Population of Dynamically Formed Triples in Dense Stellar Systems

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Summary. In dense stellar systems, frequent dynamical interactions between binaries lead to the formation of multiple systems. In this contribution we discuss the dynamical formation of hierarchically stable triples: the formation rate, main characteristics of dynamically formed population of triples and the impact of the triples formation on the population of close binaries. In particular, we estimate how much the population of blue stragglers and compact binaries could be affected.

1 Introduction

In globular clusters, the most plausible way for the dynamical formation of hierarchical triples is via binary-binary encounters. As numerical scattering calculations show, the probability of triple formation is quite substantial and for equal masses, a hierarchical triple is formed in roughly 50% of all encounters [3]. The probability is reduced by only a factor of a few when original semi-major axes of binaries are about equal [9]. The consideration of stars as non-point masses can decrease this probability further, as the physical collisions enhance strongly the destruction of close binaries during binary-binary encounters [2]. The formation of triple stars have often been noticed in numerical simulations of dense stellar systems using $N-$body codes (e.g. [10, 4]), however so far there has been no attempt to study in detail the population of triple systems as well as their effect on the close binaries and blue stragglers. In this contribution we report the preliminary results of our study of triples population.

2 Method and Assumptions

We use a *Monte Carlo* method described in detail in [6]. This method assumes a static cluster background, all relevant dynamical parameters being kept constant throughout dynamical simulation. In particular, the cluster model we consider here has central density $n_{\rm c} = 10^5$ [pc^{-3}], velocity dispersion $\sigma = 10$ [km/s], escape velocity $v_{\rm esc}$ [km/s] and half-mass relaxation time $t_{\rm rh} = 10^9$ [yr]. The code takes into account such important dynamical processes as mass segregation and evaporation, recoil, physical collisions, tidal captures, and binary–single and binary–binary encounters. For dynamical encounters that involve binaries we use `Fewbody`, a numerical toolkit for direct N-body integrations of small-N gravitational dynamics [2]. This toolkit is particularly suited to automatically recognize a hierarchical triple (formed via an encounter) using the stability criterion from Mardling & Aarseth [8]. In order to get large statistics on triples formation rate, we start with 1.25×10^6 stars, 100% are in primordial binaries; the modeled cluster has mass $\sim 250\,000 M_\odot$ at 10 Gyr and the core mass is 10-20% of the total cluster mass. This cluster model represents well a "typical" globular cluster.

There is no developed population synthesis methods for triples evolution. As a result, we can not keep the triples once they were dynamically formed and have to break a triple into a binary and a single star. This approach is suitable as it is likely that a dynamically formed hierarchical triple will be destroyed during next dynamical encounter. In our standard runs, we break a triple while conserving the energy: the energy required to eject the outer companion is acquired from shrinking of the inner binary orbit. The outer companion is released unless the required shrinkage is such that the inner binary merges. In the latter case the inner system is allowed to merge and the outer companion is kept at its new, wider orbit to form the final binary system. On the other hand, it is possible that in a triple system the inner eccentricity will be later increased via the Kozai mechanism [7]. This mechanism causes large variations in the eccentricity and inclination of the star orbits and could drive the inner binary of the triple system to merge before next interaction with other stars. To check how strongly this affects the binary population, we compare two cluster models with identical initial populations of 5×10^5 stars. In one model, we use our standard treatment for triples breaking. In the second model, we compare the Kozai time-scale $\tau_{\rm Koz}$ (taken as in [5]) with the collision time-scale $\tau_{\rm coll}$. Inner binaries in the formed triples are merged if $\tau_{\rm Koz} < \tau_{\rm coll}$ (we define these triples as Kozai triples). We expect that some of the triples can also have a secular eccentricity evolution even in the case of inclinations smaller than the Kozai angle (see e.g. [1]), but we neglect this possibility.

3 Numerical Results on Dynamically Formed Triples

3.1 Formation Rates

From our standard model, we find that a "typical" cluster has about 5000 hierarchically stable triples formed in its core throughout its evolution. As our triples are formed via binary-binary encounters, the resulting formation rate intrinsically depends on the binary fraction, the binary cross-section and the total number of binaries. Hence the obtained formation rate can be written as:

$$\Delta N_{\rm tr}/N_{\rm bin} = 0.05 f_{\rm bin}\langle m_{\rm b}\rangle\langle a\rangle \text{ per Gyr.} \tag{1}$$

Here $f_{\rm bin}$ is the binary fraction, $\langle m_{\rm b}\rangle$ is the average binary mass and $\langle a\rangle$ is the average binary separation. In particular, at 10 Gyr $\langle m_{\rm b}\rangle \approx 1M_\odot$, $\langle a\rangle \approx 10R_\odot$ and $f_{\rm bin} \approx 10\%$. The formation rate at 10 Gyr is therefore such that as many as 5% of all core binaries have sucesfully participated in the formation of hierachically stable triples during 1 Gyr. We stress that the expression above is fitted to numerical simulations; it does not include directly the expected dependence on the core number density and velocity dispersion because only one set of these parameters was used in simulations.

