

Observing Planetary Systems II

An ESO workshop to bring together both communities of solar system and extra-planetary system researches and to foster our understanding of the formation and evolution of planetary systems at large

Santiago, Chile, March 5-8, 2012

Topics and Invited Speakers

The first Myr. of planetary formation

Hilke Schlichting, UCL
Bill Dent, ESO-ALM
Sebastian Wolf, Kiel University

Nature and orbits of planetary bodies

Dave Jewitt, UCL
Willy Benz, Bern University
Caroline Terquem, Institut d'Astrophysique de Paris
Didier Queloz, Genève Observatory
Alessandro Morbidelli, Nice Observatory

Planetary atmospheres and bio-markers

Tobias Owen, University of Hawaii
Enric Pallé, Instituto de Astrofísica de Canarias
Michaël Gillon, Liège University

SPHERE: Future ESO planet-finder

David Mouillet, Institut de Planétologie et d'Astrophysique de Grenoble

Organizing Committee

- Christophe Dumas (ESO, Chile)
- Michael Sterzik (ESO, Chile)
- Claudio Melo (ESO, Chile)
- Ralf Siebenmorgen (ESO, Garching)
- David Mouillet (Observatoire de Grenoble, France)
- Members of the ESO-Chile Planetary Sciences Group

© Image source: NASA/JPL

Web page: <http://www.eso.org/sci/meetings/2012/OPSII.html>
Conference e-mail: ops2012@eso.org



Forming

Planetary Systems II

An ESO workshop to bring together both communities of solar system and extra-planetary system researches and to foster our understanding of the formation and evolution of planetary systems at large

Santiago, Chile, March 5-8, 2012

Topics and Invited Speakers

The first Myr. of planetary formation

Hilke Schlichting, UCL
Bill Dent, ESO-ALM
Sebastian Wolf, Kiel University

Nature and orbits of planetary bodies

Dave Jewitt, UCL
Willy Benz, Bern University
Caroline Terquem, Institut d'Astrophysique de Paris
Didier Queloz, Genève Observatory
Alessandro Morbidelli, Nice Observatory

Planetary atmospheres and bio-markers

Tobias Owen, University of Hawaii
Enric Pallé, Instituto de Astrofísica de Canarias
Michaël Gillon, Liège University

SPHERE: Future ESO planet-finder

David Mouillet, Institut de Planétologie et d'Astrophysique de Grenoble

Organizing Committee

- Christophe Dumas (ESO, Chile)
- Michael Sterzik (ESO, Chile)
- Claudio Melo (ESO, Chile)
- Ralf Siebenmorgen (ESO, Garching)
- David Mouillet (Observatoire de Grenoble, France)
- Members of the ESO-Chile Planetary Sciences Group

© Image source: NASA/JPL

Web page: <http://www.eso.org/sci/meetings/2012/OPSII.html>
Conference e-mail: ops2012@eso.org



Forming

Planetary Systems II

An ESO workshop to bring together both communities of solar system and extra-planetary system researches and to foster our understanding of the formation and evolution of planetary systems at large

Santiago, Chile, March 5-8, 2012

- Y. Alibert
- A. Fortier
- F. Carron
- C. Mordasini
- W. Benz
- L. Fouchet
- S. Pfyffer
- N. Cabral
- H. Meheut
- K. Dittkrist



Topics and Invited Speakers

The first Myr. of planetary formation

Hilke Schlichting, UCL
Bill Dent, ESO-ALM
Sebastian Wolf, Kiel University

Nature and orbits of planetary bodies

Dave Jewitt, UCL
Willy Benz, Bern University
Caroline Terquem, Institut d'Astrophysique de Paris
Didier Queloz, Genève Observatory
Alessandro Morbidelli, Nice Observatory

Planetary atmospheres and bio-markers

Tobias Owen, University of Hawaii
Enric Pallé, Instituto de Astrofísica de Canarias
Michaël Gillon, Liège University

SPHERE: Future ESO planet-finder

David Mouillet, Institut de Planétologie et d'Astrophysique de Grenoble

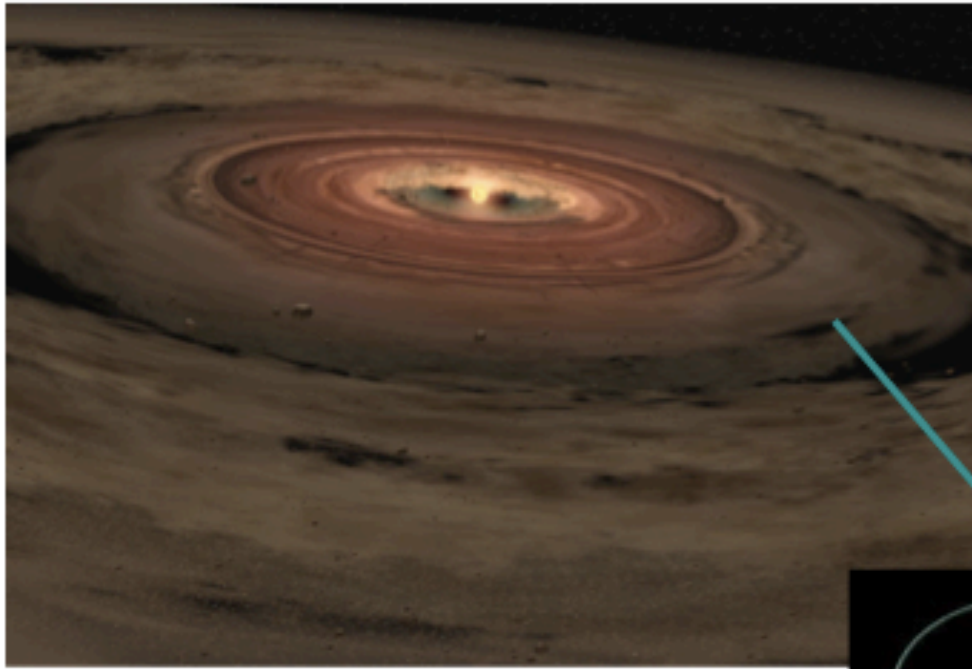
Organizing Committee

- Christophe Dumas (ESO, Chile)
- Michael Sterzik (ESO, Chile)
- Claudio Melo (ESO, Chile)
- Ralf Siebenmorgen (ESO, Garching)
- David Mouillet (Observatoire de Grenoble, France)
- Members of the ESO-Chile Planetary Sciences Group

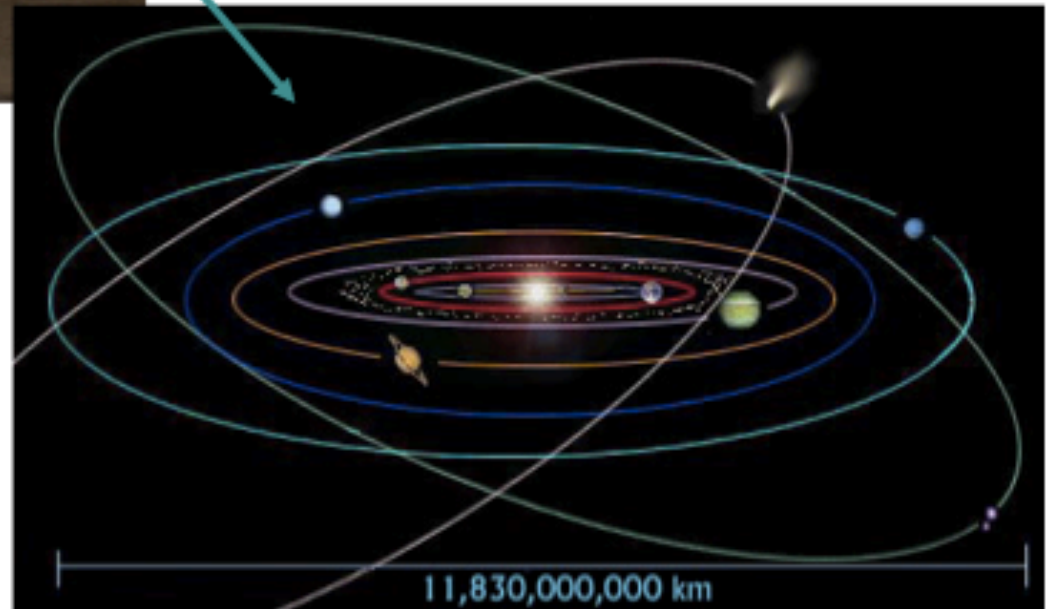
Web page: <http://www.eso.org/sci/meetings/2012/OPSII.html>
Conference e-mail: ops2012@eso.org

© Image source: NASA/JPL

Protoplanetary discs \Rightarrow Planetary system



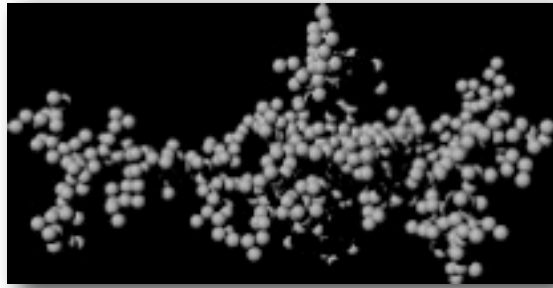
From a gas and dust-rich
disc to a planetary system



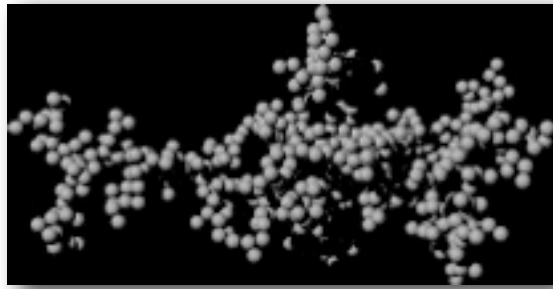
Planet formation

The nucleated instability model (I)

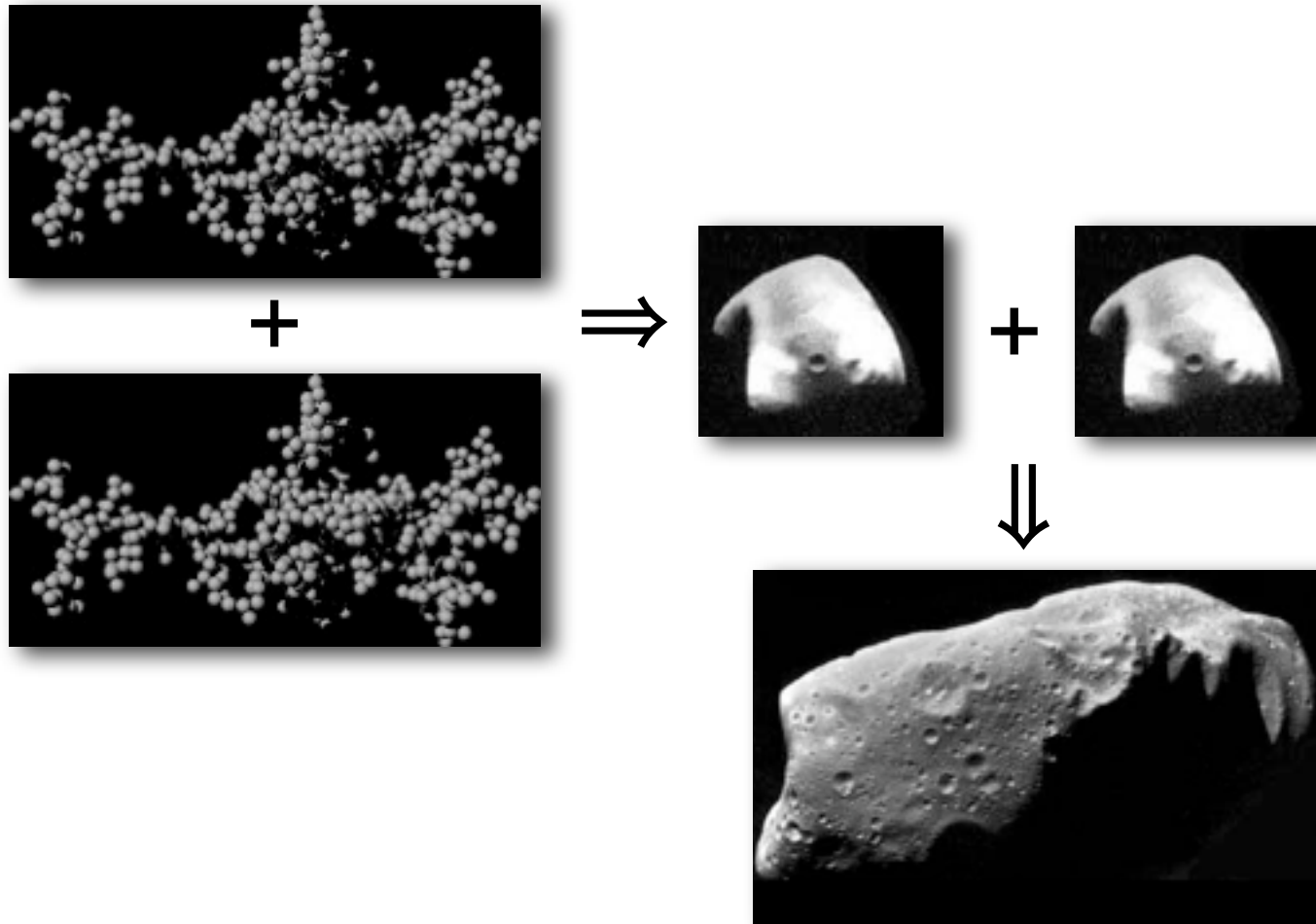
The nucleated instability model (I)



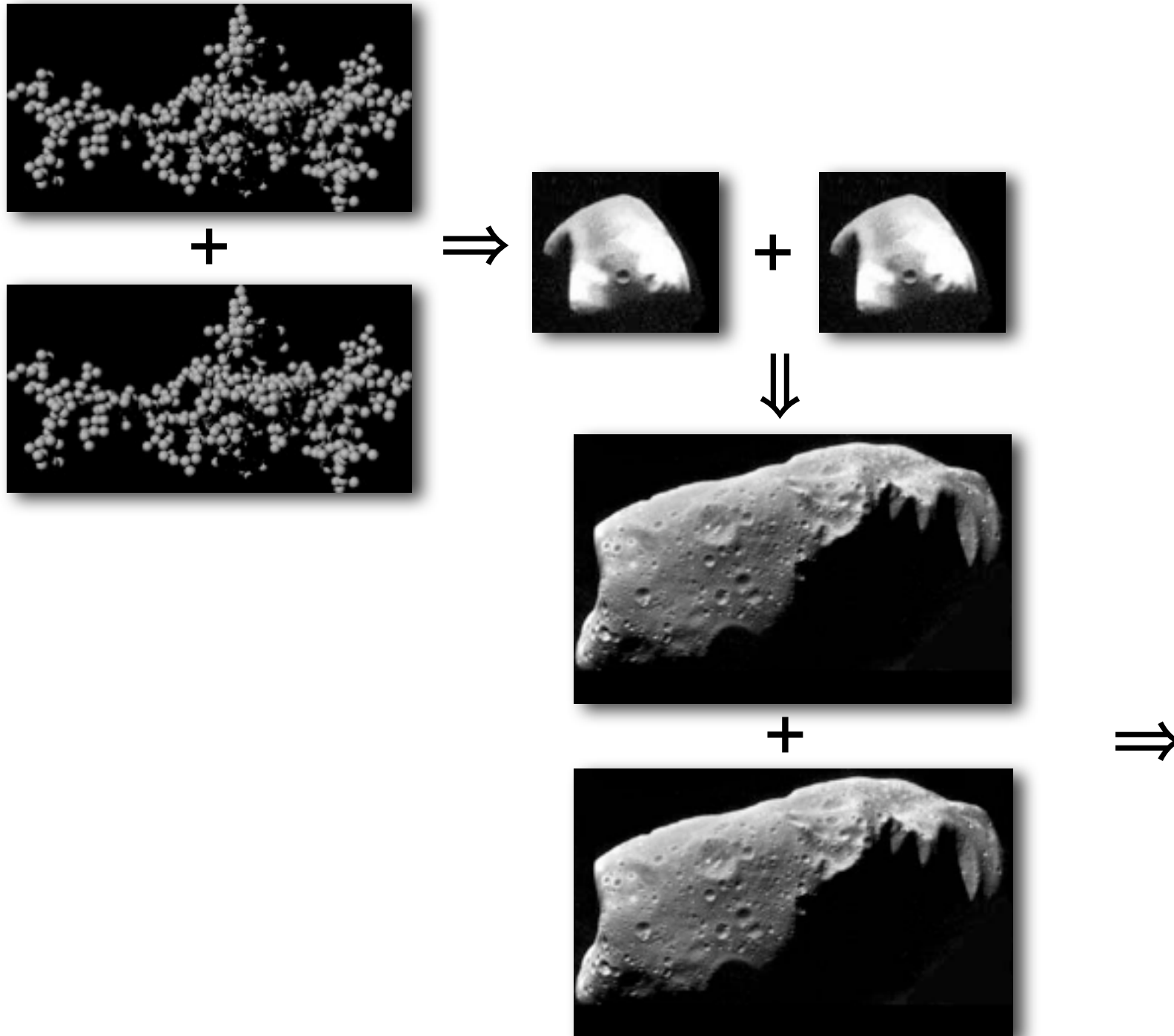
+



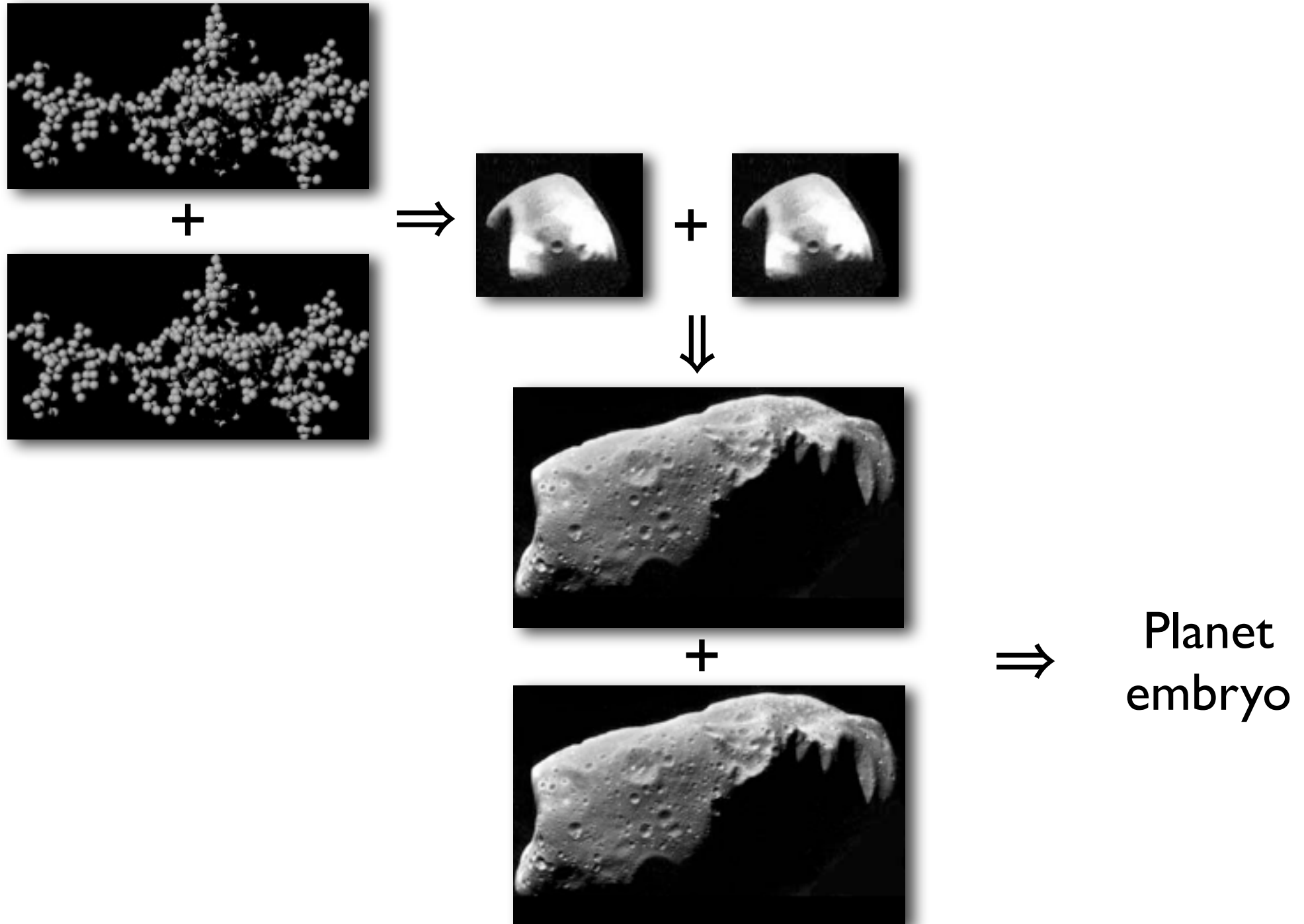
The nucleated instability model (I)



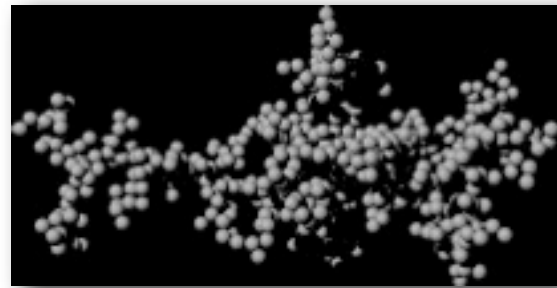
The nucleated instability model (I)



The nucleated instability model (I)



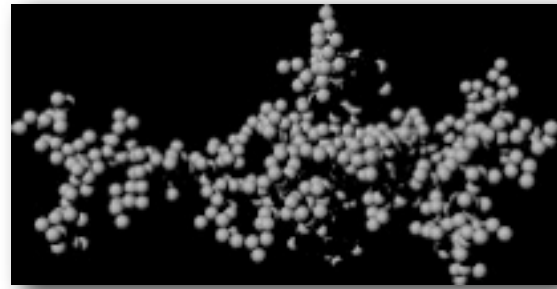
The nucleated instability model (I)



+



+



⇓ **Gas-solids coupling**

Johansen et al. 2007
Boley et al. 2010
Windmark et al. 2012



+



Planet
embryo

The nucleated instability model (2)



The nucleated instability model (2)



gas giant



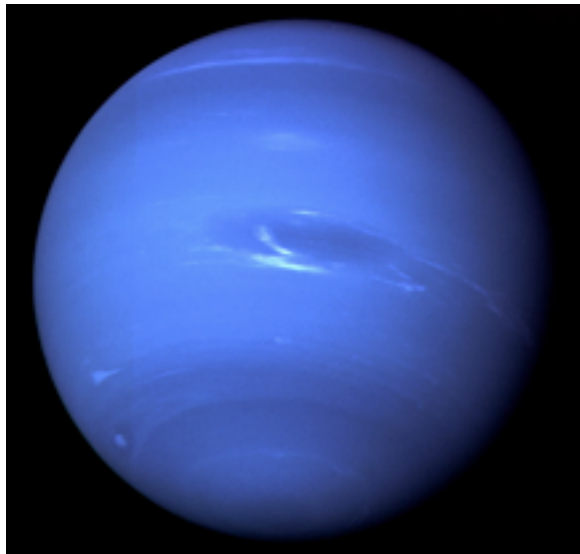
The nucleated instability model (2)



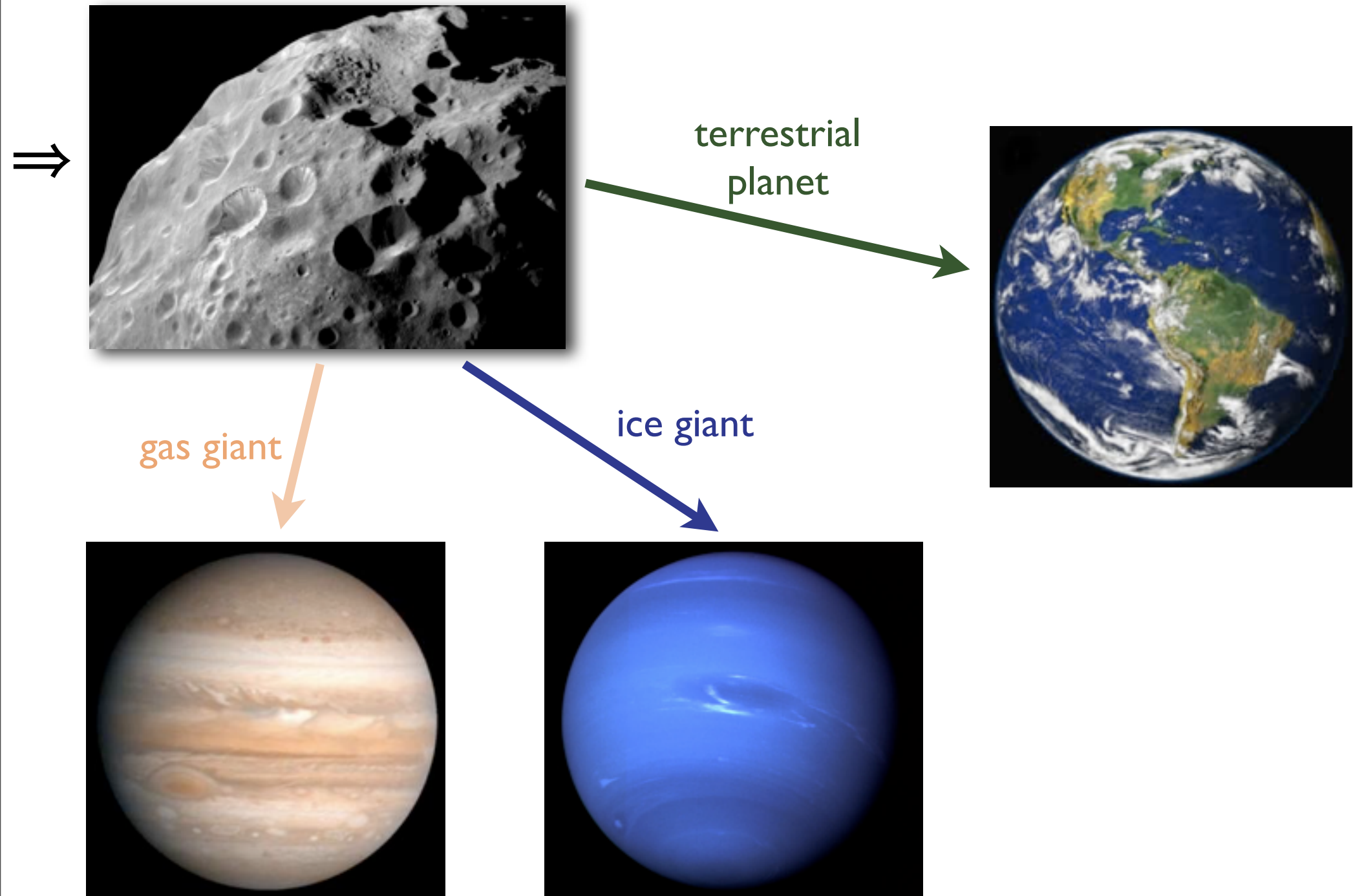
gas giant



ice giant



The nucleated instability model (2)





‘Bern II model’

‘Bern II model’

Core

Envelope

‘Bern II model’

Core

Envelope

Rad. str.

‘Bern II model’

Core

Envelope

Vert. str.

Rad. str.

‘Bern II model’

Core

Envelope

Vert. str.

Rad. str.

Solid disk

‘Bern II model’

Core

Envelope

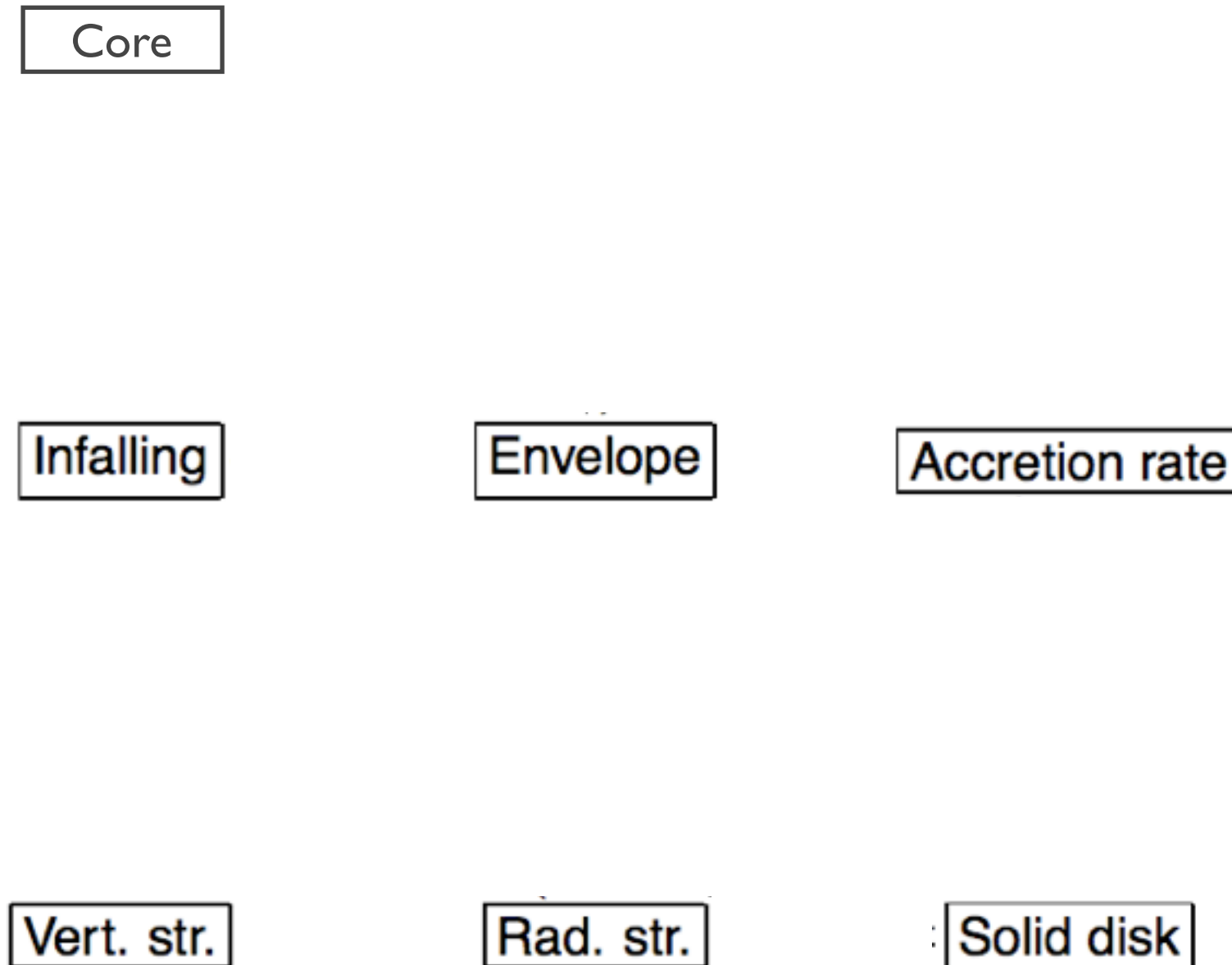
Accretion rate

Vert. str.

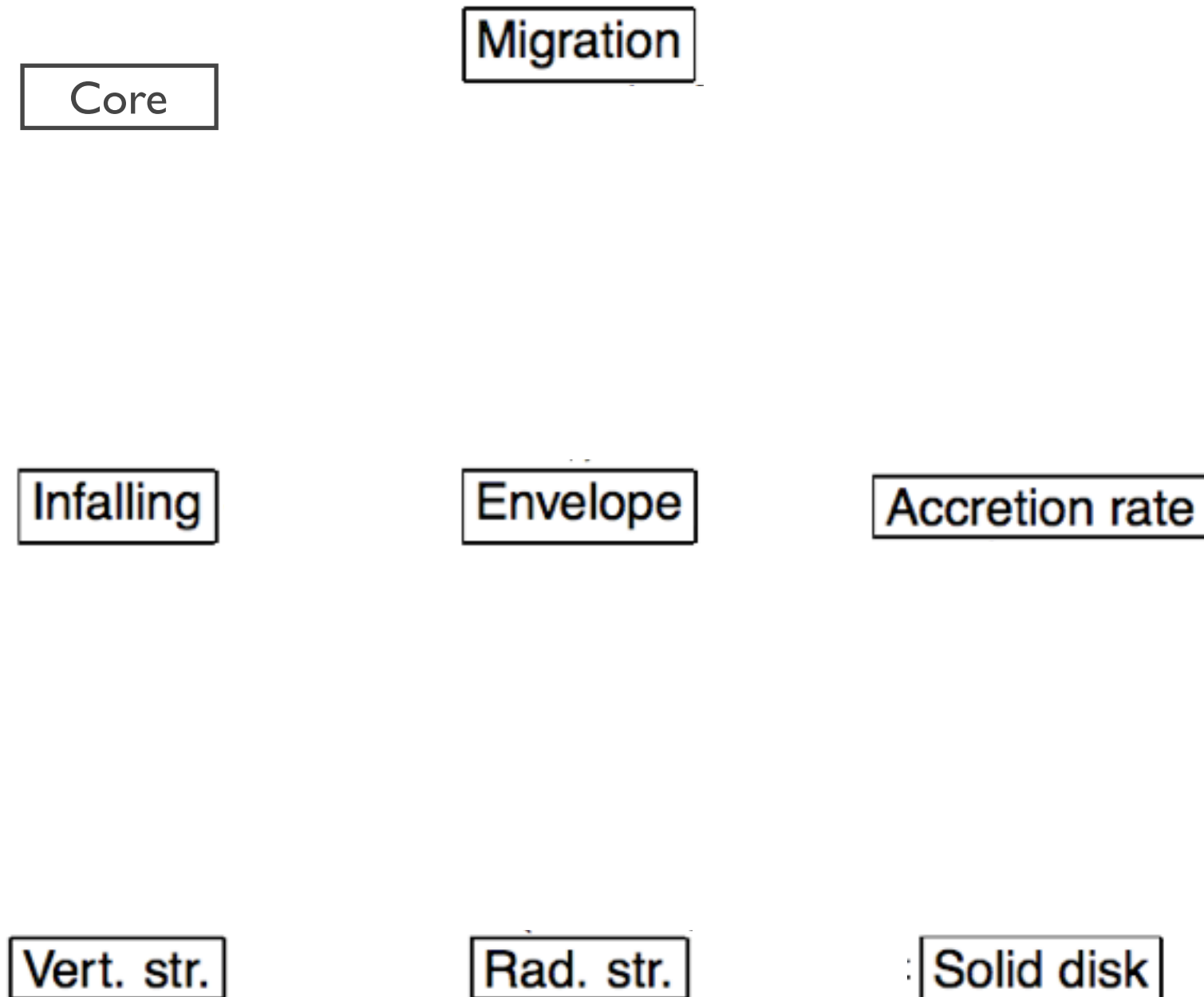
Rad. str.

