Synthetic color-magnitude diagrams: the ingredients

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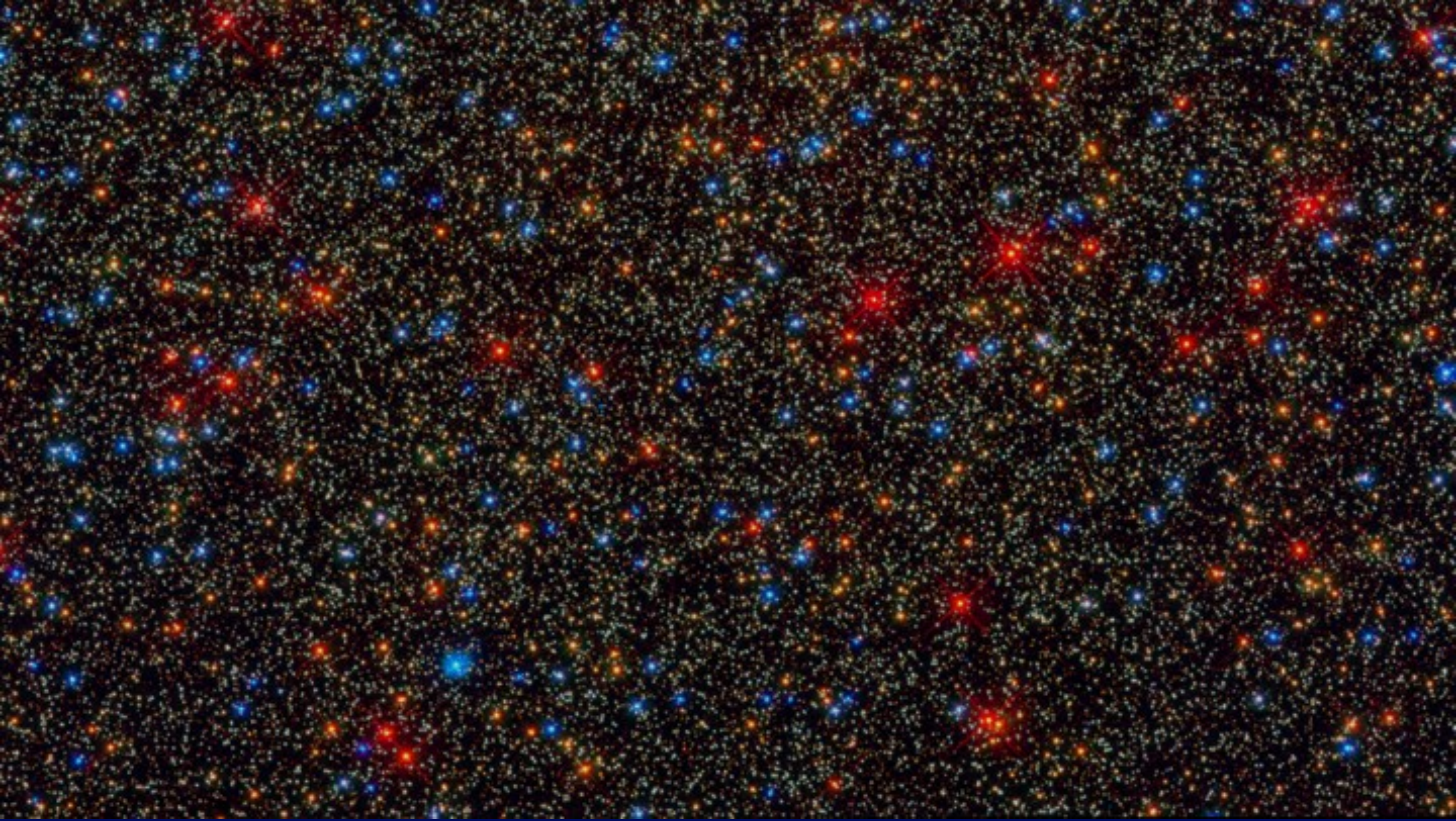
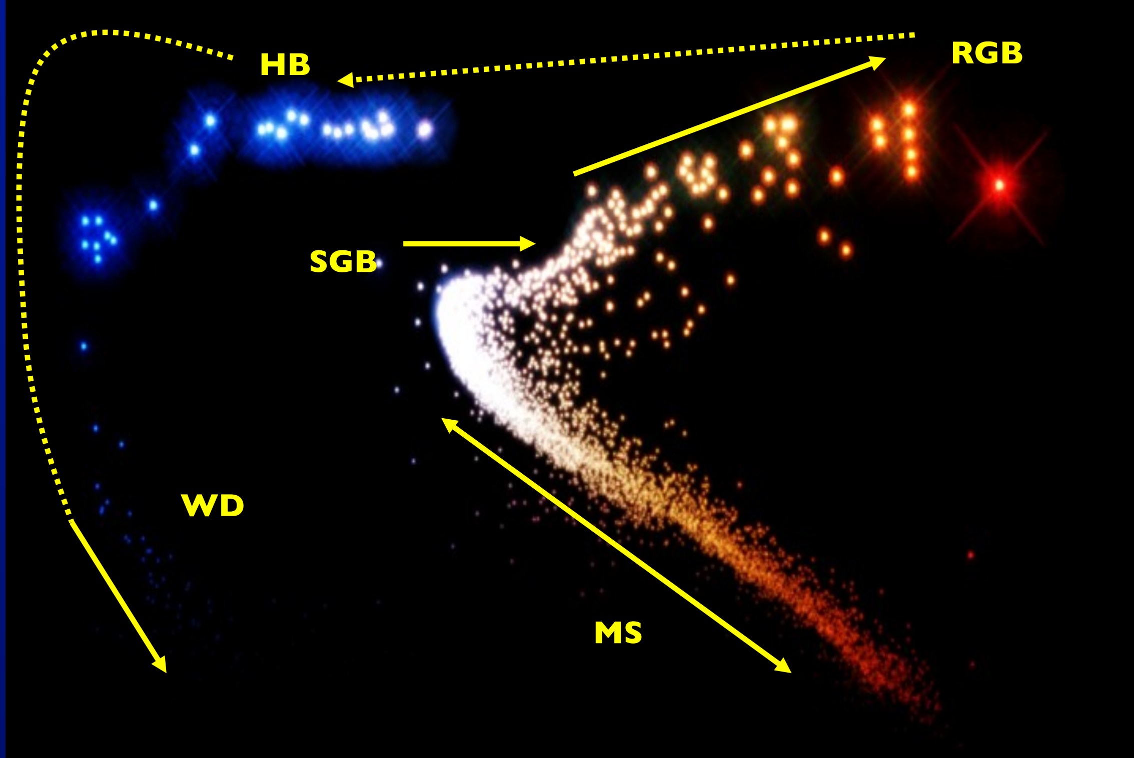


