

Cumulative theoretical uncertainties in Lithium Depletion Boundary Age

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Introduction (1/2)

Lithium Depletion Boundary: Alternative method to assign an age to young clusters

(9 clusters, see e.g. reviews of Jeffries 2006, Soderblom et al 2013)

Lithium depletion:

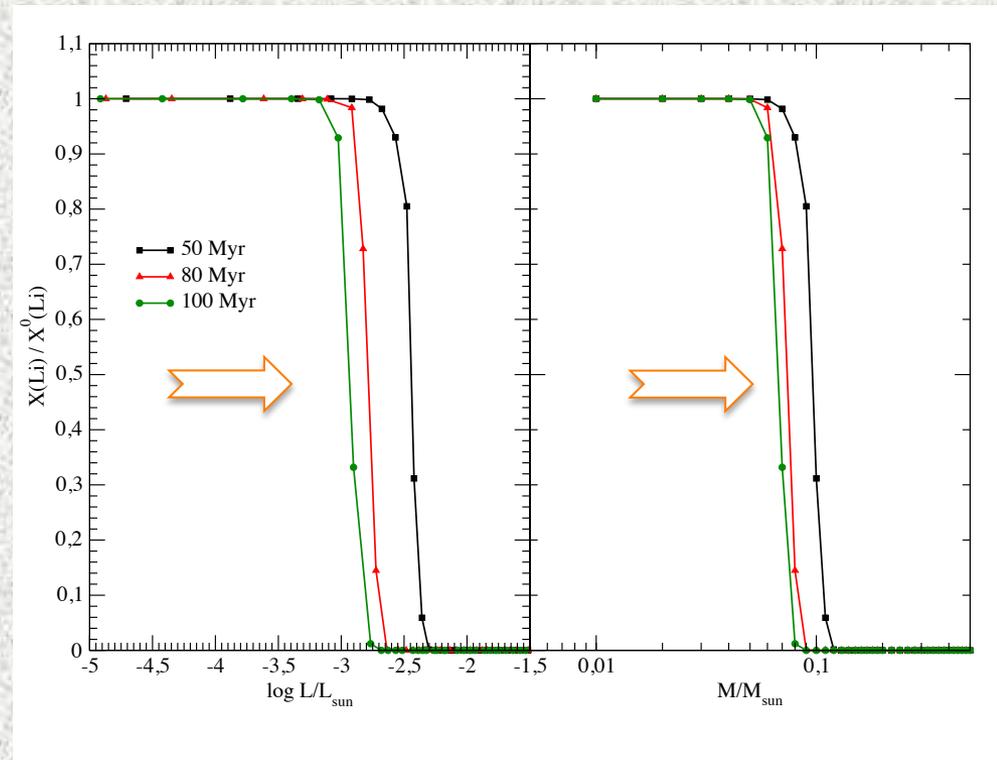
- ${}^7\text{Li}$ destroyed during the PMS ($T_c > 2 \times 10^6$ K) for stellar mass $M > 0.06 M_{\text{sun}}$
- Completely destroyed in fully convective stars: $0.06 M_{\text{sun}} < M < 0.5 M_{\text{sun}}$
- Destruction timescale (τ_{ldb}) depends on T_c : $T_c = T_c(M)$
- τ_{ldb} decreases with the stellar mass: strongly dependent on M

Introduction (2/2)

Lithium depletion boundary (LDB):

In a cluster: the smallest object with ≈ 0 surface ${}^7\text{Li}$ abundance

Mass range $\approx [0.06, 0.5] M_{\text{sun}}$
Age range $\approx [10, 300] \text{ Myr}$



Age inferred by comparing the **observed** and **theoretically** computed **LDB luminosity**. Uncertainty on the models propagates into a final age uncertainty (Bildsten et al. 1997, Burke et al. 2004)

Stellar Models (1/3)

PROSECCO: Pisa stellar evolutionary code

(Degl'Innocenti et al. 2008)

Updated input physics

(Tognelli et al. 2012, Dell'Omodarme et al. 2012, Tognelli et al. 2014)

Detailed atmospheric models: PHOENIX atmospheric code

(Brott & Haushildt 2005)

Equation of state: extension to the brown dwarf regime

(Saumon, Chabrier & VanHorn 1995)

Updated nuclear cross sections for light elements (deuterium, lithium, beryllium, and boron)

Recently updated solar metals abundances (Asplund et al. 2009)

Stellar Models (2/3)

Uncertainty analysis Input physics and initial chemical composition

$$\text{LDB} = \text{LDB}(\{p_l\}, \{x_k\})$$

$\{p_l\}$ = input physics quantity (i.e. opacity, cross section, mixing length...)

