## Mapping the PSF across Adaptive Optics images

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#### **Abstract**

- Adaptive Optics (AO) has become a key technology for all the main existing telescopes (VLT, Keck, Gemini, Subaru, LBT..) and is considered a kind of enabling technology for future giant telescopes (E-ELT, TMT, GMT).
- AO increases the energy concentration of the Point Spread Function (PSF) almost reaching the resolution imposed by the diffraction limit, but the PSF itself is characterized by complex shape, no longer easily representable with an analytical model, and by sometimes significant spatial variation across the image, depending on the AO flavour and configuration.
- The aim of this lesson is to describe the AO PSF characteristics and variation in order to provide (together with some AO tips) basic elements that could be useful for AO images data reduction.

#### What's PSF

 'The Point Spread Function (PSF) describes the response of an imaging system to a point source'

Circular aperture of diameter D at a wavelenght λ (no aberrations) → Airy

diffraction disk

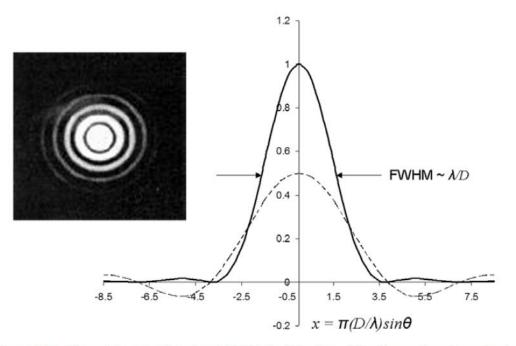


Figure 1.15. The point spread function (solid line) of the Airy diffraction pattern for a circular aperture of diameter D is illustrated both as an image and in cross-section.

$$I_{\theta} = I_0 \{ J_1(x)/x \}^2$$

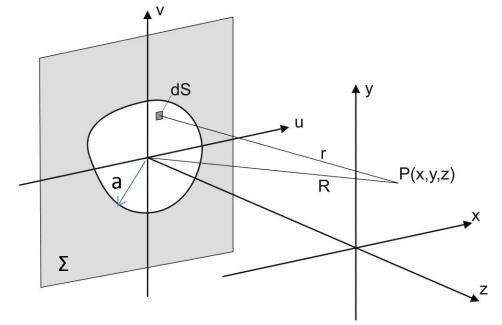
Where  $J_1(x)$  represents the **Bessel function** of order 1

$$x = \pi(D/\lambda)\sin\theta$$

 $\vartheta$  is the angular radius from the aperture center

 $\rightarrow$  First goes to 0 when  $\vartheta \sim 1.22 \ \lambda/D$ 

# Imaging of a point source through a general aperture



Consider a **plane wave** propagating in the z direction and illuminating an aperture.

The element ds = dudv becomes the sourse of a secondary spherical wave.

The complex field in *P* generated by *d*S is:

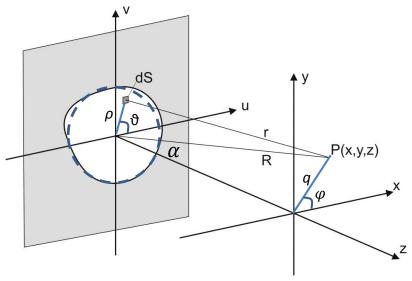
$$dE = \frac{\varepsilon_0}{r} e^{i(\omega t - kr)} dS$$

$$E(x,y) \cong \frac{\varepsilon_0}{R} e^{i(\omega t - kR)} \iint_{aperture} e^{ik(ux + vy)/R} du dv$$

The PSF is the field intesity distribution in the image plane

# Imaging of a point source through a circular aperture

Spherical coordinates



$$u = \rho \cos \theta$$
  $v = \rho \sin \theta$ 

$$x = q \cos \varphi$$
  $y = q \sin \varphi$ 

$$ds = \rho d\rho d\varphi$$

$$E(x,y) \propto \int_0^a \rho \int_0^{2\pi} e^{i\left(\frac{k\rho q}{R}\right)\cos(\vartheta-\varphi)} d\rho d\vartheta =$$
$$= 2\pi a^2 \left(\frac{R}{k\rho q}\right) J_1\left(\frac{R}{k\rho q}\right)$$

$$I(\alpha) = I(0) \left[ \frac{2J_1(ka\sin\alpha)}{ka\sin\alpha} \right]^2$$
 where  $\alpha = q/R$ 

...or we can define a function

$$A(u,v) = \begin{cases} 1 & \text{if } (u,v) \in \text{aperture} \\ 0 & \text{otherwise} \end{cases}$$

$$E(x,y) \propto \int_{-\infty-\infty}^{+\infty+\infty} A(u,v) e^{i\frac{k}{R}(ux+vy)} dudv$$

$$f_{x} = \frac{k}{R}x \qquad f_{y} = \frac{k}{R}y$$

$$E(f_{x}, f_{y}) = \iint_{+\infty} A(u, v)e^{i(f_{x}u + f_{y}v)}dudv$$

$$= FT[A(u, v)] \qquad \text{2D Fourier transform of the aperture function}$$

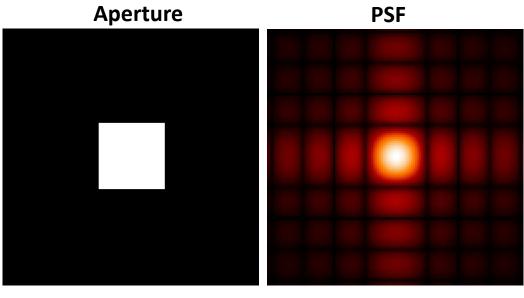
The field distribution in the image plane is the spatial frequency spectrum of the aperture function

$$I(f_x, f_y) = |E(f_x, f_y)|^2 = |FT[A(u, v)]|^2$$

### **PSF** computation using FFT

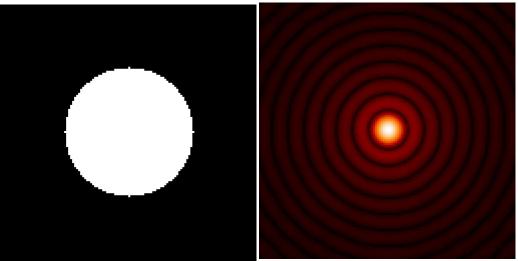
```
FUNCTION square_PSF, n, d

x = rebin(findgen(n) - n/2, n, n)
y = rebin(transpose(findgen(n)) - n/2, n, n)
a = abs(x) le d/2 and abs(y) le d/2
a = shift(a, -n/2, -n/2)
psf = abs(fft(a))^2
psf = shift(psf, n/2, n/2)
psf = psf / total(psf)
return, psf
end
```



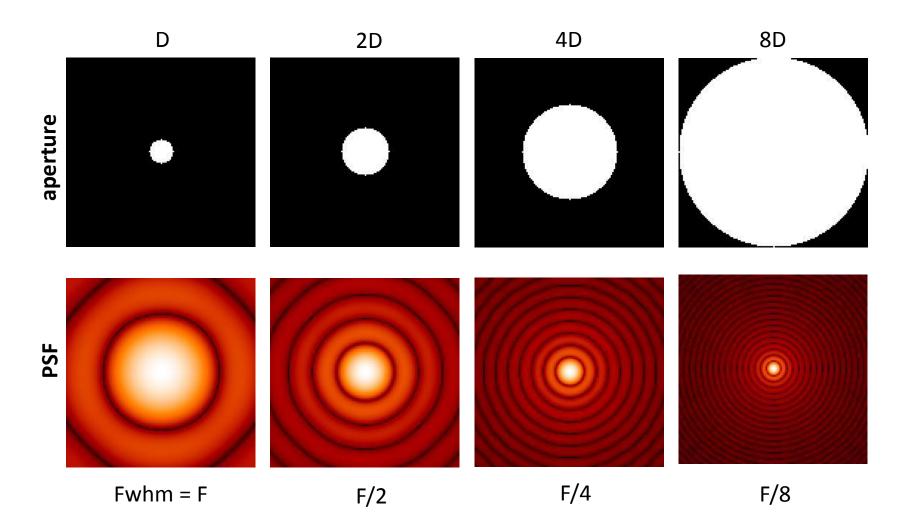
```
FUNCTION circ_PSF, n, d

x = rebin(findgen(n) - n/2, n, n)
y = rebin(transpose(findgen(n)) - n/2, n, n)
a = sqrt(x^2 + y^2) le d/2
a = shift(a, -n/2, -n/2)
psf = abs(fft(a))^2
psf = shift(psf, n/2, n/2)
psf = psf / total(psf)
return, psf
end
```

