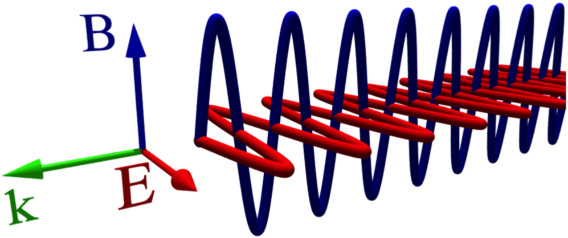
Fundamentals of Polarized Light

1. Polarized Light in the Universe
2. Descriptions of Polarized Light
3. Polarizers
4. Retarders

Polarized Light in the Universe

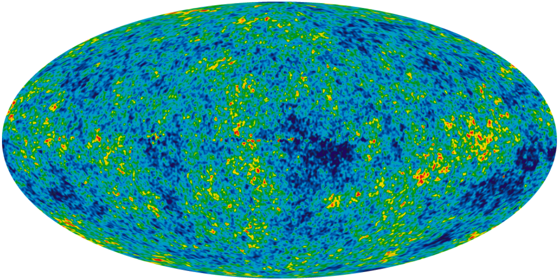


Polarization indicates *anisotropy* ⇒ not all directions are equal

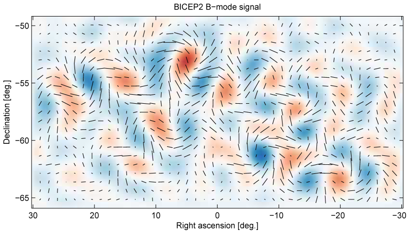
Typical anisotropies introduced by

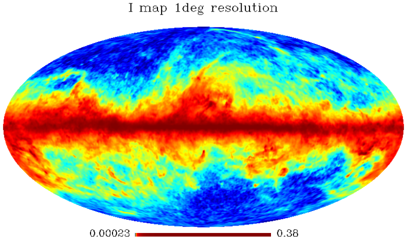
- geometry (not everything is spherically symmetric)
- temperature gradients
- density gradients
- magnetic fields
- electrical fields