The formation rate of triples can also be written as a function of the cluster age (for ages > 1 Gyr):

$$\Delta N_{\rm tr} = 600\, T_9^{-1/3} \text{ per Gyr,} \tag{2}$$

where T_9 is the cluster age in Gyr.

3.2 Masses, Orbital Periods and Eccentricities

In Fig. 1 we show the distributions of companions masses in triple systems (the mass of the inner binary and of the outer companion), of inner and outer orbital periods, and of outer eccentricities, for all dynamically formed triples throughout the entire cluster evolution. The inner eccentricities did not show strong correlation with any other parameters and are distributed rather flatly. A "typical" triple has the mass ratio $M_3/(M_1+M_2) \approx 0.5 \pm 0.1$, the total mass of the inner binary is $M_1+M_2 \approx 1.3 \pm 0.3 M_\odot$ (such a binary, if merges, will likely produce a blue straggler), $P_{\rm in} \approx 1$ day. The typical period ratio is high, $P_{\rm out}/P_{\rm in} \approx 1000$, the ratio of the orbital separations $a_{\rm out}/a_{\rm in} \approx 100$ and the outer eccentricity is very large, $e_{\rm out} \approx 0.95 \pm 0.05$.

3.3 Hardness and the Kozai Mechanism

Only those triples that have the binding energy of the inner binary with the outer companion greater than a kinetic energy of an average object in the field are stable against subsequent dynamical encounters. The ratio of these energies is the triple's hardness, η. We find that 45% of all triples have $\eta > 1$ and only 7% of all triples have $\eta > 10$. In our numerical simulation we find that for binaries, to likely survive subsequent dynamical encounters, a hardness should be about few times larger than 1. Therefore we can assume that most of the formed triples can be easily destroyed in their subsequent evolution in the dense core. However, we find also that about a third of all triples are the Kozai triples. The probability that a triple is affected by Kozai mechanism does not correlate with the triple's hardness, the orbital periods or the eccentricities. In the result, a significant fraction of all triples can evolve not as our triples-breaking treatment predicts.

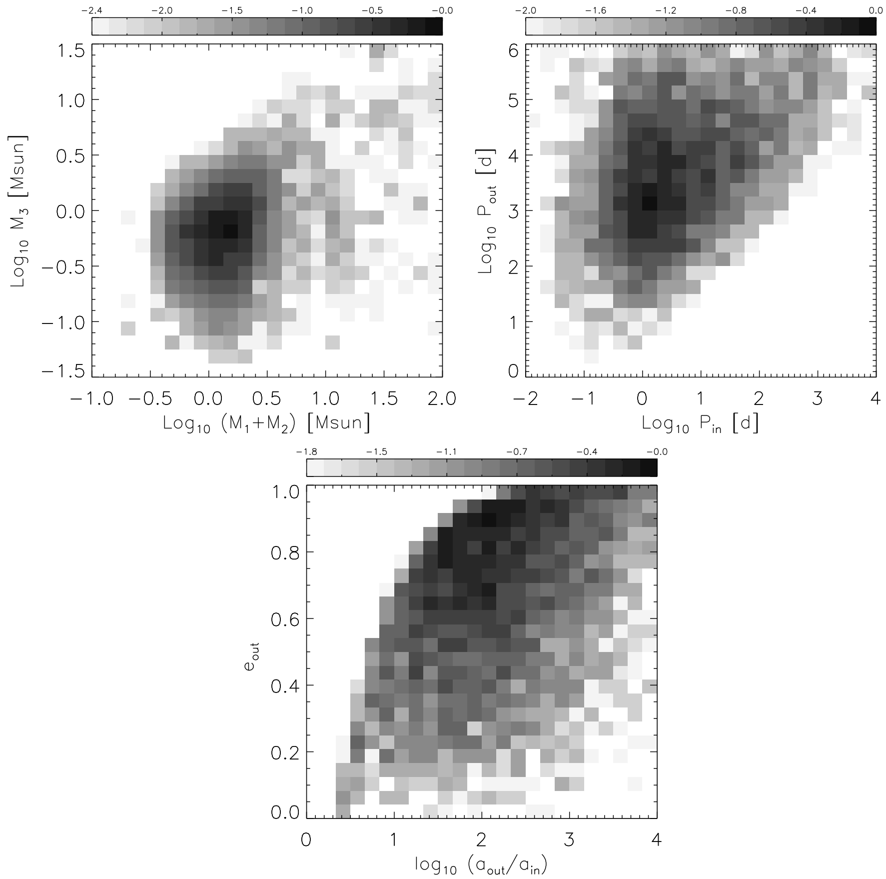

Fig. 1. The distributions of masses of inner binaries and outer companions (left panel), of inner and outer orbital periods (right panel) and the plot of outer companion's eccentricities versus the orbital separation ratio (bottom panel) in the dynamically formed hierarchically stable triples. The colors correspond to the logarithm of the probability density.