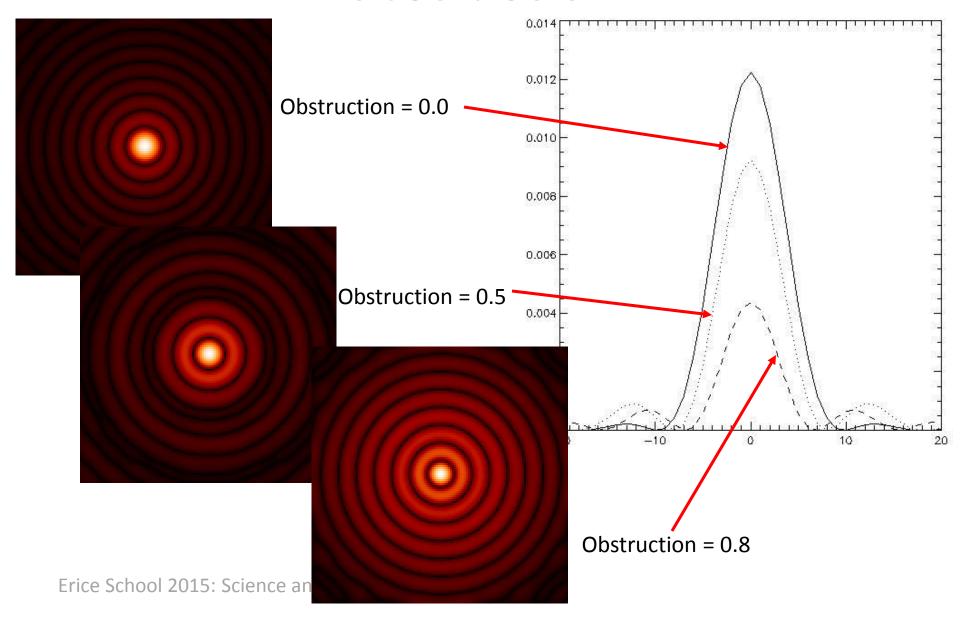


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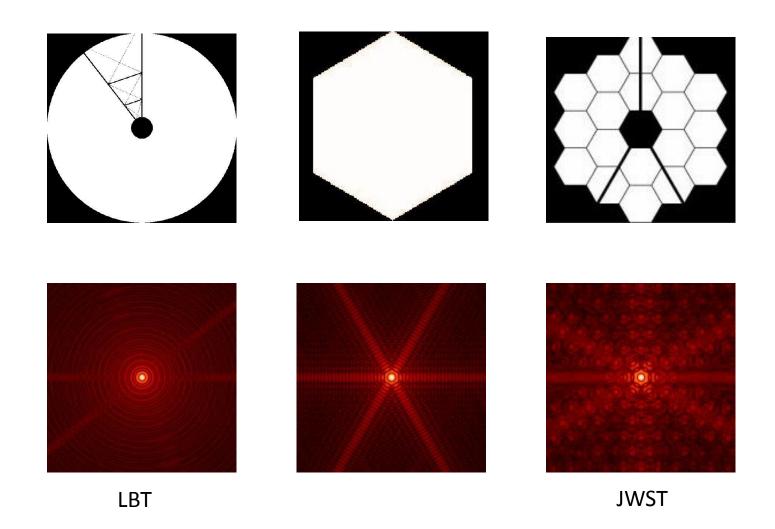
### Circular apertures of different sizes



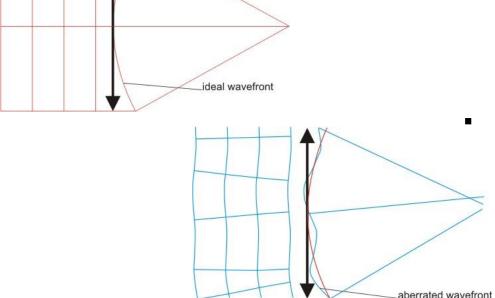
# Circular aperture with central obstruction



### **Different apertures**



# Diffraction in presence of aberrations



- In the ideal case (red), the incident wavefront is flat; the converging optical element focuses on the focal plane the Fraunhofer diffraction pattern that would be observed at a very long distance
  - In the aberrated case (blue), the wavefront is distorted and the diffraction pattern on the focal plane will be different.

NOTE that the aberrations can be introduced by the optical system itself

In presence of aberrations, the complex field of the incoming wave on the aperture can be expressed as:

$$E(u,v) = \varepsilon_0 e^{i\phi(u,v)}$$

The function  $\phi(u,v)$  describes the wavefront distortions

The intensity distribution, in Fraunhofer approximation, becomes:

$$i(f_x, f_y) = |E(f_x, f_y)|^2 = |FT[A(u, v)e^{i\phi(u, v)}]|^2$$

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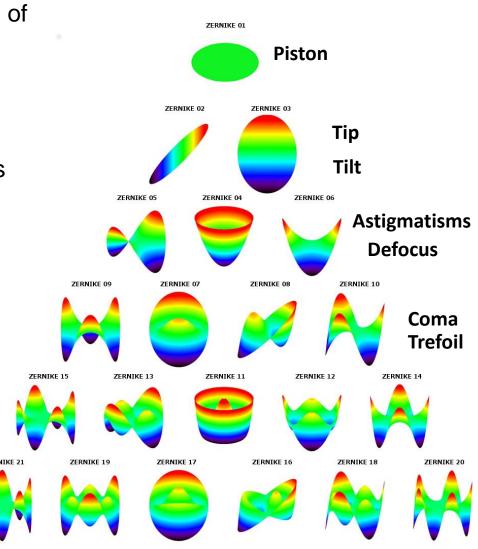
# Modal Representation of aberrations

- Wavefront distortions at the entrance of the telescope can be represented as the linear combination of a proper defined basis of functions.
- Zernike polinomial are orthogonal.
- They are defined in polar coordinates on a unit circle as functions of azimuthal frequency m and radial degree n, where m ≤ n, and n – m is even.

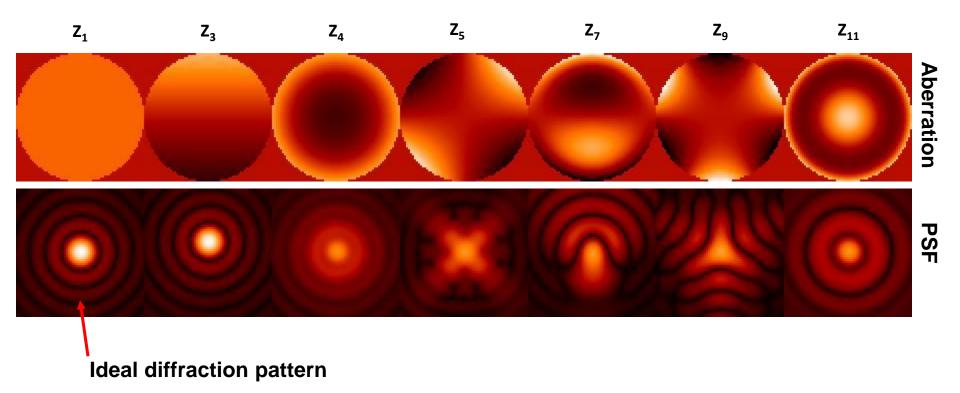
$$Z_{even,j} = \sqrt{n + R_n^m \rho} \sqrt{2} \cos m\theta$$

$$Z_{odd,j} = \sqrt{n + 1} R_n^m \rho \sqrt{2} \sin m\theta$$

$$Z_j = \sqrt{n + 1} R_n^0 (\rho)$$



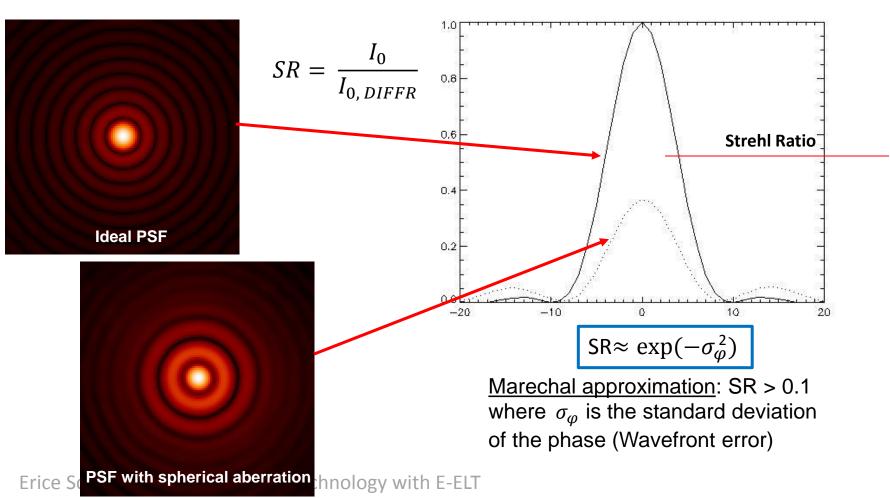
# Circular Aperture PSF in presence of aberrations (static)



