Fast new control algorithms for XAO on ELT with pyramid WFS

Iuliia Shatokhina, A. Obereder

Industrial Mathematics Institute, JKU, Linz

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Outline

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Problem

XAO

- large SCAO system - 1 NGS, 1 WFS, 1 DM
- aim – direct imaging of exoplanets (e.g., EPICS on E-ELT)
- 42 m telescope
- pyramid WFS with 200 $\times$ 200 subapertures
- P-WFS without / with linear / with circular modulaition
- frame rate 3 kHz, time for reconstruction: 0.3 ms

Task

- To determine the unknown wavefront ϕ from pyramid WFS data S_x, S_y

$$S_x = Q_x\phi, \qquad S_y = Q_y\phi. \tag{1}$$

Simplifying assumptions

- Fourier optics based models are non-linear, difficult to invert
- Instead of the full models we consider linearized models

Assumptions

- Roof WFS approximation
- infinite telescope size
- small wavefront perturbations (as in closed loop)

Approximate forward models

- without modulation – Hilbert transform

$$S_x(x,\cdot) = \phi(x,\cdot) * \frac{1}{\pi x}, \qquad (2)$$

- with linear modulation

$$S_x(x,\cdot) = \phi(x,\cdot) * \frac{\text{sinc}(\alpha_\lambda x)}{\pi x}, \qquad (3)$$

- with circular modulation

$$S_x(x,\cdot) = \phi(x,\cdot) * \frac{J_0(\alpha_\lambda x)}{\pi x}, \qquad (4)$$

where the modulation radius $\alpha_\lambda = 2\pi\alpha/\lambda$, $\alpha = b\lambda/D$,
b – non-negative integer, D – telescope diameter
J_0 – zero-order Bessel function of the first kind.

Our approach

- Linear modulation case is similar to circular modulation
- Non-modulated P-WFS is a special case of modulated
- Focus on P-WFS with linear modulation
- Depending on the frequency, measurements are either the Hilbert transform of the phase, or the wavefront derivative (as for SH-WFS) (Vérinaud'04)
- Representation of the measurements in the Fourier domain (Vérinaud'04)

Outline

Method 1: CuReD with data preprocessing

Method has two components

1. **Data preprocessing:** Transform the modulated P-WFS data to SH-like data according to the analytical relation in the Fourier domain.

2. **CuReD:** To the modified data apply the CuReD, which is a very efficient reconstructor for SH-WFS data [see talk by M. Rosensteiner].

Fourier domain filters [Verinaud'04]

- P-WFS with linear modulation

$$(\mathcal{F}S_{pyr})(u) = (\mathcal{F}\phi)(u) \cdot g_{pyr}(u) \cdot \text{sinc}(du). \quad (5)$$

$$g_{pyr} = \begin{cases} i\,\text{sign}(u), & |u| > u_{mod}, \\ iu/u_{mod}, & |u| \leqq u_{mod}, \end{cases} \quad (6)$$

where $u_{mod} = \alpha/\lambda = b/D$.

- SH-WFS

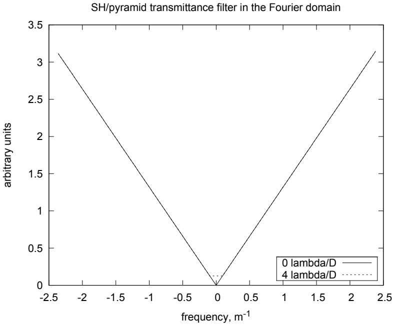
$$(\mathcal{F}S_{sh})(u) = (\mathcal{F}\phi)(u) \cdot g_{sh}(u) \cdot \text{sinc}(du). \quad (7)$$

$$g_{sh}(u) = 2i\pi du. \quad (8)$$

SH-Pyr transfer filter

- SH-Pyr transfer filter

$$g_{sh/pyr}(u) = \frac{g_{sh}(u)}{g_{pyr}(u)} = \begin{cases} 2\pi du\,\mathrm{sign}(u), & |u| > u_{mod}, \\ 2\pi du_{mod}, & |u| <= u_{mod}. \end{cases} \quad (9)$$