$\{x_k\}$ = element abundance

- **Independent variation** of each quantity.
Individual uncertainty source.

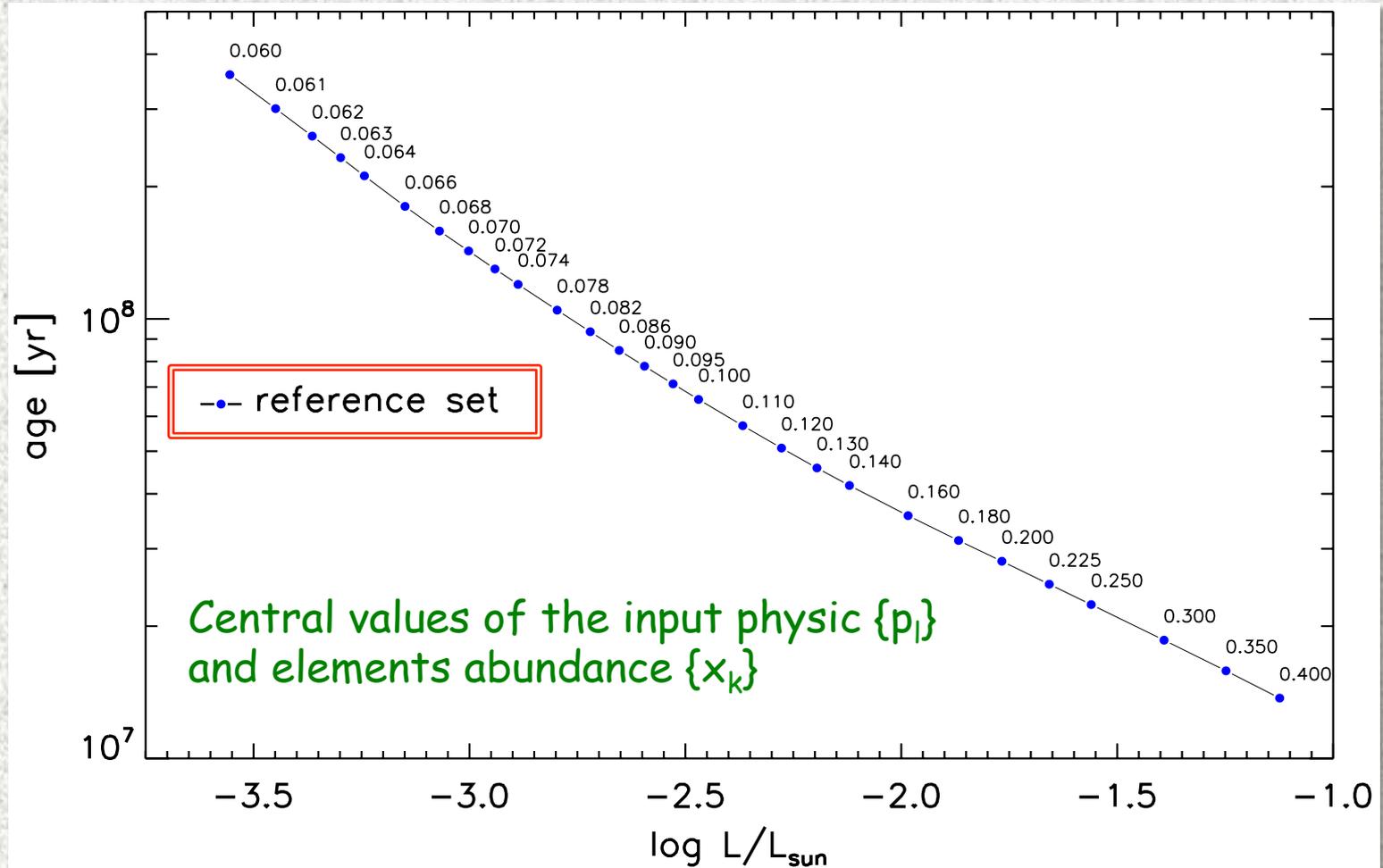
input physics: $\text{LDB} = \text{LDB}(p_j \pm \Delta p_j, \{p_{l \neq j}\}, \{x_k\})$

chemical composition: $\text{LDB} = \text{LDB}(\{p_l\}, x_j \pm \Delta x_j, \{x_{k \neq j}\})$

- **Cumulative error stripe.**
Simultaneous variation of all the analysed quantities at the same time.