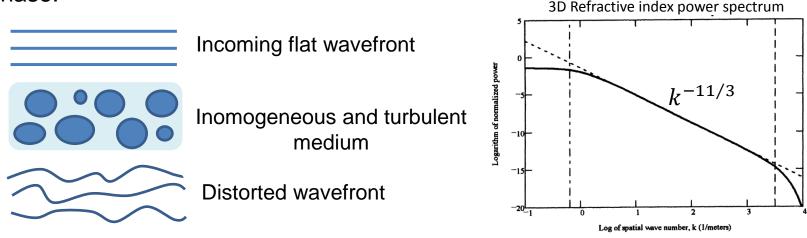
The first term (piston) does not have any effect on the image. The second term (tilt) produces a shift in the image. The higher orders terms introduce deformation in the PSF.

#### **The Strehl Ratio**

The Strehl Ratio (SR) is the ratio of the peak aberrated image intensity from a point source compared to the maximum attainable intensity using an ideal optical system limited only by diffraction over the system's aperture.



 Before entering the atmosphere, light from stars forms plane waves (flat wavefronts). Refraction index variations in space and time due to turbulent air cells along the wavefront path, produce local variations in the wavefront phase.



- From the Power Spectrum of the Refractive Index it is possible to build a 'phase map', that describe, point by point, the phase delay coused by the atmospheric refracion index variation.
- The Refractive index power spectrum follows a -11/3 power law (Kolmogorov)

### How to build a phase map (screen)

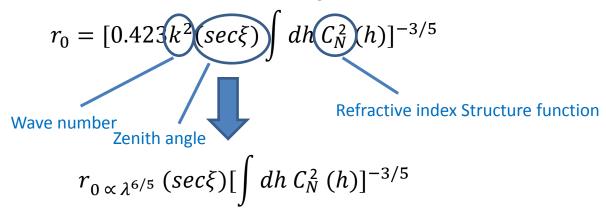
 A turbolent layer can be simulated with the computer multiplying the sqrt of power spectrum by a random phase.

```
pro makelayer, layer
                                                          Frequencies (u)
                                                                                  Power Spectrum (ps)
 nsize = 256
 npupil = 64
 d = 8.0
 pixlayer = d / npupil
 r0 = 0.5
 Dr0 = d/r0
 layer = dblarr(nsize,nsize)
 f sampling = double(pixlayer*double(nsize))
 u = frequencies(nsize,pixlayer)
                                                        Random phase (phase)
                                                                                  Layer
 ps = double(abs(u)^(-11.d/3.d))
 phase = (2.d0*randomu(Seed,nsize,nsize)-1.d0)*!Dpi
 CPS = Sqrt(ps)*DComplex(Cos(Phase), Sin(Phase))
 layer = fft(CPS, 1, /Double)
 layer = shift(layer,nsize/2,nsize/2)
 layer = layer - (moment(layer, /DOUBLE))[0]
 layer = layer * sqrt(1.0299d0*Dr0^(5.d0/3.d0))
 return
 end
```

The sqrt of the power spectrum is multiplied by sin and cos of the random phase and The fourier transform (from frequency to distance space) represents tha phase map Erice School 2015: Science and Technology with E-ELT

#### **The Fried Parameter**

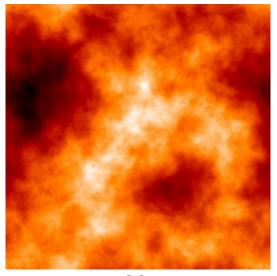
• The Fried Parameter  $r_0$  gives a measure of the strenght of the turbulence. It is defined as the following:

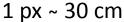


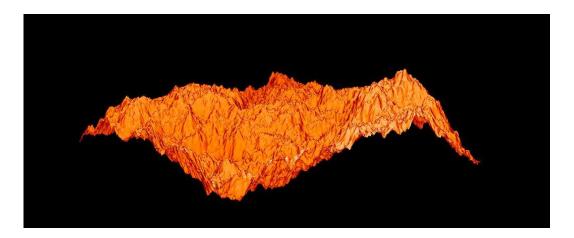
- $r_0$  gets small when turbulence is strong ( $C_N^2$  large)
- $r_0$  gets bigger at longer wavelenghts (10-20 cm @ 550 nm, 50 cm @ 2200 nm)
- $r_0$  gets smaller as telescope looks toward the horizon
- $r_0$ defines an aperture size over which the mean-square wavefront error is 1 rad<sup>2</sup>

We compute the istantaneus PSF remembering that:

$$i(f_x, f_y) = |E(f_x, f_y)|^2 = |FT[A(u, v)e^{i\phi(u, v)}]|^2$$

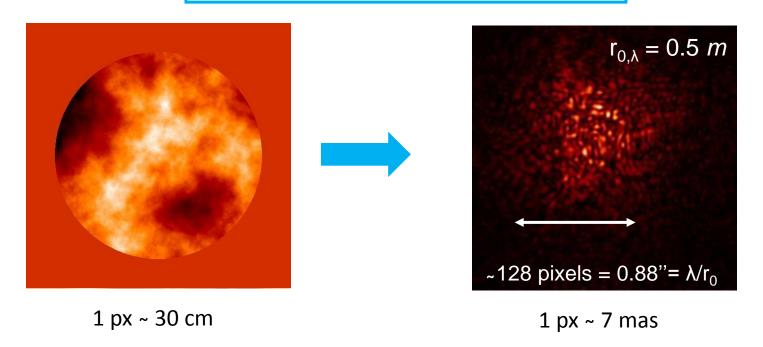






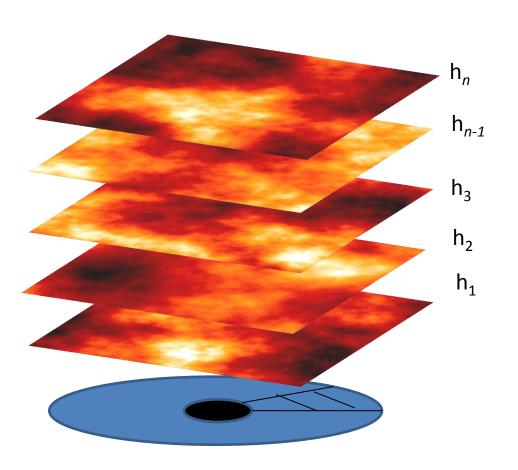
We compute the istantaneus PSF remembering that:

$$i(f_x, f_y) = |E(f_x, f_y)|^2 = |FT[A(u, v)e^{i\phi(u, v)}]|^2$$



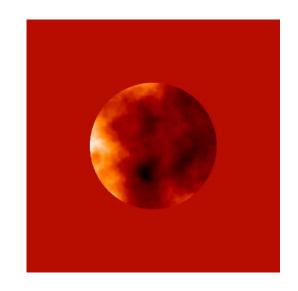
The short exposure image consists of a large number of speckles having size  $\lambda/D$ 

 To produce long exposure PSFs, we need a dynamic atmospheric model.



Each of the layer is characterized by Its altitude (h), it's altitude and its velocity.