13.7 billion year old temperature fluctuations from WMAP

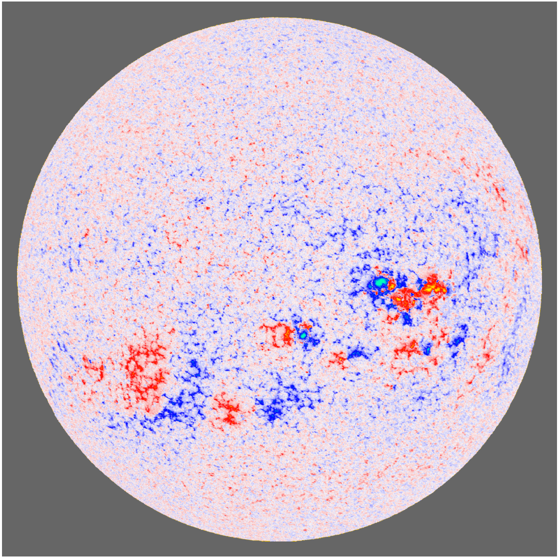


BICEP2 results and Planck Dust Polarization Map

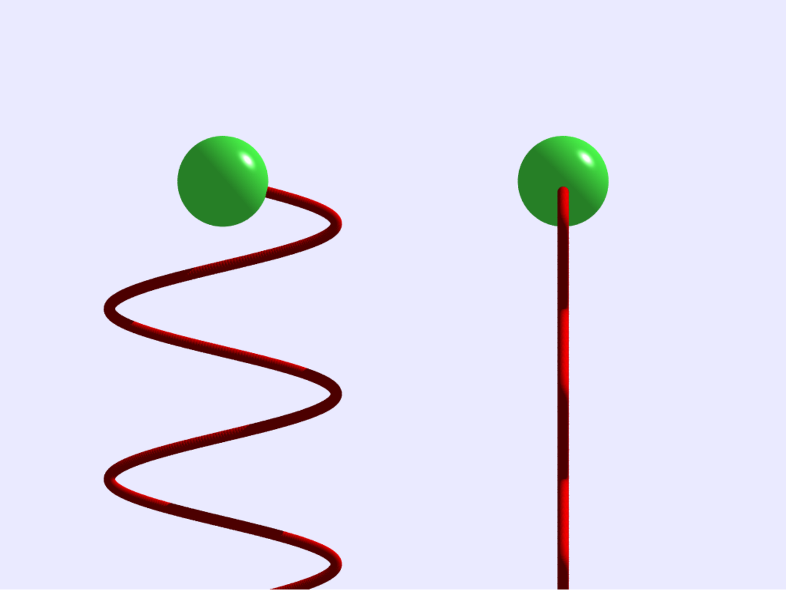




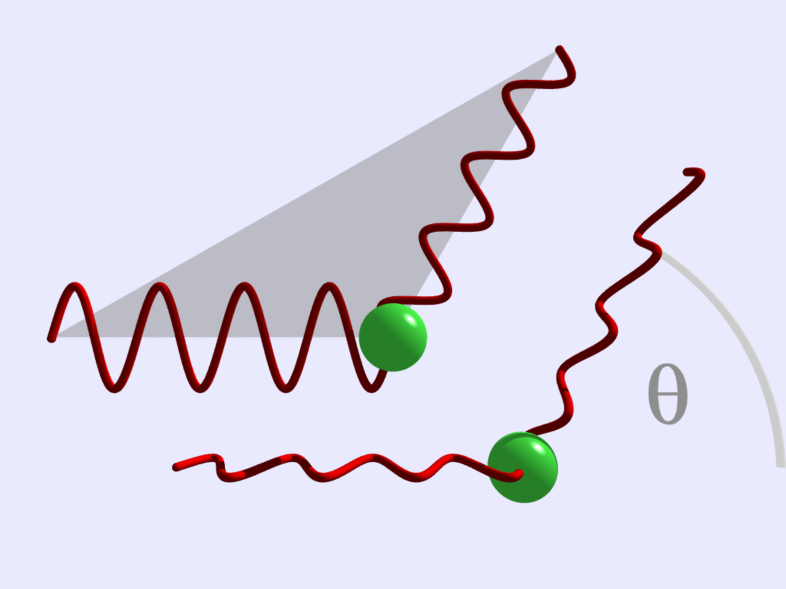
Solar Magnetic Field Maps from Longitudinal Zeeman Effect