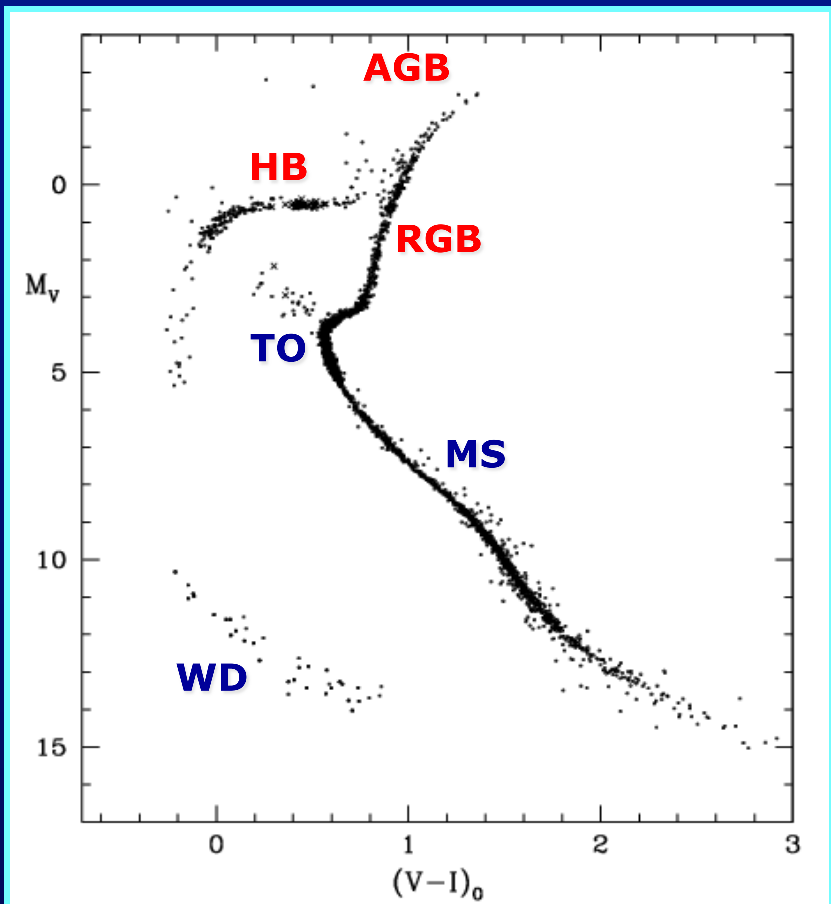
Making a "real" Color-Magnitude Diagram: the case of the Globular Cluster Omega Centauri