The total phase delay at the telescope aperture is given by the  $\sum_{1}^{n} layers$  in the star direction



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- The size of the turbulence degradated image is due to:
  - Diffraction limit of the aperture  $(\lambda/D)$
  - Short Esposure image spread  $(\lambda/r_0)$
  - Image motion due to overall tilt (determined by both D and r<sub>0</sub>)



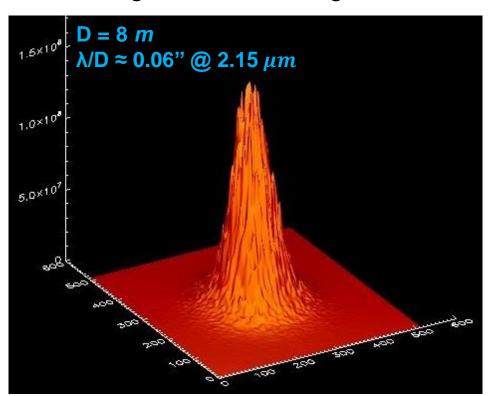
Short exposure PSF

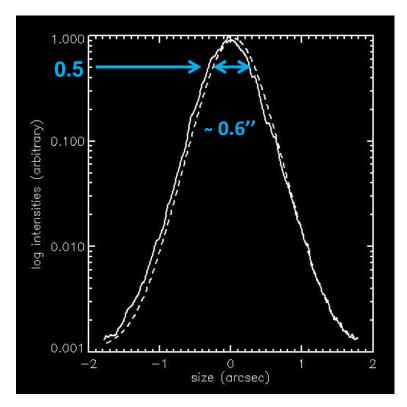
Long exposure PSF

Diffraction limited PSF

### Seeing limited PSF (D/r<sub>0</sub>~ 10)

 The long exposure PSF (K band) looks like a 2D moffat function having FWHM = seeing



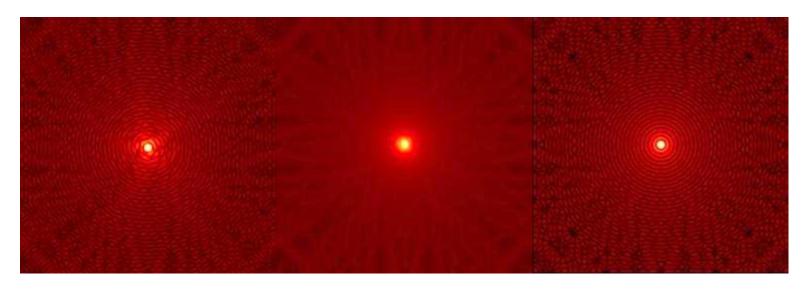


Moffat distribution:  $I(r) = \frac{1}{1 + (\frac{r}{r_m})^{2\beta}}$  [King 1971]

where  $r_m$  is the moffat radius and  $\beta$  describes the asymptotic power low of the wings

## Seeing limited PSF (D = $r_0$ )

- For very small apertures, atmospheric turbulence has little effect on the image size, which is determined by D.
- Wavefront distortion is mainly overall tilt.



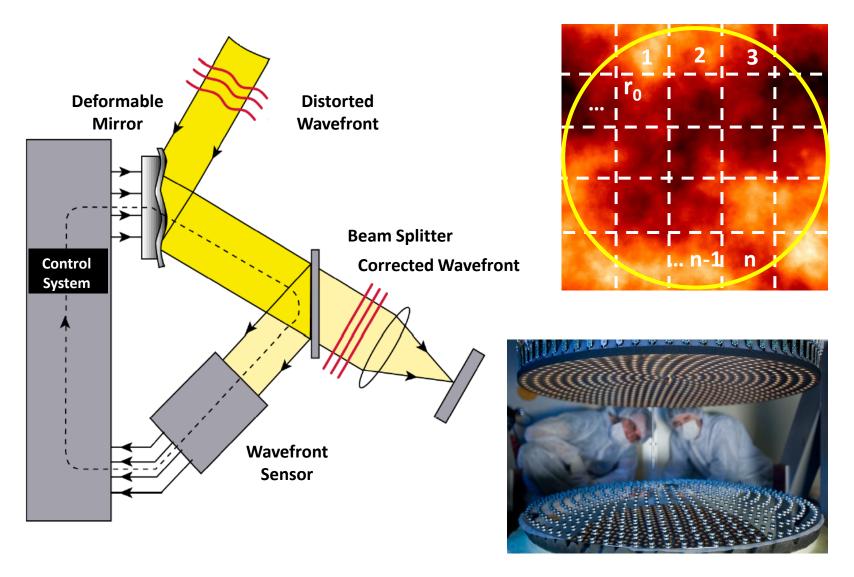
Short exposure PSF

Long exposure PSF

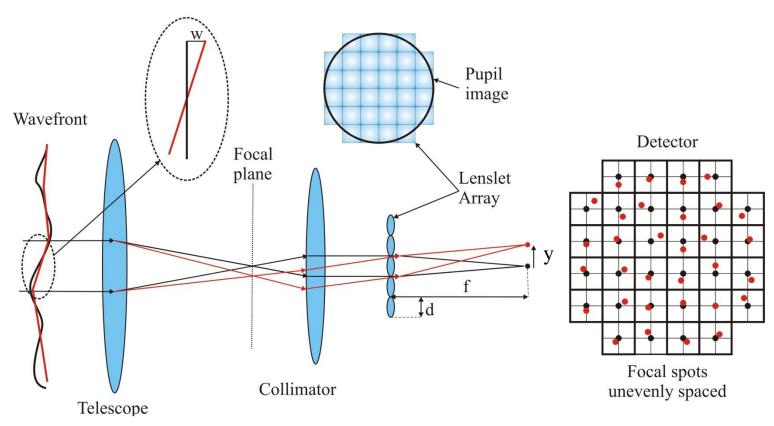
**Diffraction limited PSF** 

r<sub>0</sub> defines the diameter of a diffraction limited telescope

### **Adaptive optics concept**



### **Wavefront Sensing**

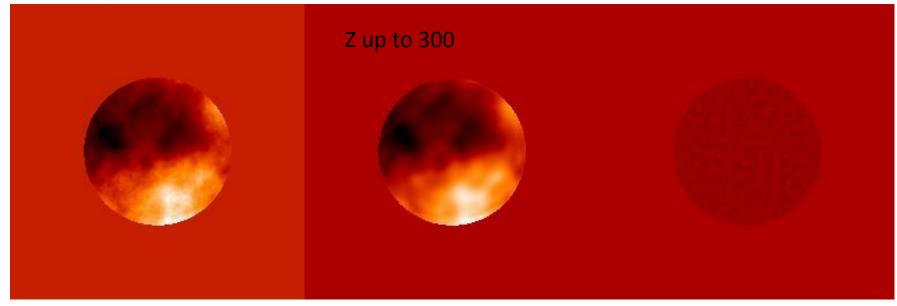


#### ☐ Shack-Hartmann WFS (SH-WFS)

- array of lenses on a pupil image (num of sub-apertures  $\propto (D/r_0)^2$ ,  $r_0$  of the science observations)
- each lens re-images the source as seen by a small telescope having size  $r_0$
- an incoming plane WF is focuses on the center of the sub-apertures
- an aberrated WF causes a spots displacement respect to the reference position prop to the local mean WF
- WF measurement performance depends on the centroiding accuracy (SNR of the spot)

### **Partial compensation**

- Zernike modes to represent the aberrated wavefront  $\phi = \sum_{i=1}^{n} a_{i}Z_{i}$
- Subtract the measured wavefront
- Compute the PSF
- Integration in time