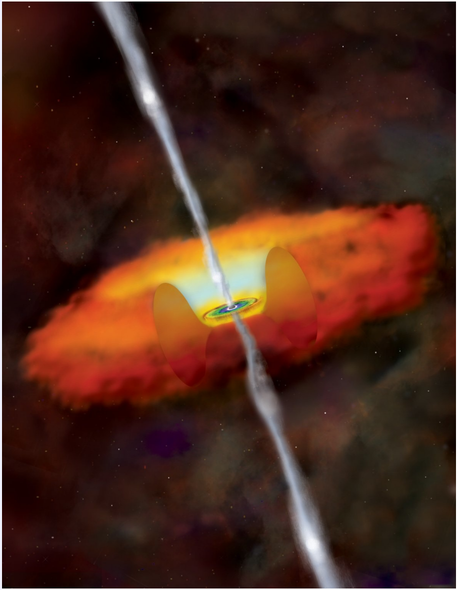
Scattering Polarization 1

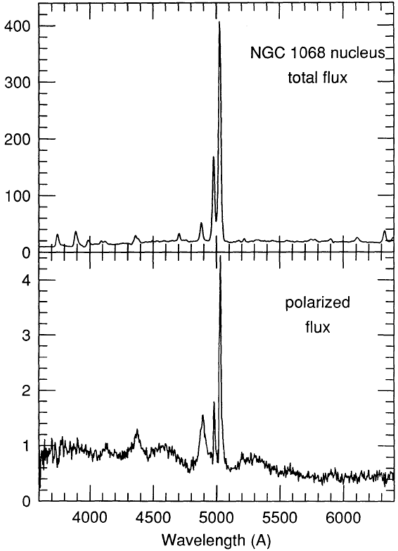


Scattering Polarization 2



Unified Model of Active Galactic Nuclei

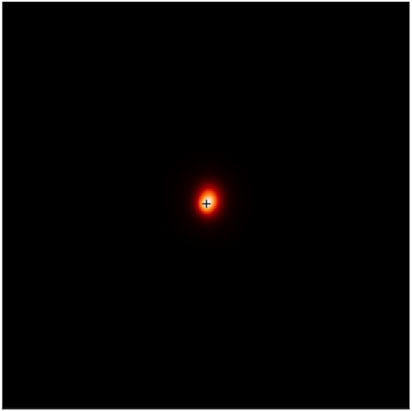




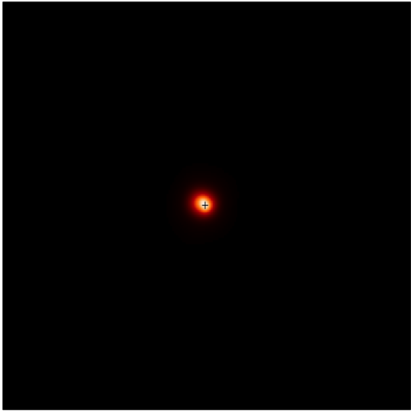
Miller et al., 1991, ApJ 378, 47-64

The Power of Polarimetry

T Tauri in intensity

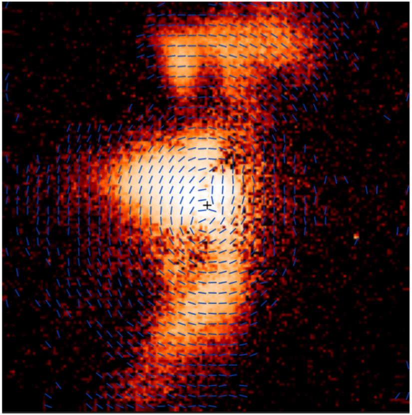


MWC147 in intensity

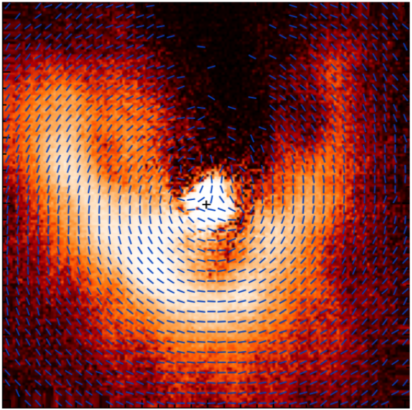


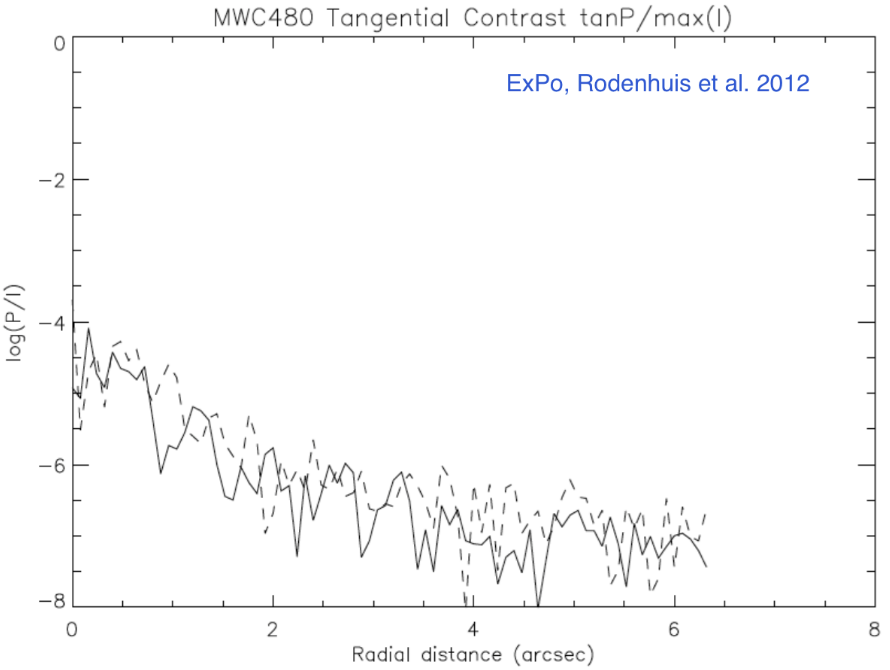
The Power of Polarimetry

T Tauri in Linear Polarization

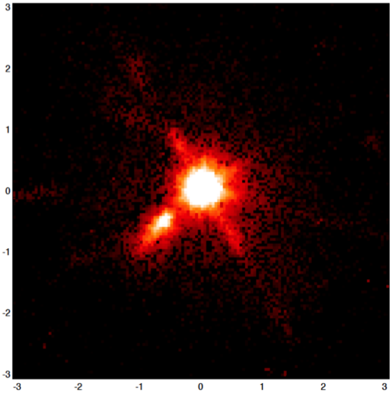