3.4 Population of Kozai Triples

About 60% of all triples have inner binaries with both components on the Main Sequence (MS-MS binaries), and 30% of them are Kozai triples. A typical total mass of such binary is $1.3 \pm 0.2\ M_\odot$ (see also Fig. 2). If Kozai mechanism leads to a merger, we find that these triples can provide 10% of all blue stragglers ever created.

About 40% of all triples have inner binaries with a compact object (WD for white dwarf and NS for neutron star) and 35% (40% for WD-WD inner binaries) of them are Kozai triples. A typical WD-MS binary affected by the Kozai evolution consists of a 0.8 $M_\odot$ WD and 0.8 $M_\odot$ MS star. It is not clear

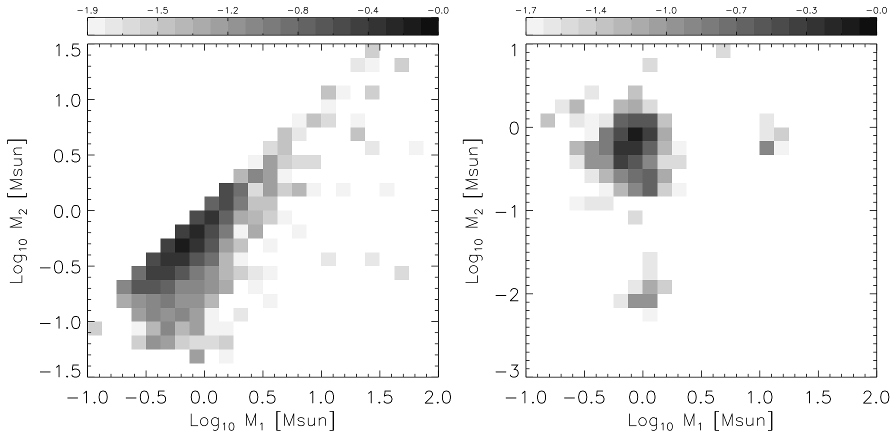

Fig. 2. The population of inner binaries in Kozai triples: MS-MS sequence binaries (left panel) and inner binaries with a compact object (right panel). The colors correspond to the logarithm of the probability density, M_1 is the mass of the more massive companion in the case of MS-MS binary or the mass of a compact companion.

Table 1. The stellar population of close binaries and triples in the cluster core at the age of 10 Gyr. MS stands for a Main Sequence star, RG for a red giant, WD for a white dwarf and NS for a neutron star.

	Binaries		Triples, inner binaries		Triples, outer companions
	MS	WD	MS	WD	
MS	79%	13%	60%	20%	79%
RG	0.7%	0.3%	2%	1%	0.7%
WD		7%		15%	20%
NS	0.3%	0.3%	0.6%	0.7%	0.3%

if the Kozai mechanism will lead to the merger or to the start of the mass transfer and will create therefore a cataclysmic variable. In the latter case, the formation rate of cataclysmic variables via Kozai mechanism is rather large. For example, in a typical globular cluster at the age of 10 Gyr, it can provide in 1 Gyr as many new cataclysmic variables as 25-50% of all cataclysmic variables that were created by all other channels and are present in the cluster at this age .

3.5 Comparison of the Close Binaries Population and the Triples Population

In Table 1 we provide the complete statistics for the stellar population of close binaries and triples in the cluster core at the age of 10 Gyr. We find that the inner binary of a triple is more likely to contain a compact object than a core binary at 10 Gyr. The statistics for outer stars follows that of the binariy populations.

3.6 Comparison of Models with and without Kozai Triples Mergers.

We find that different treatments of Kozai triples (our standard breaking of triples based on the energy balance or the enforced merge of the inner binary in a Kozai triple) do not lead to significantly different results for the binary fractions (the difference is less than 1%). Some difference is noticed for the distribution of binary periods of core binaries with a WD companion: in the model where Kozai triples, once formed, had their inner binaries merged, the relative number of binaries with the periods less than 0.3 day is smaller than in the model where Kozai triples were treated as usual, and for binaries with periods between 0.3 and 3 days the situation is reversed.

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References

1. E. B. Ford, B. Kozinsky, F.A. Rasio: ApJ **535**, 385 (2000)
2. J. M. Fregeau, P. Cheung, S. F. Portegies Zwart et al.: MNRAS **352**, 1 (2004)
3. D. Heggie, P. Hut: *The Gravitational Million-Body Problem: A Multidisciplinary Approach to Star Cluster Dynamics* (Cambridge University Press 2003)
4. J. R. Hurley, O. R. Pols, S. J. Aarseth, S. J. at al: ArXiv Astrophysics e-prints, arXiv:astro-ph/0507239 (2005)
5. K. .A. Innanen, J. Q. Zheng, S. Mikkola et al: AJ **113**, 1915 (1997)
6. N. Ivanova, K. Belczynski, J. M. Fregeau et al: MNRAS **358**, 572 (2005)
7. Y. Kozai: AJ **67**, 591 (1962)
8. R. A Mardling, S. J. Aarseth: MNRAS 2001 **321**, 398 (2001)
9. S. Mikkola: MNRAS **207**, 115 (1984)
10. S. McMillan, P. Hut, J. Makino: ApJ **372**, 111 (1991)