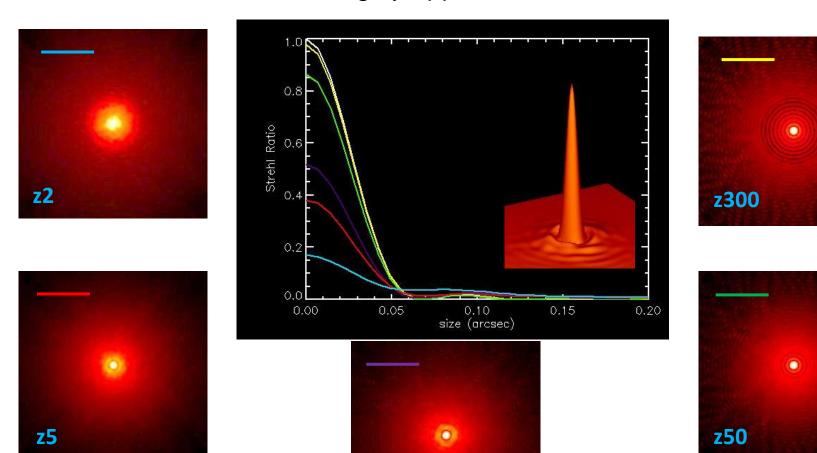
Incoming wavefront

Fit with zernike

Residual

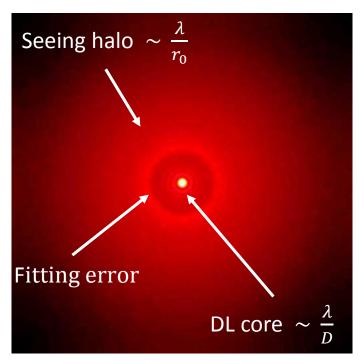
### **Partial compensation**

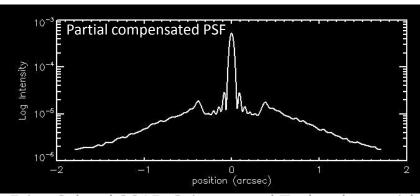
The fraction of light in the central core is related to the degree of correction and can be Roughly approximated with the Strehl Ratio

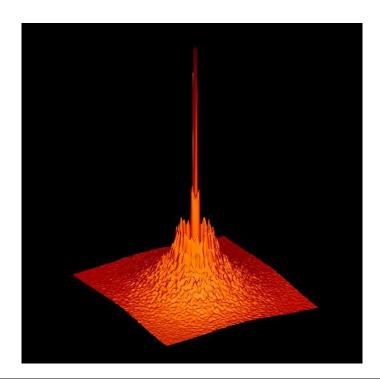


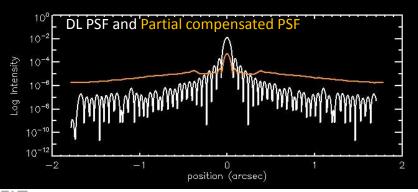
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#### **Partial compensated PSF**





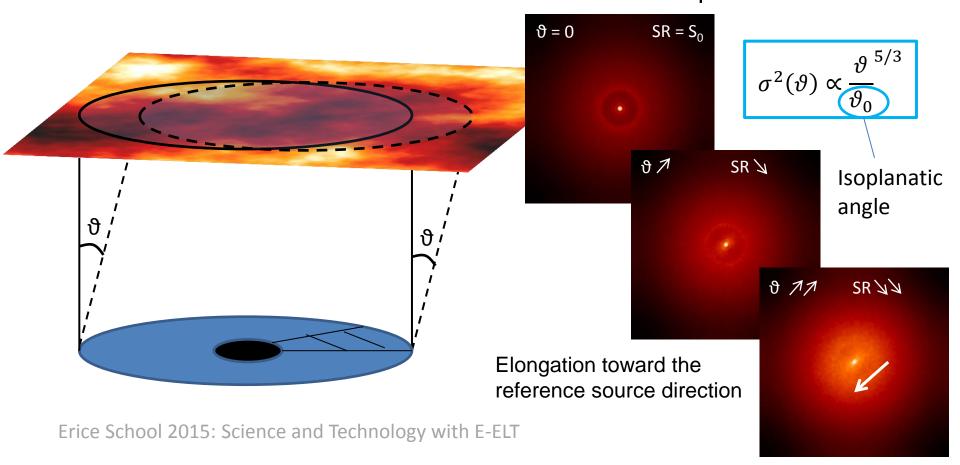




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### What happens across the FoV?

 The turbulence is measured only in the direction of the guide star, but the information is used to correct the wavefronts coming from all the directions within the FoV → This causes a degradation of the correction across the FoV. This error is called anisoplanatic error





#### MEASUREMENT OF ISOPLANATISM

#### ISOPLANATISM ANISOTROPY:

#### An Intuitive Explanation

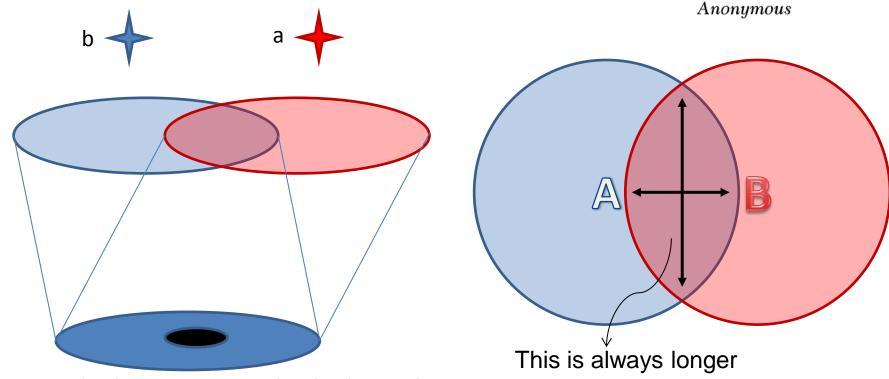
"Take an 'Oreo' cookie.

Take the wafers apart.

Stick them back together, but decentered a lot.

[McClure 1991] [Sasiela & Shelton 1993]

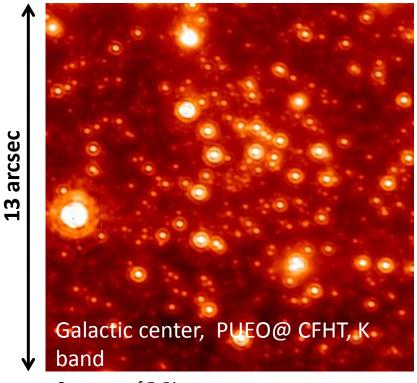
In which direction is it easiest to tip one wafer with respect to the other against the force of correlation exerted by the cream?"

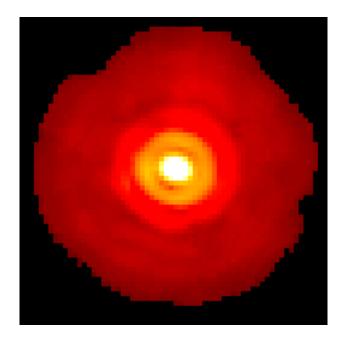


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#### How do the data look like?