MWC147 in Linear Polarization

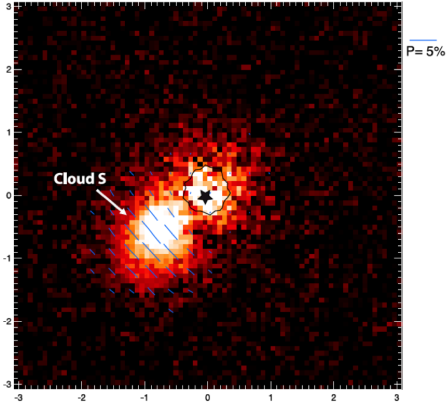



MWC480 Tangential Contrast tanP/max(I)
ExPo, Rodenhuis et al. 2012
log(P/I)
Radial distance (arcsec)
0
−2
−4
−6
−8
0
2
4
6
8

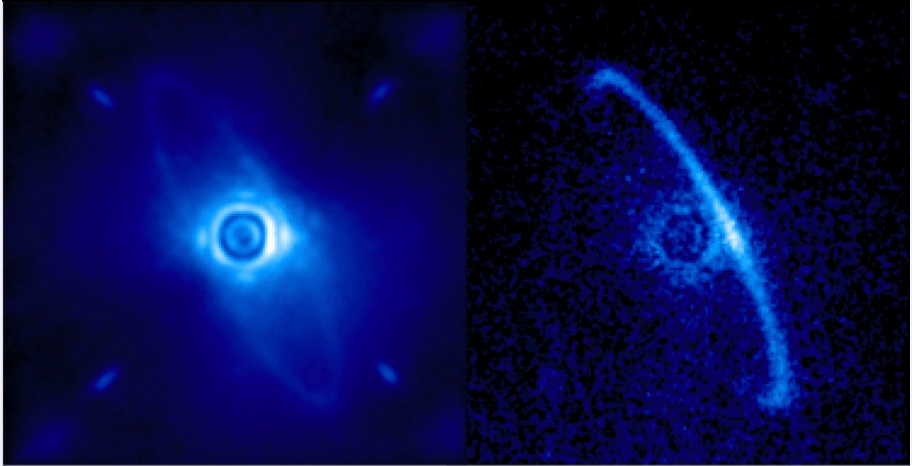
R Cr B with HST in intensity



R Cr B in Linear Polarization

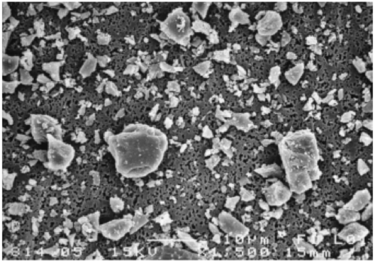


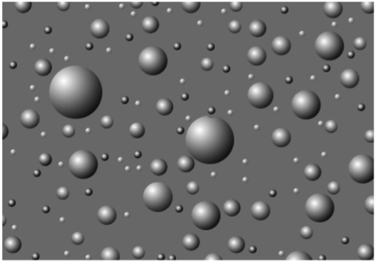
Characterization

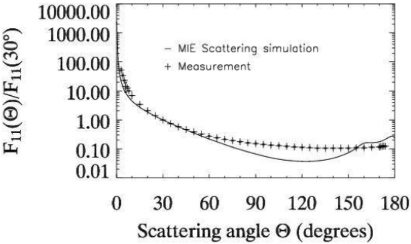


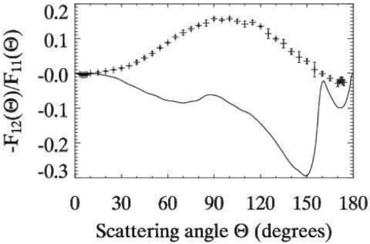
GPI First Light HR4796A (Image credit: Processing by Marshall Perrin, STScI) real dust