Solid disk

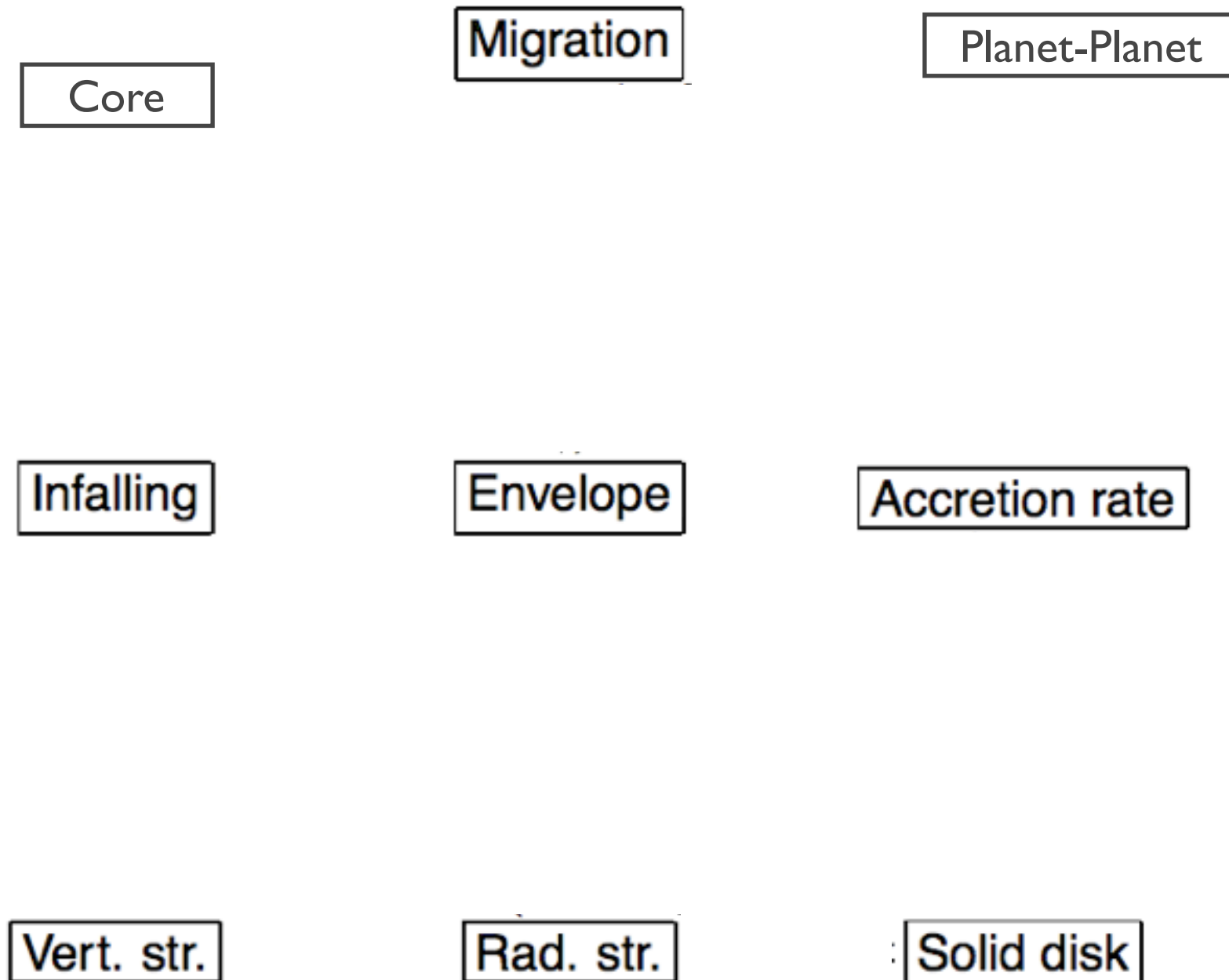
‘Bern II model’



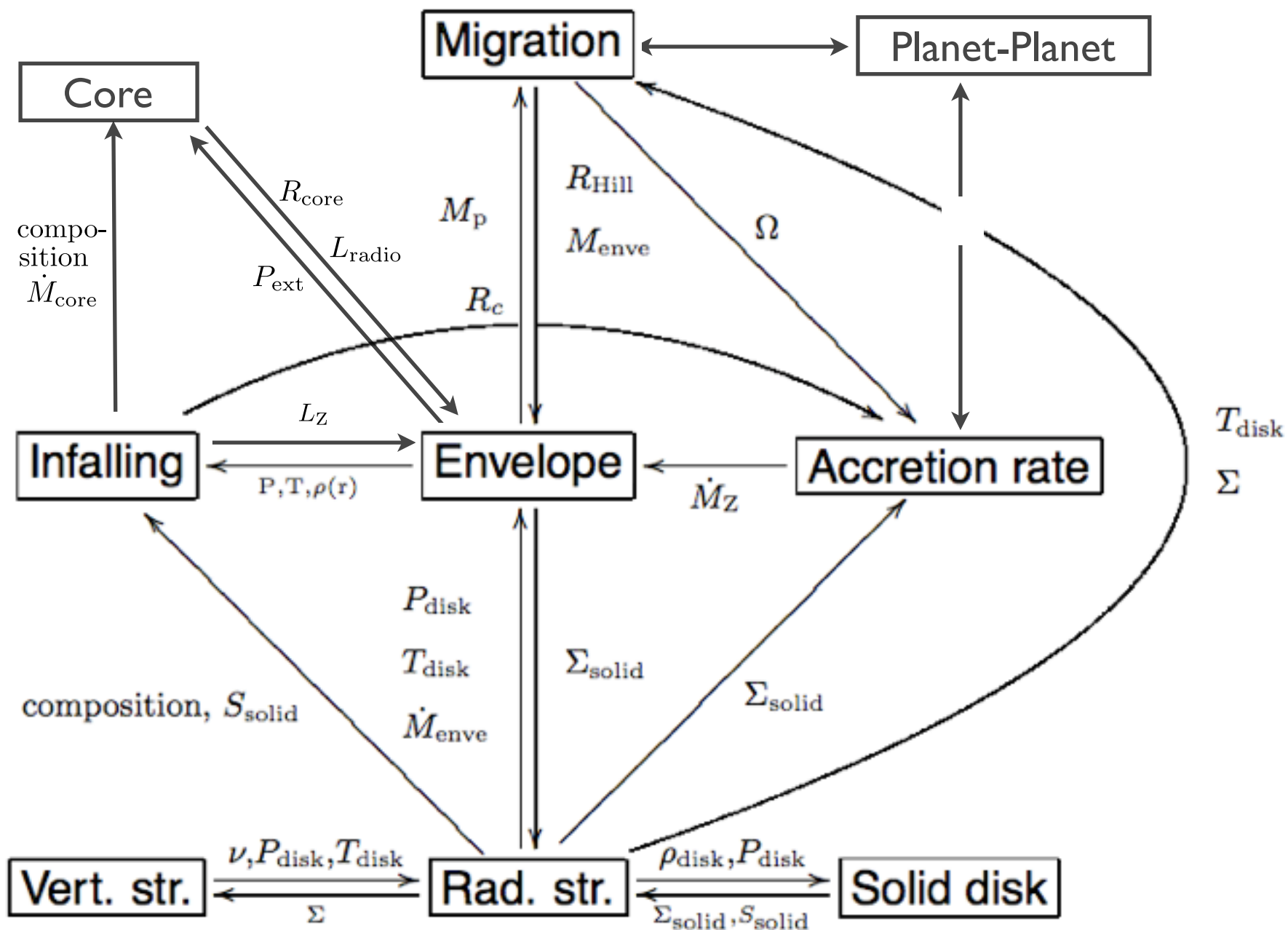
'Bern II model'



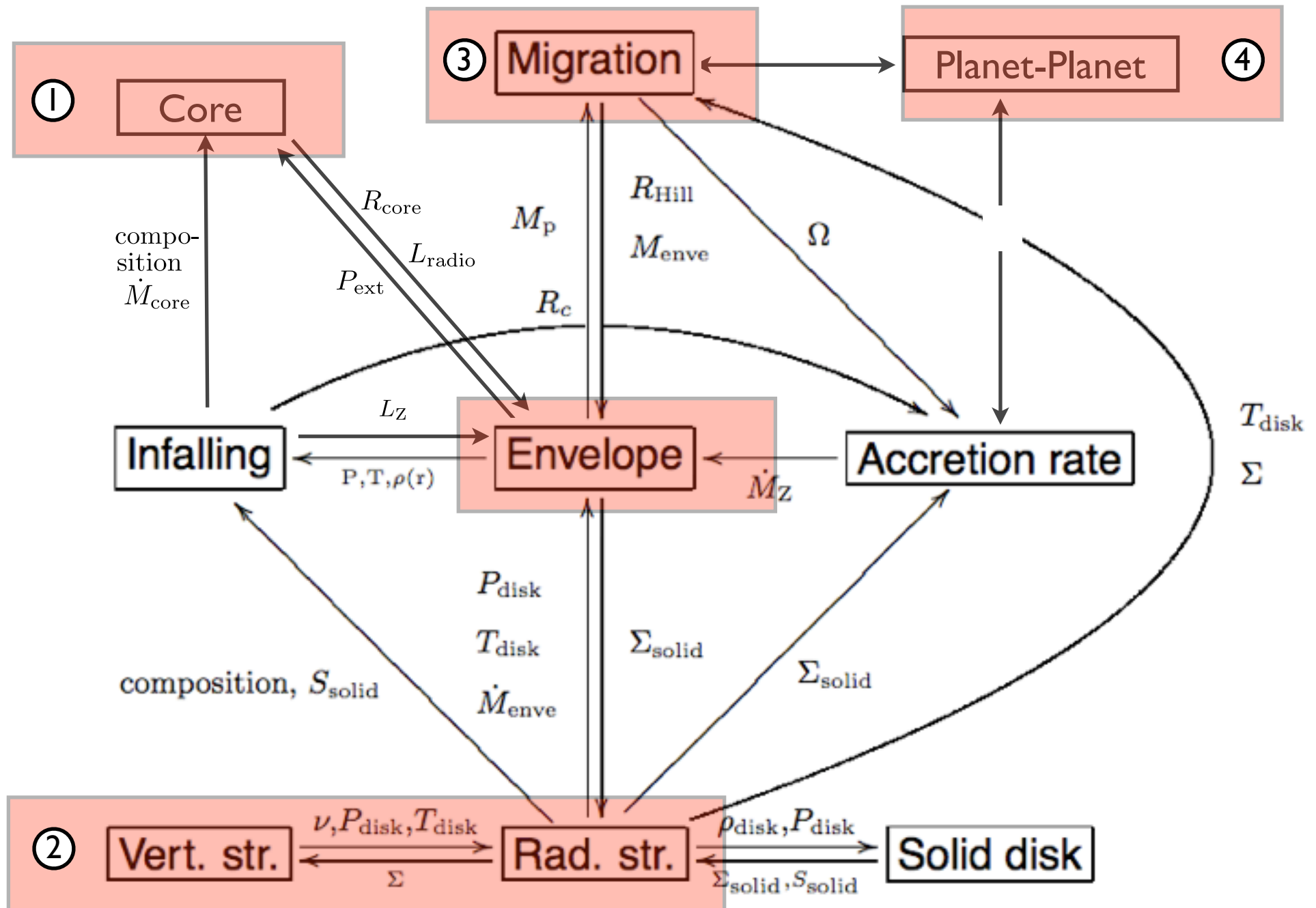
'Bern II model'



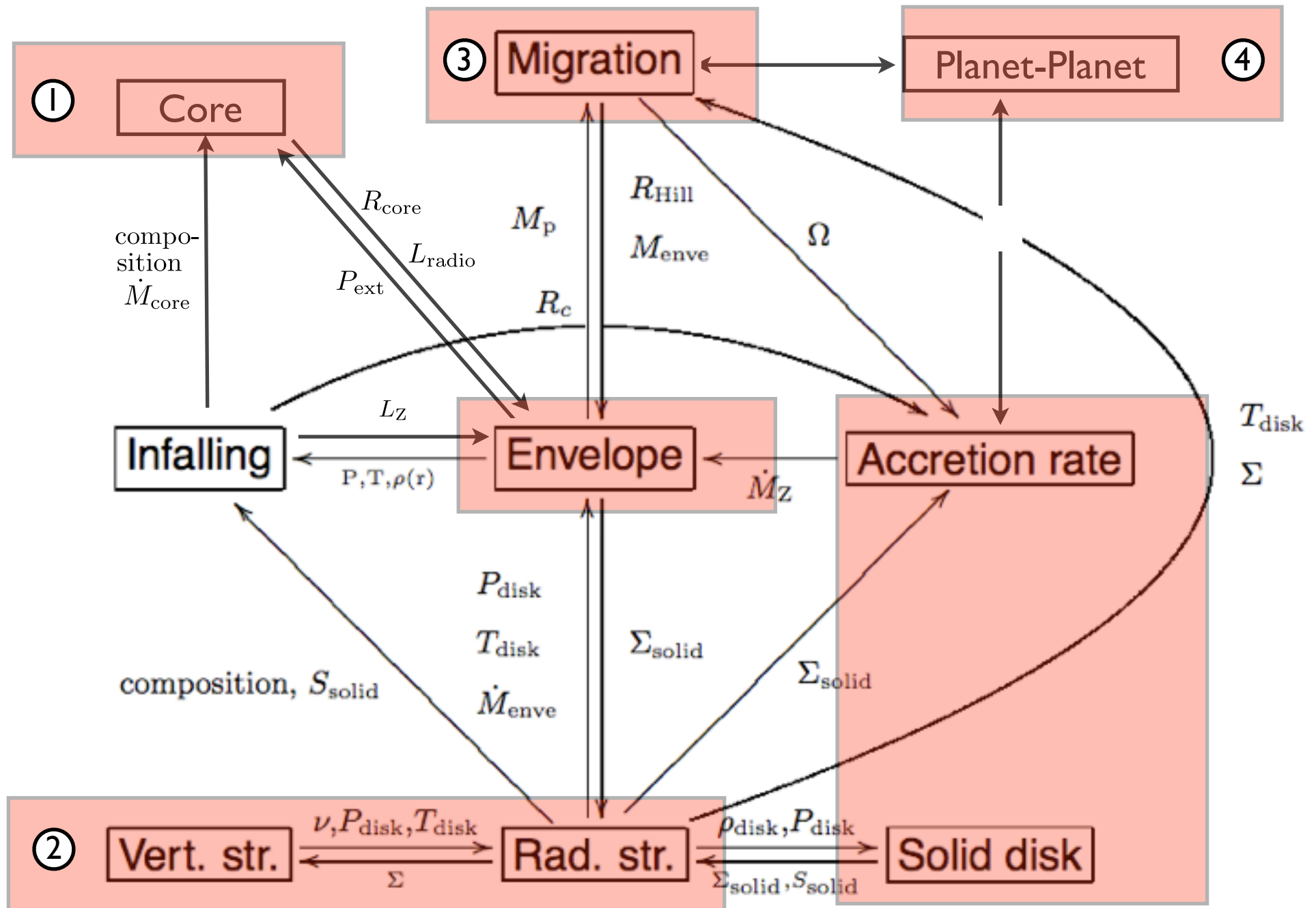
'Bern II model'



'Bern II model'



'Bern II model'



① Planet core structure

Solve **internal structure equations** for the solid core

- Differentiated** planet
 - Iron core, silicate mantle, ice layer (if)
- Simplified** EOS. No temperature dependence.

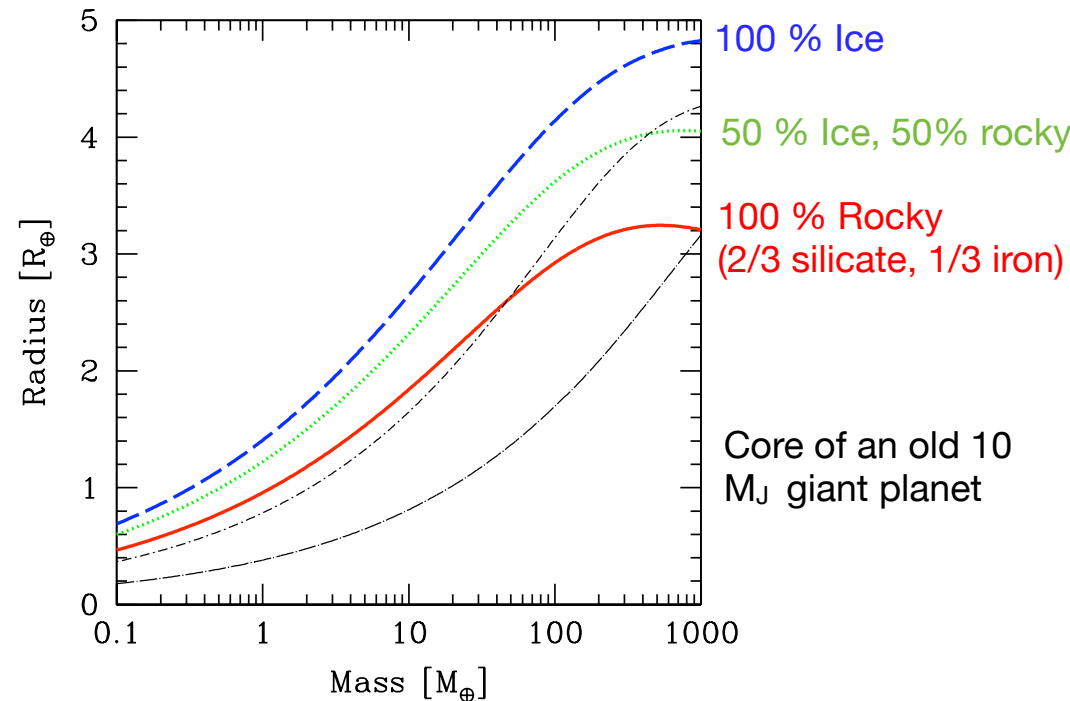
$$\rho(P) = \rho_0 + cP^n$$

Seager et al. 2007

$$\frac{\partial r}{\partial M_r} = \frac{1}{4\pi r^2 \rho} \quad \frac{\partial P}{\partial M_r} = -\frac{GM_r}{4\pi r^4}$$

- Include effect of **external** pressure

$$R_{\text{core}} = R_{\text{core}}(M_{\text{core}}, f_{\text{iron}}, f_{\text{ice}}, P_{\text{ext}})$$



① Planet core structure

Solve internal structure equations for the solid core

- Differentiated planet
 - Iron core, silicate mantle, ice layer (if)
- Simplified EOS. No temperature dependence.

$$\rho(P) = \rho_0 + cP^n \quad \text{Seager et al. 2007}$$

$$\frac{\partial r}{\partial M_r} = \frac{1}{4\pi r^2 \rho} \quad \frac{\partial P}{\partial M_r} = -\frac{GM_r}{4\pi r^4}$$

- Include effect of external pressure

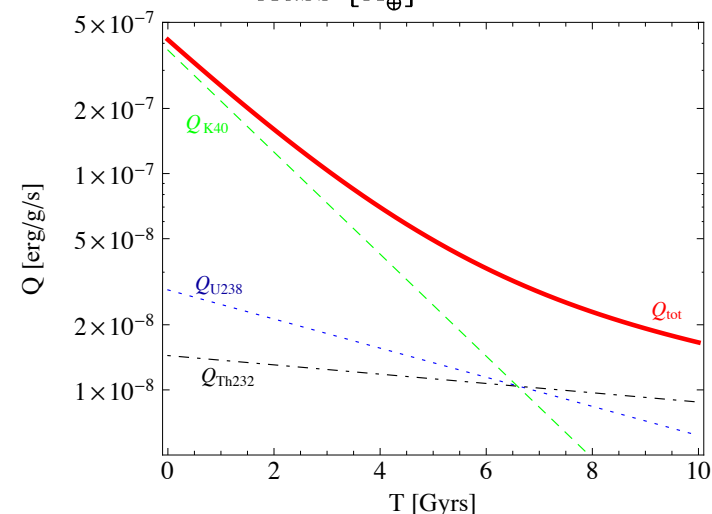
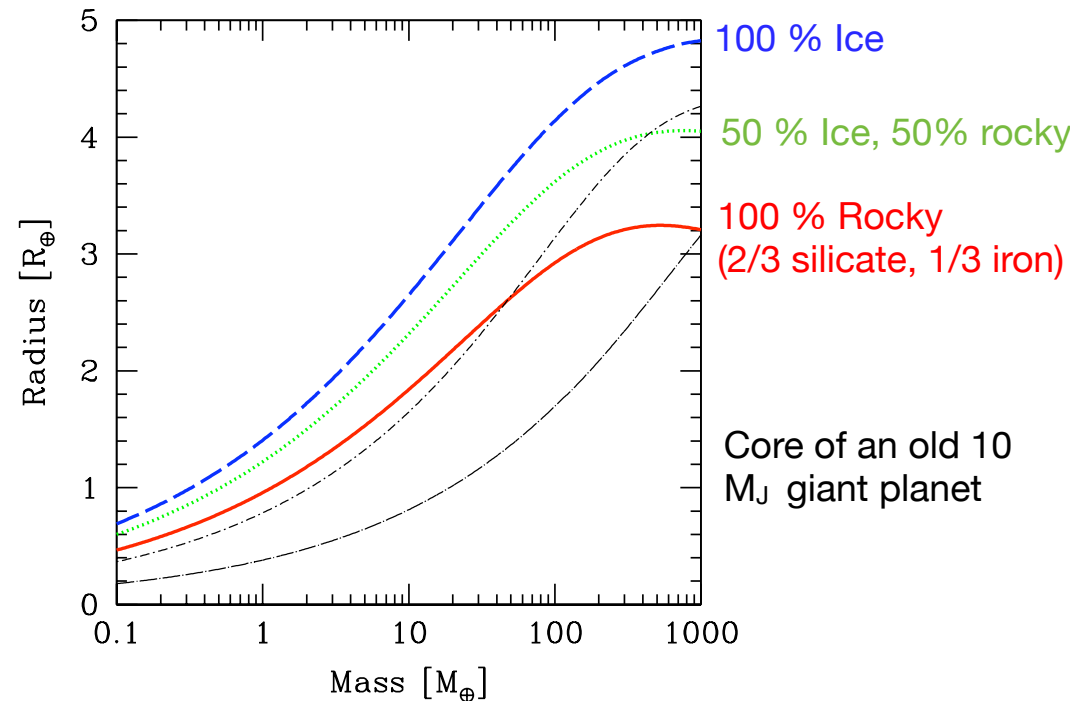
$$R_{\text{core}} = R_{\text{core}}(M_{\text{core}}, f_{\text{iron}}, f_{\text{ice}}, P_{\text{ext}})$$

Radiogenic core heating

- Assume chondritic mantle composition

$$Q_{\text{tot}}(t) = Q_{0,K}e^{-\lambda_K t} + Q_{0,U}e^{-\lambda_U t} + Q_{0,Th}e^{-\lambda_{Th} t}$$

$$L_{\text{radio}}(t) = Q_{\text{tot}}(t)f_{\text{mantle}}f_{\text{rocky}}M_Z$$



① Planet's internal structure: gas

In situ formation of Jupiter at 5.2 AU

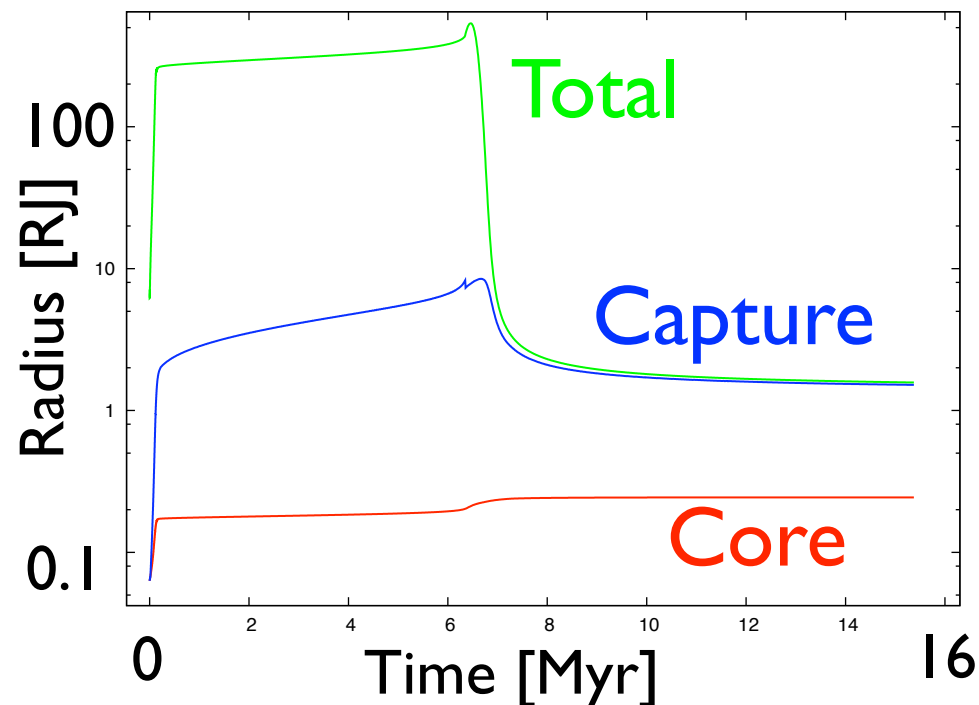
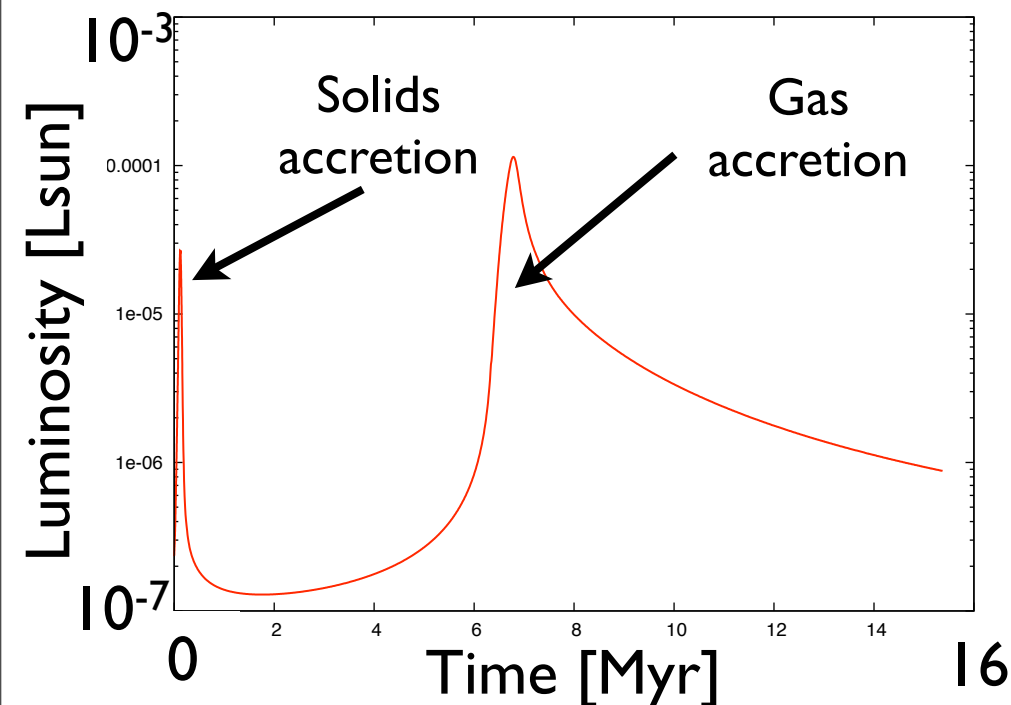
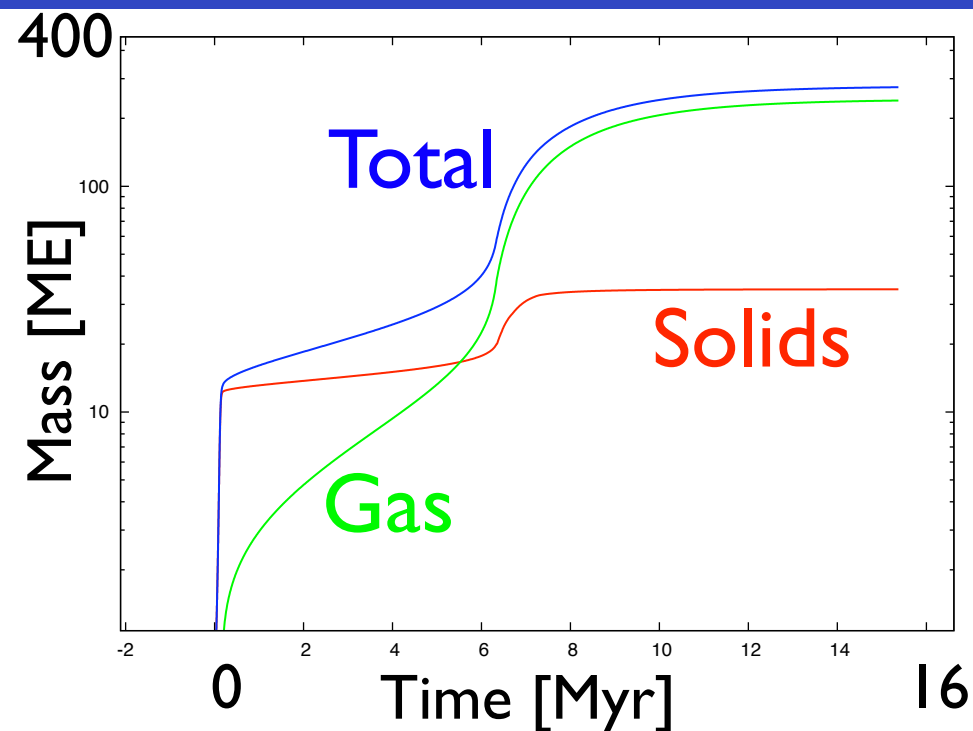
$$\Sigma_{\text{solids}} = 10 \text{g/cm}^2 @ 5.2 \text{AU}$$

$$M_{\text{disk}} \approx 0.03 M_{\odot}$$

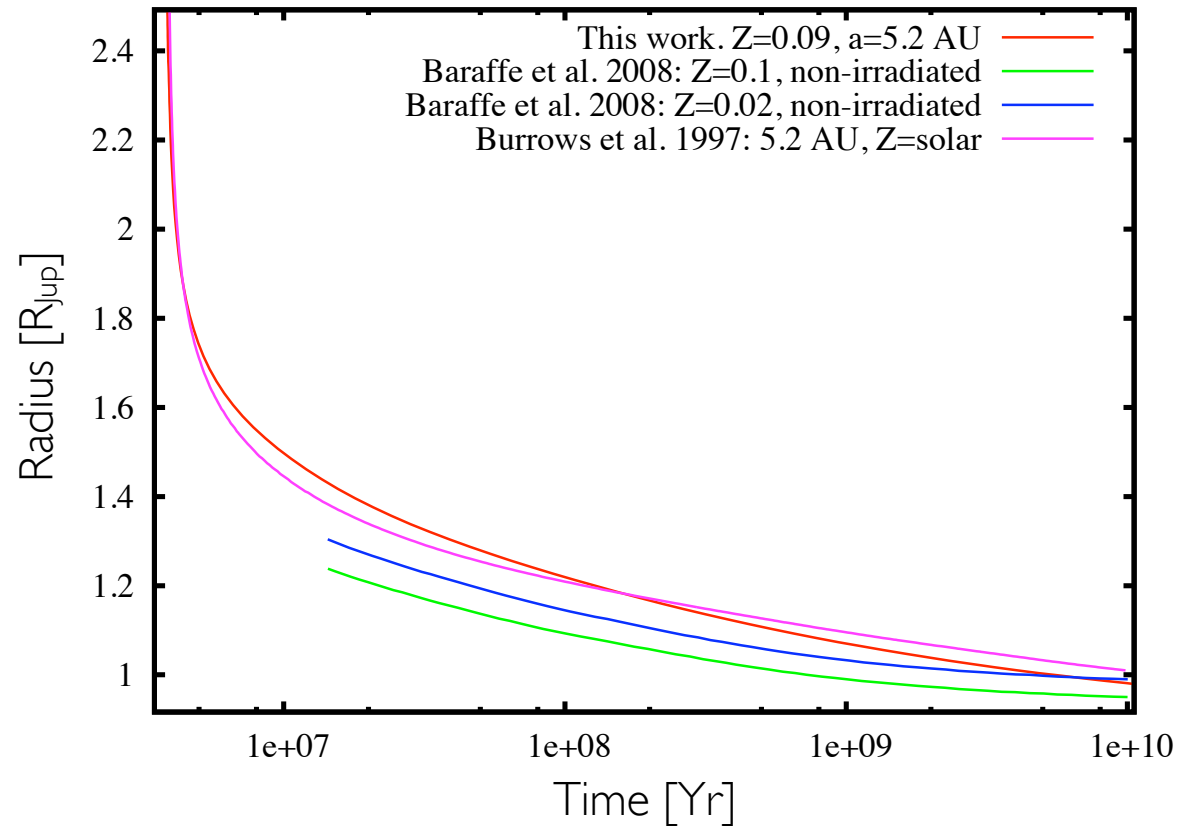
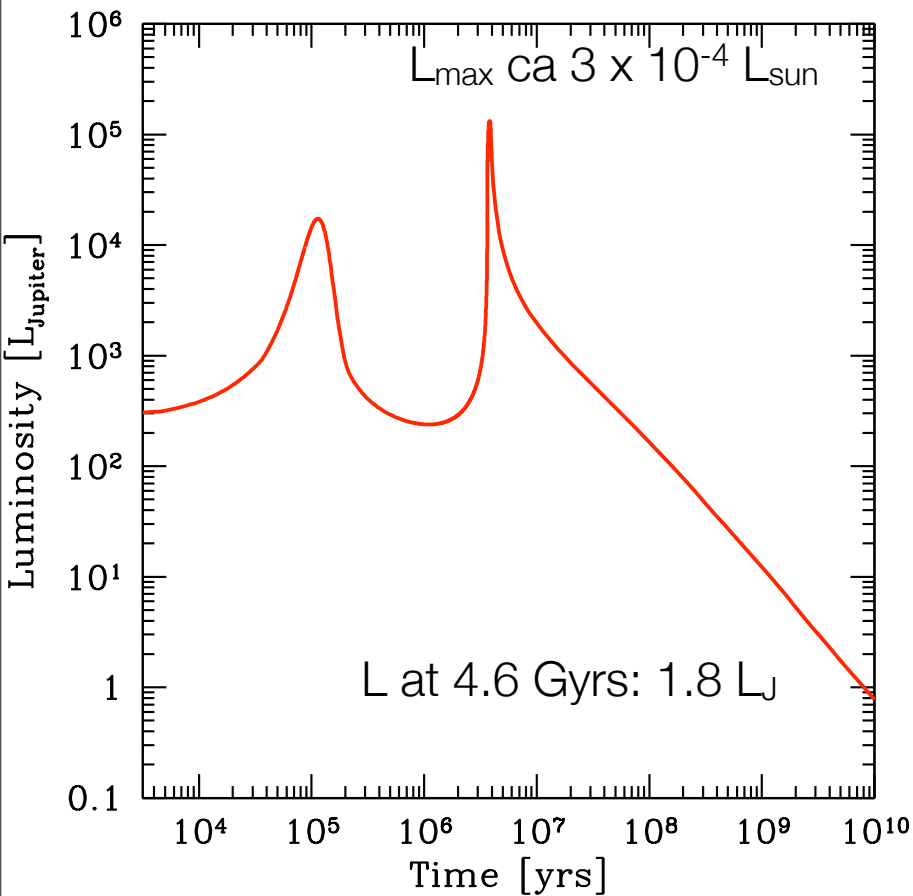
only viscosity

Planet in contact
with disk

Planet isolated

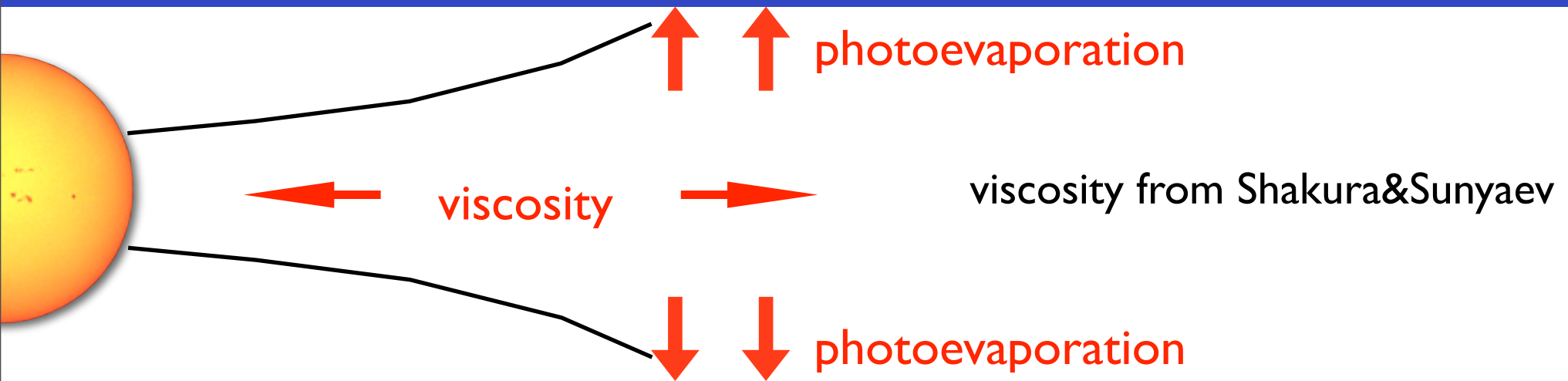


① Planet gas envelope structure - long term evolution



see talk by M. Bonnefoy (friday)

② I+ID disk model: gas



$$\frac{d\Sigma}{dt} = \frac{3}{r} \frac{\partial}{\partial r} \left[r^{1/2} \frac{\partial}{\partial r} \tilde{\nu} \Sigma r^{1/2} \right] + \dot{\Sigma}_w(r) + \dot{Q}_{\text{planet}}(r)$$

Viscosity

Photoevaporation

Planet accretion

Vertical & radial structure.

