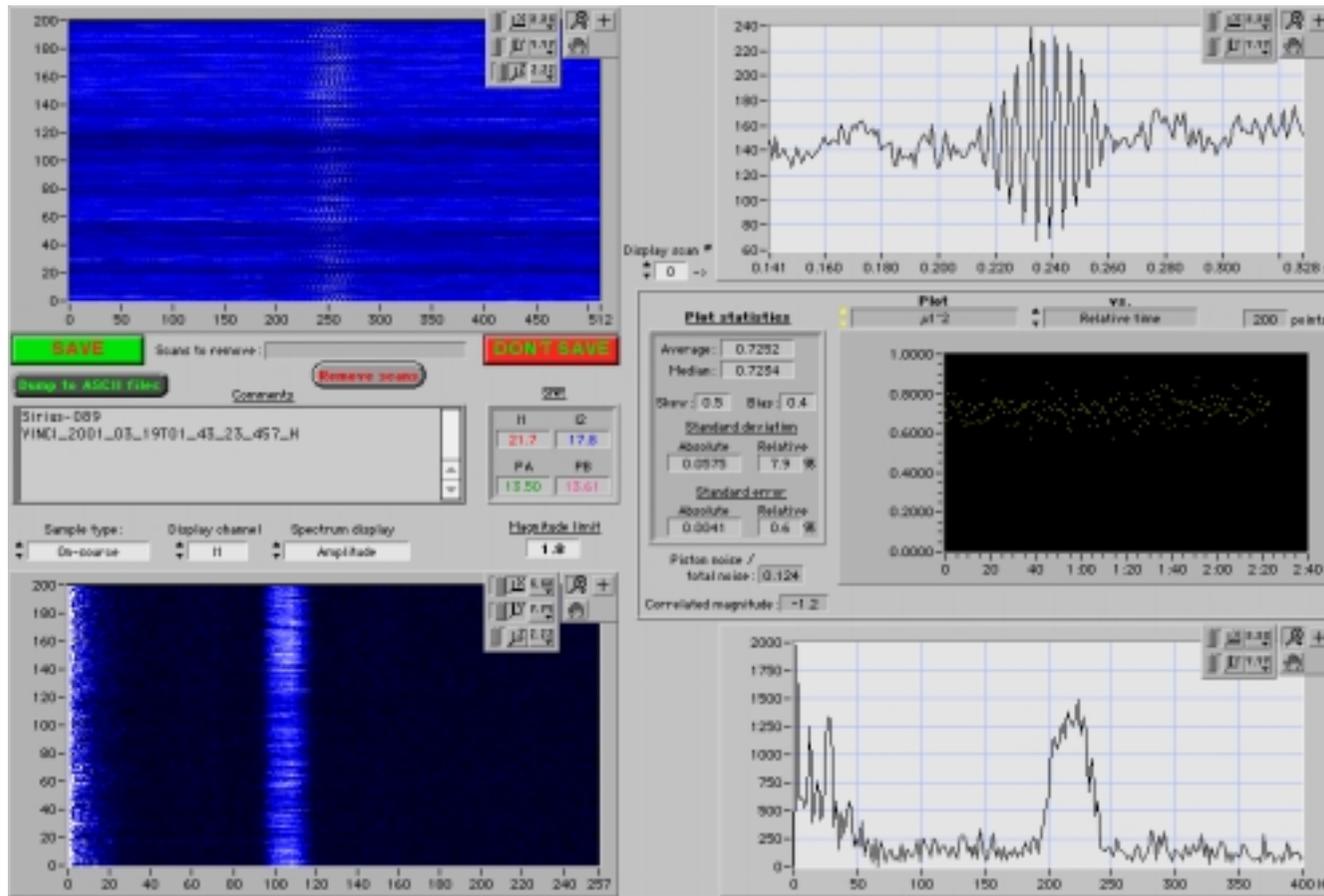
- Adaptive Optics is required to increase the sensitivity
- With a perfectly corrected Airy disk, the sensitivity is limited by the fringe motion, reducing the exposure time to some 10msec
- Remaining fringe motion has to be removed by fringe tracker
- PRIMA allows to go as faint as  $K \sim 20$  with bright guide star ( $K \sim 12-16$  depending on required accuracy) within  $< 1$  arcmin
- Conclusion: Atmospheric turbulence can be controlled, FDR is passed  $\Rightarrow$  VINCI