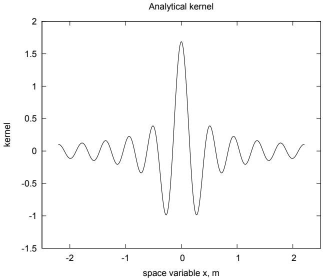


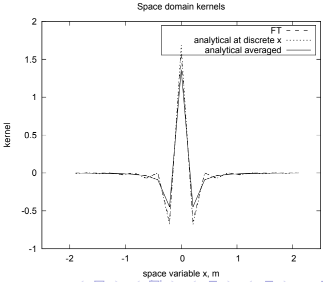
Step 1: Data preprocessing

- Convolve S_x row-wise and S_y column-wise with the 1d kernel $p_{sh/pyr}$

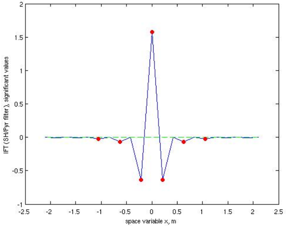
$$S_{sh}(x) = S_{pyr}(x) * \underbrace{\left(\mathcal{F}^{-1} g_{sh/pyr}\right)(x)}_{p_{sh/pyr}(x)} . \qquad (10)$$

$$p_{sh/pyr}(x) = \frac{\pi}{d} \operatorname{sinc}\left(\frac{\pi x}{d}\right) + \frac{2\pi d b^2}{D^2} \operatorname{sinc}^2\left(\frac{\pi x b}{D}\right) - \frac{\pi}{2d} \operatorname{sinc}^2\left(\frac{\pi x}{2d}\right) . \qquad (11)$$





Step 2: CuReD



- Data preprocessing step is very cheap, as the convolution kernel has only few non-zero values
- Preprocessing step is highly parallelizable and pipelinable
- To the modified data apply the CuReD – wavefront reconstructor from SH-WFS data
- CuReD has a linear complexity, is parallelizable and pipelinable, see talk by M. Rosensteiner

Method 2: CLIF

- Fourier domain representation of the P-WFS data [Verinaud'04]

$$(\mathcal{F}S_x)(u) = (\mathcal{F}\phi)(u) \cdot h(u), \tag{12}$$

$$h(u) = g_{pyr}(u) \cdot \mathrm{sinc}(du). \tag{13}$$

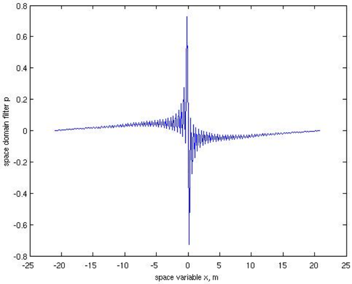
- Reconstruction formula based on the corresponding relation in the space domain

$$\phi_x^{rec}(x,\cdot) = S_x(x,\cdot) * \underbrace{(\mathcal{F}^{-1}h^{-1})(x)}_{p(x)}. \tag{14}$$

- CLIF – Convolution with Linearized Inverse Filter

Convolution

- The space domain kernel $p(x) = \mathcal{F}^{-1} h^{-1}(x)$

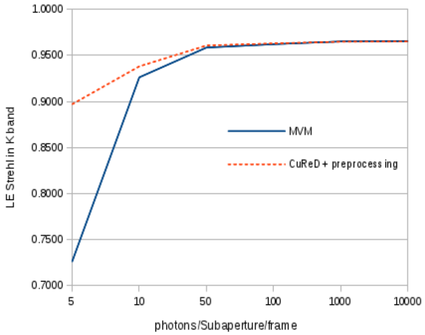


- Convolve S_x row-wise and S_y column-wise with a 1d kernel p
- Algorithm is parallelizable and pipelinable

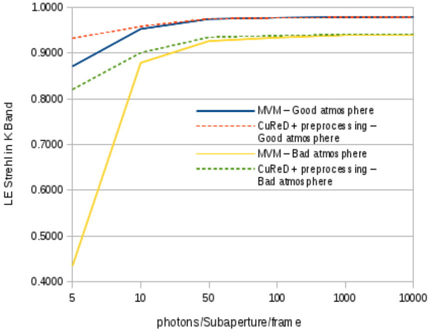
Outline

LE Strehl

Comparison of LE Strehls in K band: MVM and CuReD with a preprocessing step versus the detected NGS photon flux.