theoretical dust



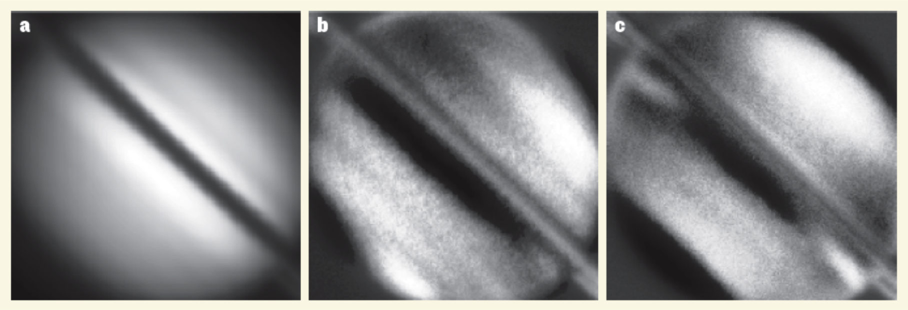






Laan et al (2007)

Planetary Scattered Light

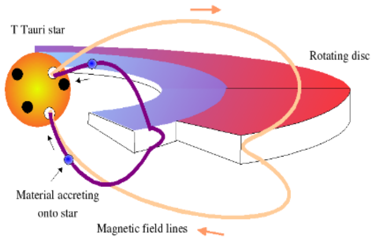


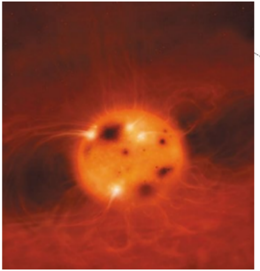
- solar-system planets show scattering polarization
- much depends on cloud height
- can be used to study exoplanets
- ExPo@WHT, SPHERE@VLT, EPICS@E-ELT

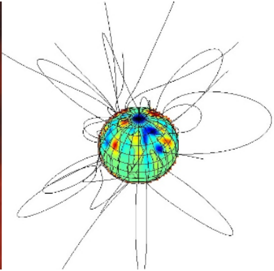
Other astrophysical applications

- interstellar magnetic field from polarized starlight
- supernova asymmetries
- stellar magnetic fields from Zeeman effect
- galactic magnetic field from Faraday rotation

Magnetic Fields of TTauri Stars (Courtesy S.V.Jeffers)







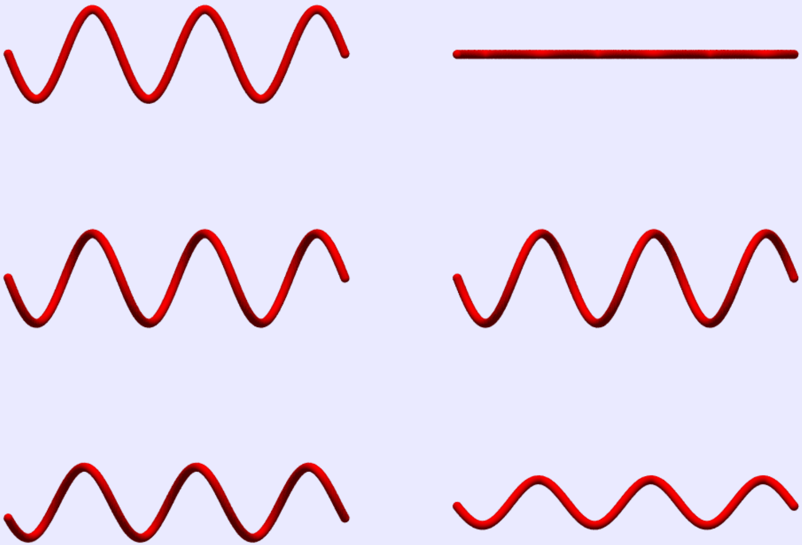
Summary of Polarization Origin

- Plane Vector Wave ansatz $\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$
- spatially, temporally constant vector $\vec{E}_0$ lays in plane perpendicular to propagation direction $\vec{k}$
- represent $\vec{E}_0$ in 2-D basis, unit vectors $\vec{e}_x$ and $\vec{e}_y$, both perpendicular to $\vec{k}$