# VINCI - Measurement principle

- Light is fed into two monomode fibers (Concept adopted from FLUOR at IOTA)
- Fiber coupler adds as beam combiner for coaxial beam combination
- Temporal fringe pattern measured in I1 and I2
- Modulation performed at fiber feed



# Data analysis software from Obs. de Meudon

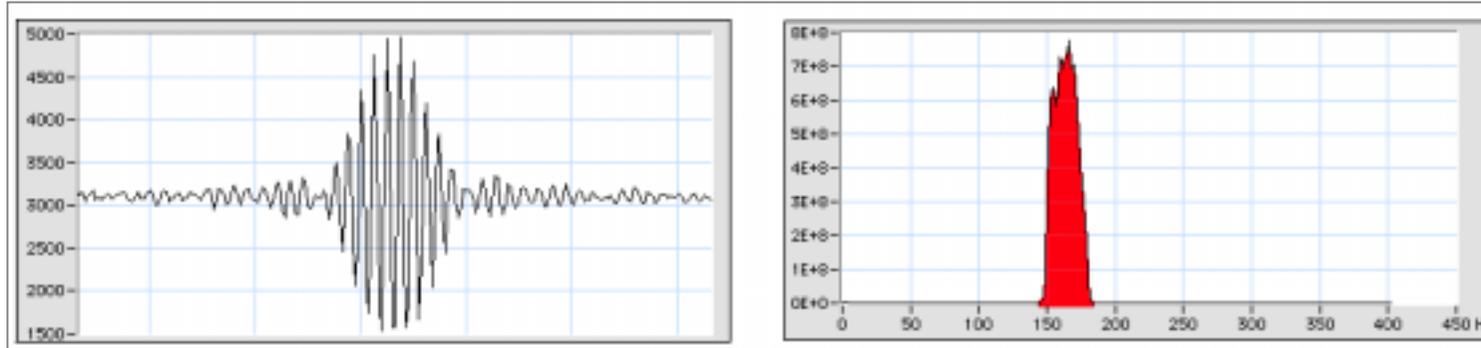


- Temporal fringe pattern left: 'waterfall display', right: individual scan

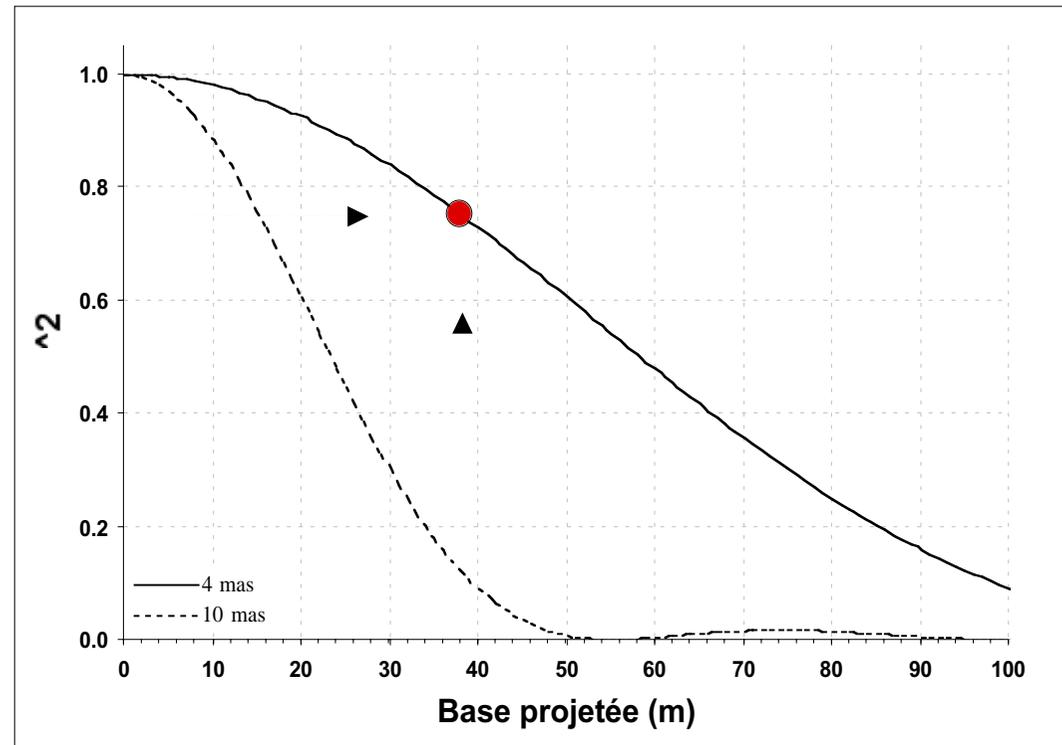
- Power spectrum with noisy DC term and K-band spectrum left: 'waterfall display', right: individual scan