-constant α $\nu = \alpha c_s H$

-stellar irradiation included for temperature

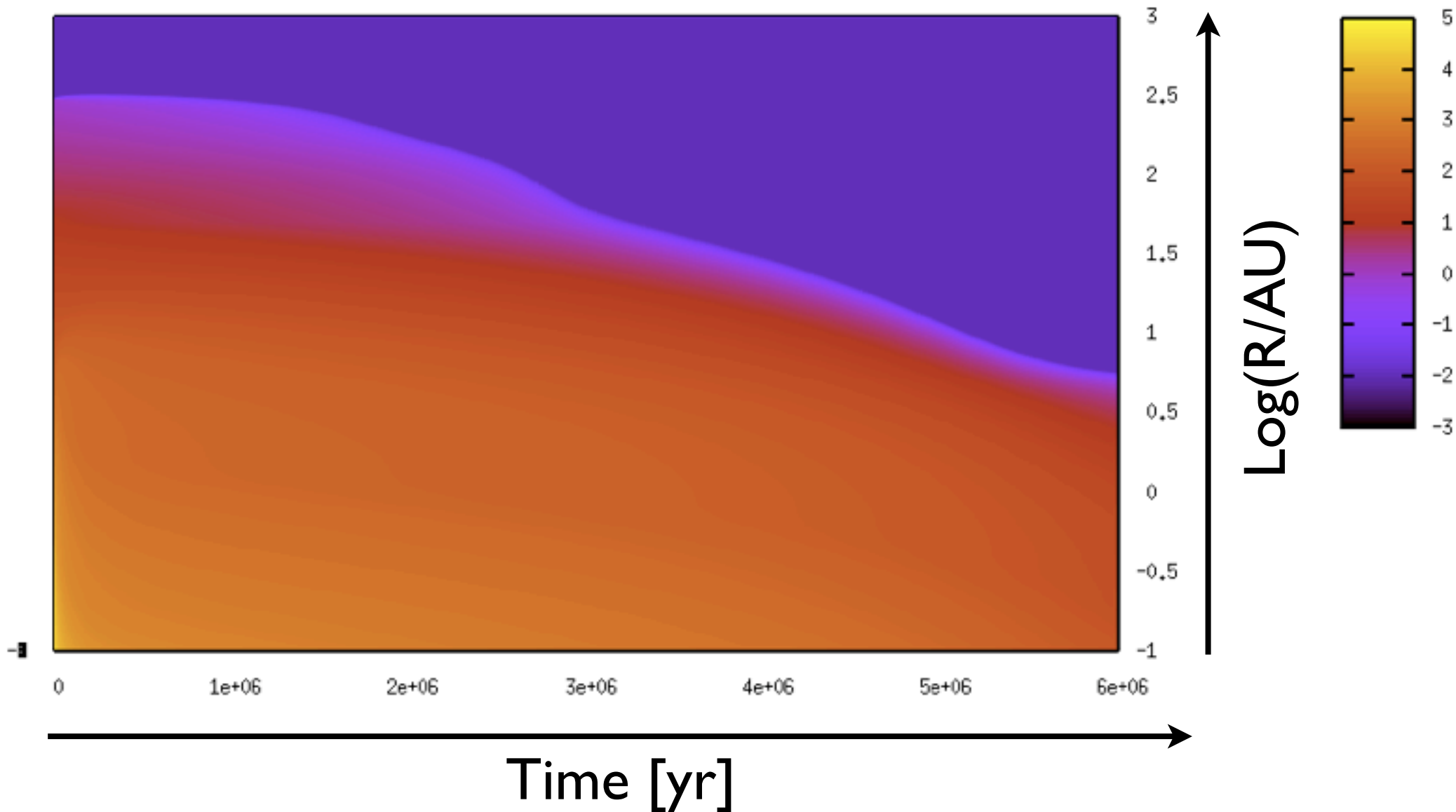
-external photoevaporation

$$\dot{\Sigma}_{w,\text{ext}} = \begin{cases} 0 & \text{for } r < \beta R_{g,I} \\ \frac{\dot{M}_{\text{wind,ext}}}{\pi(r_{\text{max}}^2 - \beta^2 R_{g,I}^2)} & \text{otherwise} \end{cases}$$

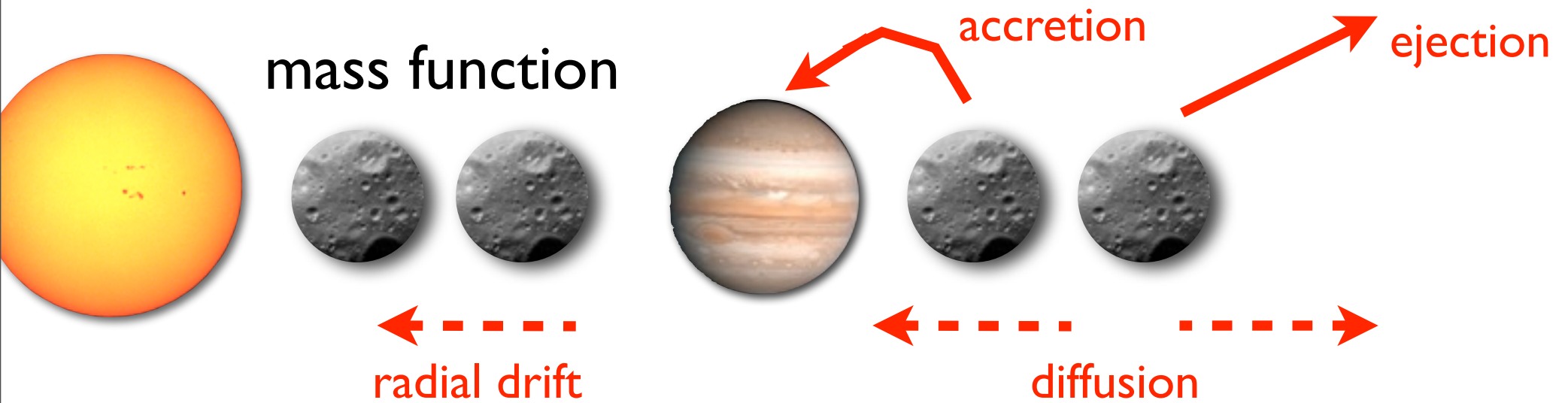
-internal photoevaporation

$$\dot{\Sigma}_{w,\text{int}} = \begin{cases} 0 & \text{for } r < R_{\text{wind}} \\ 2c_{s,II} n_0(r) u_{\text{ma}} & \text{otherwise} \end{cases}$$

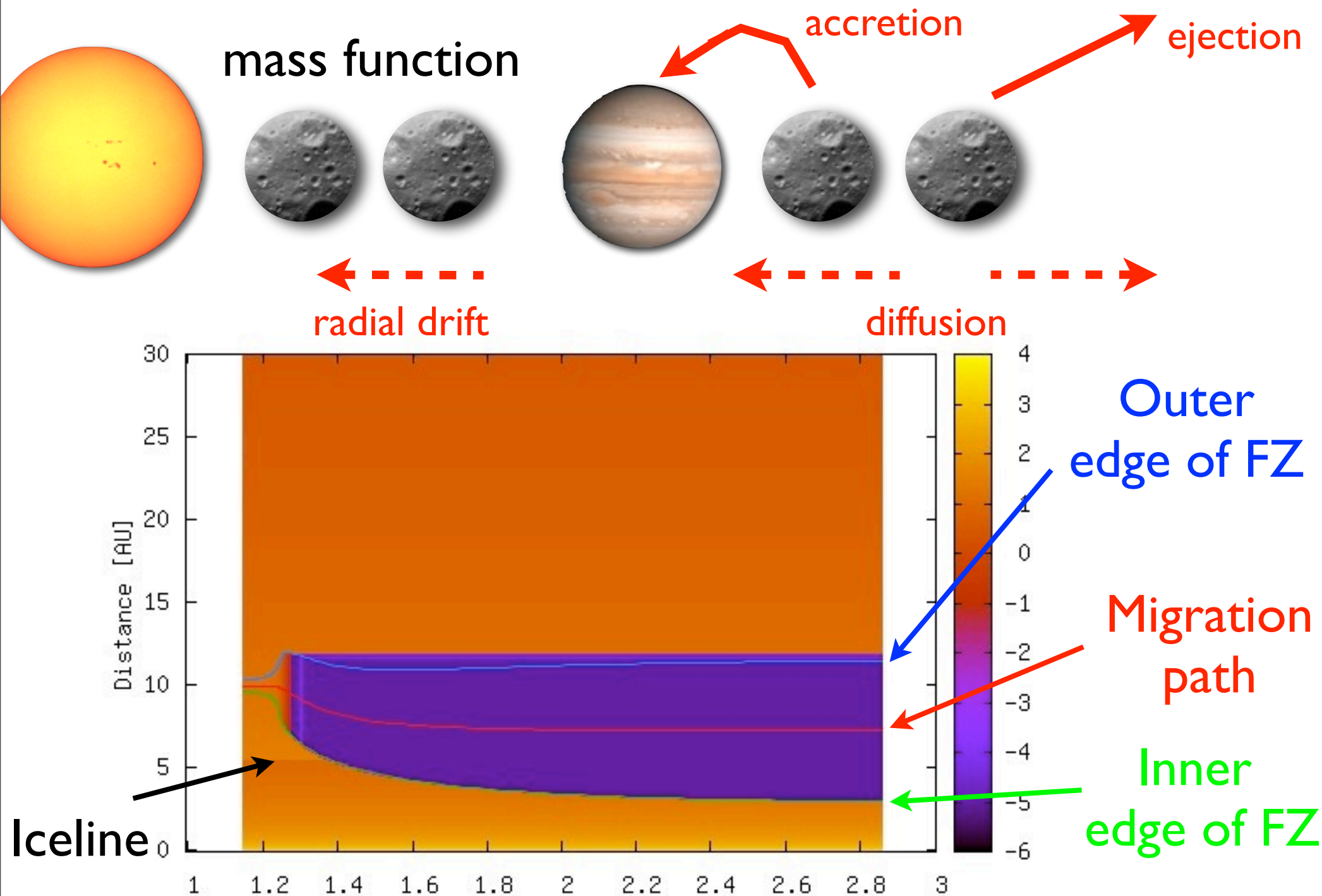
② I+ID disk model



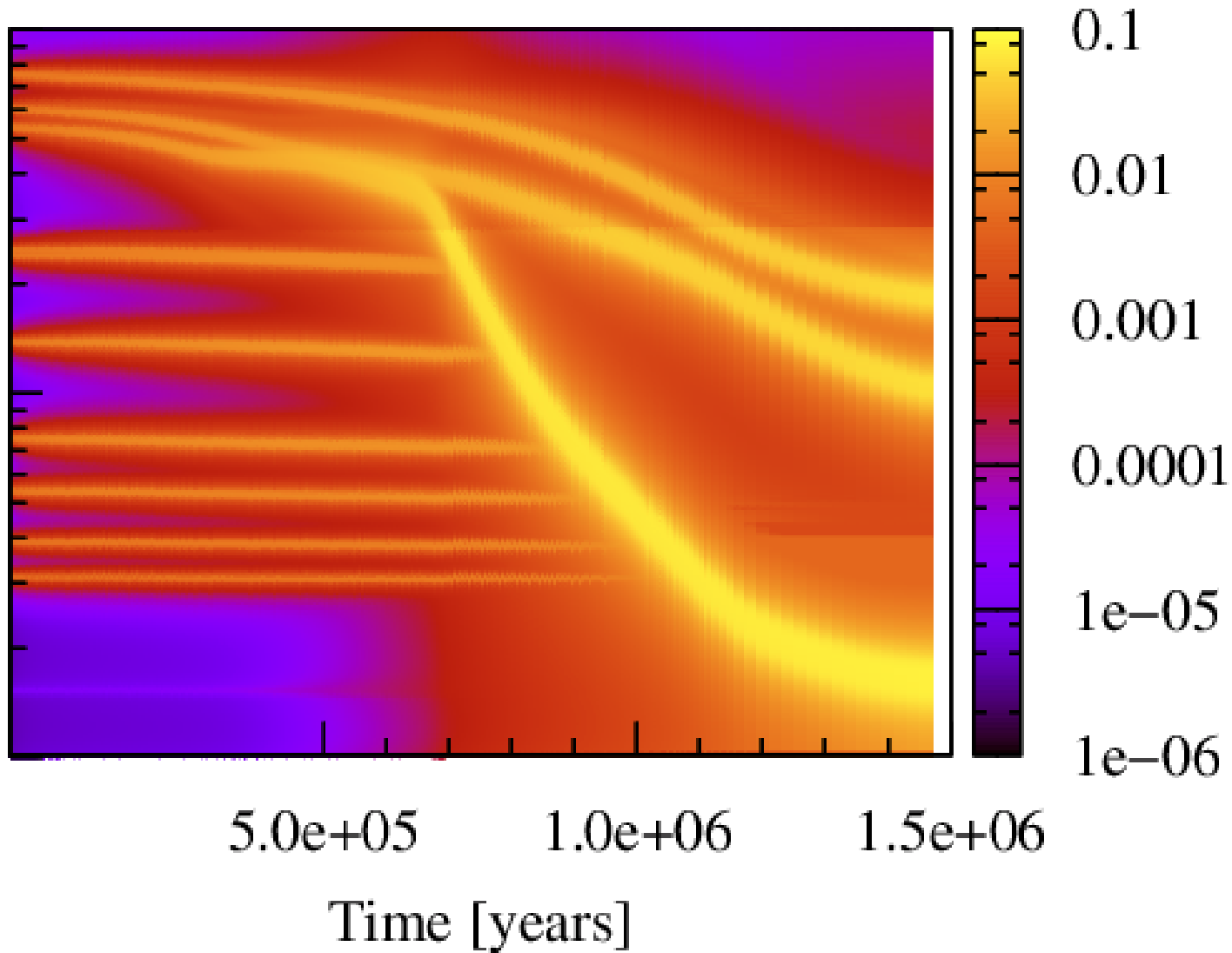
② I+ID disk model: solids



② I+ID disk model: solids



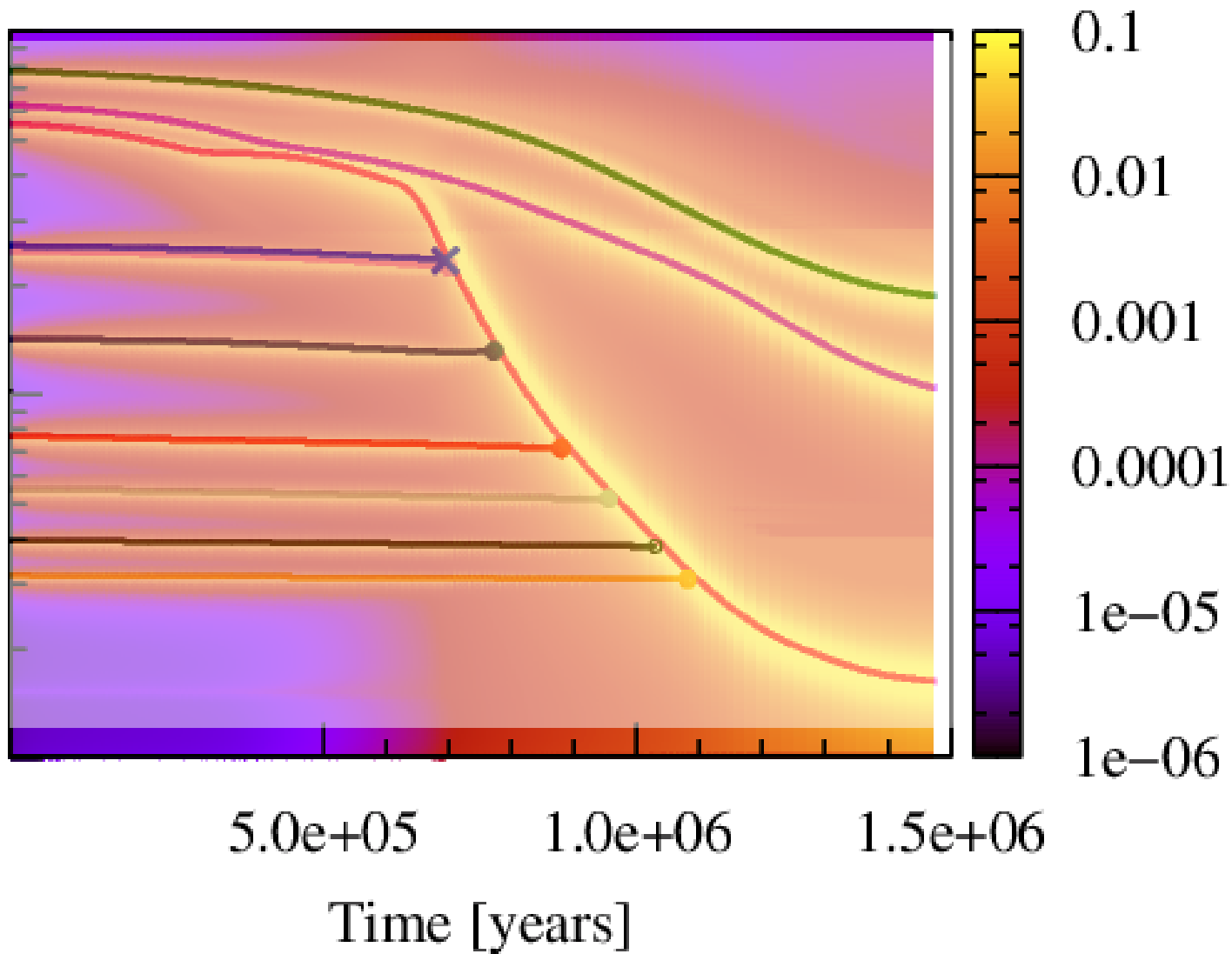
② I+ID disk model: dynamics of solids



$$\frac{de}{dt} = \left. \frac{de}{dt} \right|_{GD} + \left. \frac{de}{dt} \right|_{VS,E} + \left. \frac{de}{dt} \right|_{VS,p}$$

See Poster by A. Fortier

② I+ID disk model: dynamics of solids



$$\frac{de}{dt} = \left. \frac{de}{dt} \right|_{GD} + \left. \frac{de}{dt} \right|_{VS,E} + \left. \frac{de}{dt} \right|_{VS,p}$$

See Poster by A. Fortier

③ Migration

Low mass planets (no gap, $M < \text{ca. } 100 M_{\text{earth}}$): Type I

From [Paardekooper et al. 2009](#)

$$\Gamma_{tot} = \sum_{ILR} \Gamma_{LR} + \sum_{OLR} \Gamma_{LR} + \Gamma_{CR} \quad \frac{dr_p}{dt} = -2r_p \frac{\Gamma_{tot}}{J_p} \quad J_p = M_p (GM_* r_p)^{1/2}$$

$$\Gamma_{tot} = \frac{1}{\gamma} (C_0 + C_1 \alpha + C_2 \beta) \Gamma_0 \quad \Gamma_0 = \left(\frac{q}{h}\right)^2 \Sigma_p^2 r_p^2 \Omega_p^2 \quad \alpha = \frac{d \log \Sigma}{d \log r} \quad \beta = \frac{d \log T}{d \log r}$$

[Baruteau & Masset. 2008](#)

③ Migration

Low mass planets (no gap, $M < \text{ca. } 100 M_{\text{earth}}$): Type I

From [Paardekooper et al. 2009](#)

$$\Gamma_{\text{tot}} = \sum_{ILR} \Gamma_{LR} + \sum_{OLR} \Gamma_{LR} + \Gamma_{CR} \quad \frac{dr_p}{dt} = -2r_p \frac{\Gamma_{\text{tot}}}{J_p} \quad J_p = M_p (GM_* r_p)^{1/2}$$

$$\Gamma_{\text{tot}} = \frac{1}{\gamma} (C_0 + C_1 \alpha + C_2 \beta) \Gamma_0 \quad \Gamma_0 = \left(\frac{q}{h}\right)^2 \Sigma_p^2 r_p^2 \Omega_p^2 \quad \alpha = \frac{d \log \Sigma}{d \log r} \quad \beta = \frac{d \log T}{d \log r}$$

[Baruteau & Masset. 2008](#)

Giant planets (with gap): Type II

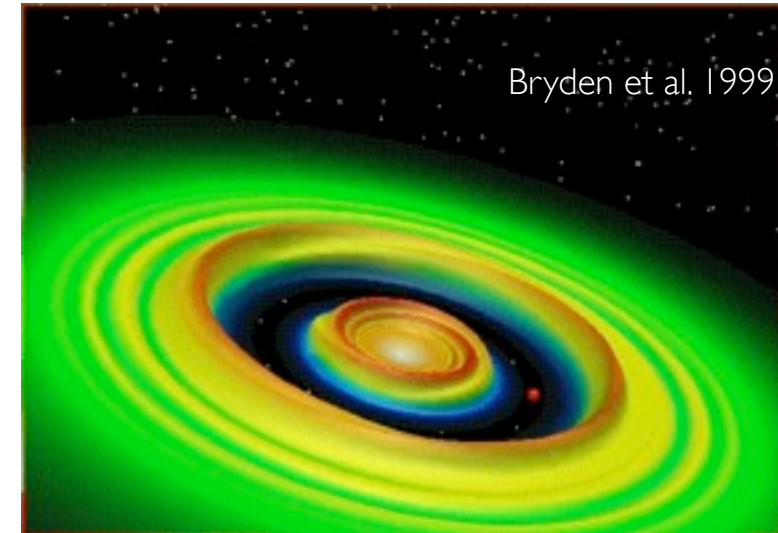
- Disk dominated $M_p < 2\Sigma a^2$

$$\frac{da_{\text{planet}}}{dt} = v_{r,\text{gas}}$$

- Planet dominated $M_p > 2\Sigma a^2$

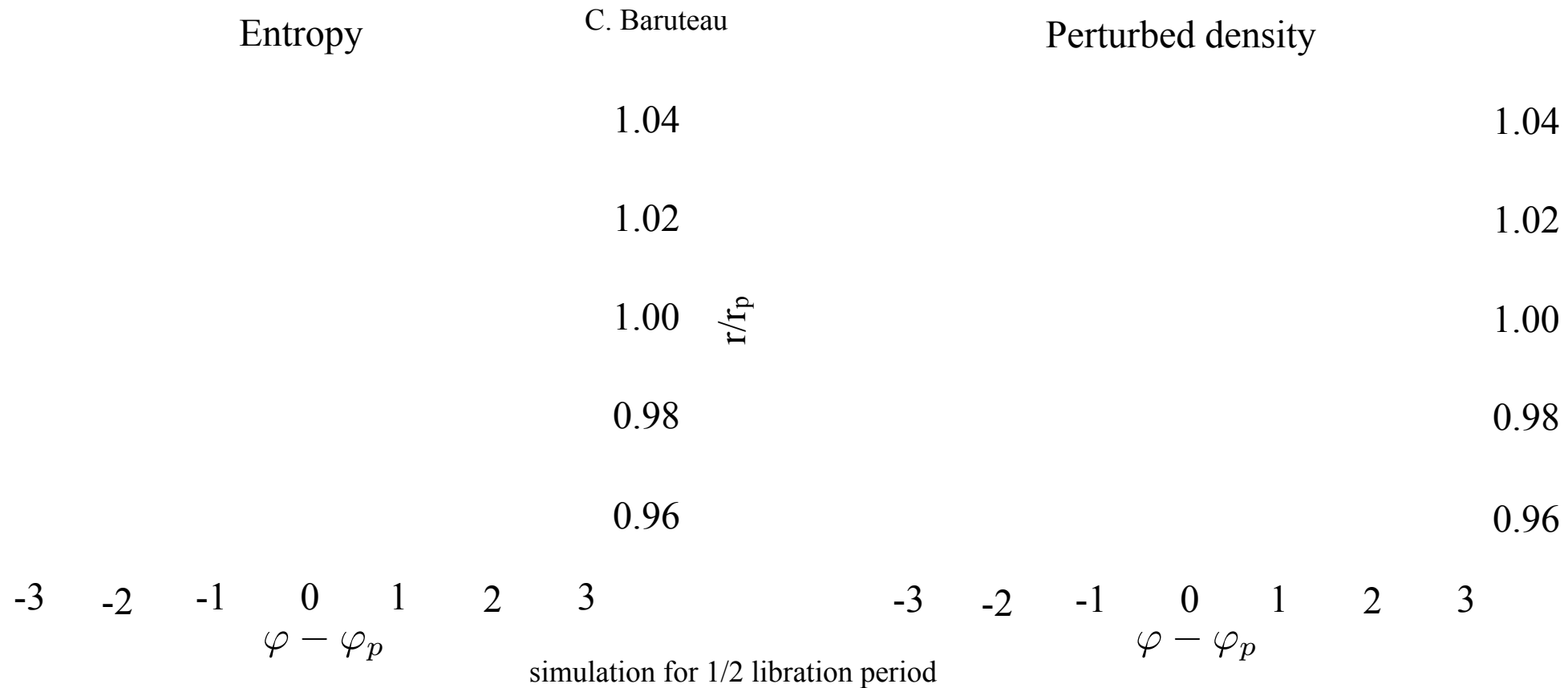
$$\frac{da_{\text{planet}}}{dt} = \left(\frac{2\Sigma a^2}{M_{\text{planet}}}\right)^{k_p} v_{r,\text{gas}} \quad k_p = \begin{cases} 1 & \text{"fully suppressed"} \\ 1/2 & \text{"partially suppressed"} \end{cases}$$

Slow-down



Bryden et al. 1999

③ Migration: why thermodynamics matter?

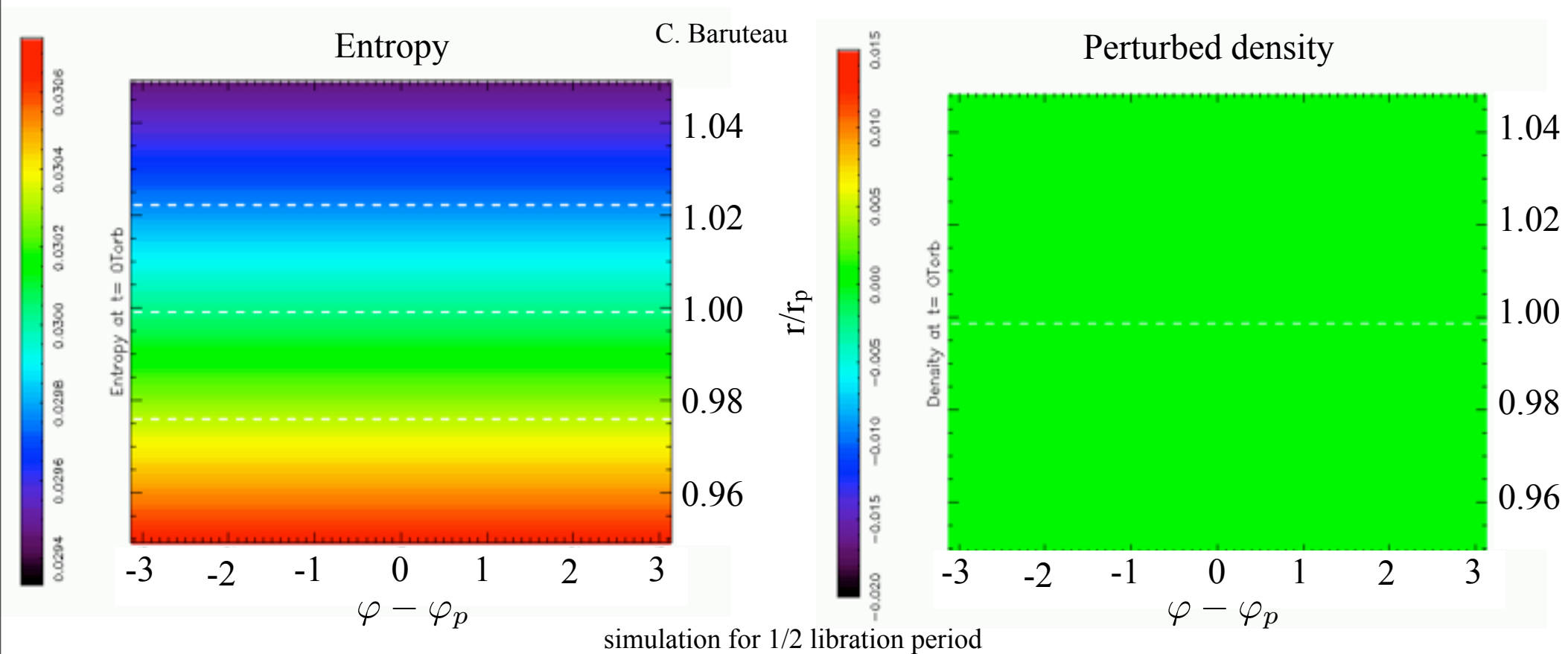


The exchange of fluid elements lead to an overdensity at shorter radii. This translates into a increased torque pushing the planet outwards...

For this mechanism to work, the fluid has to remain adiabatic during the exchange process. In other words: $\tau_{cool} \gg \tau_{u-turn}$

$$\tau_{cool} \approx \frac{\Sigma c_V T}{Q} = \frac{\Sigma c_V T}{2\sigma T_{eff}^4} \quad \tau_{u-turn} \approx 1.16 \sqrt{\frac{h^3}{\gamma q}} \frac{64}{9\Omega_p}$$

③ Migration: why thermodynamics matter?

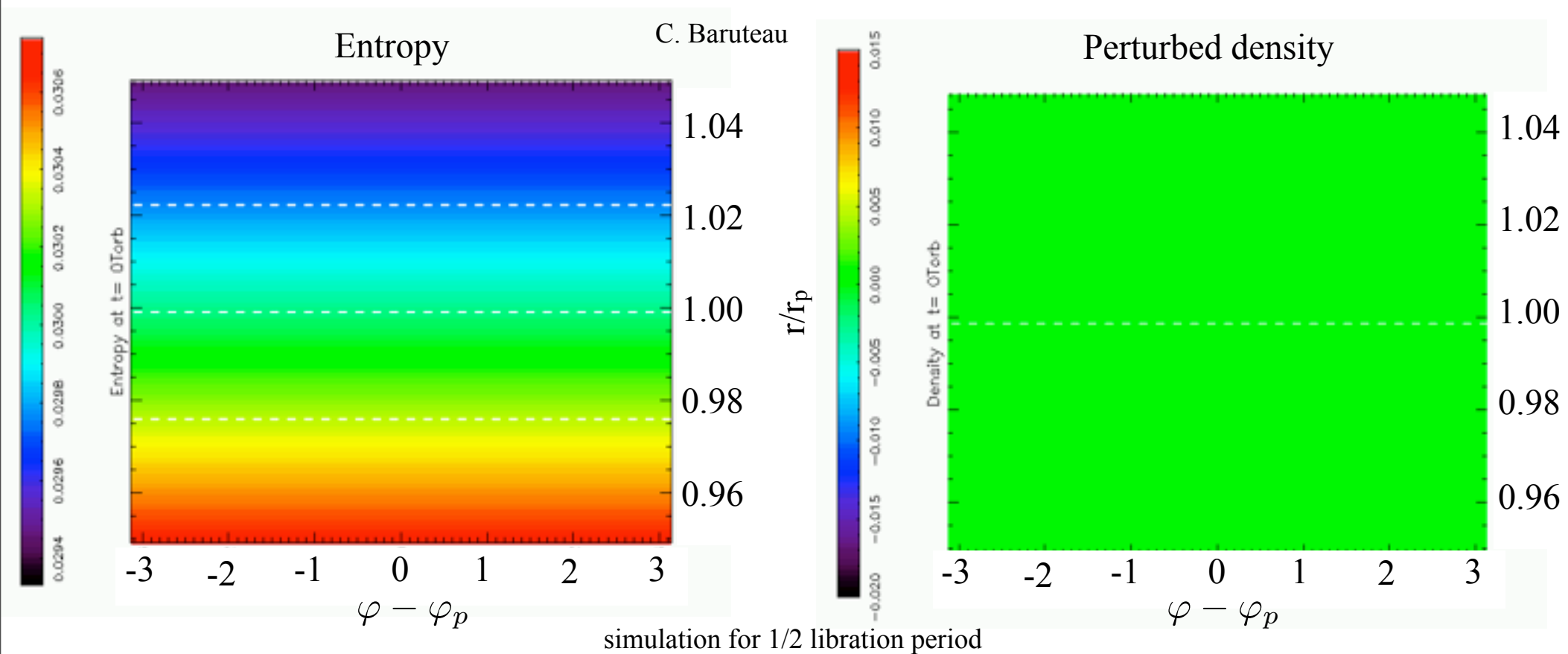


The exchange of fluid elements lead to an overdensity at shorter radii. This translates into a increased torque pushing the planet outwards...