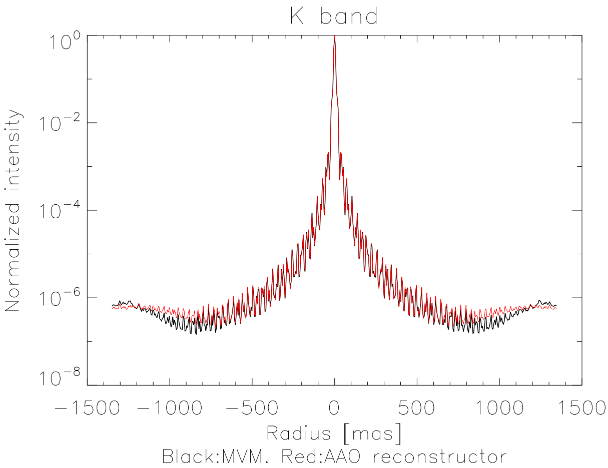
ESO median atmosphere



ESO bad/good atmospheres

LE PSF

LE PSF in the K-band for median atmosphere and high-flux case:
MVM and CuReD with data preprocessing.



Complexity

CuReD with data preprocessing:

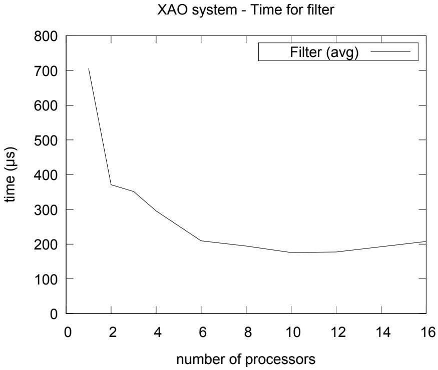
- Data preprocessing requires $26N$ operations.
- CuReD requires $20N$ operations.
- Parallelizable and pipelinable.

CLIF:

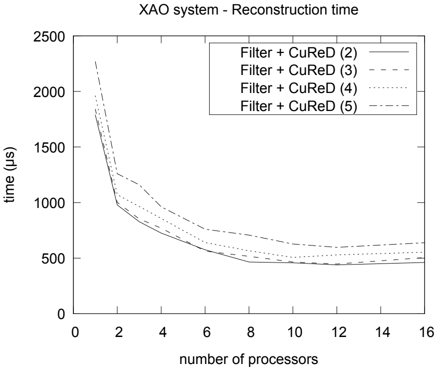
- CLIF requires $4N\sqrt{N}$ operations.
- Parallelizable and pipelinable.

Speed: CuReD with data preprocessing

The reconstruction speed of the C-prototype was measured on a computer with 2 Intel hexacore processors (X5650), running at a speed of 2.66 GHz.



Data preprocessing step.



CuReD(n) with data preprocessing step.

Comparison: CuReD with data preprocessing vs CLIF vs MVM

	CLIF		*CuReD with data preprocessing*	*MVM*
	full kernel	*cut kernel 50%*		
Strehl, 500 it, median atm., high flux	0.945	0.940	0.965	0.965
Complexity	$4N\sqrt{N}$	$2N\sqrt{N}$	$46N$	$4N^2$
Number of operations for ~29600 active subapertures	2.04E+007	1.02E+007	1.36E+006	3.50E+009
Percentage of MVM flops	0.6%	0.3%	0.04%	100%
Parallelizability	yes		yes	yes
Pipelinability	yes		yes	yes

Summary

- Two fast new reconstruction methods for P-WFS with/without modulation.
- CuReD with data preprocessing has a linear complexity $\mathcal{O}(N)$.
- CLIF has a complexity of $\mathcal{O}(N\sqrt{N})$.
- Strehl ratios are the same as / better than MVM.
- PSF is suitable for extrasolar planet search.
- Method suits the XAO requirements on ELTs.

Thanks for your attention!

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Thanks for your attention!