Note: In the meantime, a similar package has been written for the ESO pipeline

# Measurement of the Visibility function with VINCI



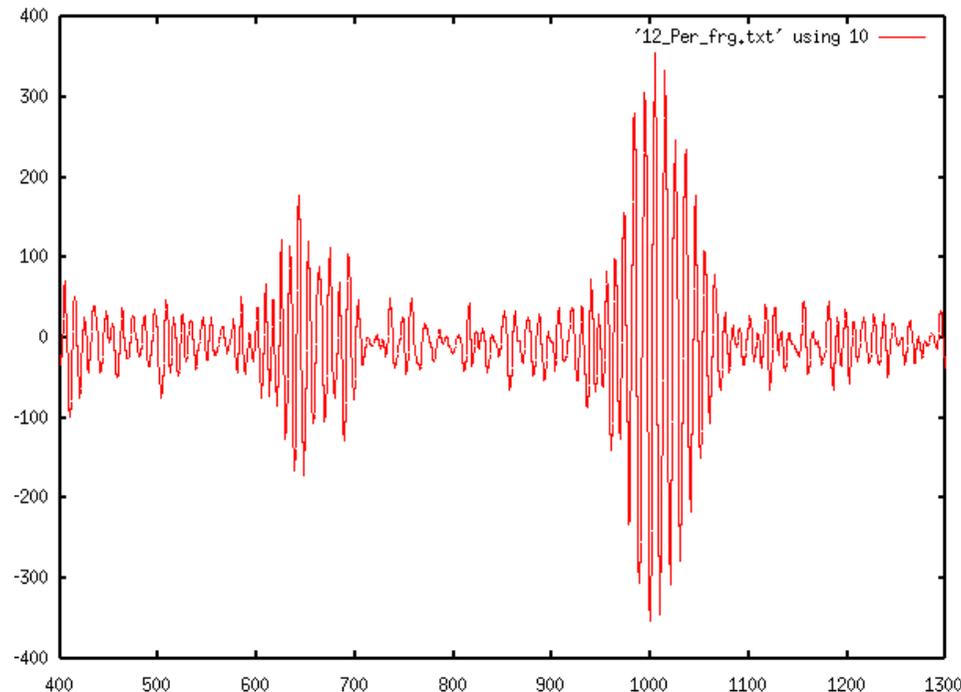
- The power spectrum is masked with the K-band spectrum
- The integrated power determines  $V^2$  the square of the visibility



# Binary observation in a co-axial fringe pattern

The two fringe pattern are completely separated due to separation  $\delta$  times baseline  $B >$  coherence length