For this mechanism to work, the fluid has to remain adiabatic during the exchange process. In other words: $\tau_{cool} \gg \tau_{u-turn}$

$$\tau_{cool} \approx \frac{\Sigma c_V T}{Q} = \frac{\Sigma c_V T}{2\sigma T_{eff}^4} \quad \tau_{u-turn} \approx 1.16 \sqrt{\frac{h^3}{\gamma q}} \frac{64}{9\Omega_p}$$

③ Migration: why thermodynamics matter?



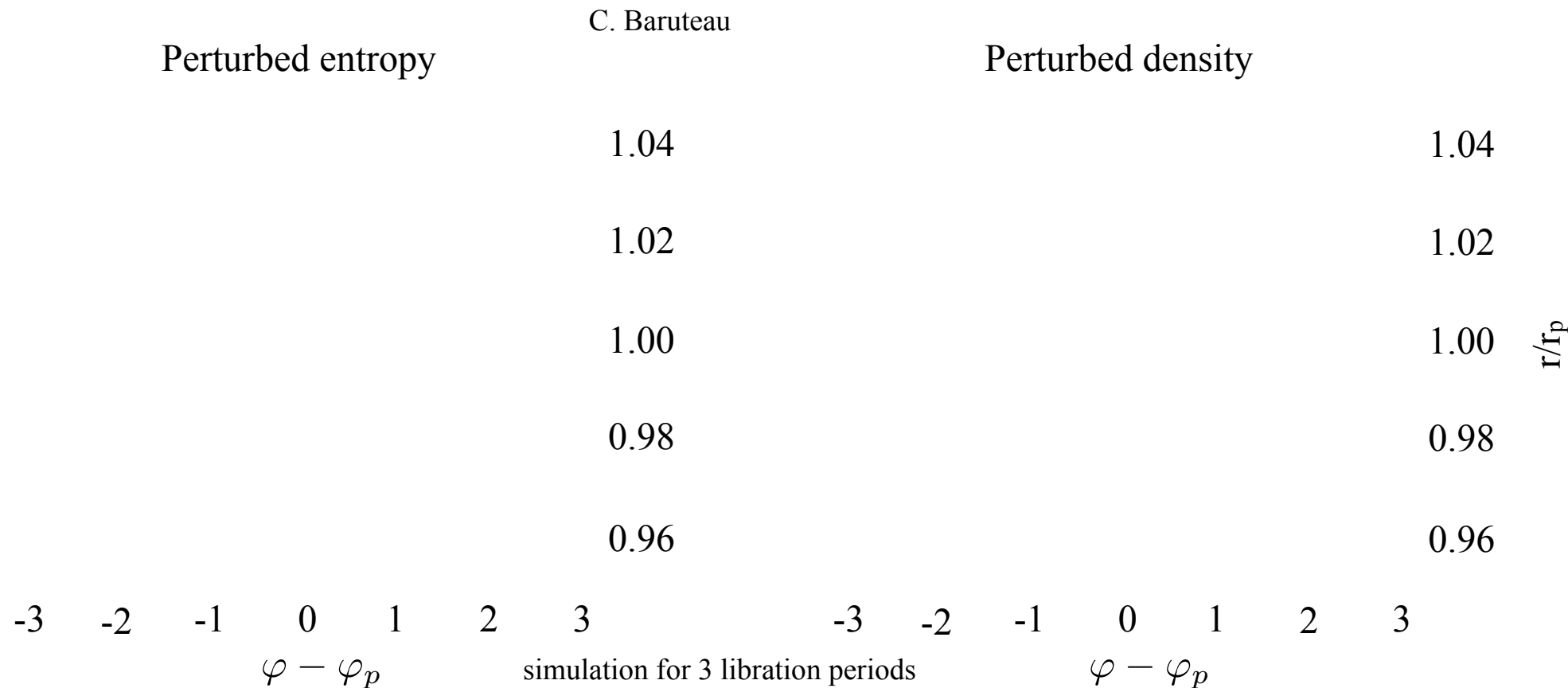
The exchange of fluid elements lead to an overdensity at shorter radii. This translates into a increased torque pushing the planet outwards...

For this mechanism to work, the fluid has to remain adiabatic during the exchange process. In other words: $\tau_{cool} \gg \tau_{u-turn}$

$$\tau_{cool} \approx \frac{\Sigma c_V T}{Q} = \frac{\Sigma c_V T}{2\sigma T_{eff}^4} \quad \tau_{u-turn} \approx 1.16 \sqrt{\frac{h^3}{\gamma q}} \frac{64}{9\Omega_p}$$

③ Migration: why thermodynamics matter?

Unfortunately, after some time, the situation becomes much less clear and the torques begin to saturate...



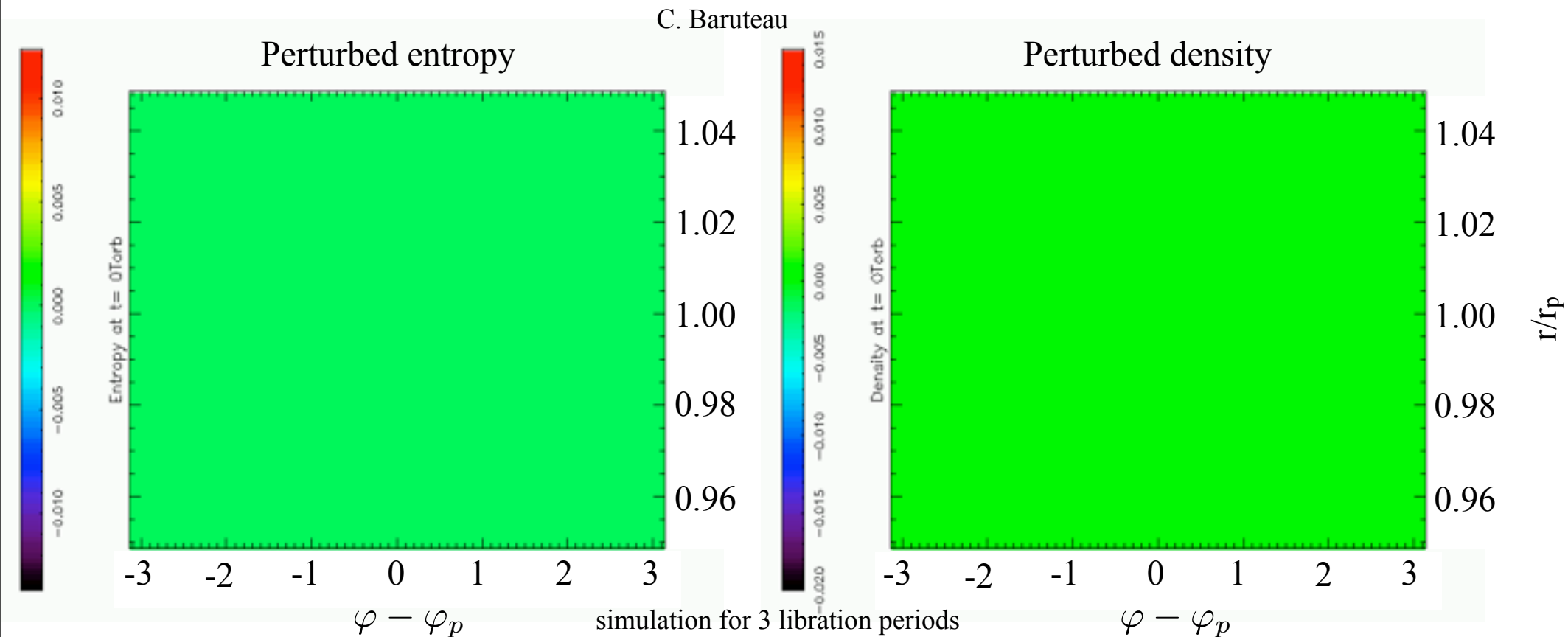
In other words, unless the viscosity re-establishes the original entropy profile before the torques saturate, the outward migration will not last... The condition for a sustainable outward migration is therefore given by:

$$\tau_{lib} \gg \tau_{visc}$$

$$\tau_{lib} = \frac{8\pi r_p}{3\Omega_p x_s} \quad \tau_{visc} = \frac{(2x_s)^2}{\nu}$$

③ Migration: why thermodynamics matter?

Unfortunately, after some time, the situation becomes much less clear and the torques begin to saturate...



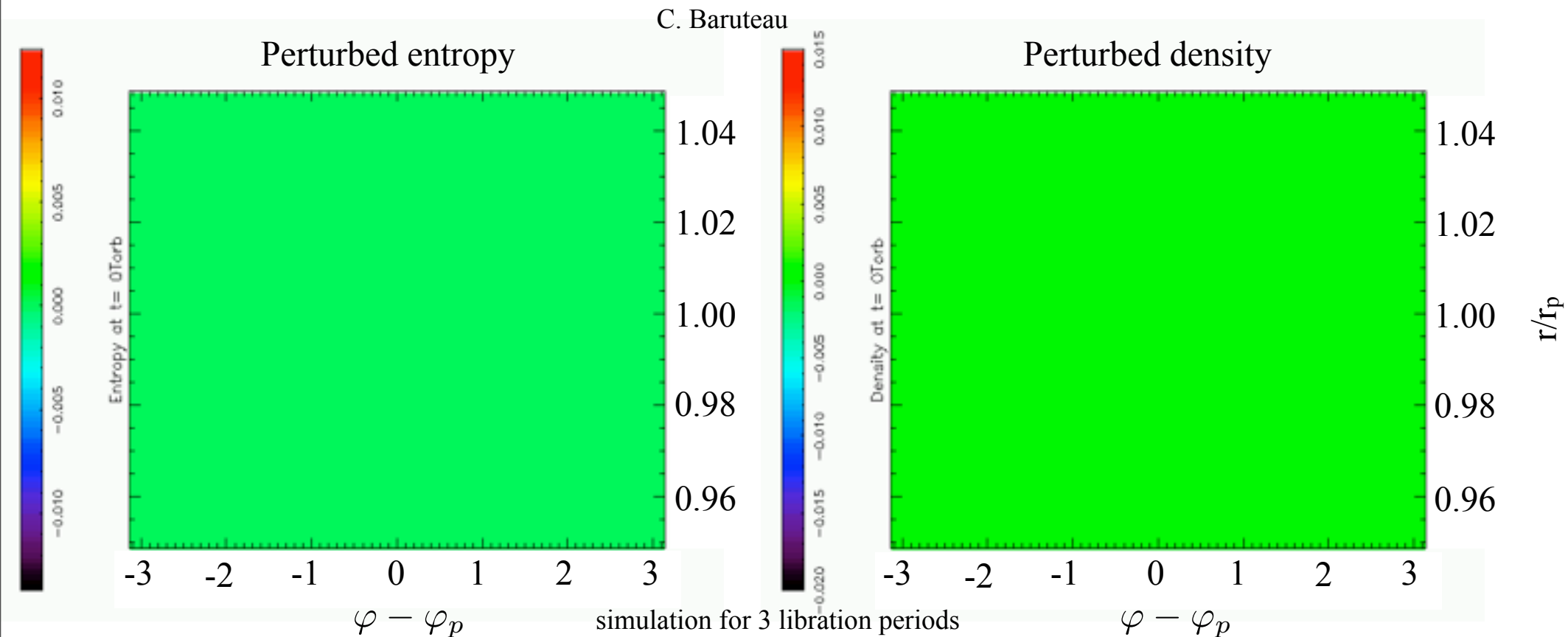
In other words, unless the viscosity re-establishes the original entropy profile before the torques saturate, the outward migration will not last... The condition for a sustainable outward migration is therefore given by:

$$\tau_{lib} \gg \tau_{visc}$$

$$\tau_{lib} = \frac{8\pi r_p}{3\Omega_p x_s} \quad \tau_{visc} = \frac{(2x_s)^2}{\nu}$$

③ Migration: why thermodynamics matter?

Unfortunately, after some time, the situation becomes much less clear and the torques begin to saturate...



In other words, unless the viscosity re-establishes the original entropy profile before the torques saturate, the outward migration will not last... The condition for a sustainable outward migration is therefore given by:

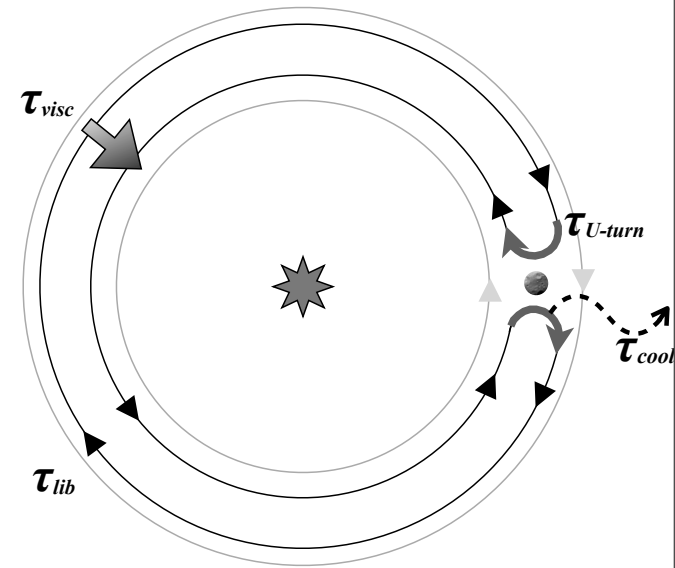
$$\tau_{lib} \gg \tau_{visc}$$

$$\tau_{lib} = \frac{8\pi r_p}{3\Omega_p x_s} \quad \tau_{visc} = \frac{(2x_s)^2}{\nu}$$

③ Migration and timescales

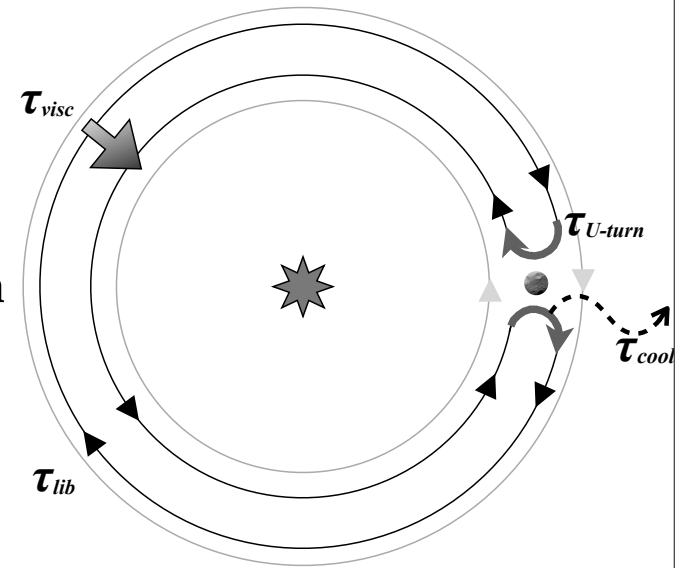
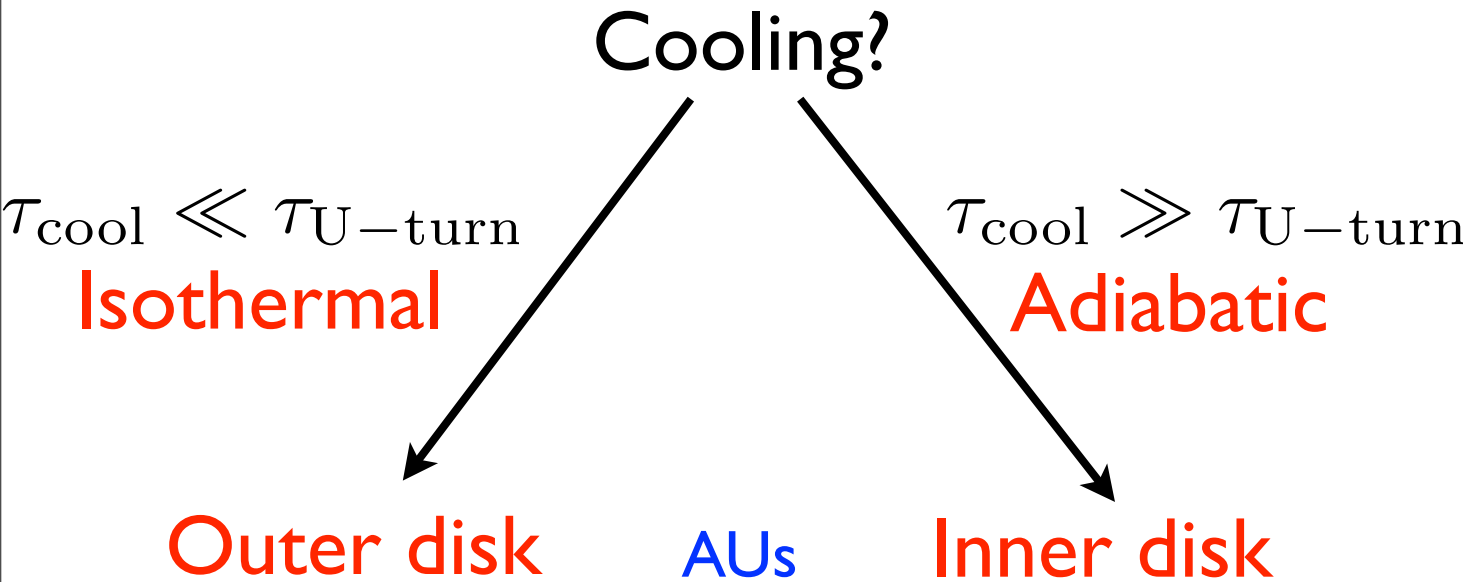
From [Paardekooper et al. 2009](#)

Cooling?



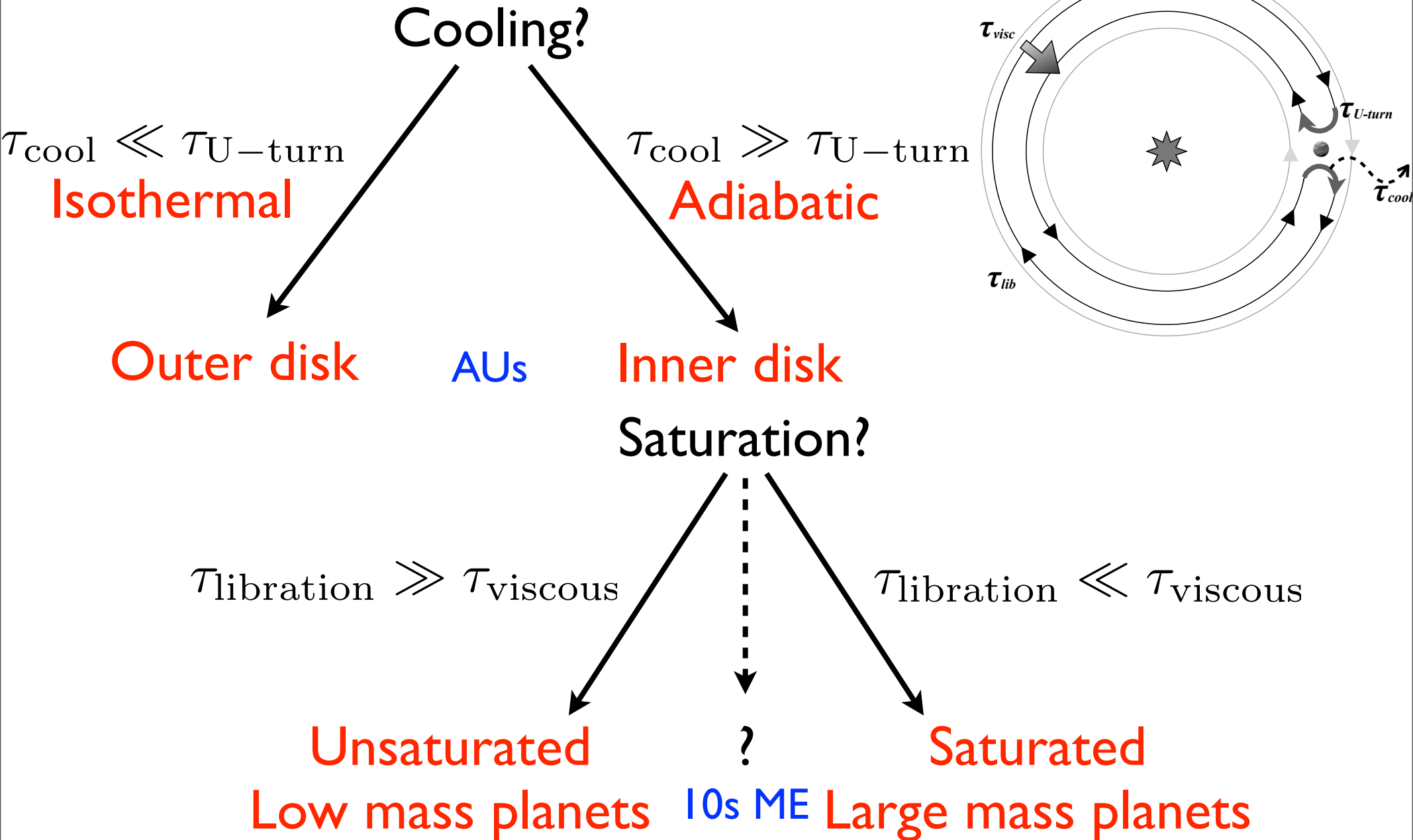
③ Migration and timescales

From [Paardekooper et al. 2009](#)



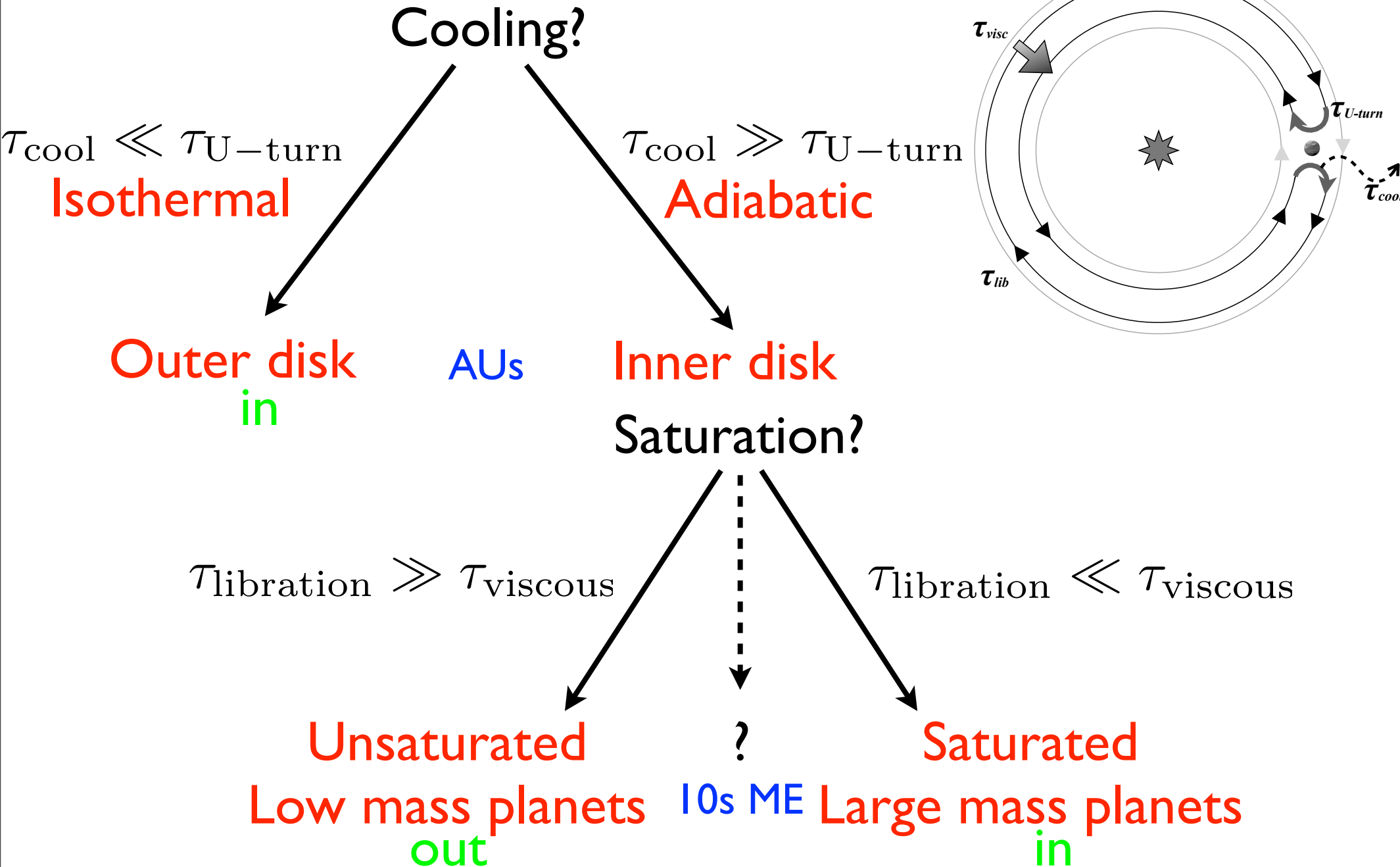
③ Migration and timescales

From [Paardekooper et al. 2009](#)



③ Migration and timescales

From [Paardekooper et al. 2009](#)



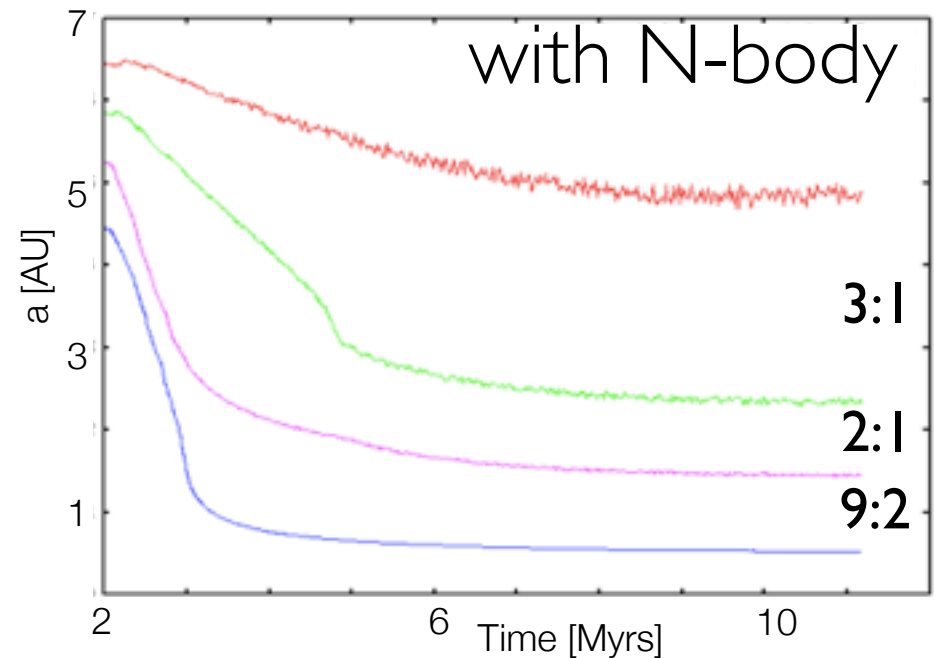
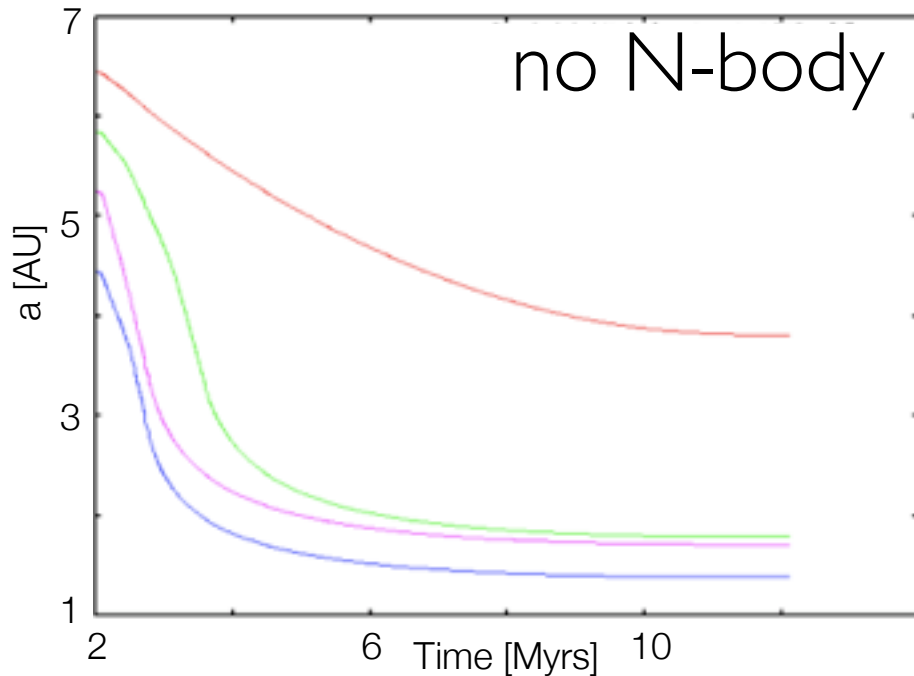
④ Planet-planet interactions

④ Planet-planet interactions

- Explicit **N-body** between planets with disk-planet interaction and collisions of planets.
- Eccentricity **damping** of planets (Nelson& Fogg 07), planetesimal ecc. as in Pollack et al. (96).
- Uniform** planetesimal density in overlapping feeding zones.

④ Planet-planet interactions

- Explicit **N-body** between planets with disk-planet interaction and collisions of planets.
- Eccentricity **damping** of planets (Nelson& Fogg 07), planetesimal ecc. as in Pollack et al. (96).
- Uniform** planetesimal density in overlapping feeding zones.