Individual Uncertainty source (1/4)

reference set



Individual Uncertainty source (2/4)

Table 3. Input physics varied in the computation of perturbed stellar models and their assumed uncertainty or range of variation. The flag "yes" specifies the quantities taken into account in the cumulative uncertainty calculation

quantity	individual	
	error	global
$^2\text{H}(p,\gamma)^3\text{He}$ reaction rate	$\pm 3\%$	no
$^2\text{H}(^2\text{H},n)^3\text{He}$ reaction rate	$\pm 5\%$	no
$^2\text{H}(^2\text{H},p)^3\text{H}$ reaction rate	$\pm 5\%$	no
$^7\text{Li}(p,\alpha)\alpha$ reaction rate	$\pm 10\%$	yes
electron screening($p+^7\text{Li}$) ^(a)	+50%, +100%	no
BCs ^(b)	BH05, AHF11, KS66	no
τ_{ph} ^(a)	2/3, 100	yes
EOS ^(b)	OPAL06, FreeEOS08, SCVH95	no
$\bar{\kappa}_{\text{rad}}$	$\pm 5\%$	no

Input Physics

(Bildsten et al. 1997, Burke et al. 2004)

Uncertainty estimation not available

Chemical Composition (analysed for the first time)

$$Y = Y_{\text{P}} + \frac{\Delta Y}{\Delta Z} Z$$

$$Z = \frac{(1 - Y_{\text{P}})(Z/X)_{\odot}}{10^{-[\text{Fe}/\text{H}]} + (1 + \Delta Y/\Delta Z)(Z/X)_{\odot}}$$

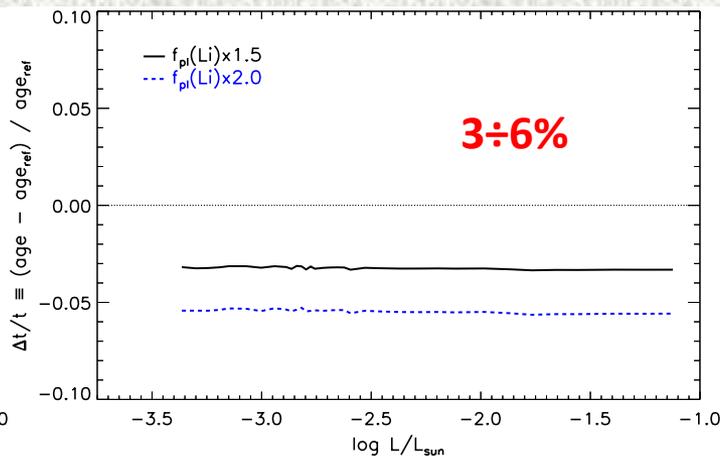
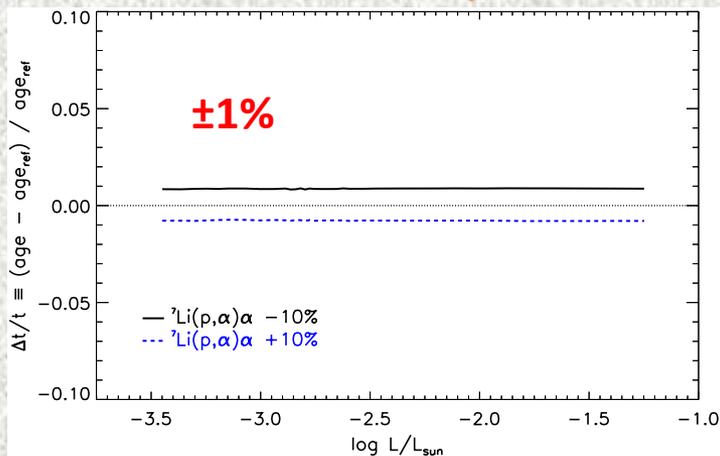
(Gennaro et al. 2010)

Table 4. Chemical composition parameters varied in the computation of perturbed stellar models and their assumed uncertainty. The flag "yes" in the last column specifies the quantities taken into account in the cumulative uncertainty calculation