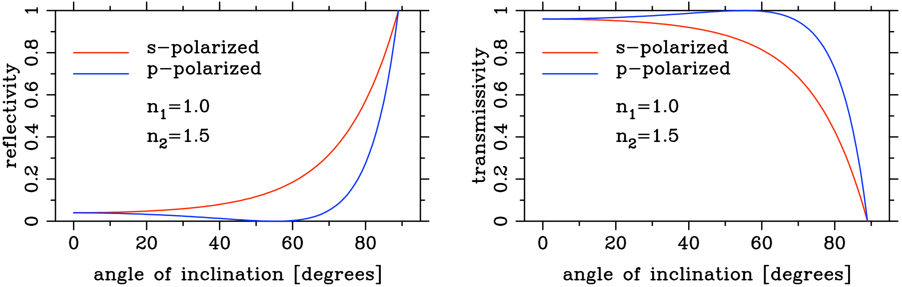
$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y.$$

  E_x, E_y: arbitrary complex scalars

- damped plane-wave solution with given ω, $\vec{k}$ has 4 degrees of freedom (two complex scalars)
- additional property is called *polarization*
- many ways to represent these four quantities
- if E_x and E_y have identical phases, $\vec{E}$ oscillates in fixed plane

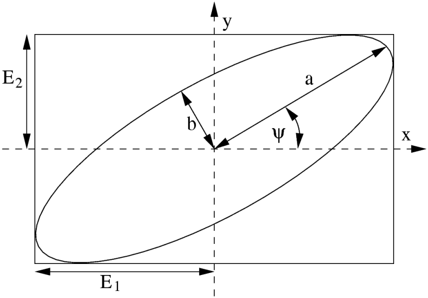


Polarization at Non-Normal Incidence



- transmitted light is p-polarized (electric field in plane of incidence)
- reflected light is s-polarized (electric field perpendicular to plane of incidence)
- at Brewster angle, reflected light is 100% polarized
- transmitted light is moderately polarized

Polarization Ellipse



Polarization

$$\vec{E}(t) = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

$$\vec{E}_0 = |E_x| e^{i\delta_x} \vec{e}_x + |E_y| e^{i\delta_y} \vec{e}_y$$

- wave vector in z-direction
- $\vec{e}_x$, $\vec{e}_y$: unit vectors in x, y
- $|E_x|$, $|E_y|$: (real) amplitudes
- $\delta_{x,y}$: (real) phases

Polarization Description

- 2 complex scalars not the most useful description
- at given $\vec{x}$, time evolution of $\vec{E}$ described by *polarization ellipse*
- ellipse described by axes a, b, orientation ψ