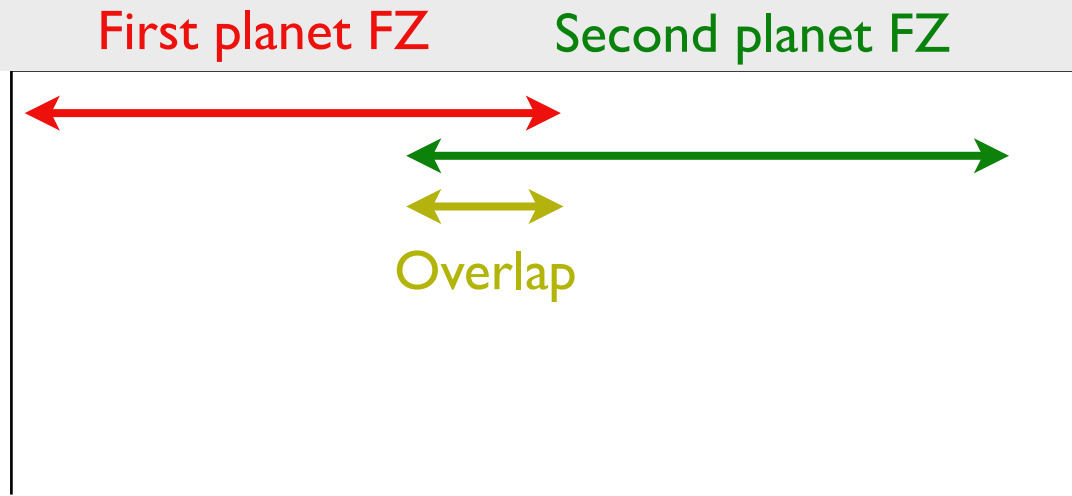
Final mass [M_{Earth}]	without N body	with N body
Planet 1	970	890
Planet 2	975	970
Planet 3	750	430
Planet 4	7	4

Study

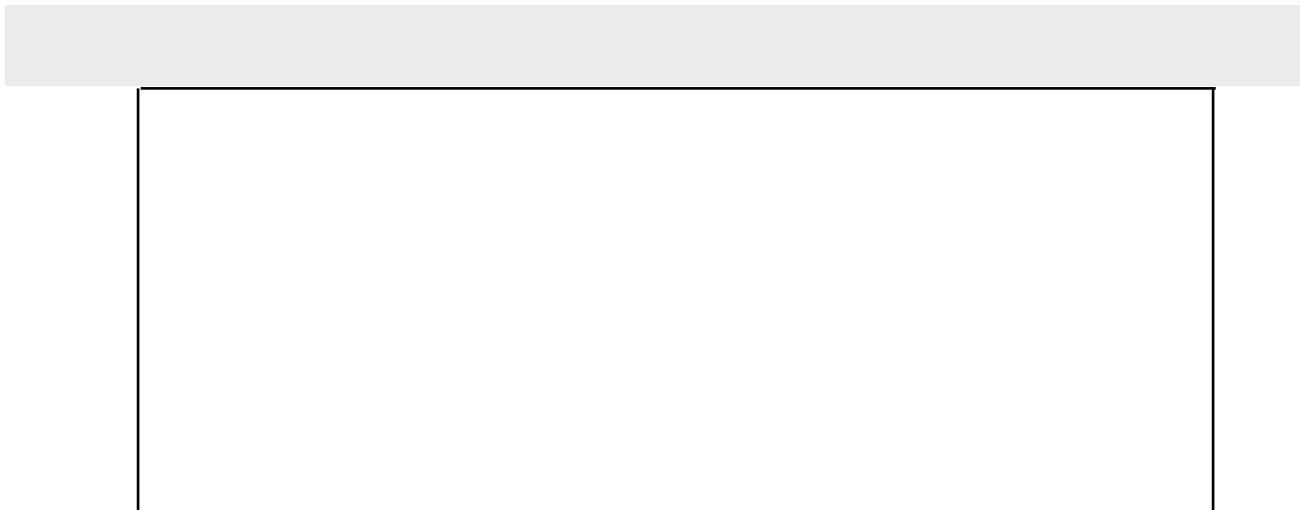
- solar system
- resonant systems
- ejection (far out planets)
- population synthesis

④ Planet-planet interactions

N-body simulations by S. Pfytter



Uniform surface density
and excitation in the
common FZ



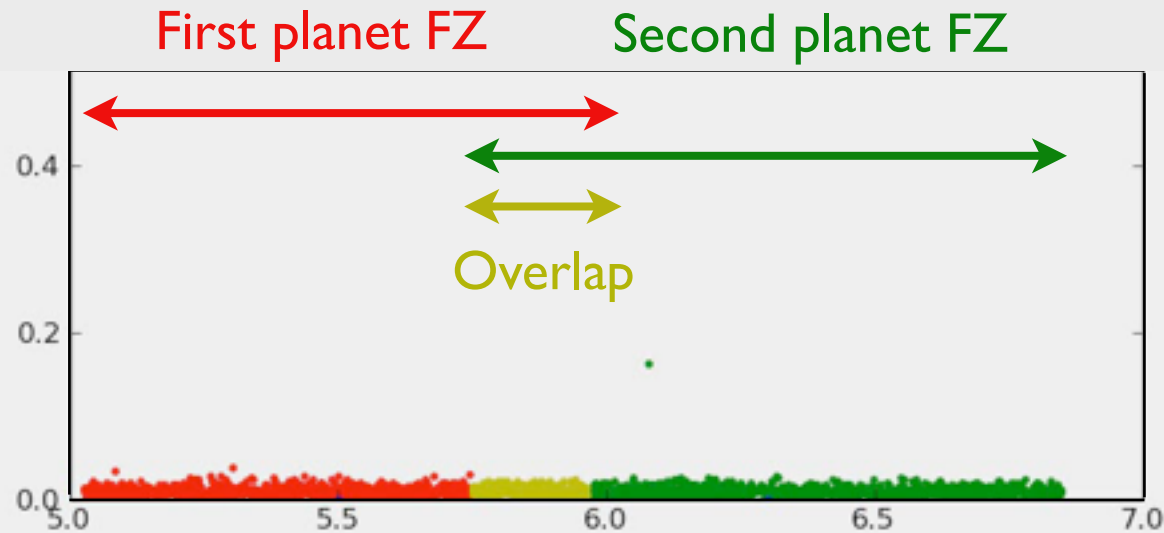
Planetesimal transport
to the outermost planet

The internal structures of the two planets are no more independent

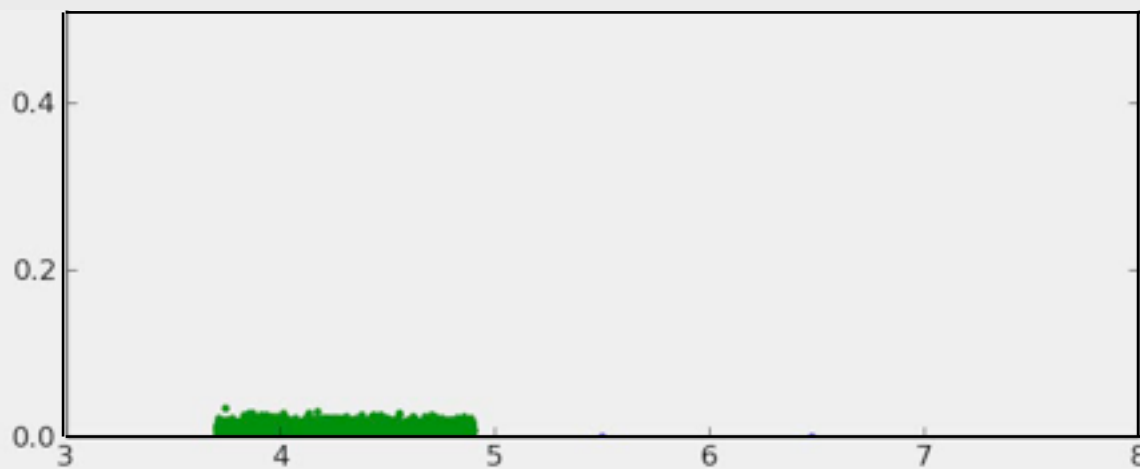
④ Planet-planet interactions

N-body simulations by S. Pfyster

Uniform surface density
and excitation in the
common FZ



Planetesimal transport
to the outermost planet

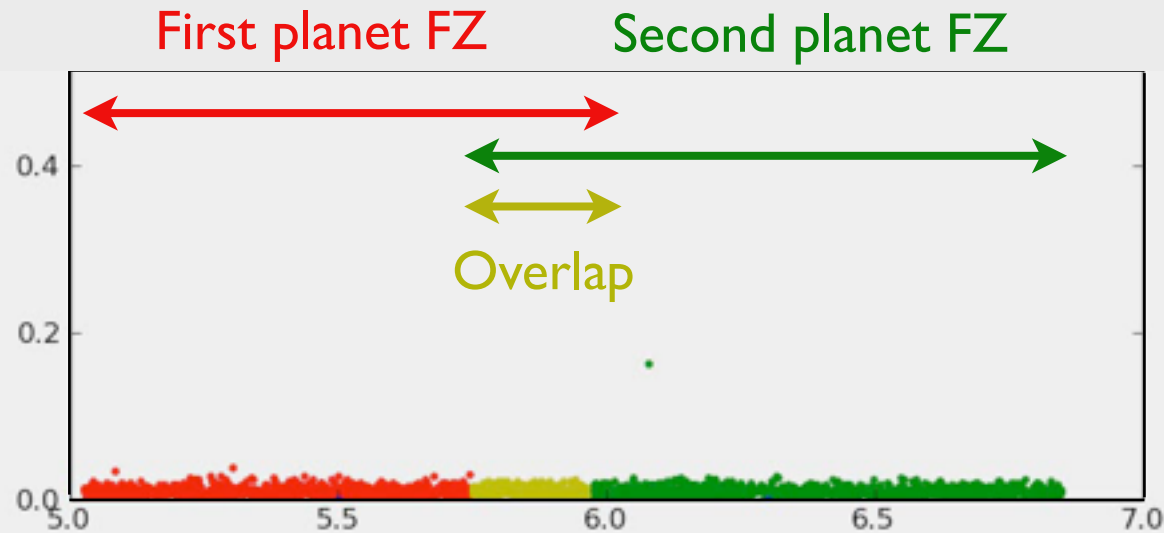


The internal structures of the two planets are no more independent

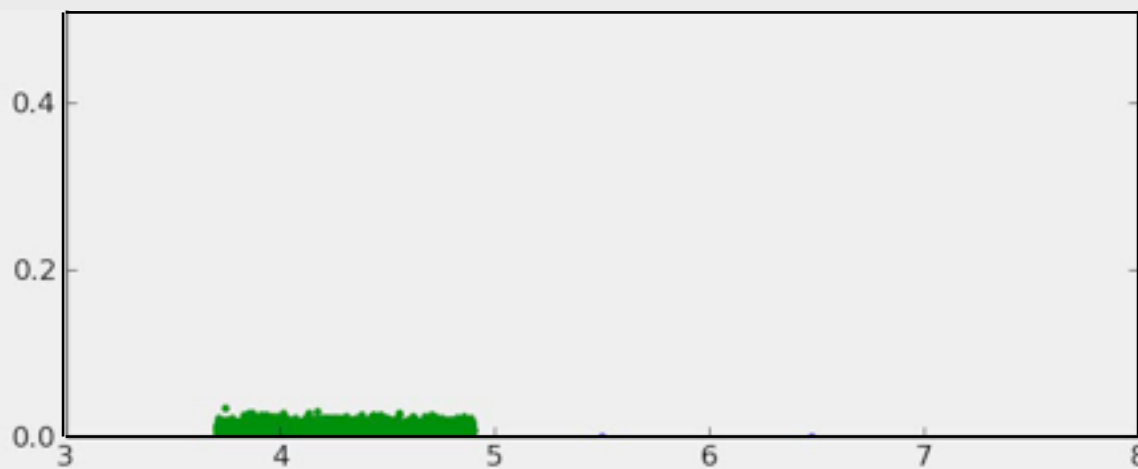
④ Planet-planet interactions

N-body simulations by S. Pfytter

Uniform surface density
and excitation in the
common FZ



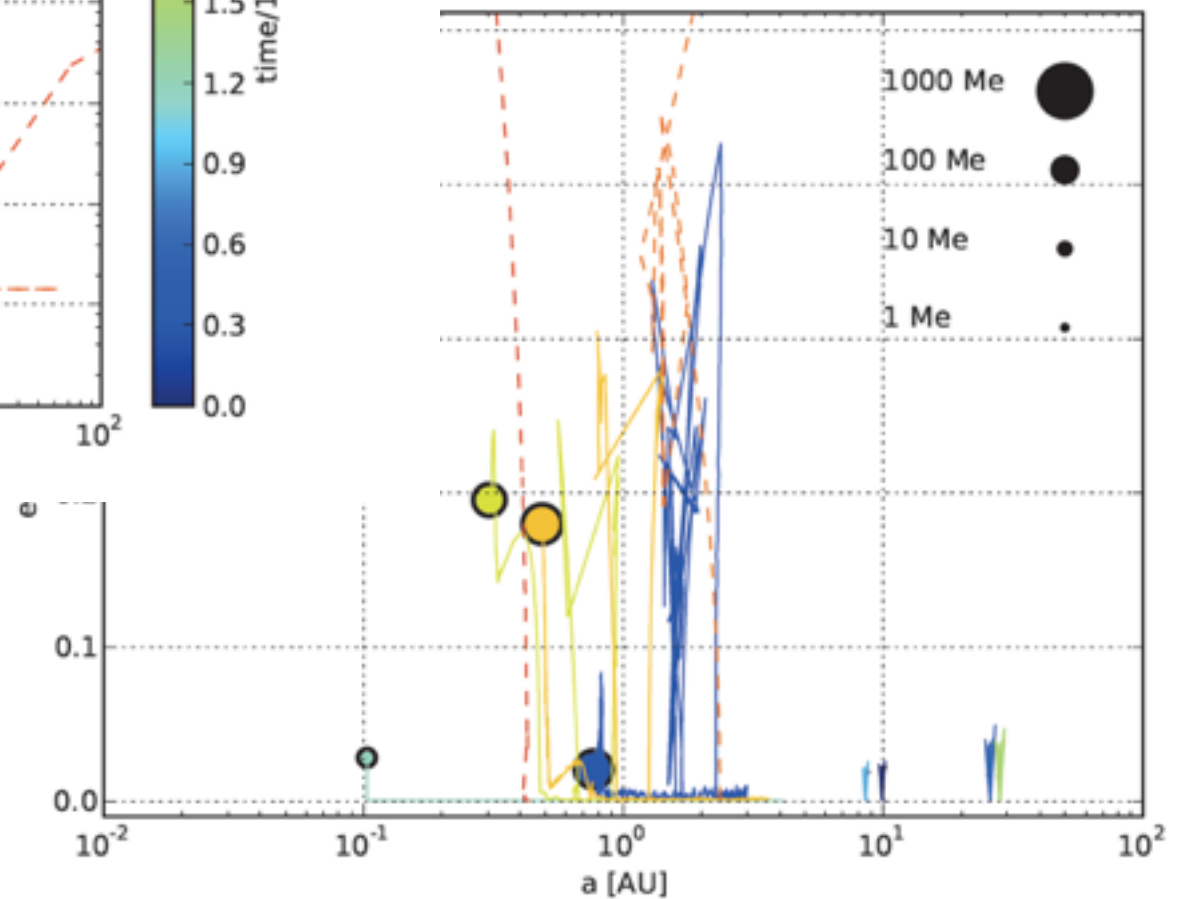
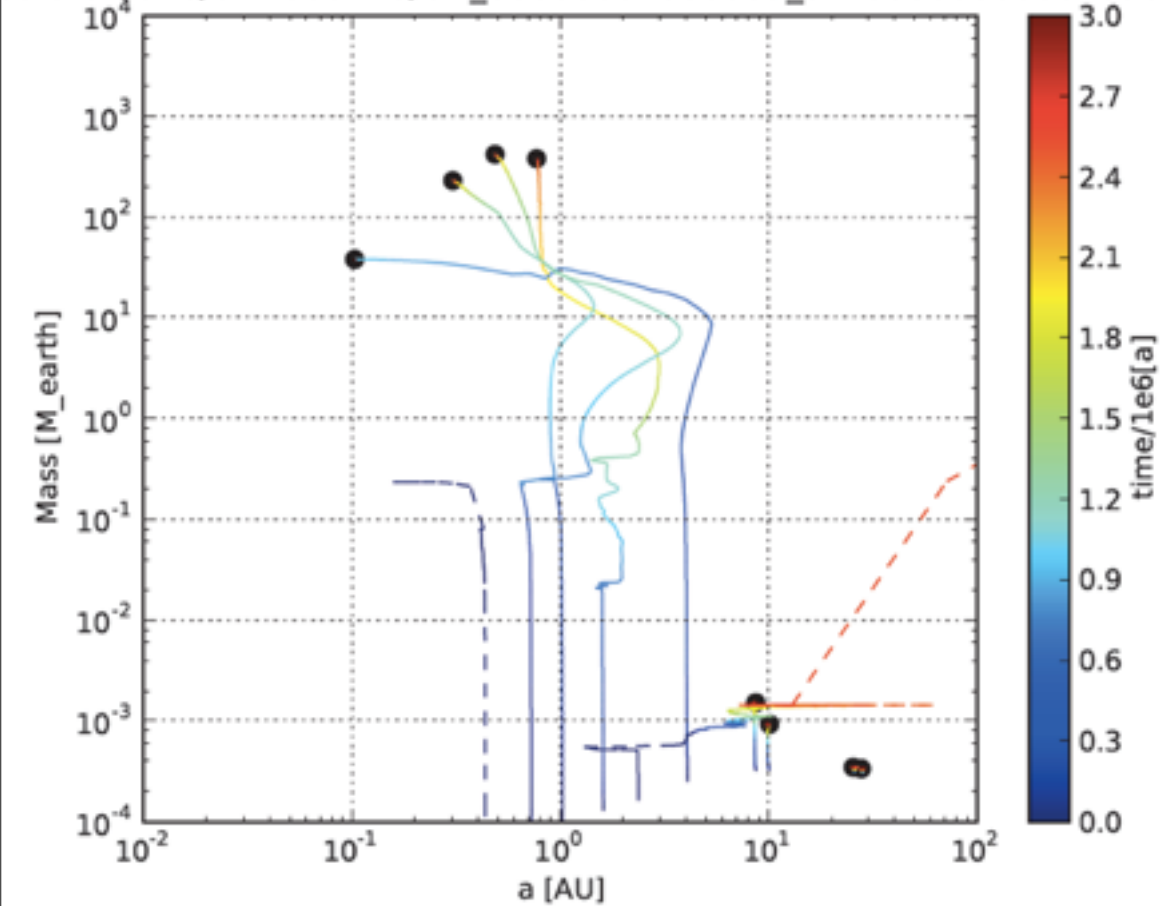
Planetesimal transport
to the outermost planet



The internal structures of the two planets are no more independent

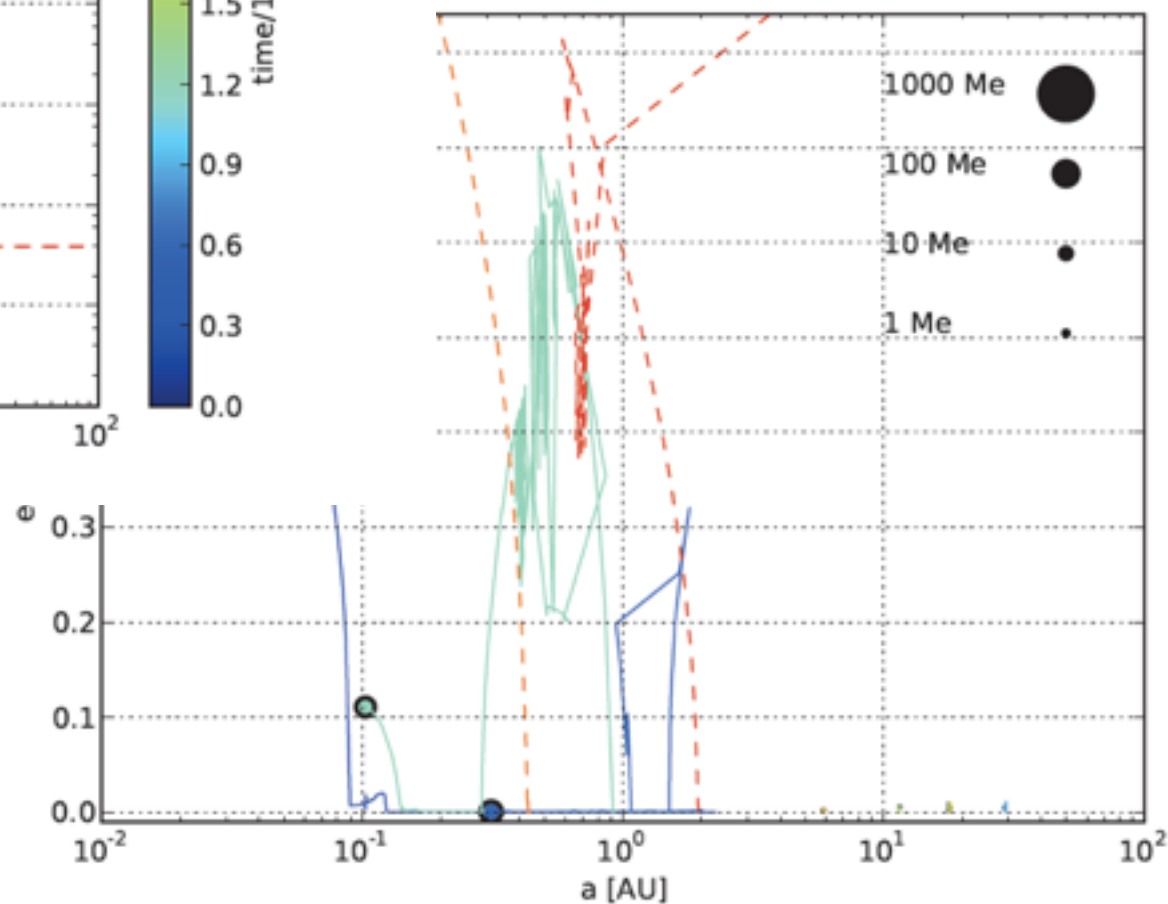
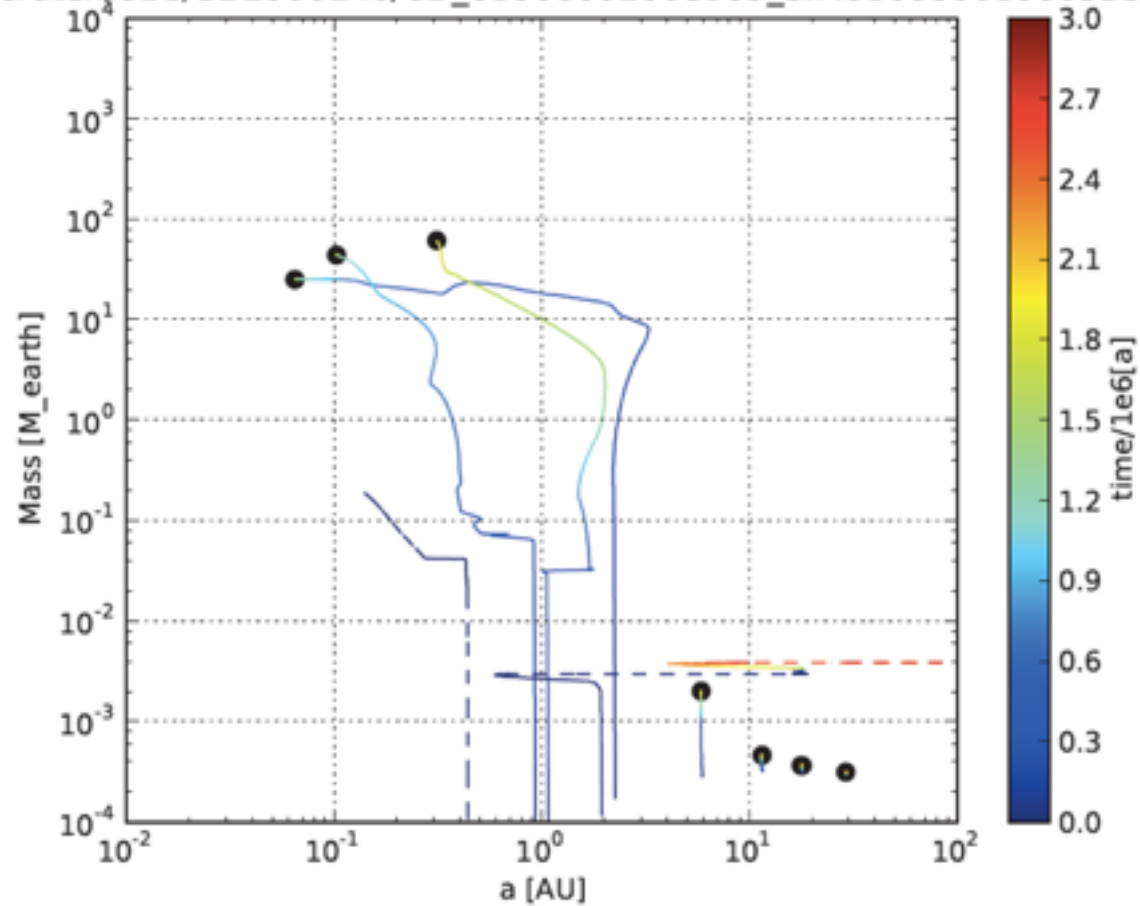
a system with three giants and one HN

scratch/CDs/CD2000146/CD_00000001000009_SIM000000000000038



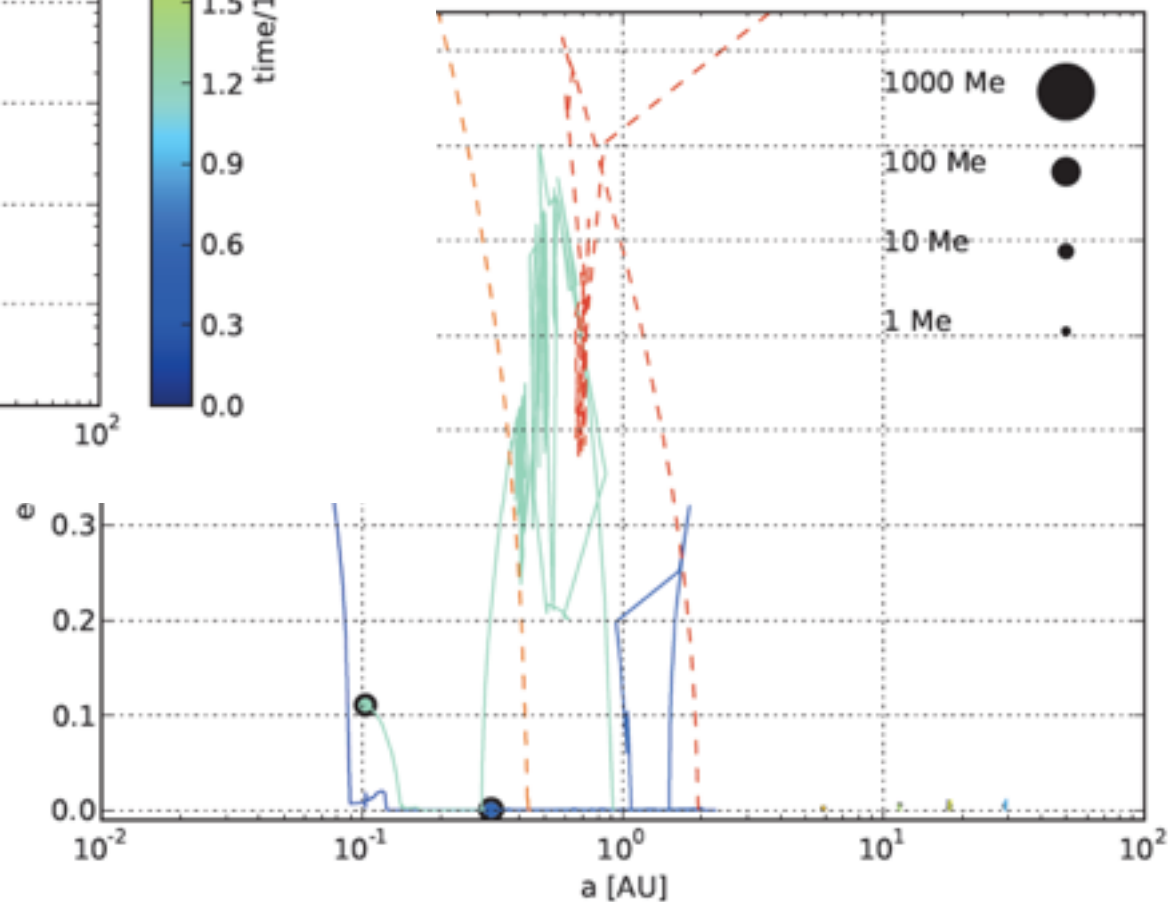
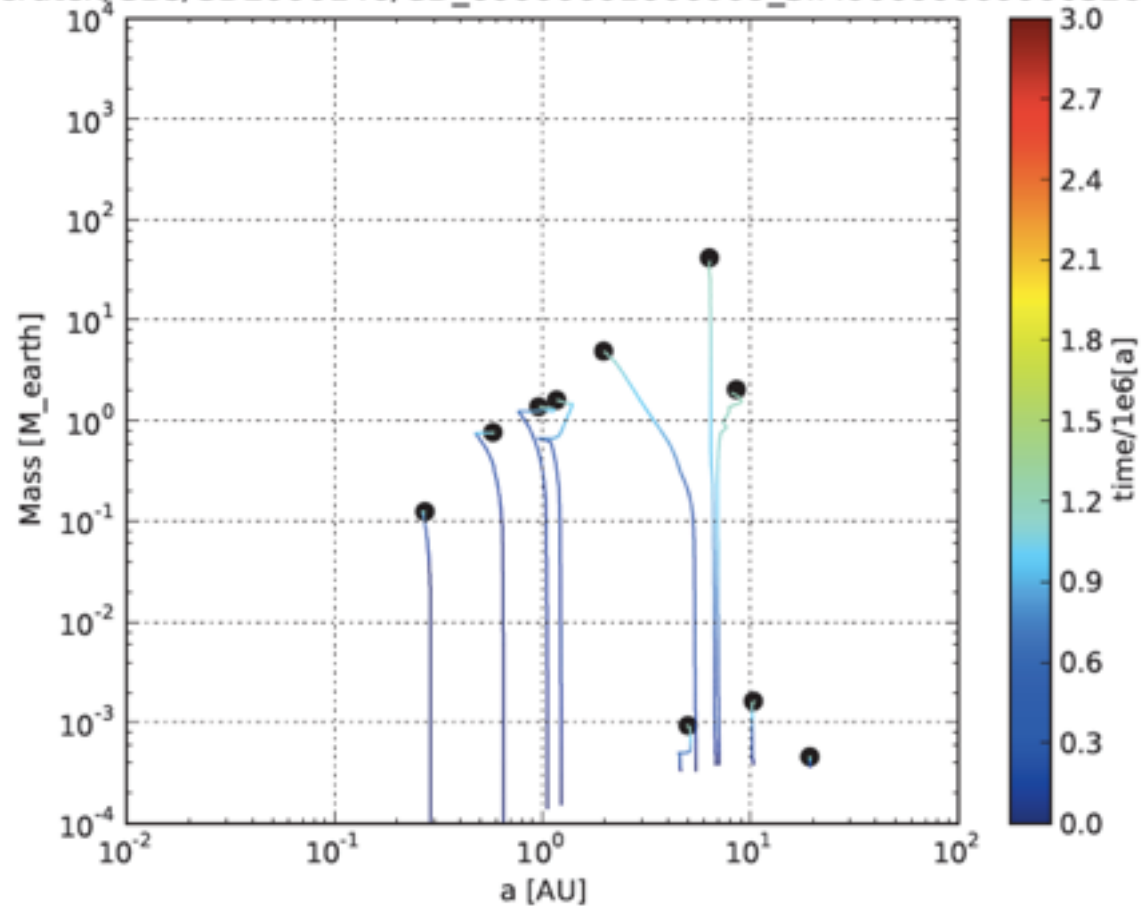
a system with three HN

scratch/CDs/CD2000146/CD_00000001000009_SIM000000000000338



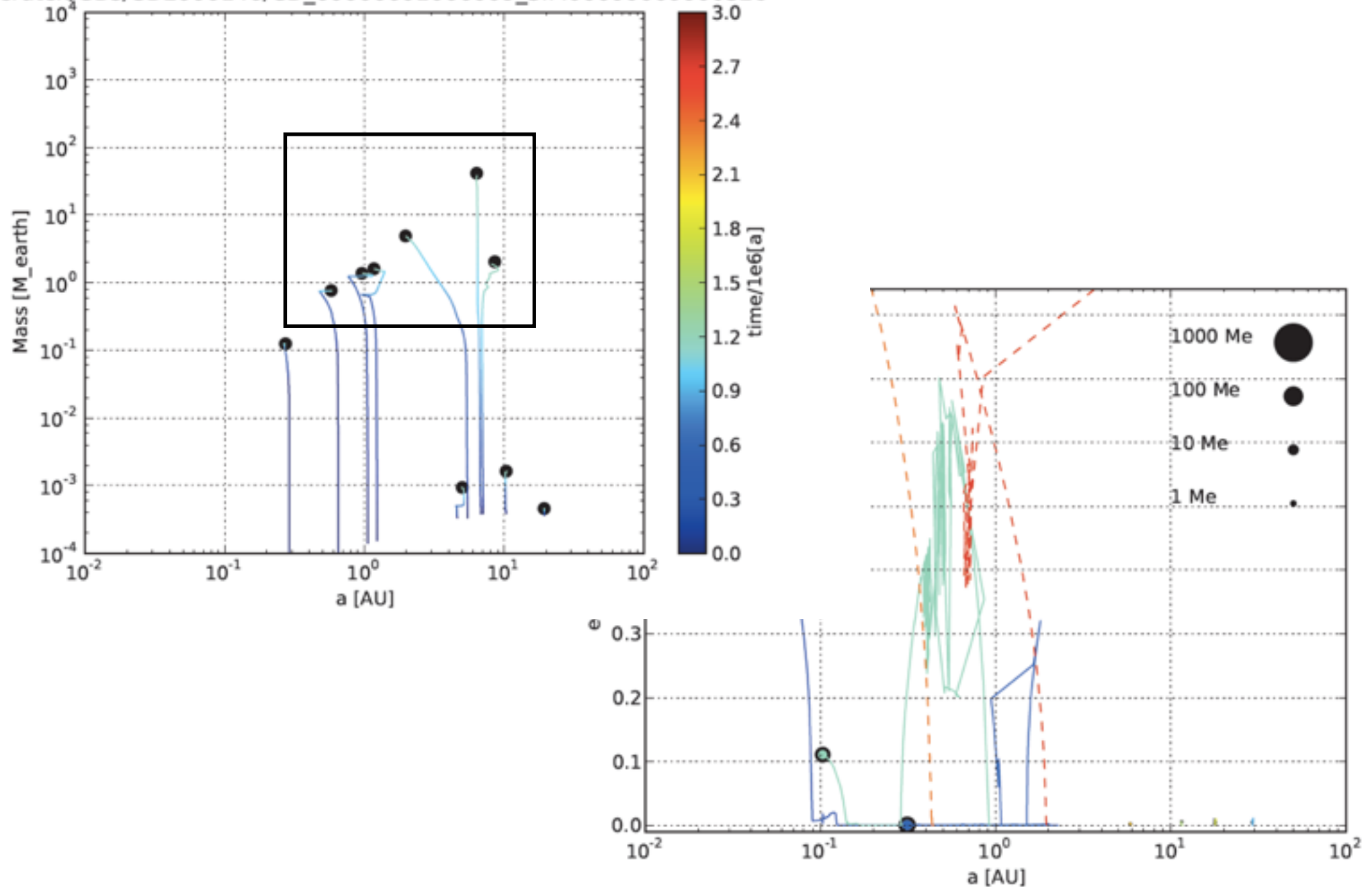
a system with TWO Earths at 1 AU

scratch/CDs/CD2000146/CD_00000001000009_SIM000000000000326

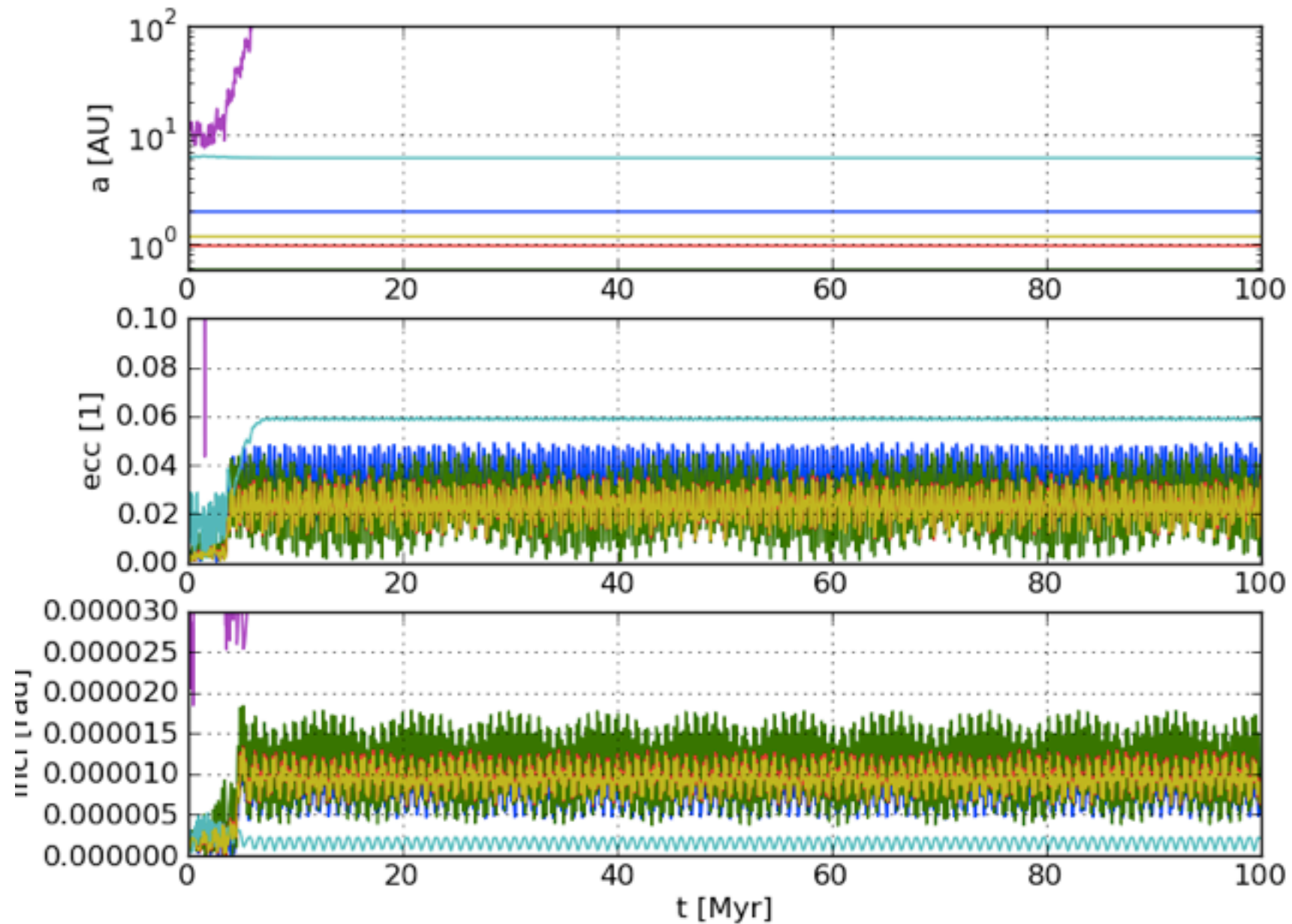


a system with TWO Earths at 1 AU

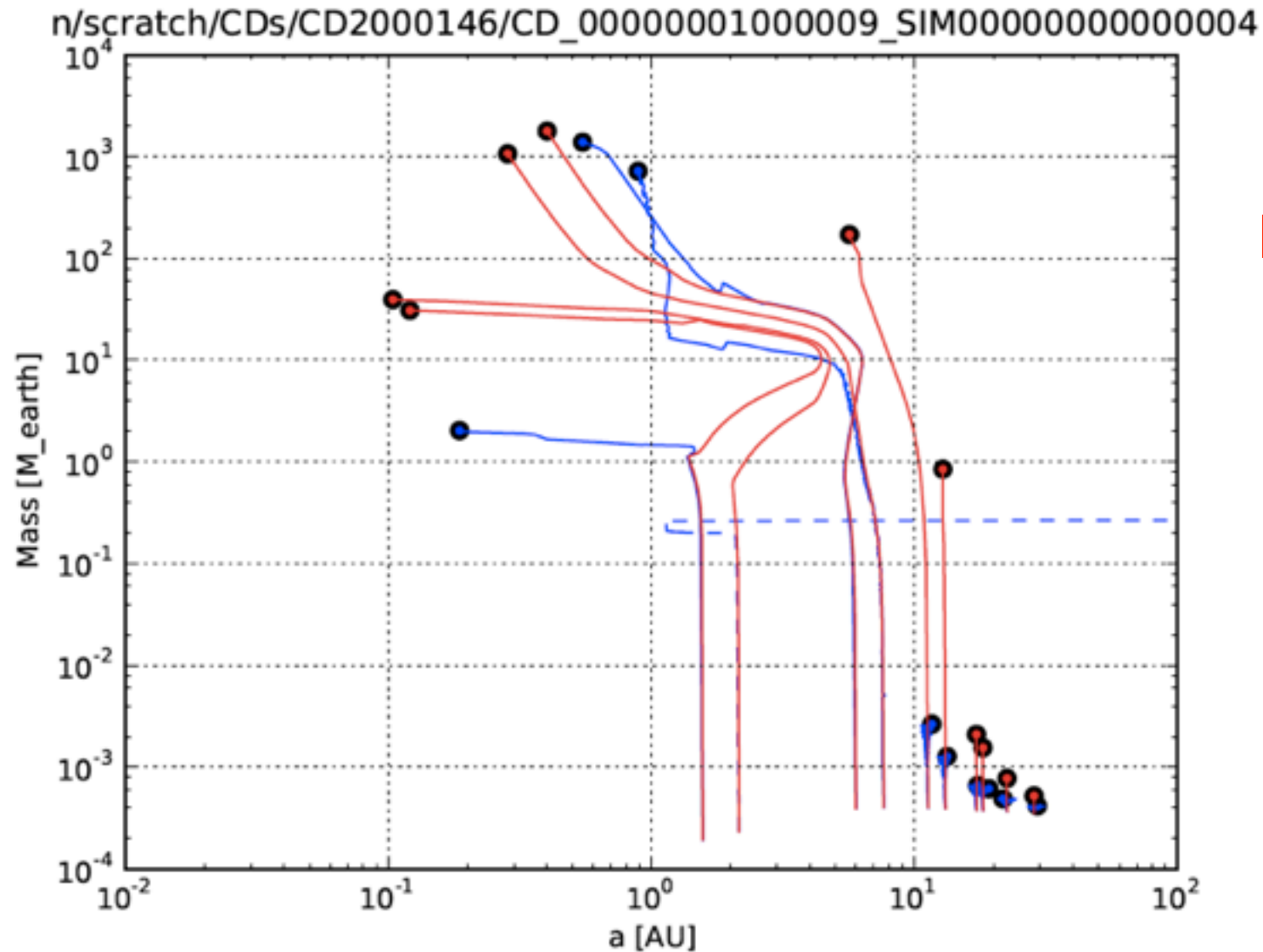
scratch/CDs/CD2000146/CD_00000001000009_SIM000000000000326



long term stability?