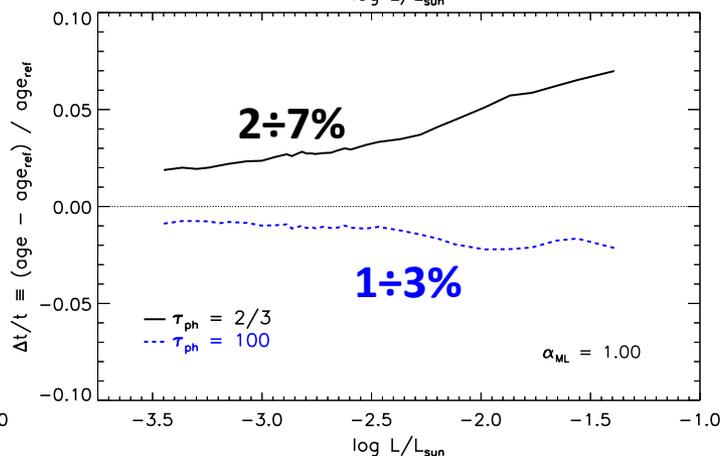
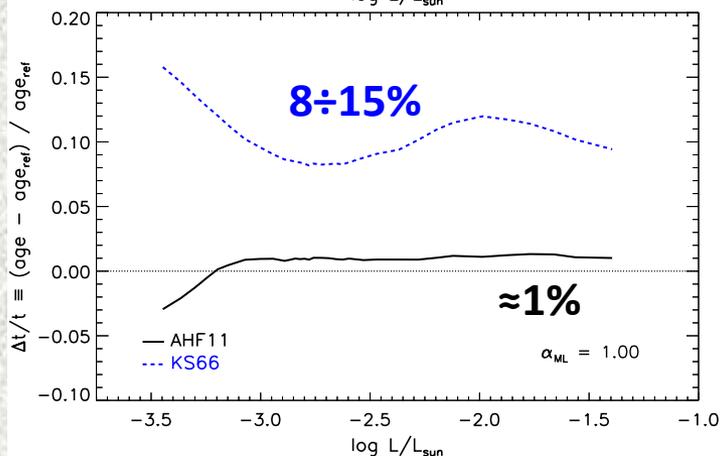
quantity	individual	
	error	global
[Fe/H]	$\pm 0.1\text{dex}$	yes
$\Delta Y/\Delta Z$	± 1	yes
$(Z/X)_{\odot}$	$\pm 15\%$	yes
X_{d}	$\pm 1 \times 10^{-5}$	yes

Individual Uncertainty source: INPUT PHYSICS (3/4)

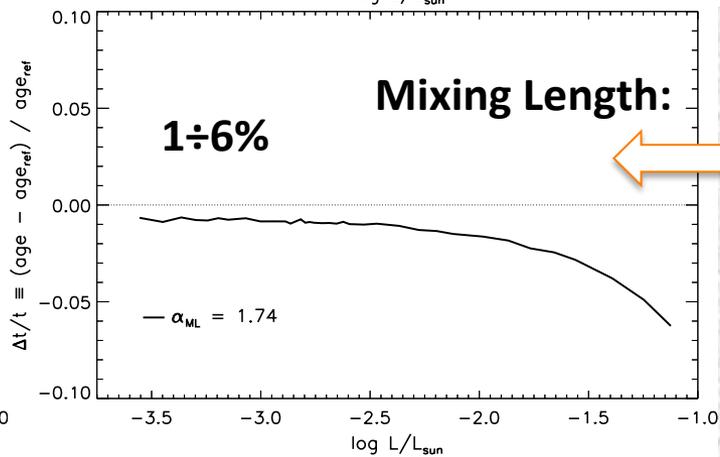
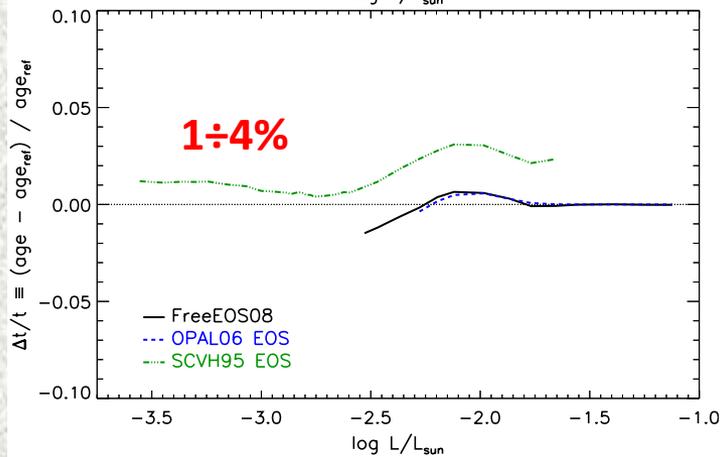
Reaction Rate:



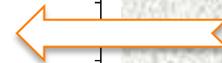
Surface Boundary Conditions:



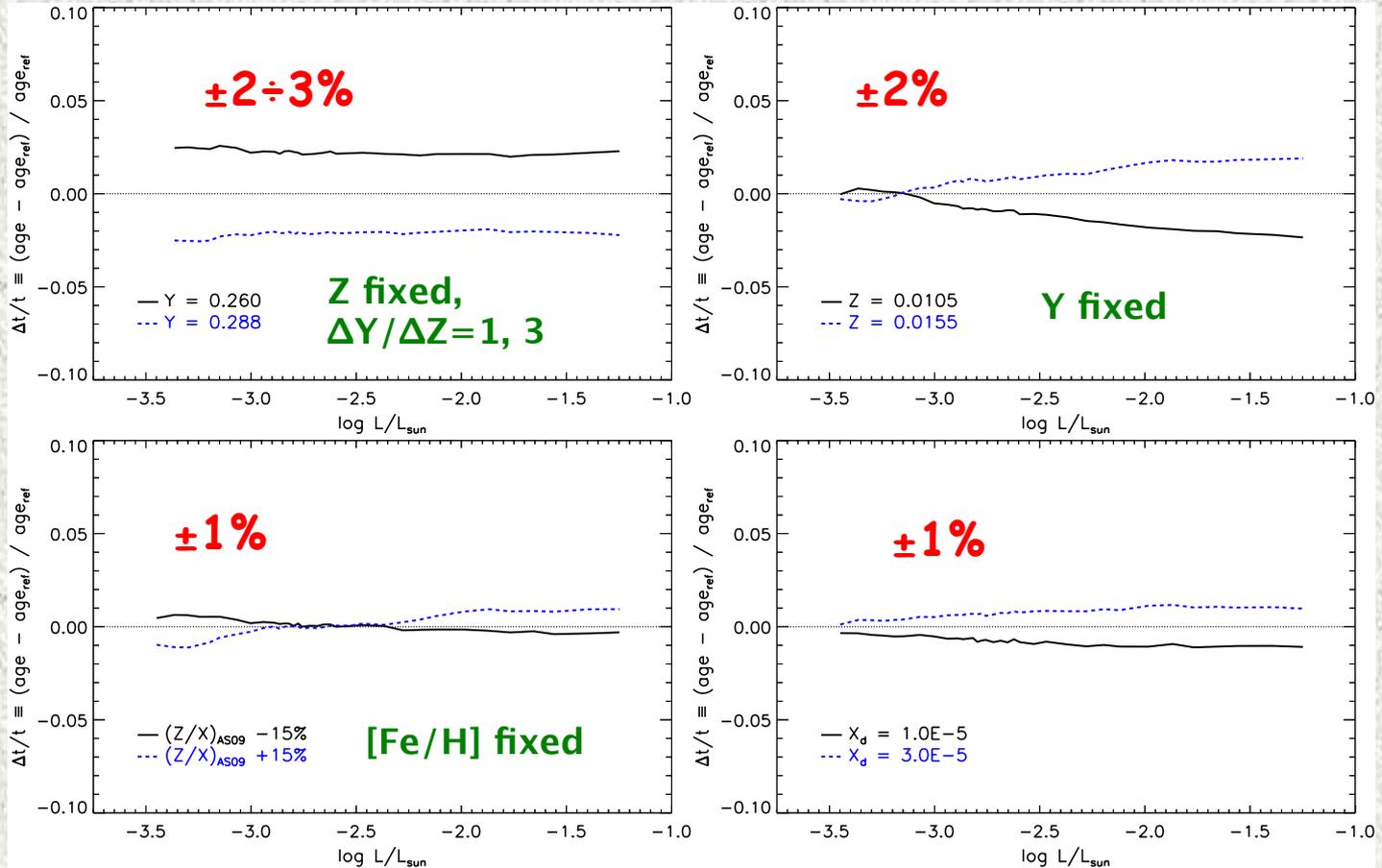
EOS:



Mixing Length:



Individual Uncertainty source: CHEMICAL COMPOSITION (4/4)



$$Y = Y_{\text{P}} + \frac{\Delta Y}{\Delta Z} Z$$

$$Z = \frac{(1 - Y_{\text{P}})(Z/X)_{\odot}}{10^{-[\text{Fe}/\text{H}]} + (1 + \Delta Y/\Delta Z)(Z/X)_{\odot}}$$

Cumulative Error Stripe (1/2)

Input physics and chemical composition quantities/parameters can vary at the same time

Input physics

$$p_j \Rightarrow \begin{cases} p_j + \Delta p_j \\ p_j \\ p_j - \Delta p_j \end{cases}$$

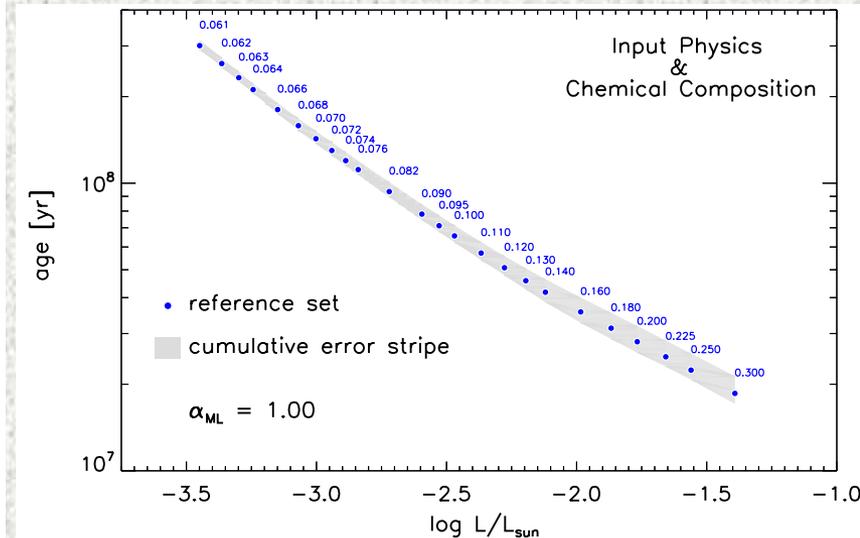
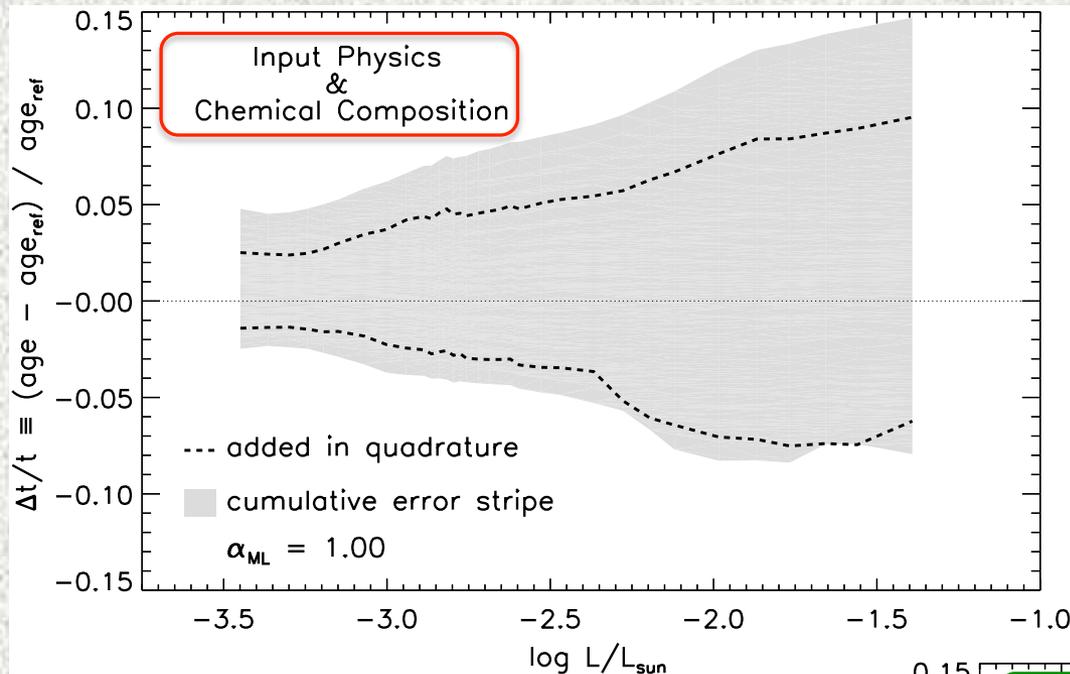
Chemical composition

$$x_k \Rightarrow \begin{cases} x_k + \Delta x_k \\ x_k \\ x_k - \Delta x_k \end{cases}$$

To obtain the error stripe we computed all the possible permutations of the perturbed $\{p_j\}$ and $\{x_k\}$

For a total of ≈ 300 sets of models!