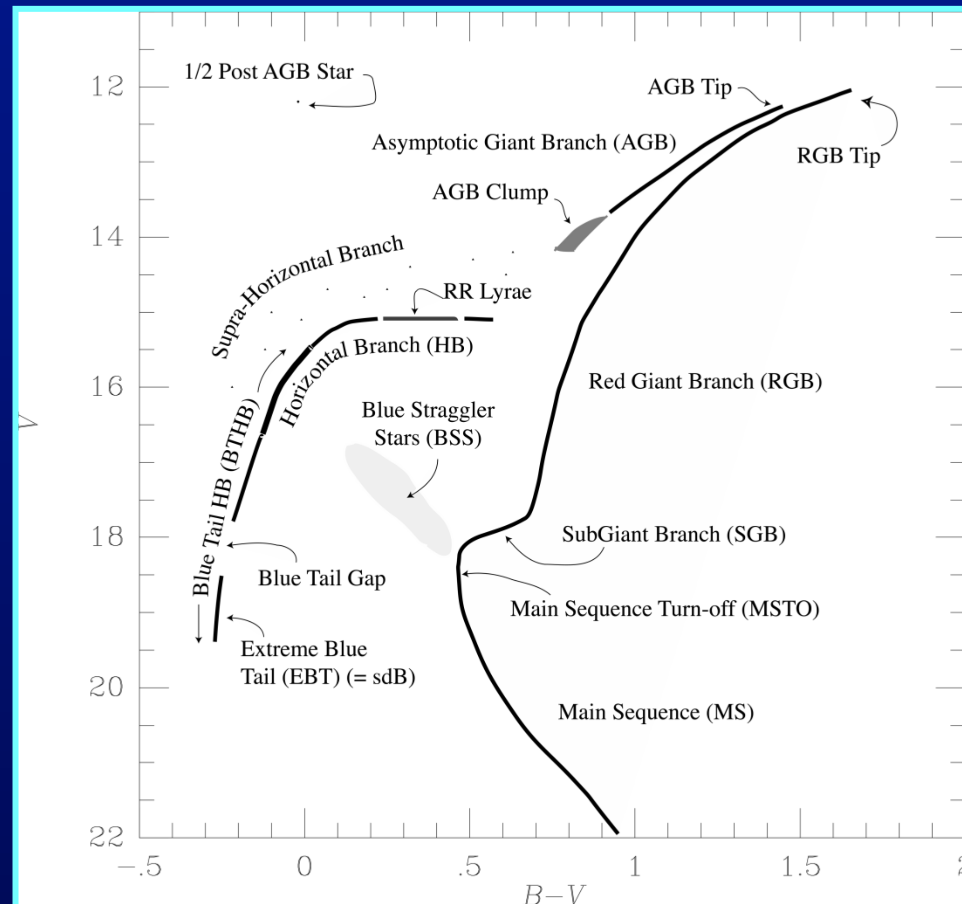


Since a star's color and brightness tell us its evolutionary phase, we can easily identify stars by phase in the image.

Color-Magnitude diagrams of star clusters:

laboratories of low- & intermediate mass stellar evolution





Rood et al. (1998)

The ingredients for "cooking" a synthetic CMD

- ✓ Stellar model predictions
- ✓ Color-Teff transformations and Bolometric Corrections
- ✓ Initial Mass Function
- ✓ Binary fraction and their mass ratio distribution
- ✓ Star formation rate
- ✓ Chemical evolution law

What is a "Simple Stellar Population"?