Single Conjugate AO → Highly structured PSF, small FoV

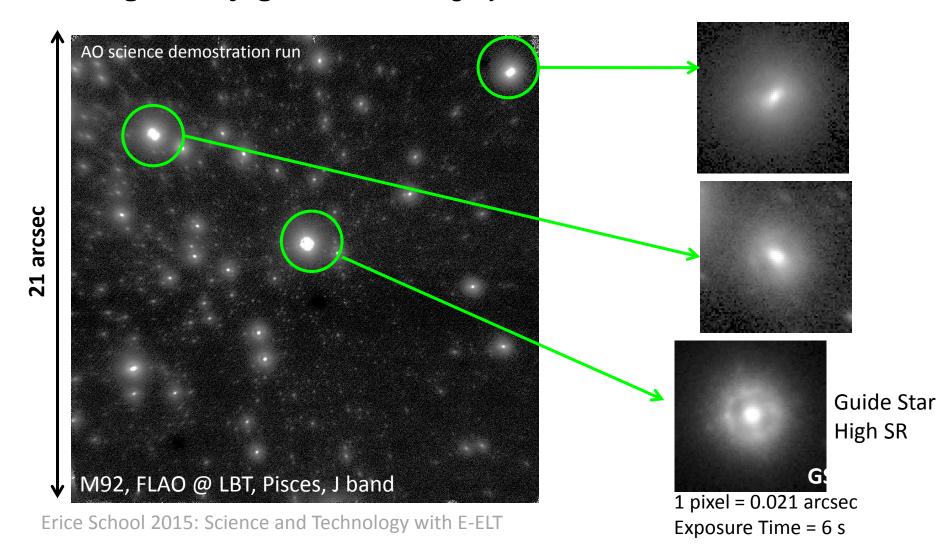




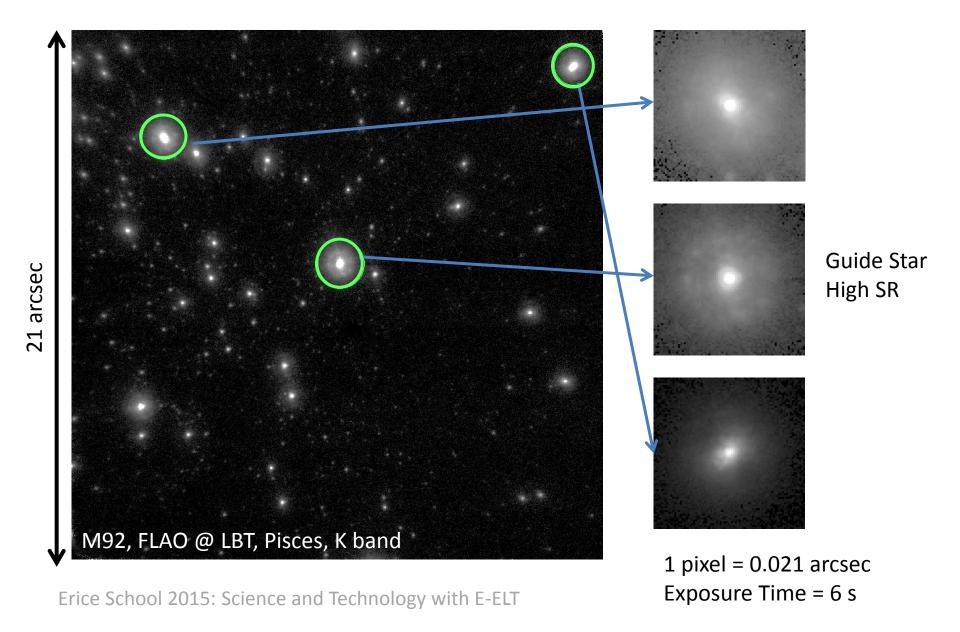
Courtesy of F. Rigaut

#### How do the data look like?

Single Conjugate AO → Highly structured and variable PSF

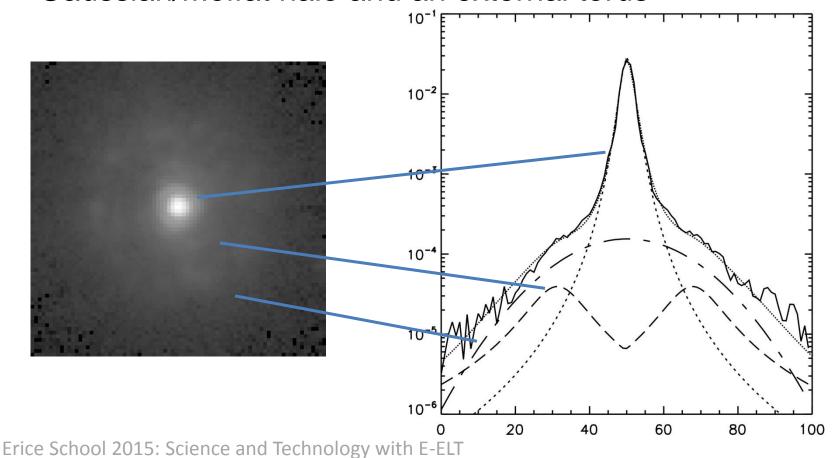


#### How do the data look like?



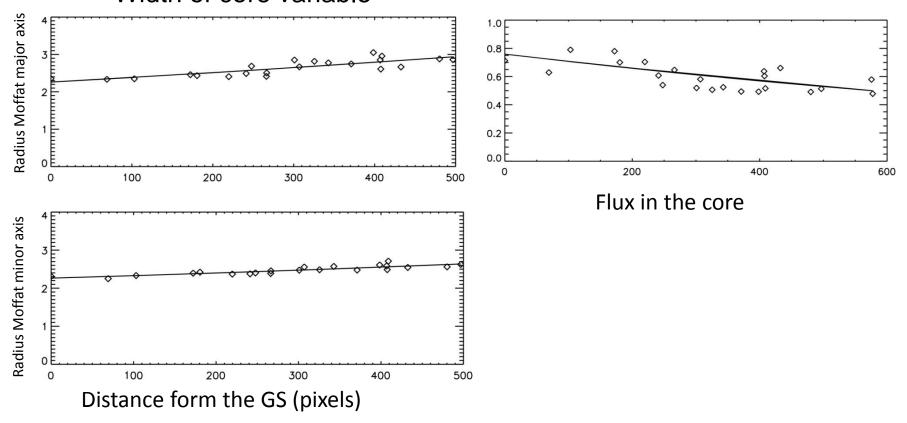
#### **PISCES M92 PSF model**

 The simplest analytical model that better represents the PSF is given by a narrow Moffat core, a broader Gaussian/Moffat halo and an external torus



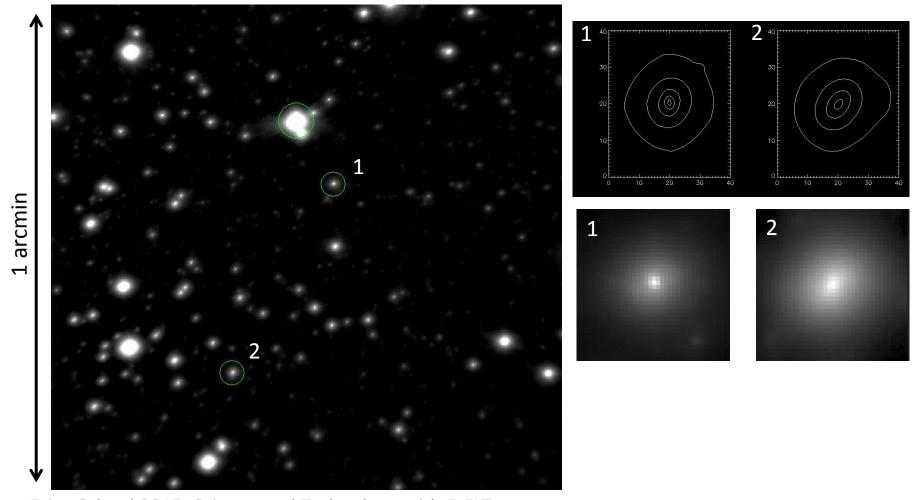
#### **PSF** model

- Variation of the PSF parameters across the FoV
  - Width of halo constant across the FoV
  - Width of core variable



### **Another example: NACO**

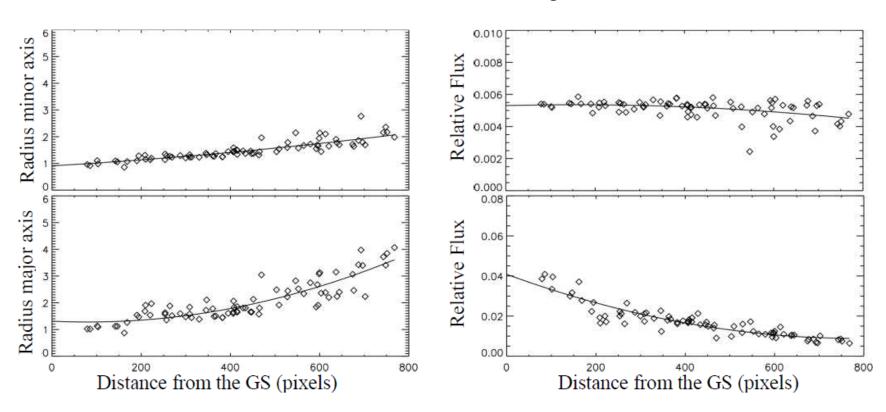
Image NACO@VLT of NGC 6440 GC [Origlia 2008]