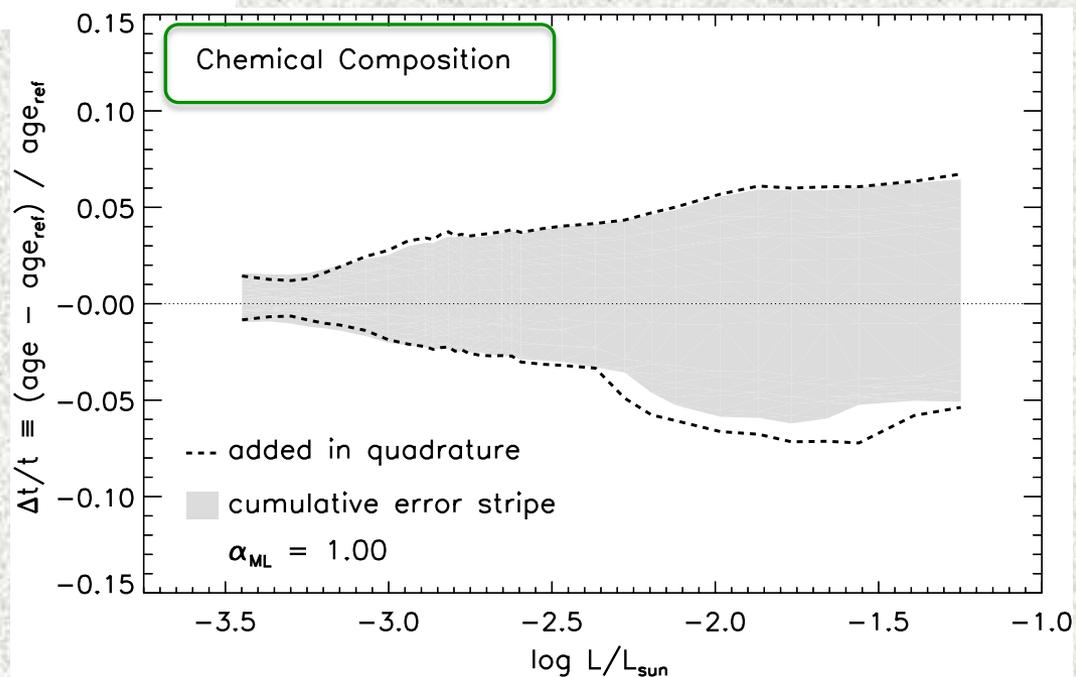
Cumulative Error Stripe (2/2)



Error stripe:
from 5% up to 15%

Burke et al (2004): \approx from 3% to 8%
(only input physics + errors added in quadrature)