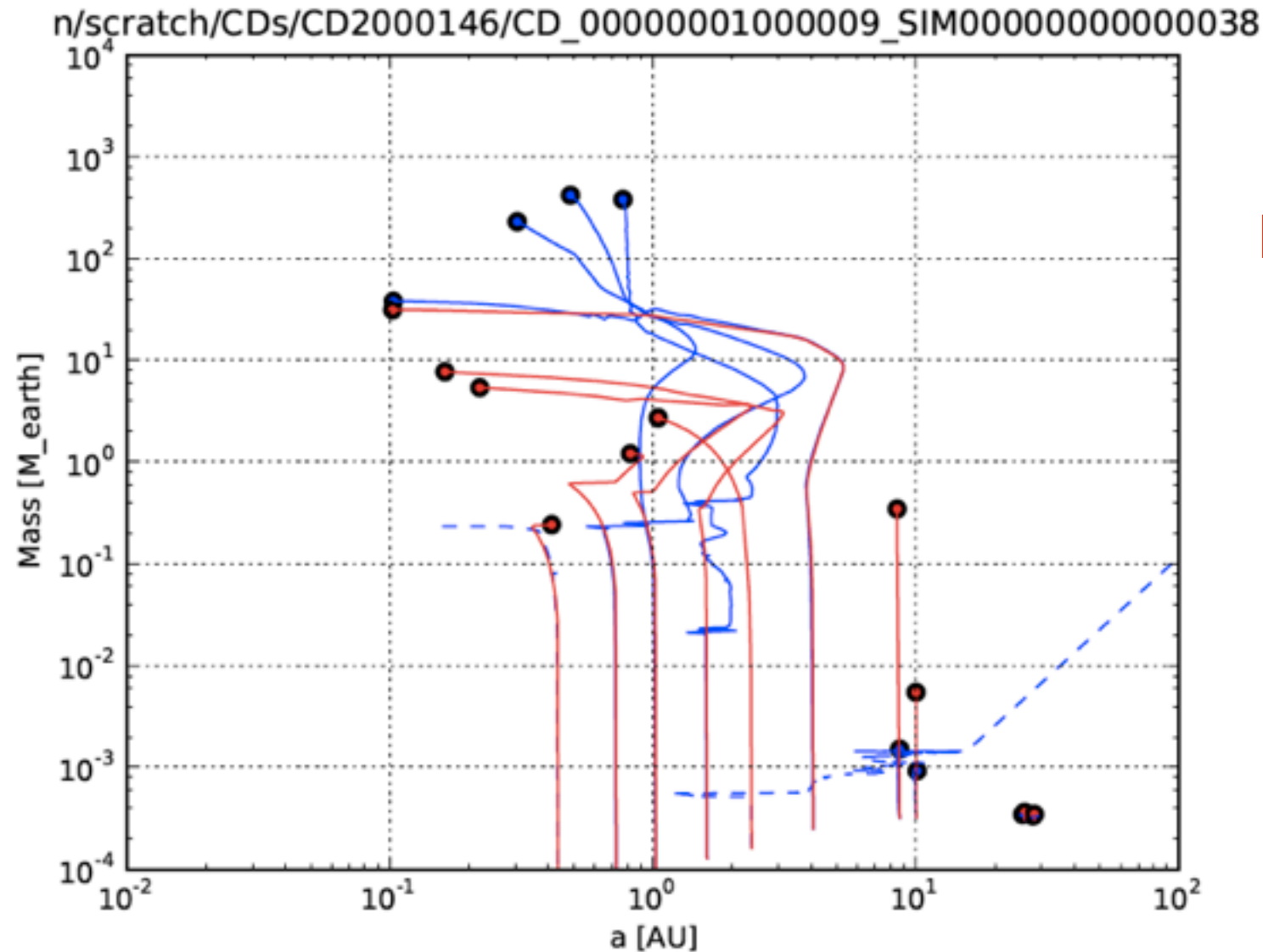


System or no system?



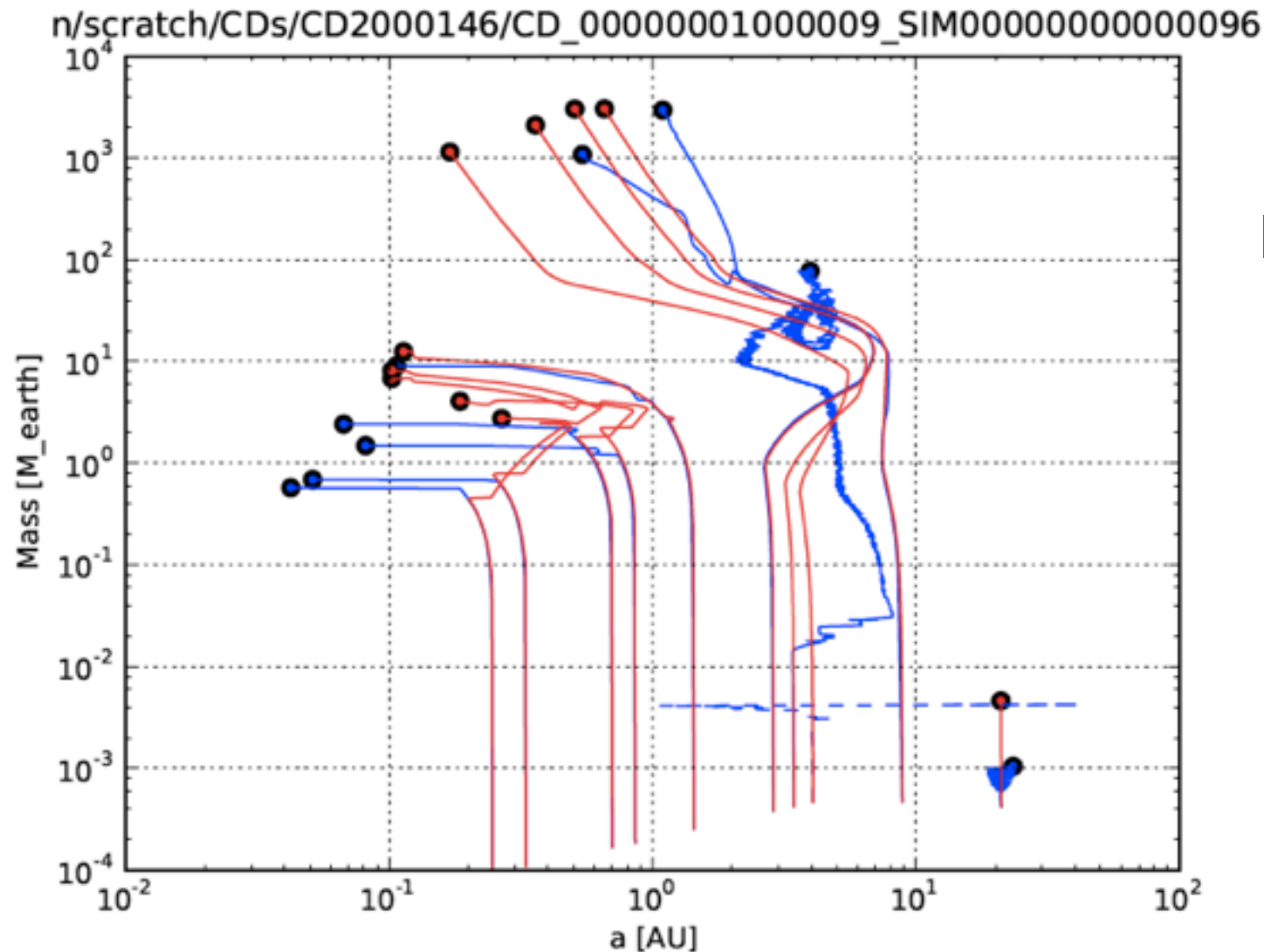
Blue planets are less massive than red ones (competition)

System or no system?



Blue planets can be more massive than red ones (sometimes)

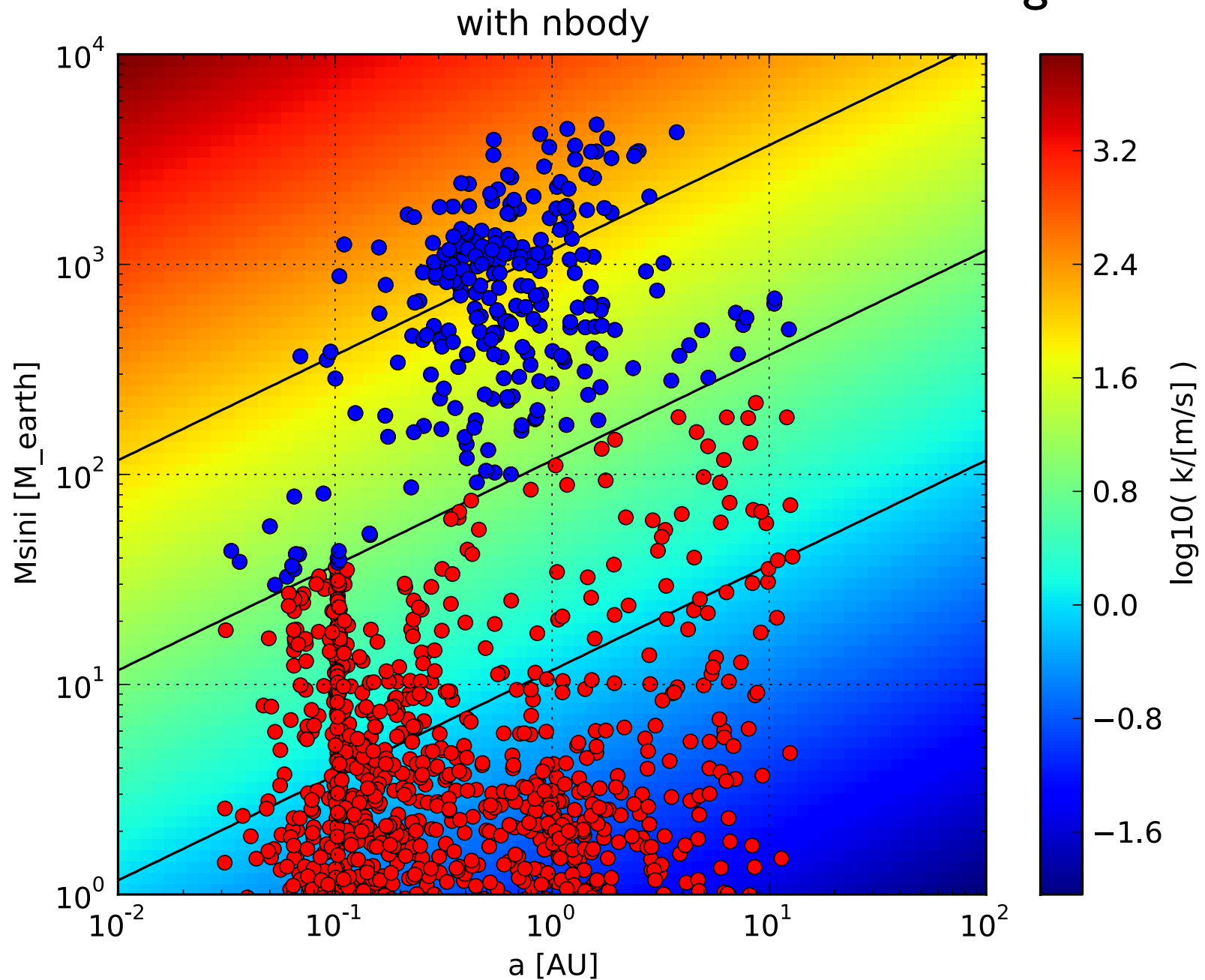
System or no system?



Massive red planets are in general closer than blue ones
Low mass blue planets are in general closer than red ones

a-M diagram

starting with 10 planets



Observing Planetary Systems II

An ESO workshop to bring together both communities of solar system and extra-planetary system researches and to foster our understanding of the formation and evolution of planetary systems at large

Santiago, Chile, March 5-8, 2012

Topics and Invited Speakers

The first Myr. of planetary formation

Hilke Schlichting, UCL
Bill Dent, ESO-ALM
Sebastian Wolf, Kiel University

Nature and orbits of planetary bodies

Dave Jewitt, UCL
Willy Benz, Bern University
Caroline Terquem, Institut d'Astrophysique de Paris
Didier Queloz, Genève Observatory
Alessandro Morbidelli, Nice Observatory

Planetary atmospheres and bio-markers

Tobias Owen, University of Hawaii
Enric Pallé, Instituto de Astrofísica de Canarias
Michaël Gillon, Liège University

SPHERE: Future ESO planet-finder

David Mouillet, Institut de Planétologie et d'Astrophysique de Grenoble

Organizing Committee

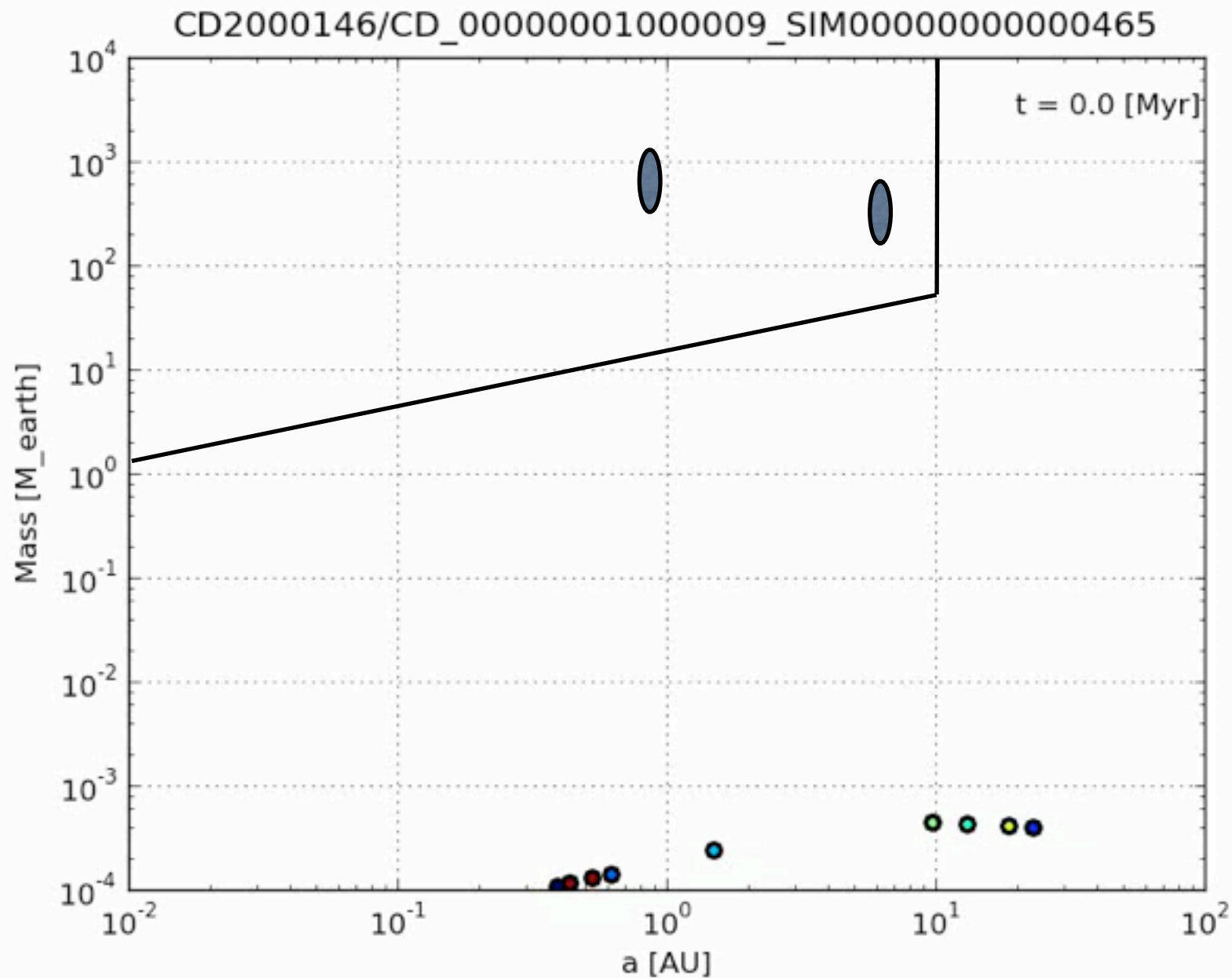
- Christophe Dumas (ESO, Chile)
- Michael Sterzik (ESO, Chile)
- Claudio Melo (ESO, Chile)
- Ralf Siebenmorgen (ESO, Garching)
- David Mouillet (Observatoire de Grenoble, France)
- Members of the ESO-Chile Planetary Sciences Group

© Image source: NASA/JPL

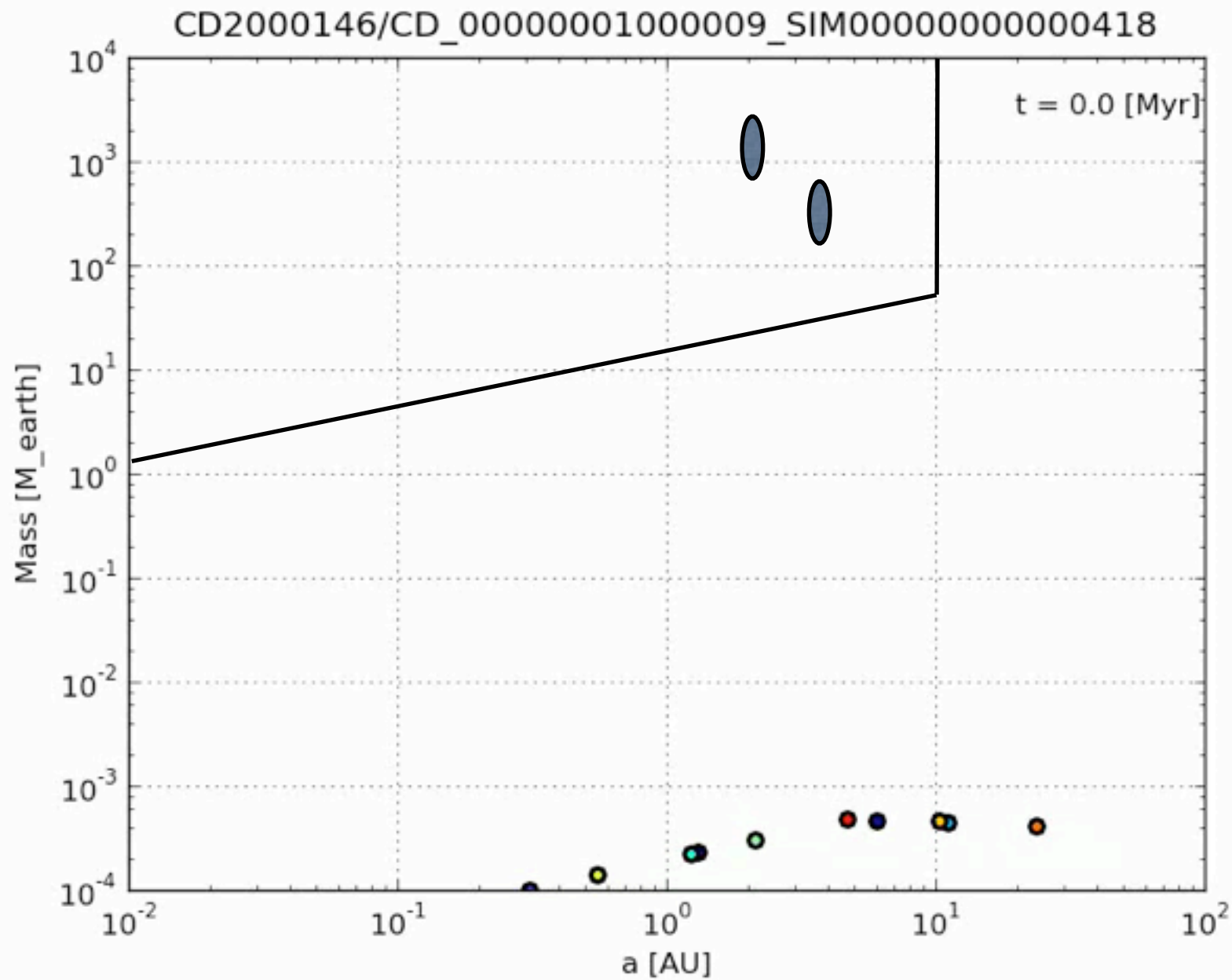
Web page: <http://www.eso.org/sci/meetings/2012/OPSII.html>
Conference e-mail: ops2012@eso.org