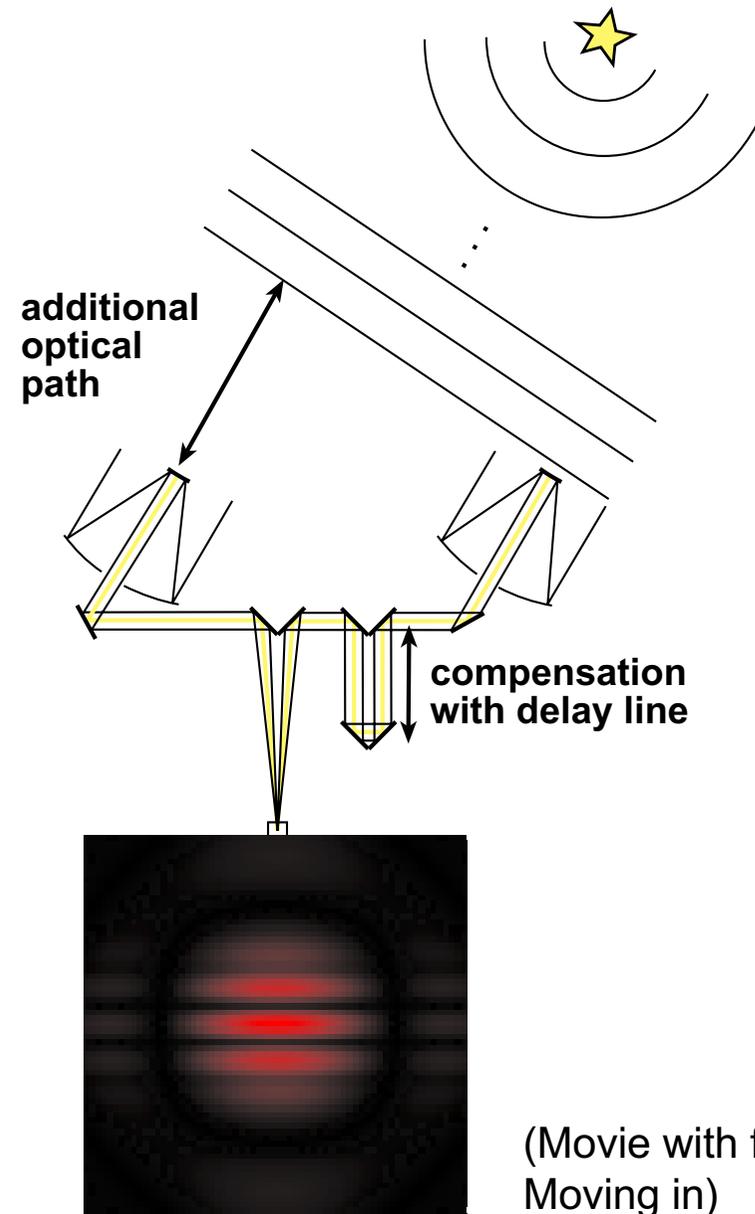
The separation is determined by the distance of the white light fringes, not by the reduced contrast described by the visibility function!



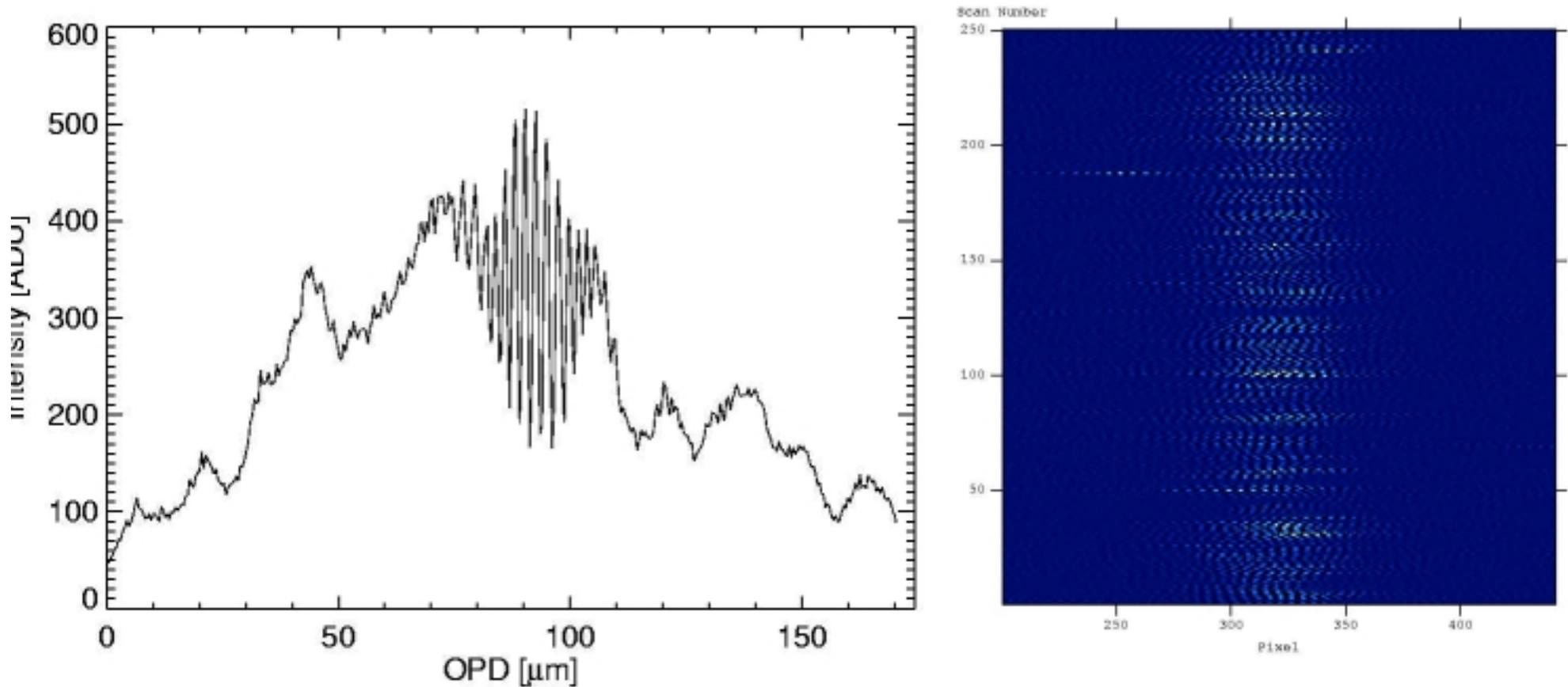
12 Persei observed on Oct 9, 2001 with the CHARA Array, K'-band, 330m baseline, separation 40 marcsec