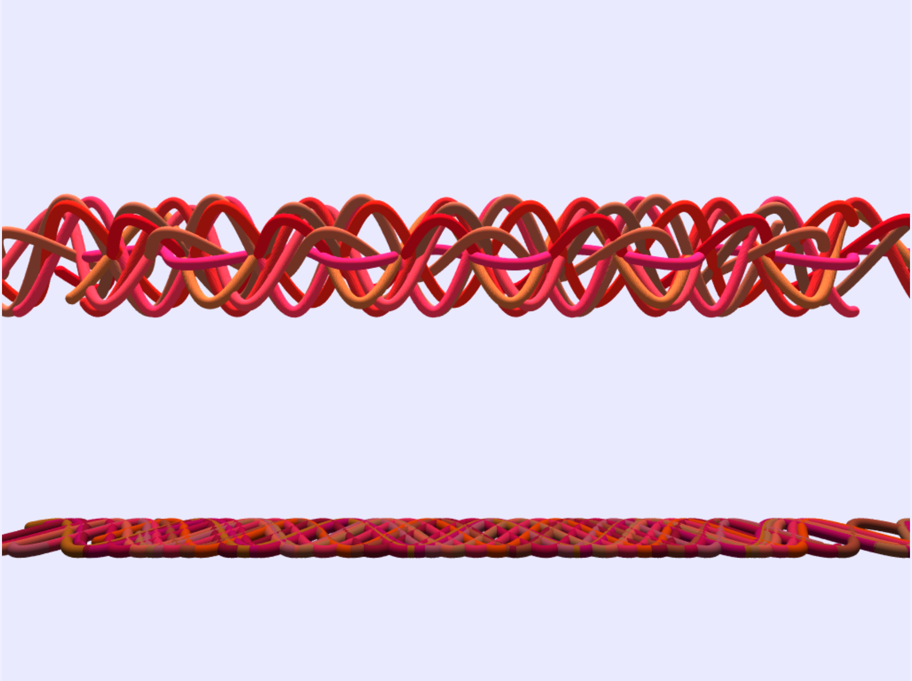
Stokes and Mueller Formalisms

Stokes Vector

- formalism to describe polarization of quasi-monochromatic light
- directly related to measurable intensities
- Stokes vector fulfills these requirements

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_x E_x^* + E_y E_y^* \\ E_x E_x^* - E_y E_y^* \\ E_x E_y^* + E_y E_x^* \\ i\left(E_x E_y^* - E_y E_x^*\right) \end{pmatrix} = \begin{pmatrix} |E_x|^2 + |E_y|^2 \\ |E_x|^2 - |E_y|^2 \\ 2|E_x||E_y|\cos\delta \\ 2|E_x||E_y|\sin\delta \end{pmatrix}$$

  Jones vector elements $E_{x,y}$, real amplitudes $|E_{x,y}|$, phase difference $\delta = \delta_y - \delta_x$
- $I^2 \geq Q^2 + U^2 + V^2$
- can describe unpolarized ($Q = U = V = 0$) light



Stokes Vector Interpretation

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left} - \text{right} \end{pmatrix}$$

- *degree of polarization*

  $$P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

  1 for fully polarized light, 0 for unpolarized light
- summing of Stokes vectors = *incoherent* adding of quasi-monochromatic light waves

Linear Polarization

- horizontal: $\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$
- vertical: $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$
- 45°: $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

Circular Polarization

- left: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
- right: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$

Mueller Matrices

- 4×4 real Mueller matrices describe (linear) transformation between Stokes vectors when passing through or reflecting from media

$$\vec{I}' = \mathrm{M}\vec{I}\,,$$

$$\mathrm{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}$$

- N optical elements, combined Mueller matrix is

$$\mathrm{M}' = \mathrm{M}_N \mathrm{M}_{N-1} \cdots \mathrm{M}_2 \mathrm{M}_1$$

Polarizers

- polarizer: optical element that produces polarized light from unpolarized input light
- linear, circular, or in general elliptical polarizer, depending on type of transmitted polarization
- linear polarizers by far the most common
- large variety of polarizers

Vertical Linear Polarizer

$$M_{\text{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

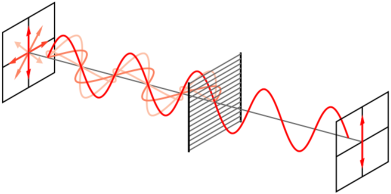
Horizontal Linear Polarizer

$$M_{\text{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Mueller Matrix for Ideal Linear Polarizer at Angle θ

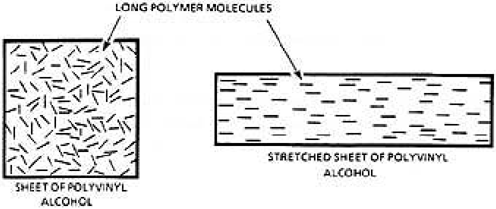
$$M_{\text{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Wire Grid Polarizers



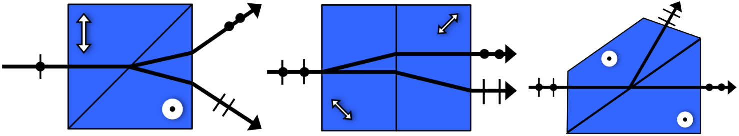
- parallel conducting wires, spacing $d \lesssim \lambda$ act as polarizer
- polarization perpendicular to wires is transmitted
- rule of thumb:
  - $d < \lambda/2 \Rightarrow$ strong polarization
  - $d \gg \lambda \Rightarrow$ high transmission of both polarization states (weak polarization)
- mostly used in infrared