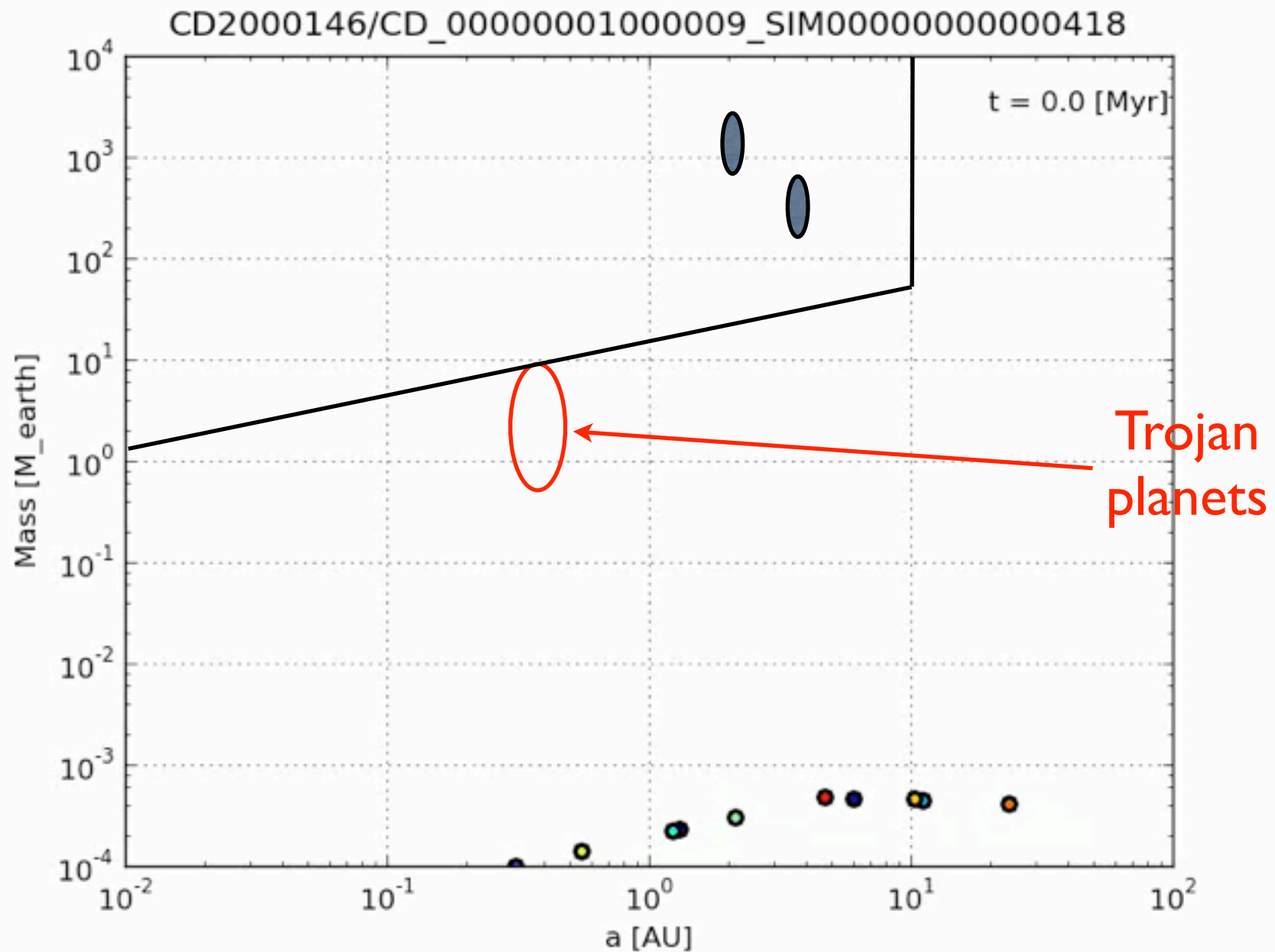
HD 134987



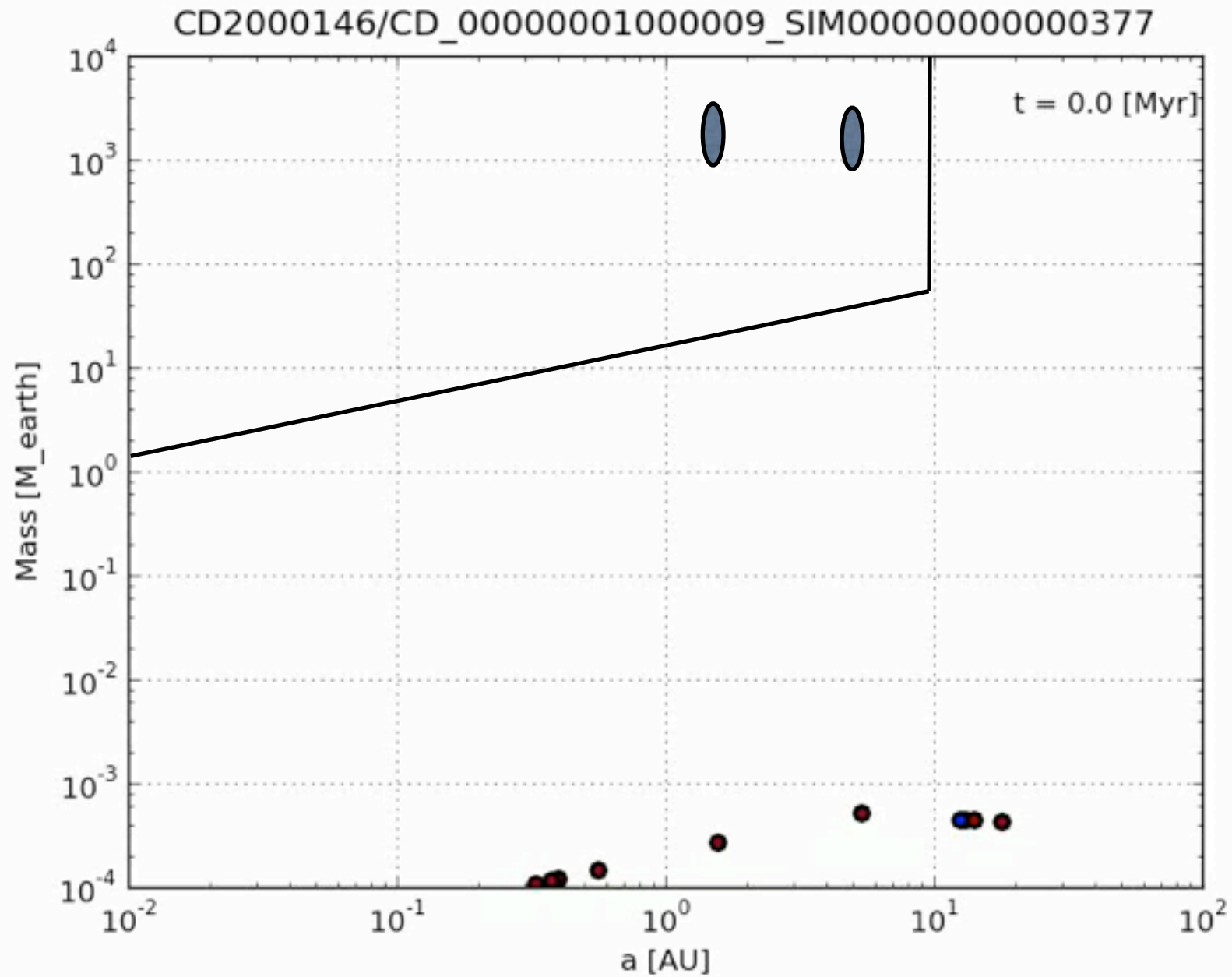
47 UMa



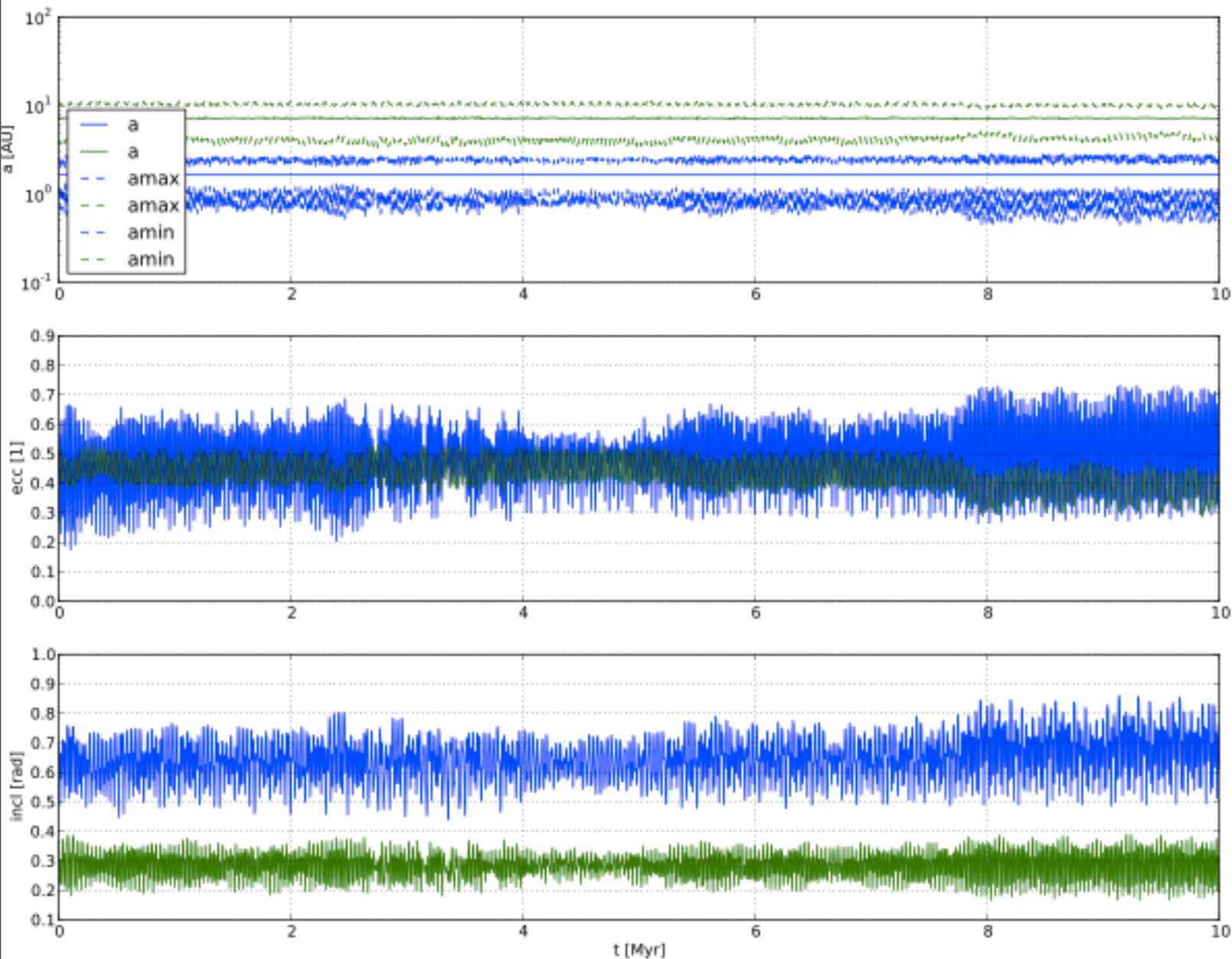
47 UMa



HD 183263



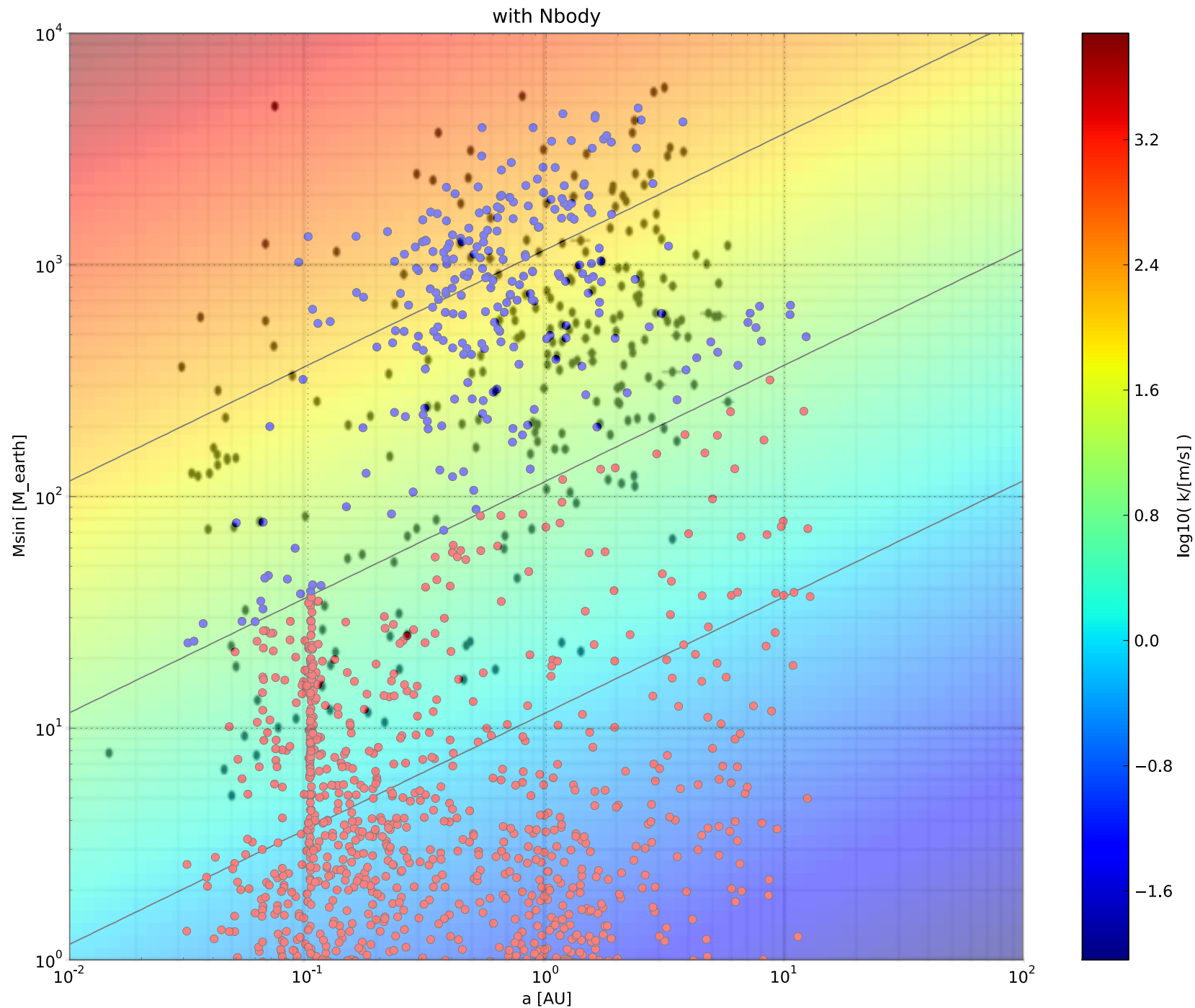
HD 183263 long term evolution - 10 Myr



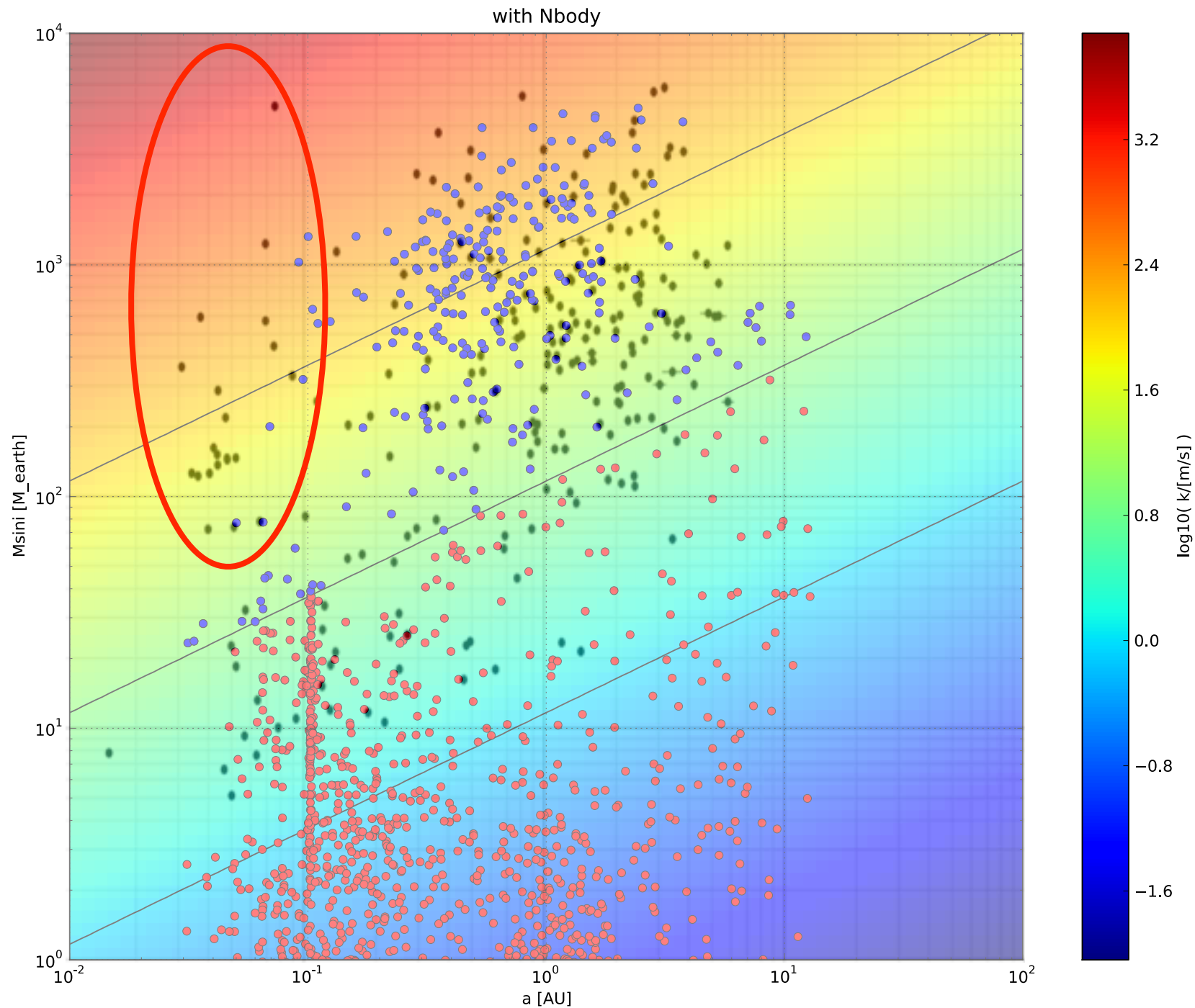
← HD 183263c
← HD 183263b

← HD 183263b
← HD 183263c

a-M diagram compared with all planets



a-M diagram compared with all planets



Conclusions

Conclusions

Planet formation models have to include

- planets internal structure
- disk (solids and gas) modelisation
- their interactions

Conclusions

Planet formation models have to include

- planets internal structure
- disk (solids and gas) modelisation
- their interactions

$N_{\text{planets}} \gg N \times \text{planet}$

- competition
- grav. interactions
- global planetesimal excitation

Conclusions

Planet formation models have to include

- planets internal structure
- disk (solids and gas) modelisation
- their interactions

$N_{\text{planets}} \gg N \times \text{planet}$

- competition
- grav. interactions
- global planetesimal excitation

What about long term evolution?

Conclusions

Future models will have to include

- particular approach for planetesimals
 - shepherding effect
 - gap in planetesimal disk
- interactions induced through the disk
 - common gap
 - modified migration

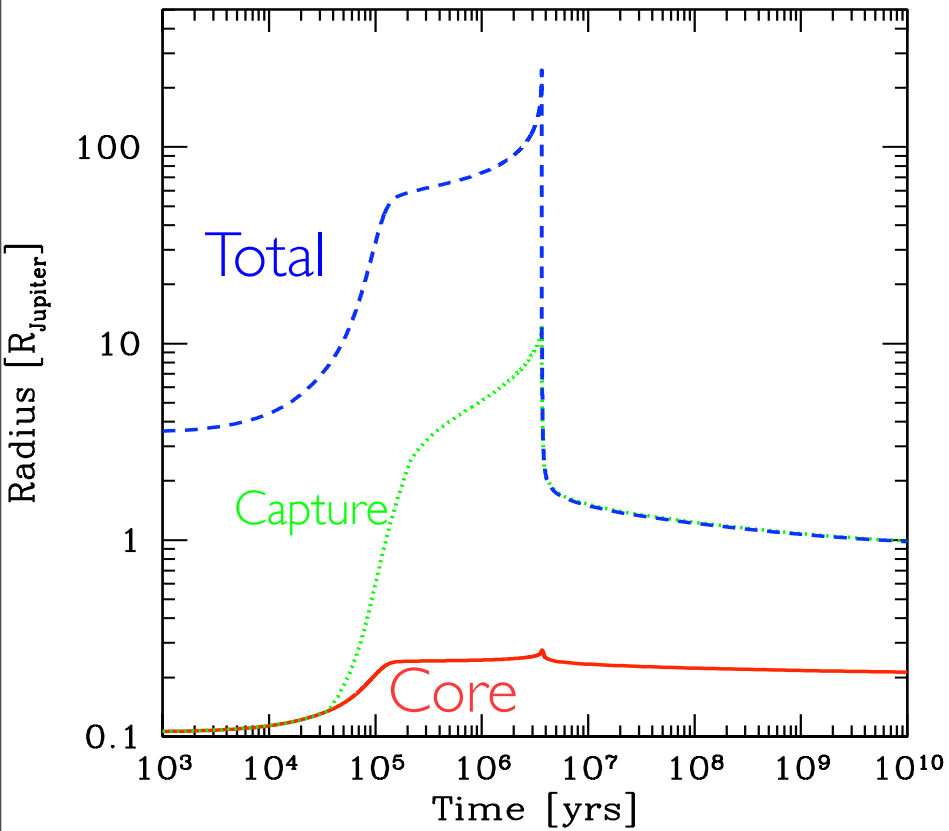
Conclusions

Future models will have to include

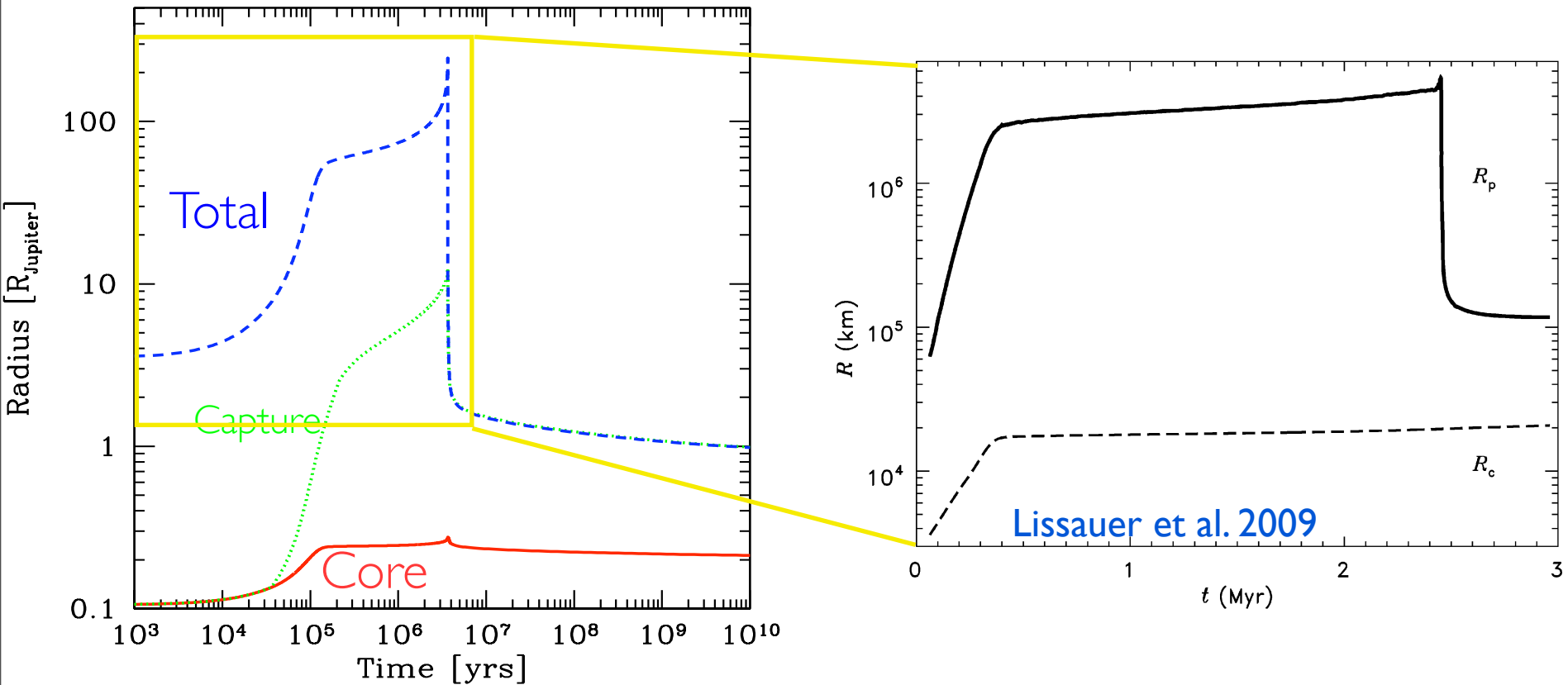
- particular approach for planetesimals
 - shepherding effect
 - gap in planetesimal disk
- interactions induced through the disk
 - common gap
 - modified migration

POPULATION SYNTHESIS

① Planet gas envelope structure - long term evolution

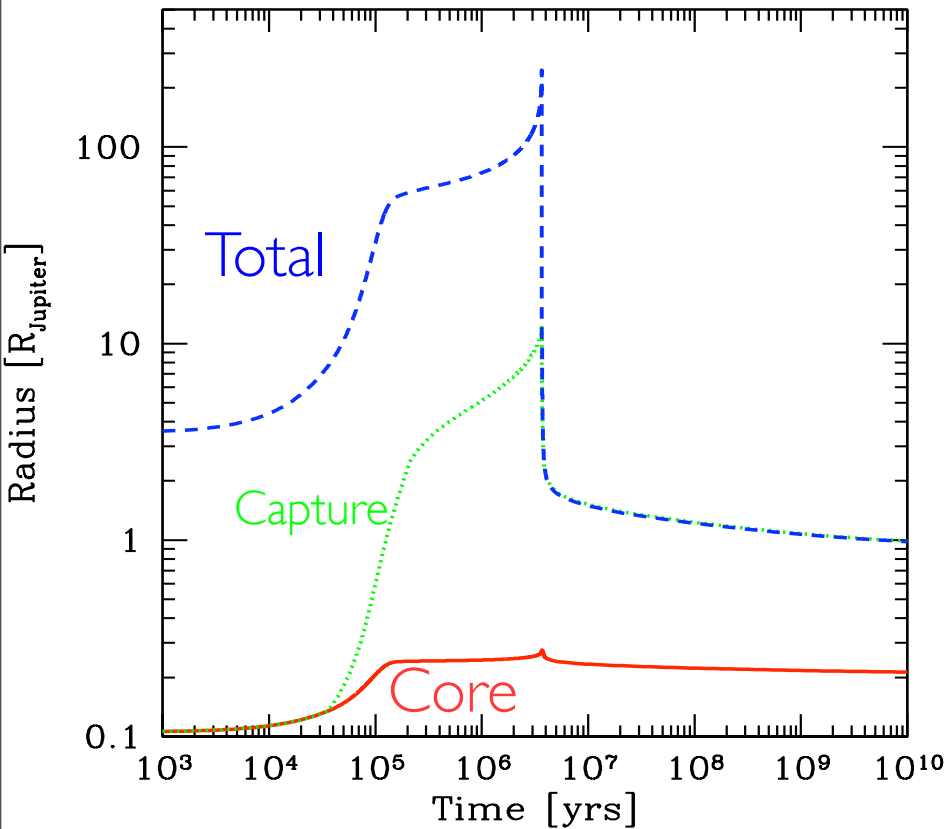


① Planet gas envelope structure - long term evolution



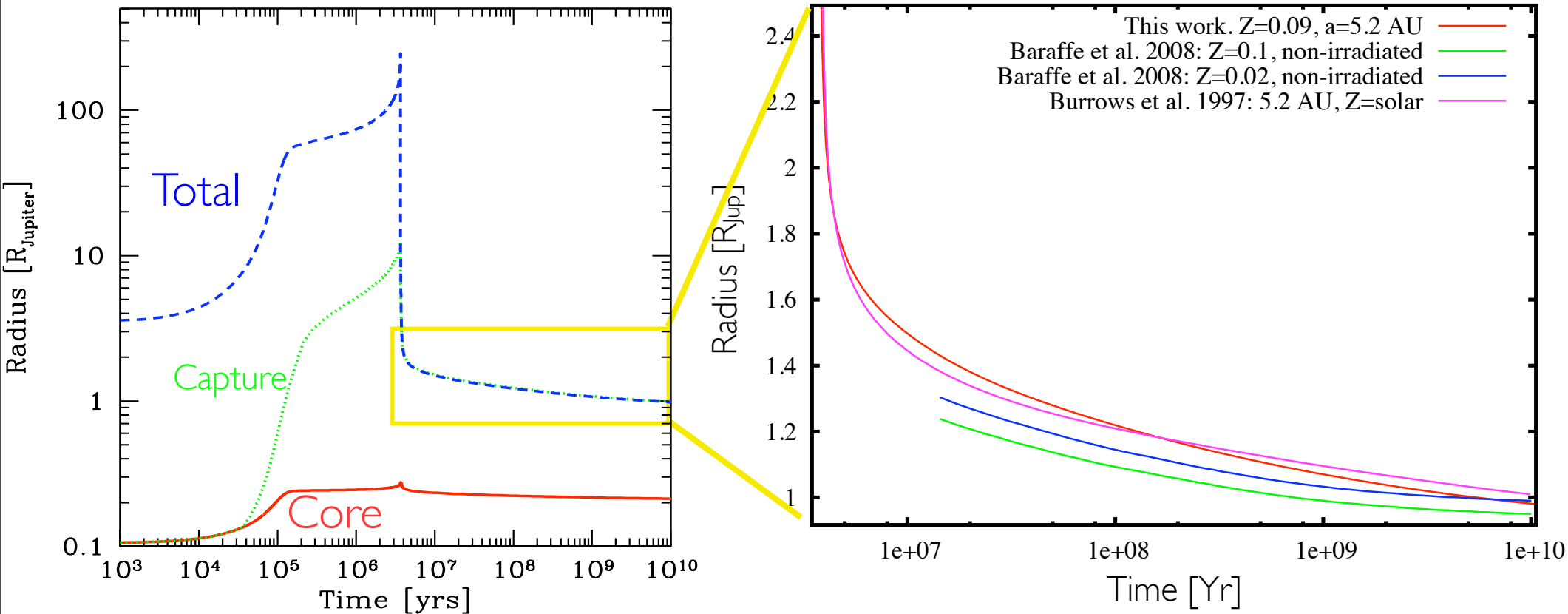
- Attached & detached (collapse) phase: Lissauer et al. 2009 & Broeg et al. in prep well reproduced.

① Planet gas envelope structure - long term evolution



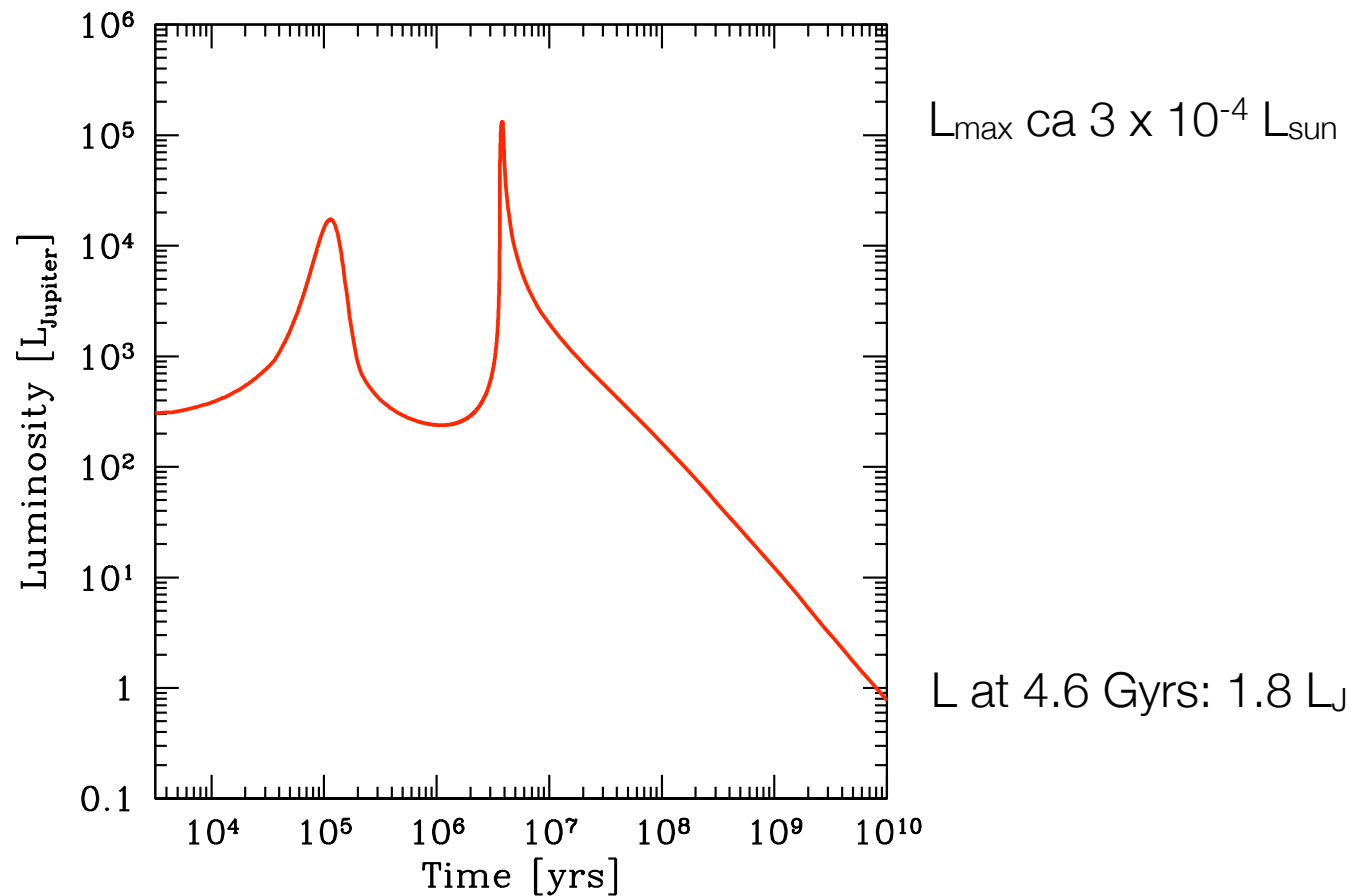
- Attached & detached (collapse) phase: Lissauer et al. 2009 & Broeg et al. in prep well reproduced.

① Planet gas envelope structure - long term evolution



- **Attached & detached (collapse) phase:** Lissauer et al. 2009 & Broeg et al. in prep well reproduced.
- **Long term** evolution of **radii** agree to typically 10 % compared to Baraffe et al. or Burrows et al. models.

① Planet gas envelope structure - long term evolution



- **Attached & detached (collapse) phase:** Lissauer et al. 2009 & Broeg et al. in prep well reproduced.
- **Long term** evolution of **radii** agree to typically 10 % compared to Baraffe et al. or Burrows et al. models.
- Agreement for **luminosities** to factor 2. Tendency to too high R and L at late times.

① Planet gas envelope structure - equations

e.g. Bodenheimer & Pollack 1986

1-D radial **structure** equations (similar to **stellar** structure)

$$\begin{aligned}\frac{dm}{dr} &= 4\pi r^2 \rho & \frac{dP}{dr} &= -\frac{Gm}{r^2} \rho \\ \frac{dl}{dr} &= 4\pi r^2 \rho \left(\epsilon - T \frac{\partial S}{\partial t} \right) & \frac{dT}{dr} &= \frac{T}{P} \frac{dP}{dr} \nabla\end{aligned}$$

Mass conservation
Hydrostat. equilibrium
Energy conservation
Energy transport

$$\nabla = \frac{d \ln T}{d \ln P} = \min(\nabla_{\text{ad}}, \nabla_{\text{rad}}) \quad \nabla_{\text{rad}} = \frac{3}{64\pi\sigma G} \frac{\kappa l P}{T^4 m}$$

Additional energy source:
impacting planetesimals

Gas accretion rate given by ability to **radiate** away **energy** (T_{KH})

① Planet gas envelope structure - equations

e.g. Bodenheimer & Pollack 1986

1-D radial **structure** equations (similar to **stellar** structure)

$$\begin{aligned}\frac{dm}{dr} &= 4\pi r^2 \rho & \frac{dP}{dr} &= -\frac{Gm}{r^2} \rho \\ \frac{dl}{dr} &= 4\pi r^2 \rho \left(\epsilon - T \frac{\partial S}{\partial t} \right) & \frac{dT}{dr} &= \frac{T}{P} \frac{dP}{dr} \nabla\end{aligned}$$

Mass conservation
Hydrostat. equilibrium
Energy conservation
Energy transport

$$\nabla = \frac{d \ln T}{d \ln P} = \min(\nabla_{\text{ad}}, \nabla_{\text{rad}}) \quad \nabla_{\text{rad}} = \frac{3}{64\pi\sigma G} \frac{\kappa l P}{T^4 m}$$

Additional energy source:
impacting planetesimals

Gas accretion rate given by ability to **radiate** away **energy** (T_{KH})

Gas accretion rate in runaway ($M_{\text{core}} > \sim 10 M_E$)

Accretion rate in the **disk**

(flow of gas usually towards the star)

$$\dot{M}_{\text{disk}} = 3\pi \tilde{v} \Sigma + 6\pi r \frac{\partial \tilde{v} \Sigma}{\partial r}$$

Planet **cannot accrete more**
than disk gives

$$\frac{dM_{XY}}{dt} = \text{Min} \left[\frac{dM_{\text{struct}}}{dt}, k_{\text{Lub}} \dot{M}_{\text{disk}} \right]$$

① Planet gas envelope structure - boundary conditions

1. Attached phase

- low mass planets ($M_{\text{core}} < \text{ca. } 10\text{-}20 M_{\text{Earth}}$)
- pre gas runaway accretion
- structure goes smoothly to Hill or accretion radius
- boundary conditions: background nebula

$$R = \frac{R_A}{1 + R_A/R_H}$$

$$\tau = \max(\rho_{\text{neb}} \kappa_{\text{neb}} R, 2/3)$$

$$T^4 = T_{\text{neb}}^4 + T_{\text{int}}^4$$

$$P = P_{\text{neb}}$$

$$T_{\text{int}}^4 = \frac{3\tau L_{\text{int}}}{8\pi\sigma R^2}$$

$$l(R) = L_{\text{int}}.$$

① Planet gas envelope structure - boundary conditions

1. Attached phase

- low mass planets ($M_{\text{core}} < \text{ca. } 10\text{-}20 M_{\text{Earth}}$)
- pre gas runaway accretion
- structure goes smoothly to Hill or accretion radius
- boundary conditions: background nebula

$$R = \frac{R_A}{1 + R_A/R_H}$$

$$P = P_{\text{neb}}$$

$$\tau = \max(\rho_{\text{neb}} \kappa_{\text{neb}} R, 2/3) \quad T_{\text{int}}^4 = \frac{3\tau L_{\text{int}}}{8\pi\sigma R^2}$$

$$T^4 = T_{\text{neb}}^4 + T_{\text{int}}^4$$

$$l(R) = L_{\text{int}}.$$

2. Detached phase

- gas runaway accretion
- structure has a free outer radius
- rapid collapse of radius to $\sim 2 R_J$
- upper boundary: accretion shock
- high mass planets
- disk and gap formation regulate dM_{XY}/dt

$$\dot{M}_{\text{XY}} = \dot{M}_{\text{max}}$$

$$v_{\text{ff}} = [2GM (1/R - 1/R_H)]^{1/2}$$

$$P = P_{\text{neb}} + \frac{\dot{M}_{\text{XY}}}{4\pi R^2} v_{\text{ff}} + \frac{2g}{3\kappa}$$

$$\tau = \max(\rho_{\text{neb}} \kappa_{\text{neb}} R, 2/3)$$

$$T_{\text{int}}^4 = \frac{3\tau L_{\text{int}}}{8\pi\sigma R^2}$$

$$T^4 = (1 - A)T_{\text{neb}}^4 + T_{\text{int}}^4$$

① Planet gas envelope structure - boundary conditions

1. Attached phase

- low mass planets ($M_{\text{core}} < \text{ca. } 10\text{-}20 M_{\text{Earth}}$)
- pre gas runaway accretion
- structure goes smoothly to Hill or accretion radius
- boundary conditions: background nebula

$$\begin{aligned} R &= \frac{R_A}{1 + R_A/R_H} & P &= P_{\text{neb}} \\ \tau &= \max(\rho_{\text{neb}} \kappa_{\text{neb}} R, 2/3) & T_{\text{int}}^4 &= \frac{3\tau L_{\text{int}}}{8\pi\sigma R^2} \\ T^4 &= T_{\text{neb}}^4 + T_{\text{int}}^4 & l(R) &= L_{\text{int}}. \end{aligned}$$

2. Detached phase

- gas runaway accretion
- structure has a free outer radius
- rapid collapse of radius to $\sim 2 R_J$
- upper boundary: accretion shock
- high mass planets
- disk and gap formation regulate dM_{XY}/dt

$$\begin{aligned} \dot{M}_{\text{XY}} &= \dot{M}_{\text{max}} & v_{\text{ff}} &= [2GM (1/R - 1/R_H)]^{1/2} \\ P &= P_{\text{neb}} + \frac{\dot{M}_{\text{XY}}}{4\pi R^2} v_{\text{ff}} + \frac{2g}{3\kappa} & \tau &= \max(\rho_{\text{neb}} \kappa_{\text{neb}} R, 2/3) \\ T_{\text{int}}^4 &= \frac{3\tau L_{\text{int}}}{8\pi\sigma R^2} & T^4 &= (1 - A)T_{\text{neb}}^4 + T_{\text{int}}^4 \end{aligned}$$

3. Evolution $M=\text{cst.}$

- Eddington approximation (gray atmosphere)

$$\begin{aligned} P &= \frac{2g}{3\kappa} & T_{\text{int}}^4 &= \frac{L_{\text{int}}}{4\pi\sigma R^2} \\ T_{\text{equi}} &= 280 \text{ K } \left(\frac{a}{1\text{AU}}\right)^{-\frac{1}{2}} \left(\frac{M_*}{M_{\odot}}\right) & T^4 &= (1 - A)T_{\text{equi}}^4 + T_{\text{int}}^4 \end{aligned}$$