## Finding fringes

- Adjust star on fiber
- Follow trajectory with Delay Lines
- Scan starts, sweeping around calculated 0 OPD position (scanning 10mm takes about 5min)
- After first few observations calculate new OPD model  
⇒ Fringes found within <1mm
- Observations executed by BOB



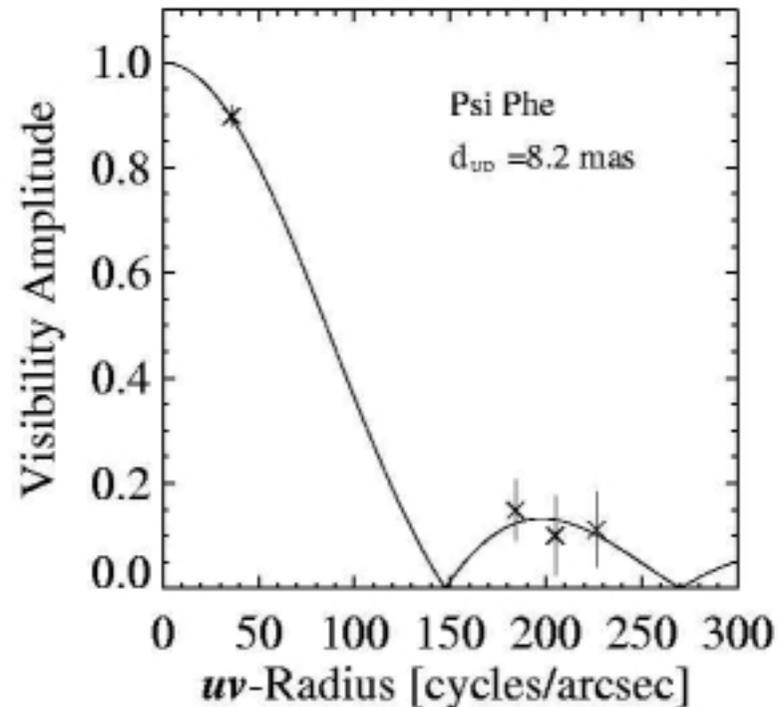
## First Fringes with the UTs



Achernard on Oct 30, 2001, at 1 am,  
scan on the left chosen from 'waterfall' display on the right

# The diameter of a red giant

- Psi Phoenicis observed with siderostats (baseline 16m), and with UT1 and UT3 (baseline 102m)
- Differences in projected baselines provide different baseline vectors
- Preliminary result for diameter: 8.21 marcsec



## Conclusion

- Stellar interferometry is not (that) complicated
- Manifold of interferometer types is limited
- Measurement principles allow to go (rather) faint, large and detailed
- VLTI started working in its literal sense, as the  
**Very Large Telescope Interferometer**