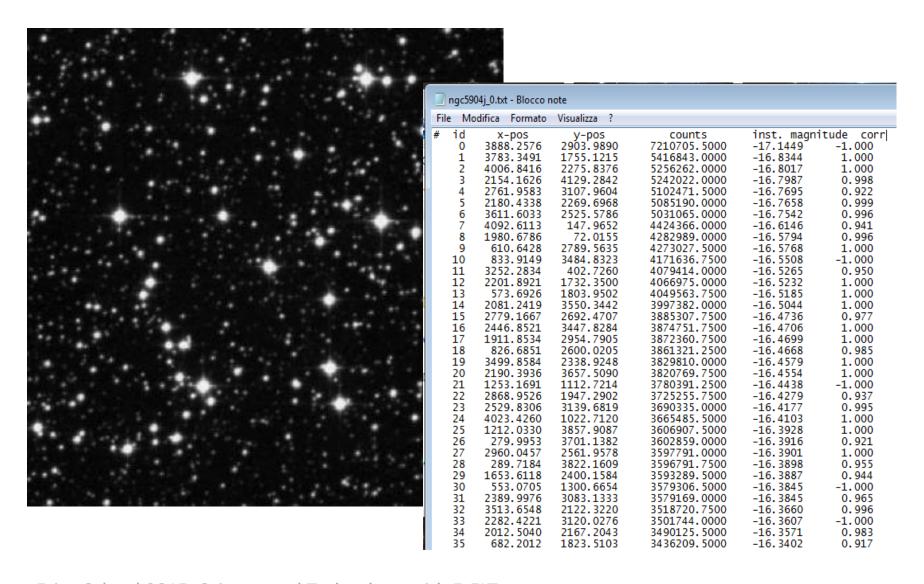
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### **Another example: NACO**

- Adopted PSF model:
  - PSF core: Elongated Moffat with axis varying over the FoV
  - Halo: round Moffat with radius = seeing constant over the FoV

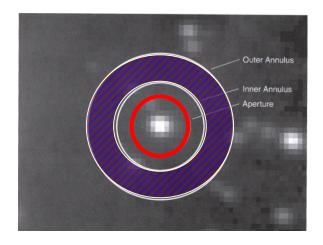


- Photometry: is the process of obtaining accurate numerical values for the brightness of objects (aperture phot./ PSF fitting).
  - Time variability of individual sources
  - Flux ratios or luminosity functions of multiple systems [Harayama et al. 2008]
  - Color Magnitude Diagrams of resolved stars (GC age, stellar population, stellar evolution, SFH) [...]
- Astrometry: precise measurements of the relative positions of objects and their variations (parallax and proper motion)
  - Dynamical masses of brown dwarfs [Dupuy et al 2009]
  - Our Galaxy's supermassive black hole [Ghez et al 2005]
  - Formation and evolution of young star clusters [Stolte et al 2008]



#### Aperture Photometry

- Measurement of the image volume within an 'appropriate' aperture. The background is estimated in an annular outer region an subtracted.
- The optimal aperture size depends on the PSF FWHM, on the S/N and on the crowding of the field
- The star position can be computed as a simple center of gravity
- Robust and precise for isolated stars
- Risk of contamination between sources
- Sextractor [Bertin 1996], ...

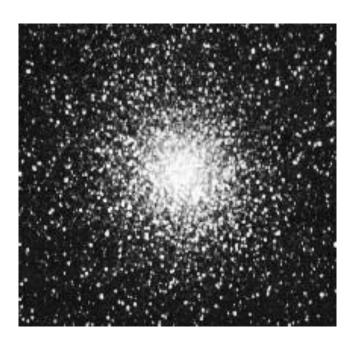


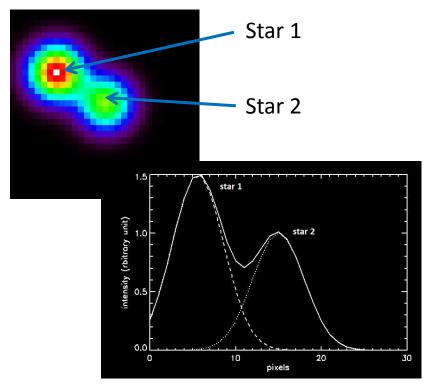
This method does not give us the possibility to take advantage from the resolution in crowded fields

It can be used in crowded fields after deconvolution for the PSF...

#### PSF fitting photometry

- Fit of the sources in the image with a PSF model
- The result depends on the model accuracy and on the background estimation (in case of source contamination and/or PSF variability)
- Suitable for dense stellar fields
- Need for isolated and bright stars to model the PSF





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### **PSF** estimation

#### PSF estimation from data:

- Analytical PSF (constant or variable)
- Numerical PSF (constant over the entire frame or in subdomains)
- Hybrid PSF (analytical model + numerical residual map)
- Product of the Blind deconvolution

#### Implemented in image analysis softwares:

- DAOPHOT (analytical/hybrid/smoothly variable) [Stetson 1987]
- Romafot (Purely analytic) [Buonanno 1983]
- DoPHOT (Analytical) [Schecter 1993]
- PSFex (analytical, linear combination of basis vectors) [Bertin 2010]
- STARFINDER (numerical/analytical/hybrid, possible hacking)
   [Diolaiti 2000]
- Dolphot (HSTPhot) [Dolphine 2000], ...

#### **SCAO** data reduction

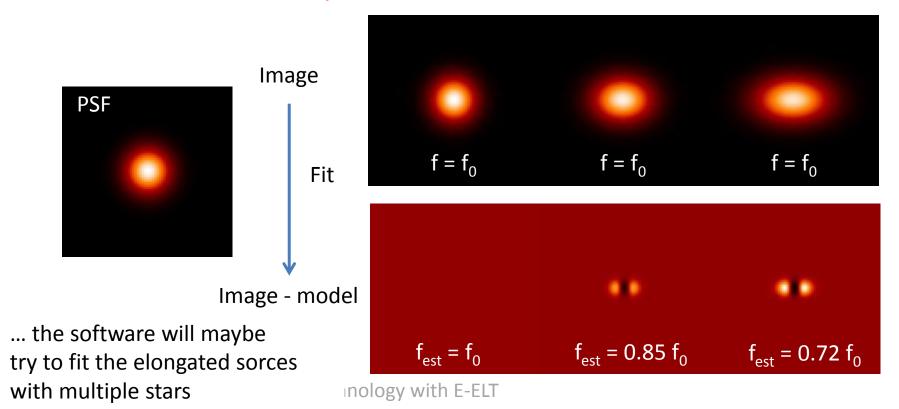
- SCAO → small corrected FoV, PSF spatial variation, high SR
  - Crowded-field AO astrometry appears to be limited by the inaccurate modeling of the Point Spread Function (PSF) [Shoedel 2010]
  - astrometry of faint sources is biased by residuals due to the incorrect subtraction of the PSF of brighter stars [Fritz 2009]
  - photometric accuracy is limited by the SNR and by the knowledge of the PSF [Shoedel 2010]
  - detection of elongated sources
  - False detections

Astrometric and photometric measurements with AO systems are mainly limited by errors in the PSF modeling and fitting.

Many 'exotic' solutions have been found to reduce data...

## **PSF fitting with constant PSF**

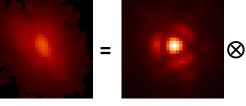
- When the PSF is invariant across the FoV, the photometric error is due mainly to SNR
- If the PSF elongation is field dependent, fitting the stars with a constant PSF causes the introduction of another (unknown) error source that is field dependent



### **SCAO** data reduction

- SCAO → small corrected FoV, PSF spatial variation, high SR
  - Galactic center (NACO): Image is first Wiener-filter-deconvolved using a suitable PSF (GS psf). Local variations in PSF kernels and ringing is taken care with locally extracted PSF fitting. [Schoedel 2010]
  - M15 GC (FLAO): Modiefied Romafot software. PSFfitting with variable moffat (no parameters fixed). [Monelli 2015]
  - NGC6440 GC (NACO): PSFfitting with starfinder using an analitical model composed by 3 gaussian components. [Origlia 2008]
  - Usage of calibration images [Steinbring et al. (2002)]
  - Usage of calibration HST fields
  - Galaxy Survey (NACO): Estimate local PSF around guide star image and model the PSF in the field as the convolution of the GS PSF and a blurring kernel. [Diolaiti 2000,

a blurring kernel. [Diolaiti 2000, Cresci 2006]



off-axis PSF

guide star



e star blurring kernel (e.g. Gaussian)

## **Multi-reference Adaptive Optics**

 Using more reference stars from different directions to analyze the wavefront, one can reduce the PSF variation across the FoV.