Polaroid-type Polarizers



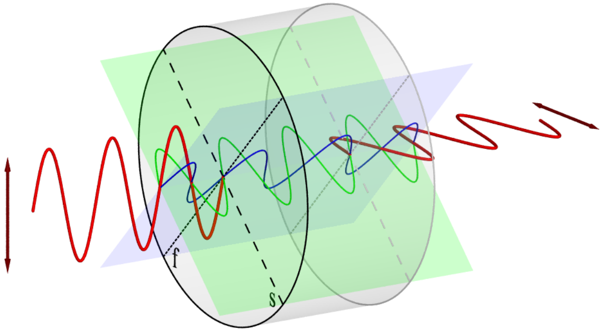
- developed by Edwin Land in 1938 $\Rightarrow$ Polaroid
- sheet polarizers: stretched polyvynil alcohol (PVA) sheet, laminated to sheet of cellulose acetate butyrate, treated with iodine
- PVA-iodine complex analogous to short, conducting wire
- cheap, can be manufactured in large sizes

Crystal-Based Polarizers



- crystals are basis of highest-quality polarizers
- precise arrangement of atom/molecules and anisotropy of index of refraction separate incoming beam into two beams with precisely orthogonal linear polarization states
- work well over large wavelength range
- many different configurations
- calcite most often used in crystal-based polarizers because of large birefringence, low absorption in visible
- many other suitable materials

Retarders or Wave Plates



- retards (delays) phase of one electric field component with respect to orthogonal component
- anisotropic material (crystal) has index of refraction that depends on polarization direction

Retarder Properties

- does not change intensity or degree of polarization
- characterized by two Stokes vectors that are not changed ⇒ *eigenvectors* of retarder
- depending on polarization described by eigenvectors, retarder is
  - *linear retarder*
  - *circular retarder*
  - *elliptical retarder*
- linear retarders by far the most common type of retarder

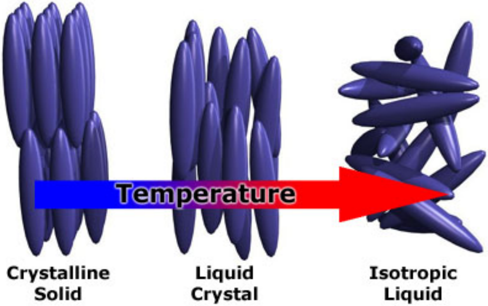
Mueller Matrix for Linear Retarder

$$\mathrm{M}_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & -\sin\delta \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix}$$

Variable Retarders

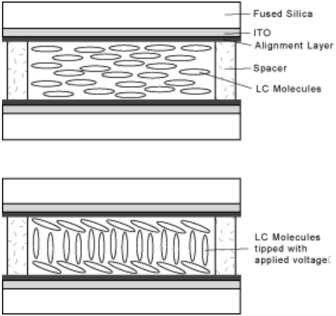
- sensitive polarimeters requires retarders whose properties (retardance, fast axis orientation) can be varied quickly (*modulated*)
- retardance changes (change of birefringence):
  - liquid crystals
  - Faraday, Kerr, Pockels cells
  - piezo-elastic modulators (PEM)
- fast axis orientation changes (change of *c*-axis direction):
  - rotating fixed retarder
  - ferro-electric liquid crystals (FLC)

Liquid Crystals



- liquid crystals: fluids with elongated molecules
- at high temperatures: liquid crystal is isotropic
- at lower temperature: molecules become ordered in orientation and sometimes also space in one or more dimensions
- liquid crystals can line up parallel or perpendicular to external electrical field

Liquid Crystal Retarders



- dielectric constant anisotropy often large $\Rightarrow$ very responsive to changes in applied electric field
- birefringence δn can be very large (larger than typical crystal birefringence)
- liquid crystal layer only a few μm thick
- birefringence shows strong temperature dependence