40% of the total error budget



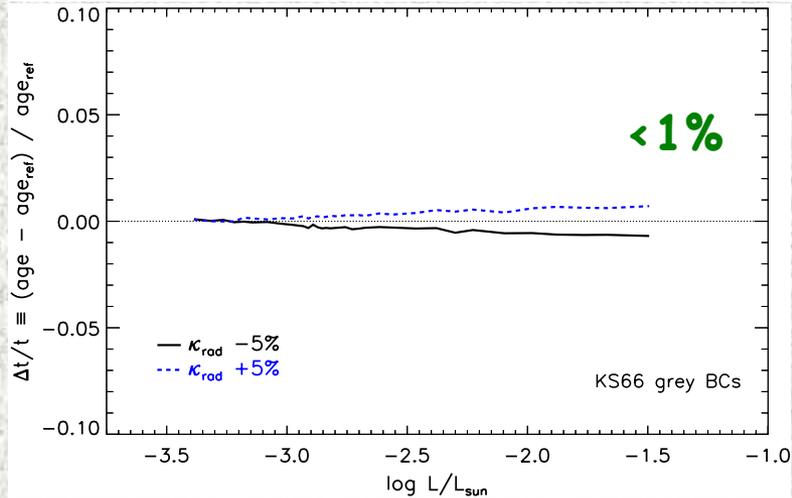
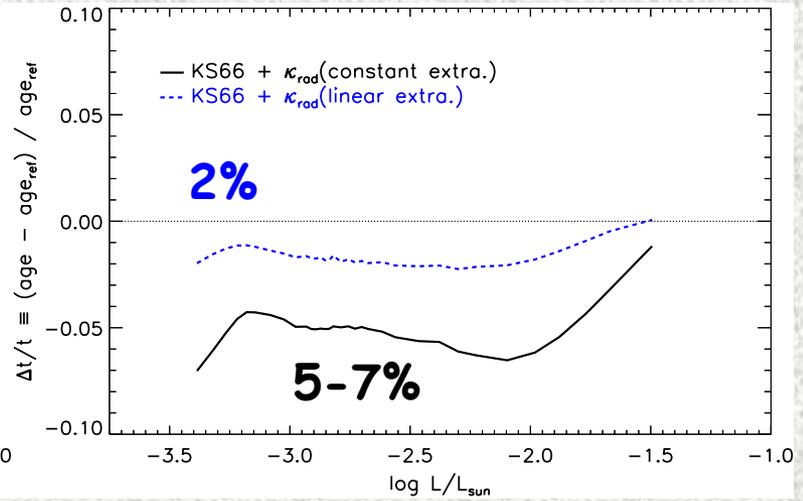
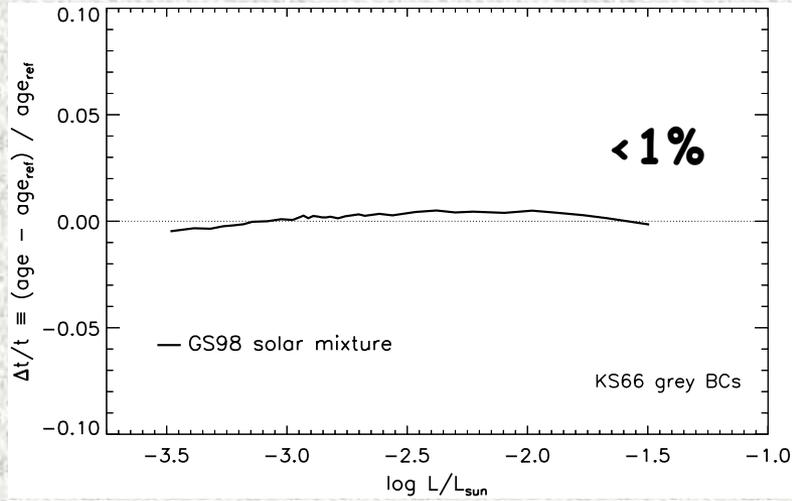
Conclusions

- Analysis of the main uncertainty sources affecting theoretical LDB age determination
- Individual uncertainty sources: input physics and chemical composition
- For the first time cumulative error stripe: simultaneous variation of the input physics and chemical composition quantities/parameter
- Error stripe: 40% of the total error budget due to the uncertainty on the initial chemical composition
- Age uncertainty: from 3-5% (100 Myr) to 8-15% (20 Myr)

"[...] after all it's been written in the stars [...]"
John Lennon, Woman, Double Fantasy, 1980

...The End!!!!

Backups 1



Opacity variation in the outer boundary conditions

Backups 2

Table 5. Pairs of (Y, Z) values used for the computations of the models with the perturbed chemical composition.

(Y, Z)	$[\text{Fe}/\text{H}]$	$\Delta Y/\Delta Z$	$(Z/X)_{\odot}$
$Y = 0.2740$ $Z = 0.0130$	+0.0	2	AS09
$Y = 0.2790$ $Z = 0.0100$	-0.1	3	AS09
$Y = 0.2650$ $Z = 0.0160$	+0.1	1	AS09
$Y = 0.2750$ $Z = 0.0090$	-0.1	3	AS09-15%
$Y = 0.2670$ $Z = 0.0190$	+0.1	1	AS09+15%

$$Z = \frac{(1 - Y_{\text{P}})(Z/X)_{\odot}}{10^{-[\text{Fe}/\text{H}]} + (1 + \Delta Y/\Delta Z)(Z/X)_{\odot}}$$

$$Y = Y_{\text{P}} + \frac{\Delta Y}{\Delta Z} Z$$