# **GLAO:** Ground Layer Adaptive Optics

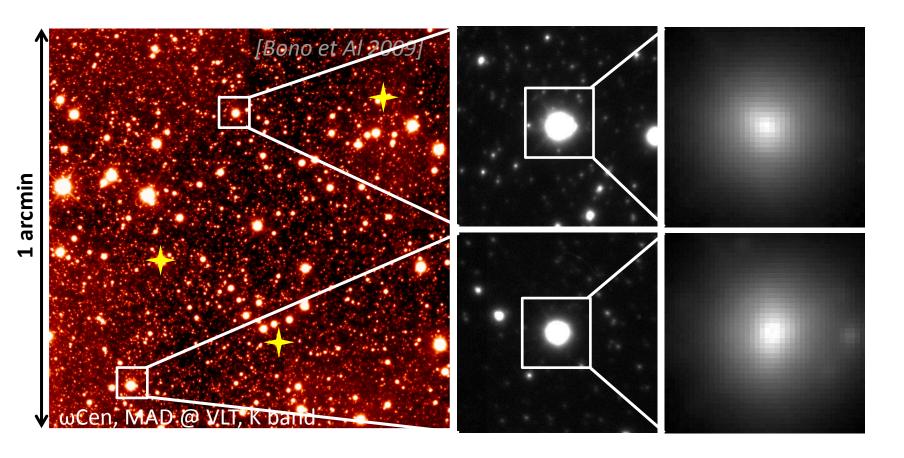
Using multiple reference sources it is possible to retrive the average contribution of the atmospheric tubulence within the guide stars directions → low SR, big FoV (some arcmin)

# MCAO: Multi-Conjugate Adaptive Optics

Using multiple reference sources AND multiple deformable mirrors conjugated ad different altitudes, it is possible to measure the turbulent wavefronts at specific altitudes and to apply the correction directly where they are generated→ medium SR, medium FoV (~1 arcmin)

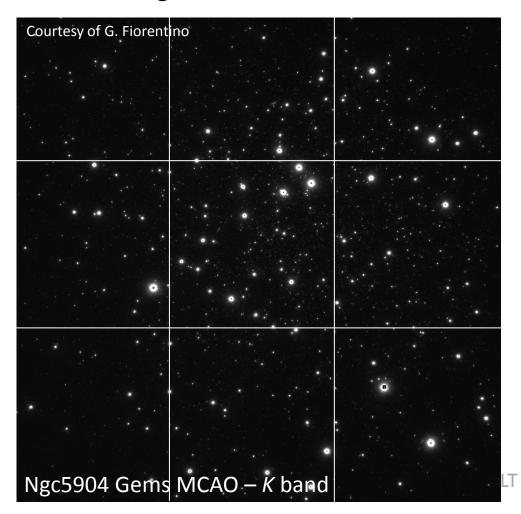
### How do the AO data look like?

 Multi Conjugate AO → Improved PSF uniformity across a larger FoV



### How do the AO data look like?

 Multi Conjugate AO → Improved PSF uniformity across a larger FoV



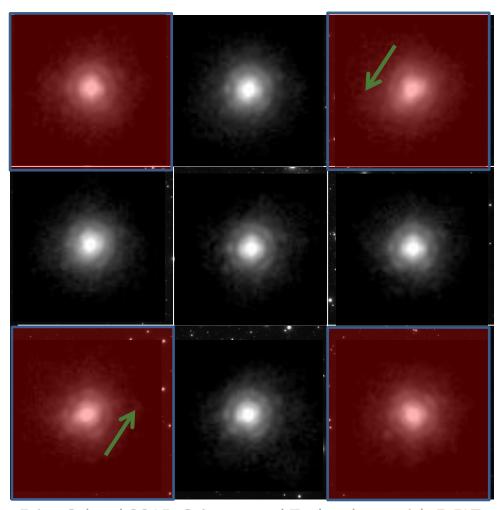
FoV = 2 arcmin

We extract numerically the psf from differen subregions of the field to analyze its variation

→ The accuracy of the local PSF depends on the local crowding and on the presence of bright local stars

#### How do the AO data look like?

K – band image



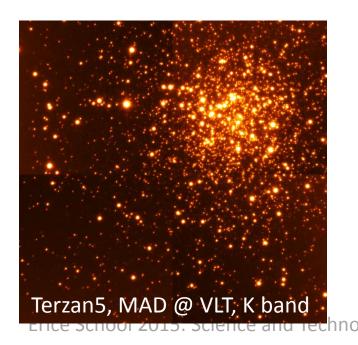
Erice School 2015: Science and Technology with E-ELT

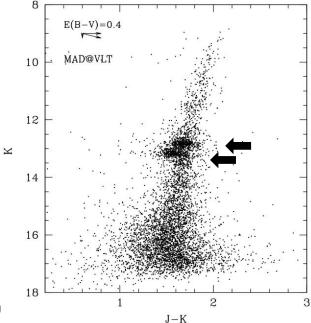
Local extracted PSFs

→ Small variation occurs: slightly elongated at the field corners

### **MCAO** data reduction

- MCAO → To improve the PSF uniformity across the FoV
  - Suitable to study dense stellar field, galaxy morphology
  - MAD: Many papers have been pubblished [Melnick SPIE 2012 for a review]
  - GeMs: First papers are coming out
  - Most diffused software for image analysis, not optimized for PSF variation across the FoV, has been used

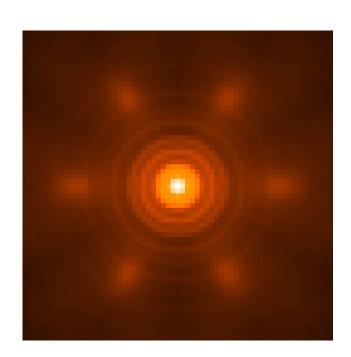


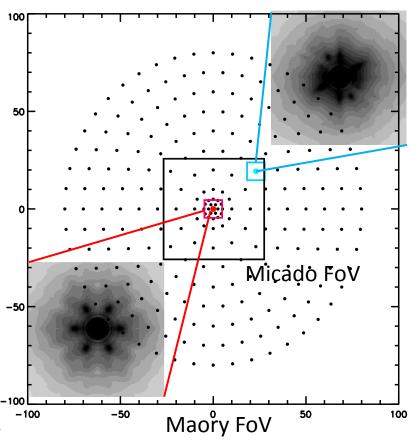


The presence of two red clumps implies the presens of two different stellar populations. [Ferraro et Al, Nature, 2009]

### **MAORY phase A PSF**

- Multi conjugate Adaptive Optics RelaY for the E-ELT
- Wavefront sensing based on 6 Sodium LGS and 3 NGS
- Uniform AO correction on a large FoV (2')



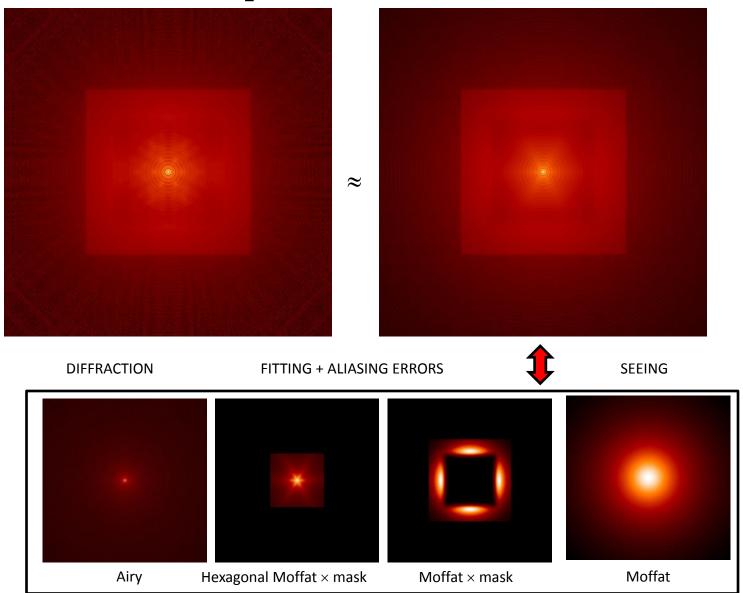


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### **MAORY phase A PSF**

K band PSF  $SR \approx 0.6$ 

Image size = 2.7"



### **Conclusions**

- If you want to gain in spatial resolution you need a big telescope
- If you want a big telescope, you have to stay on the ground
- If you want a big telescope on the ground and gain really in resolution, you need adaptive optics
- If you want to take full advantage of the scientific information encoded in AO images, you need to manage the data in the proper way
- If you want to manage data in the proper way, you need to know the PSF structure
- If you want to know the PSF structure, you need a flouring on Adaptive Optics techniques