① Planet gas envelope structure - boundary conditions

1. Attached phase

- low mass planets ($M_{\text{core}} < \text{ca. } 10\text{-}20 M_{\text{Earth}}$)
- pre gas runaway accretion
- structure goes smoothly to Hill or accretion radius
- boundary conditions: background nebula

$$R = \frac{R_A}{1 + R_A/R_H}$$

$$P = P_{\text{neb}}$$

$$\tau = \max(\rho_{\text{neb}} \kappa_{\text{neb}} R, 2/3) \quad T_{\text{int}}^4 = \frac{3\tau L_{\text{int}}}{8\pi\sigma R^2}$$

$$T^4 = T_{\text{neb}}^4 + T_{\text{int}}^4$$

$$l(R) = L_{\text{int}}$$

2. Detached phase

- gas runaway accretion
- structure has a free outer radius
- rapid collapse of radius to $\sim 2 R_J$
- upper boundary: accretion shock
- high mass planets
- disk and gap formation regulate dM_{XY}/dt

$$\dot{M}_{\text{XY}} = \dot{M}_{\text{max}}$$

$$v_{\text{ff}} = [2GM (1/R - 1/R_H)]^{1/2}$$

$$P = P_{\text{neb}} + \frac{\dot{M}_{\text{XY}}}{4\pi R^2} v_{\text{ff}} + \frac{2g}{3\kappa}$$

$$\tau = \max(\rho_{\text{neb}} \kappa_{\text{neb}} R, 2/3)$$

$$T_{\text{int}}^4 = \frac{3\tau L_{\text{int}}}{8\pi\sigma R^2}$$

$$T^4 = (1 - A)T_{\text{neb}}^4 + T_{\text{int}}^4$$

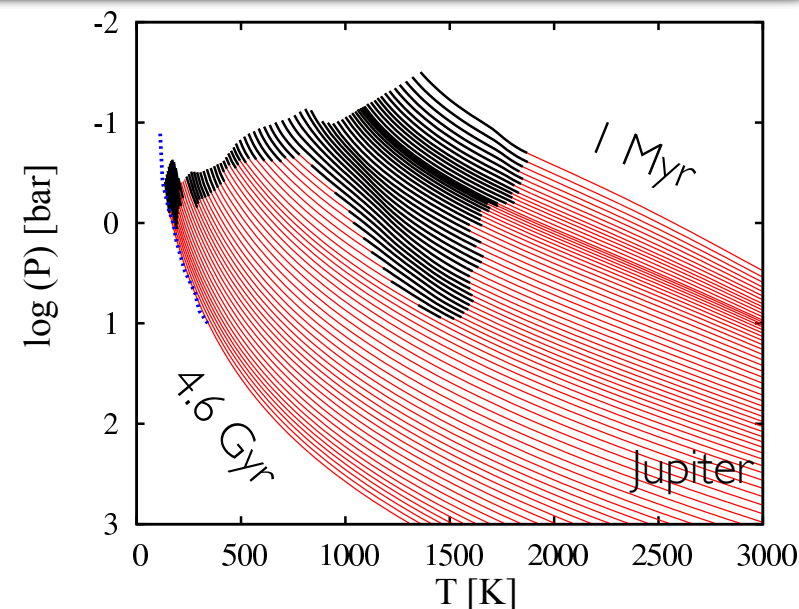
3. Evolution $M=\text{cst.}$

- Eddington approximation (gray atmosphere)

$$P = \frac{2g}{3\kappa}$$

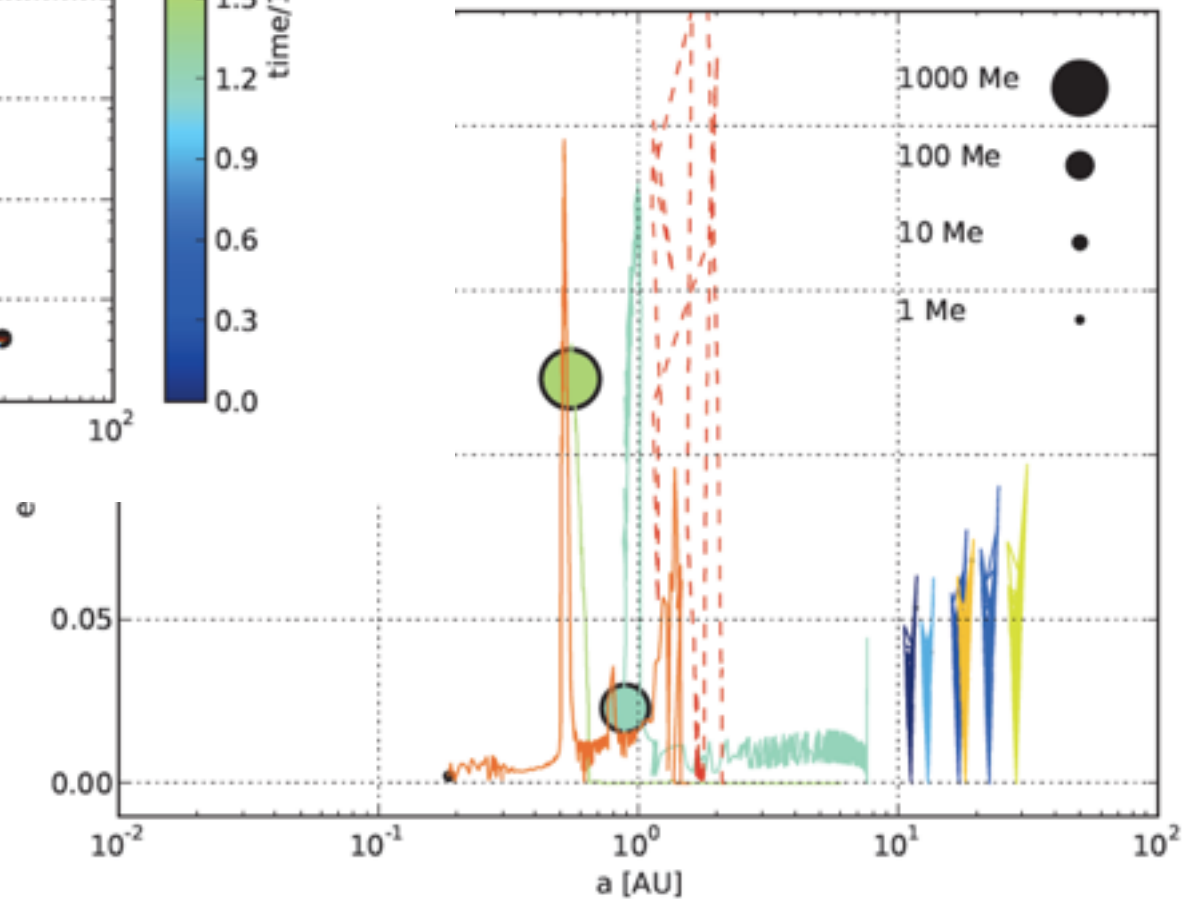
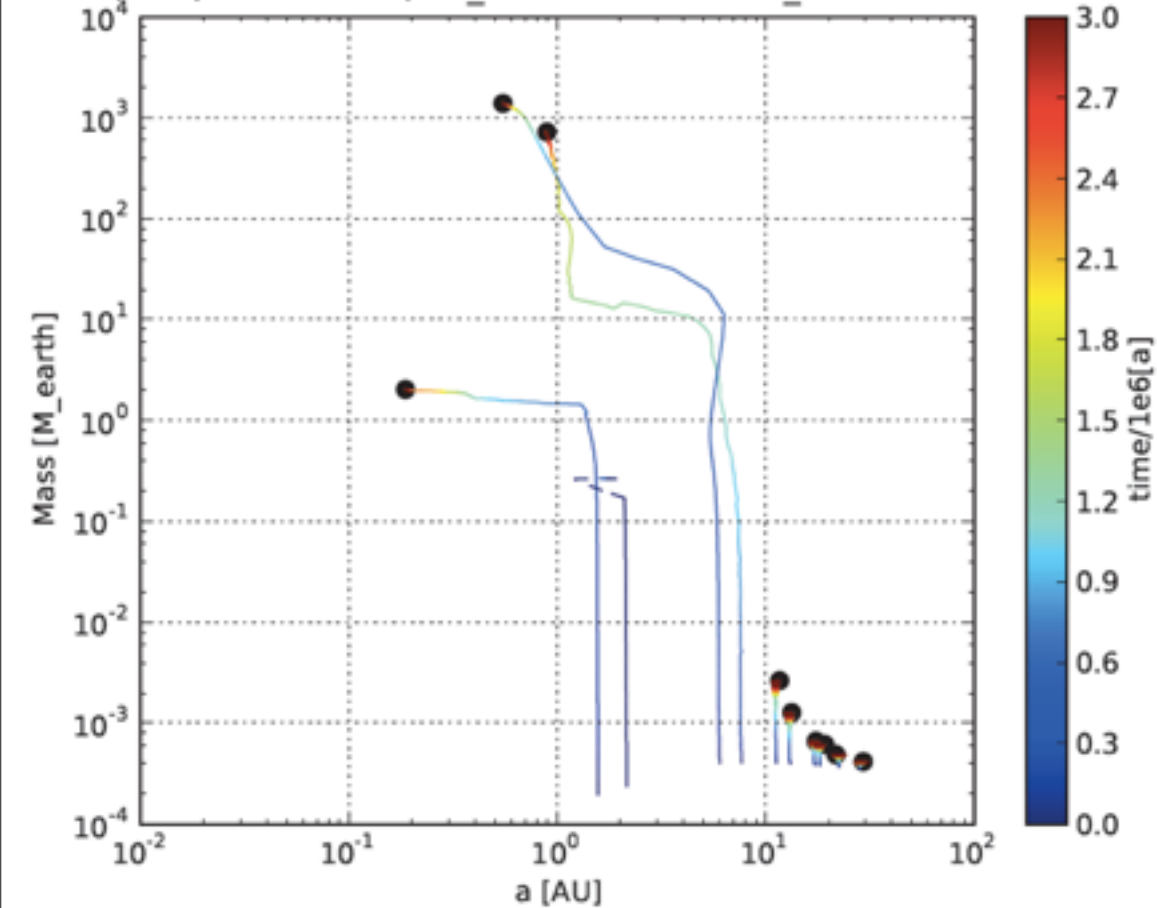
$$T_{\text{int}}^4 = \frac{L_{\text{int}}}{4\pi\sigma R^2}$$

$$T_{\text{equi}} = 280 \text{ K} \left(\frac{a}{1 \text{ AU}} \right)^{-\frac{1}{2}} \left(\frac{M_*}{M_{\odot}} \right) \quad T^4 = (1 - A)T_{\text{equi}}^4 + T_{\text{int}}^4$$



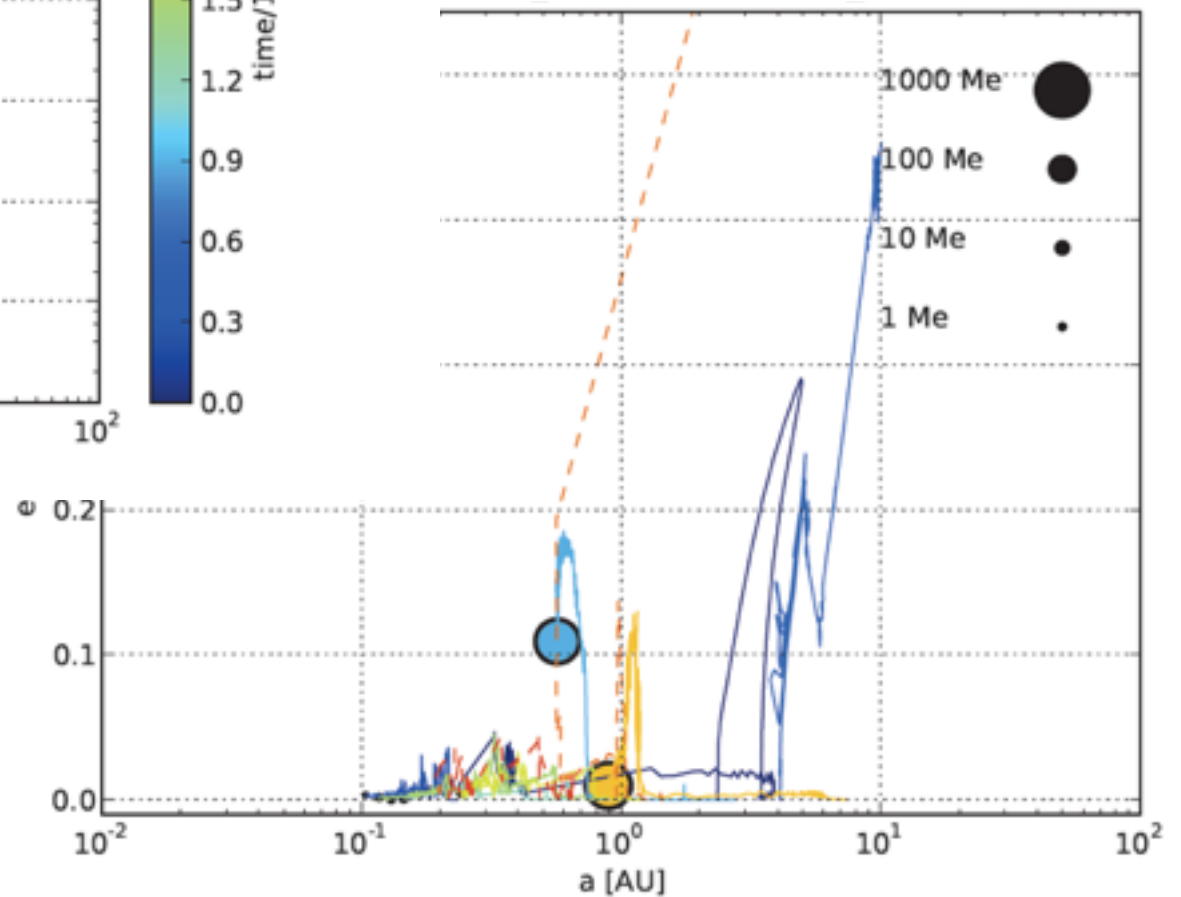
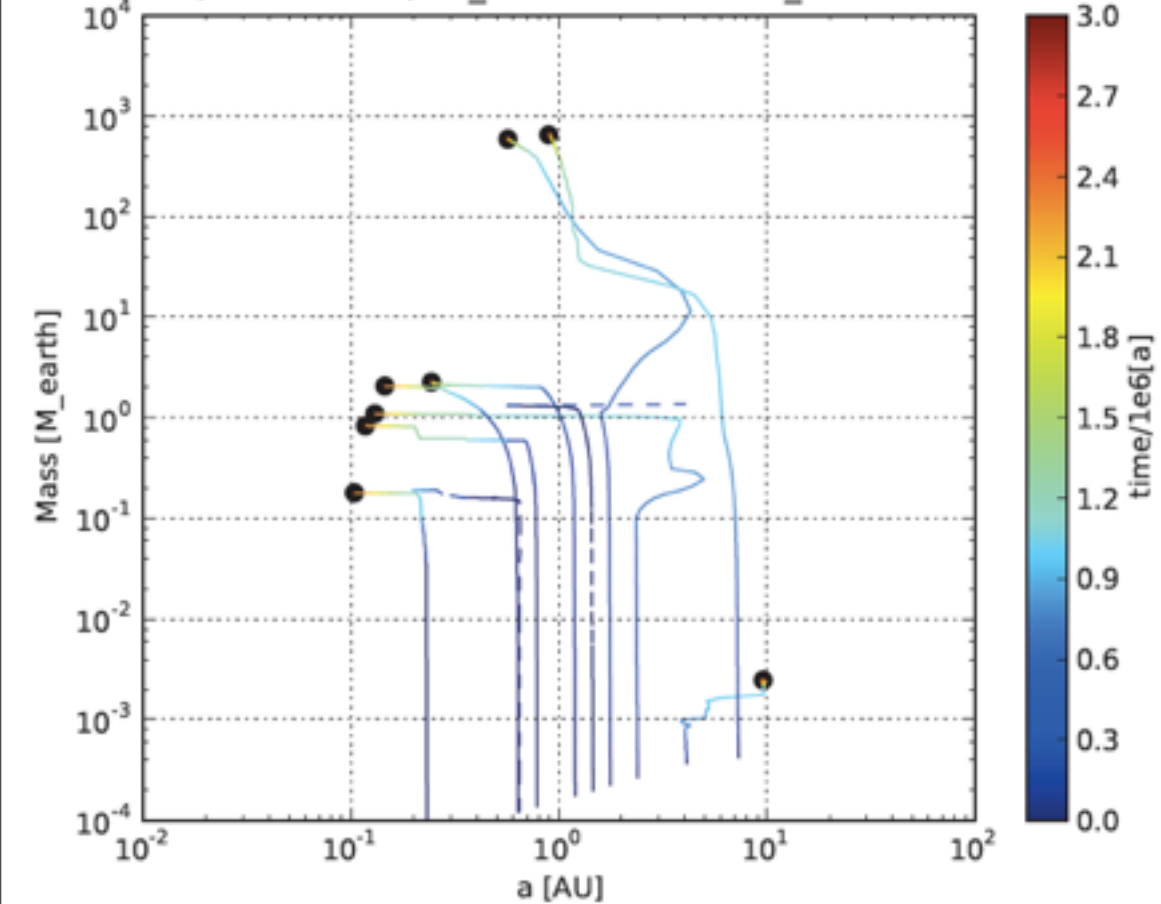
a system with two giants and one SE

scratch/CDs/CD2000146/CD_00000001000009_SIM0000000000000004



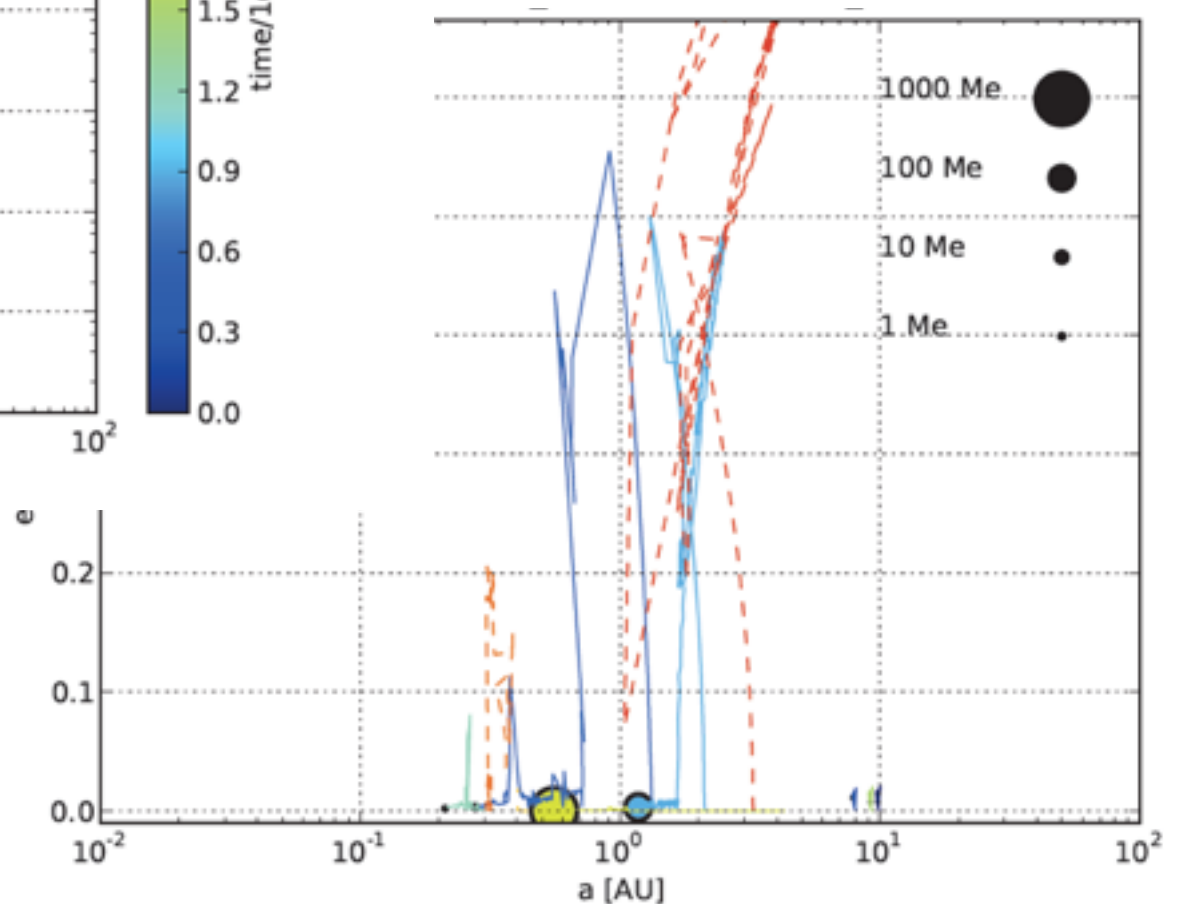
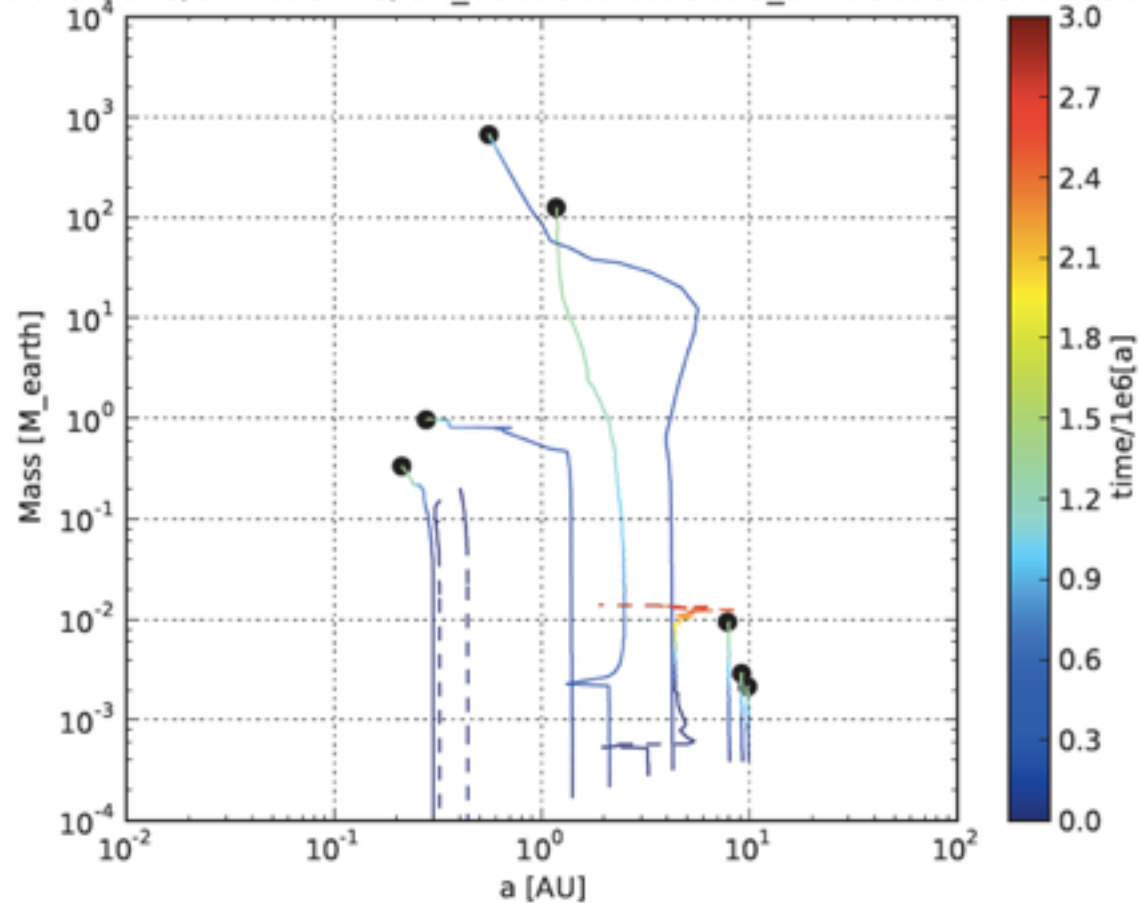
a system with two giants and many HSE

scratch/CDs/CD2000146/CD_00000001000009_SIM0000000000000030



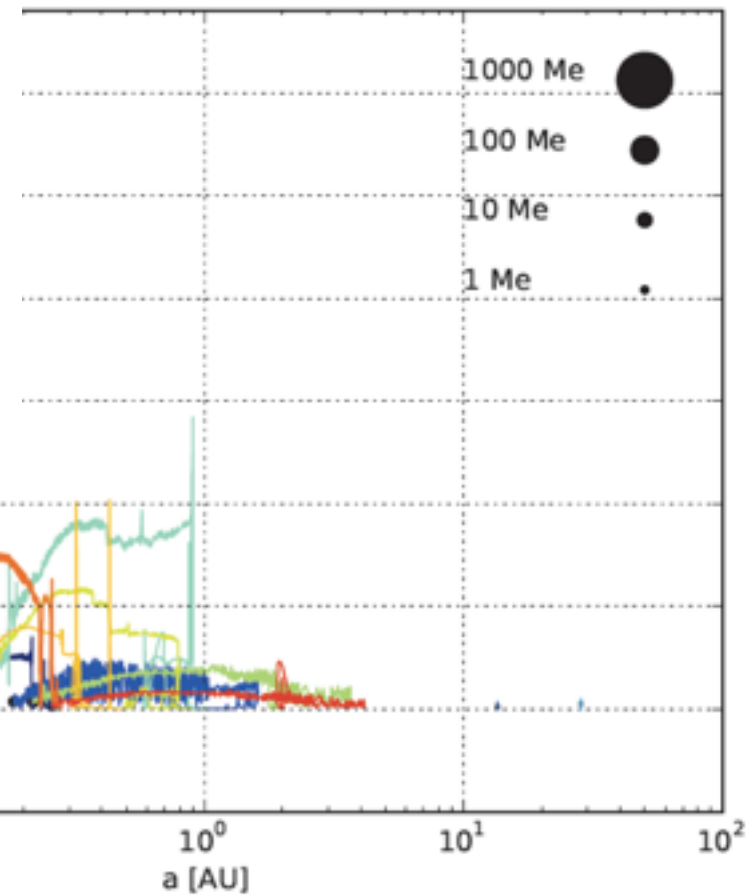
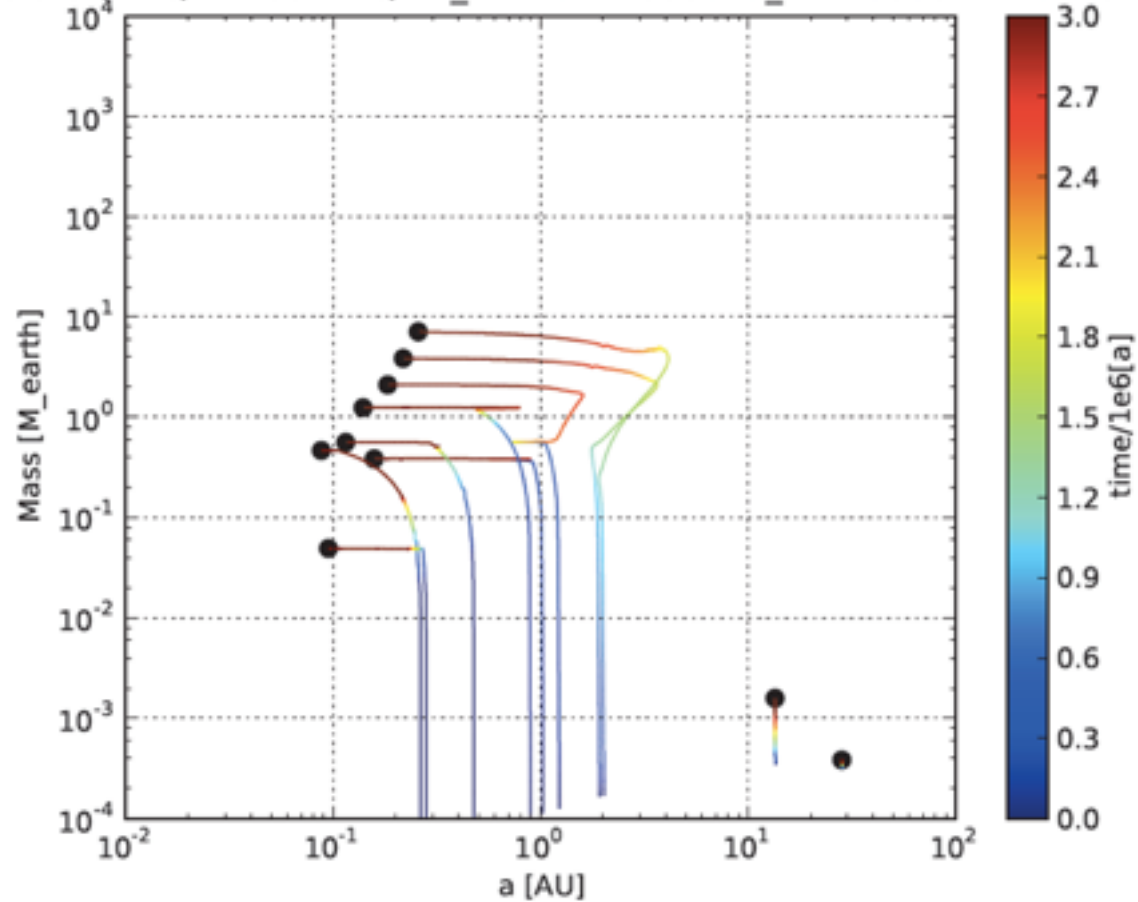
a system with two giants, no SE, no HN

scratch/CDs/CD2000146/CD_00000001000009_SIM000000000000089



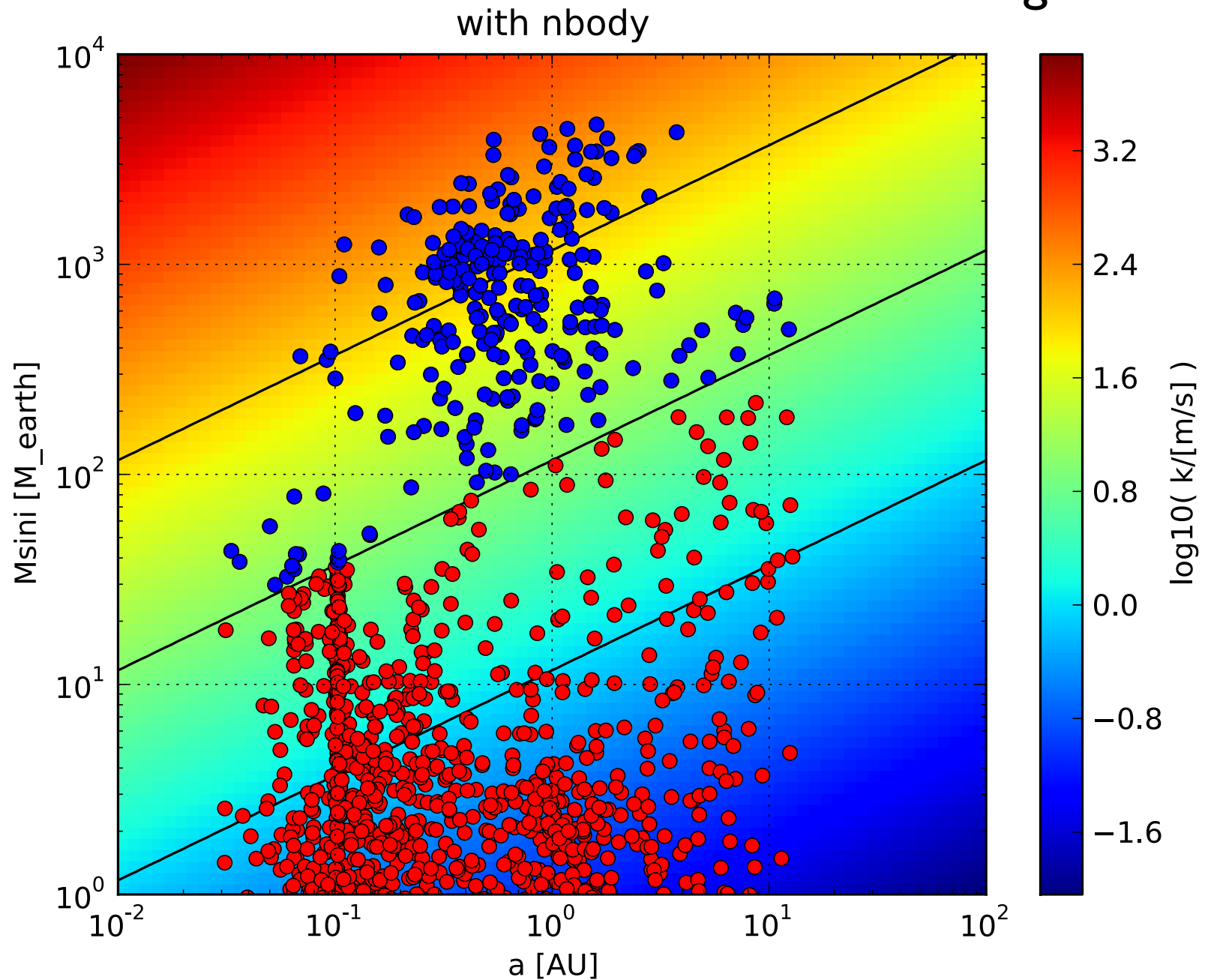
a system with four SE

scratch/CDs/CD2000146/CD_00000001000009_SIM000000000000232



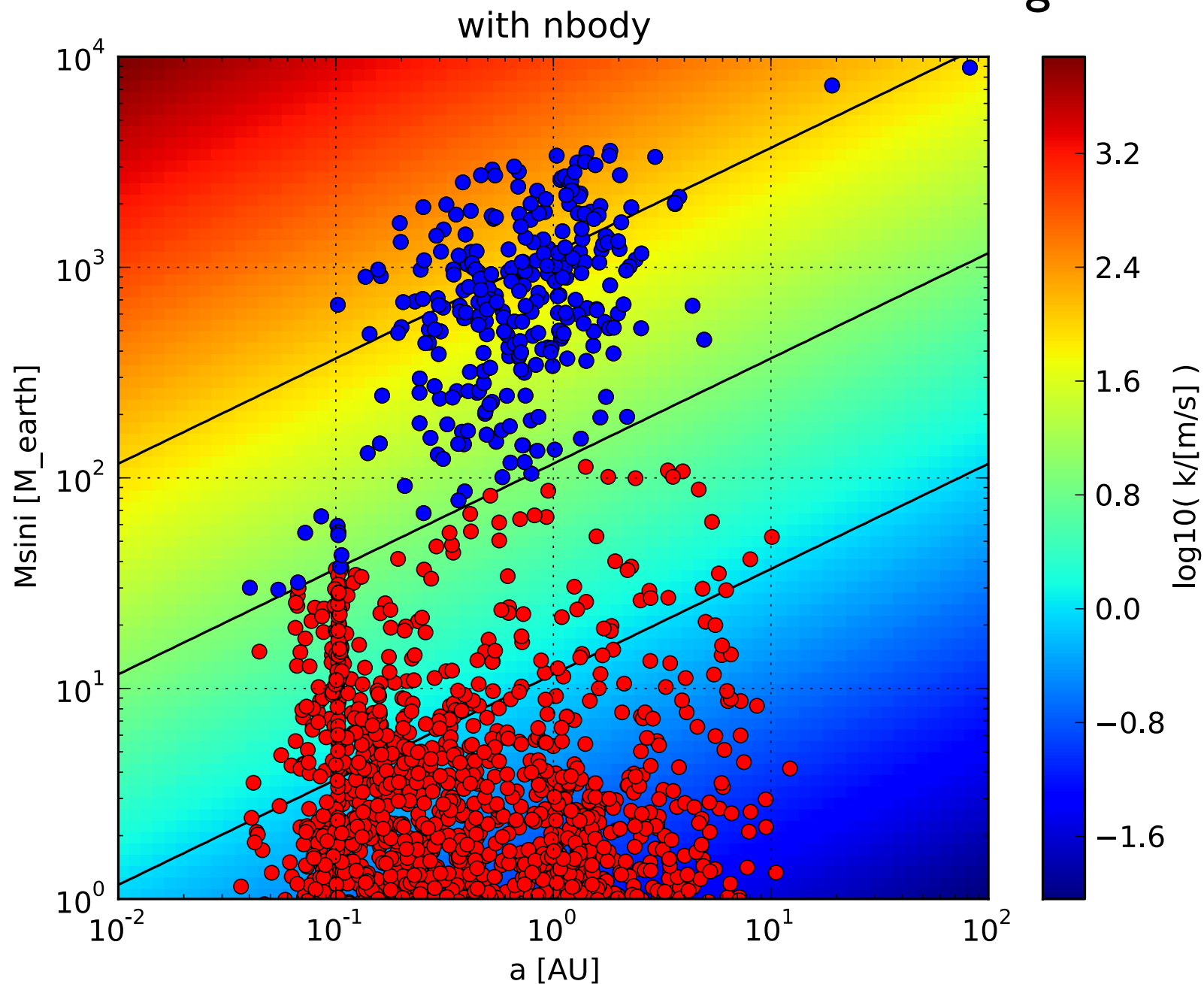
a-M diagram

starting with 10 planets



10 vs 20 planets

starting with 20 planets



stability of the Trojan planets