"A Simple Stellar Population (SSP) is defined as an assembly of coeval, initially chemically homogeneous, single stars.

Four main parameters are required to describe a SSP, namely its age, composition (Y, Z) and initial mass function.

In nature, the best examples of SSP's are the star clusters…."

Alvio Renzini & Alberto Buzzoni
1986, ASSL, 122, 195

The conditions for being a SSP are:

- The stars must be coeval

Single star formation burst

- The stars must be chemically homogenous

All stars were BORN with the same chemical composition

When ONE of these two conditions is not fulfilled, the stellar population under scrutiny is not a SSP, but it is a COMPLEX stellar population formed by various sub-populations

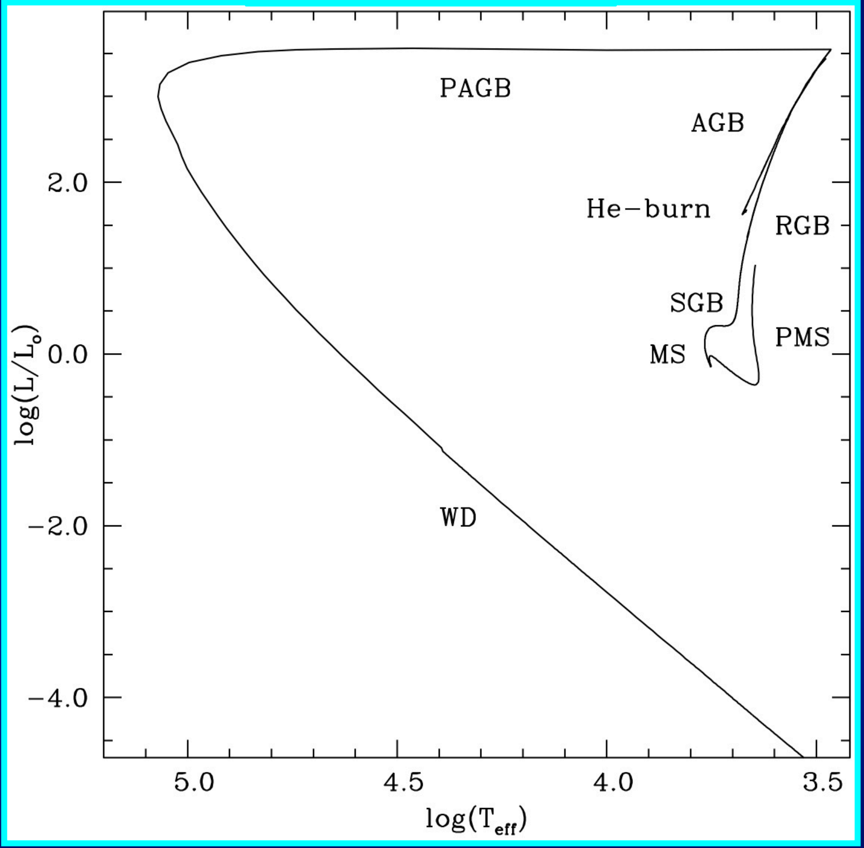
The theoretical evolutionary framework

Evolutionary stellar models provide:

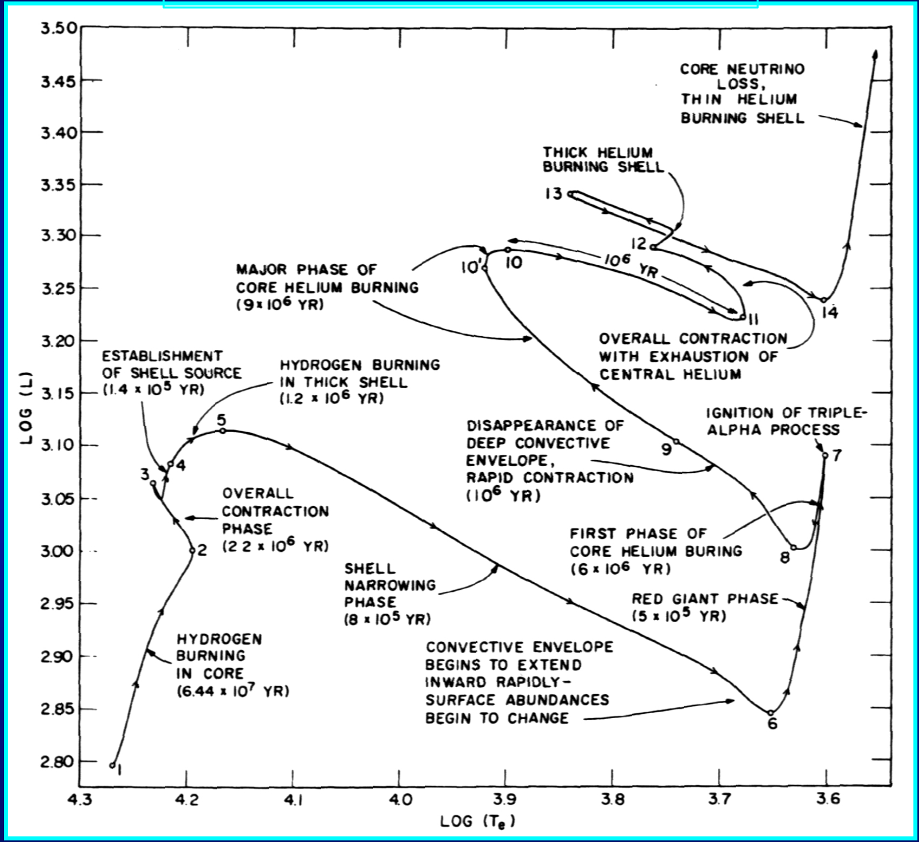
- Evolutionary lifetimes ⇒ **Star counts**
- Bolometric luminosity
- Effective temperature
- Mass – different than the initial one

Evolutionary tracks

low-mass star



intermediate-mass star

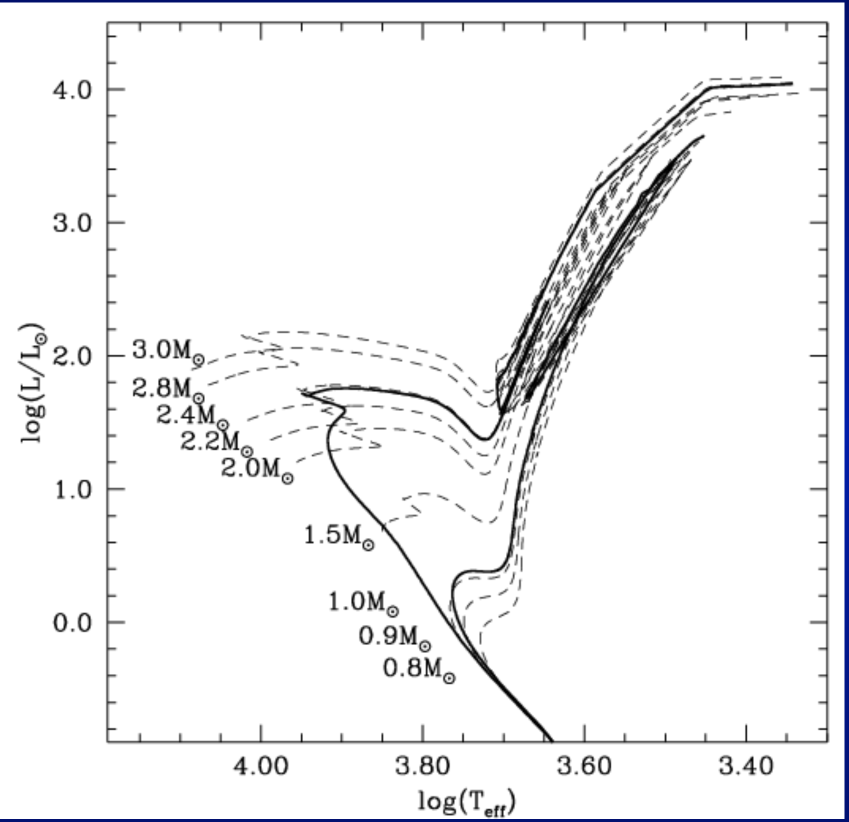


Isochrones: a fundamental tool for SPSs

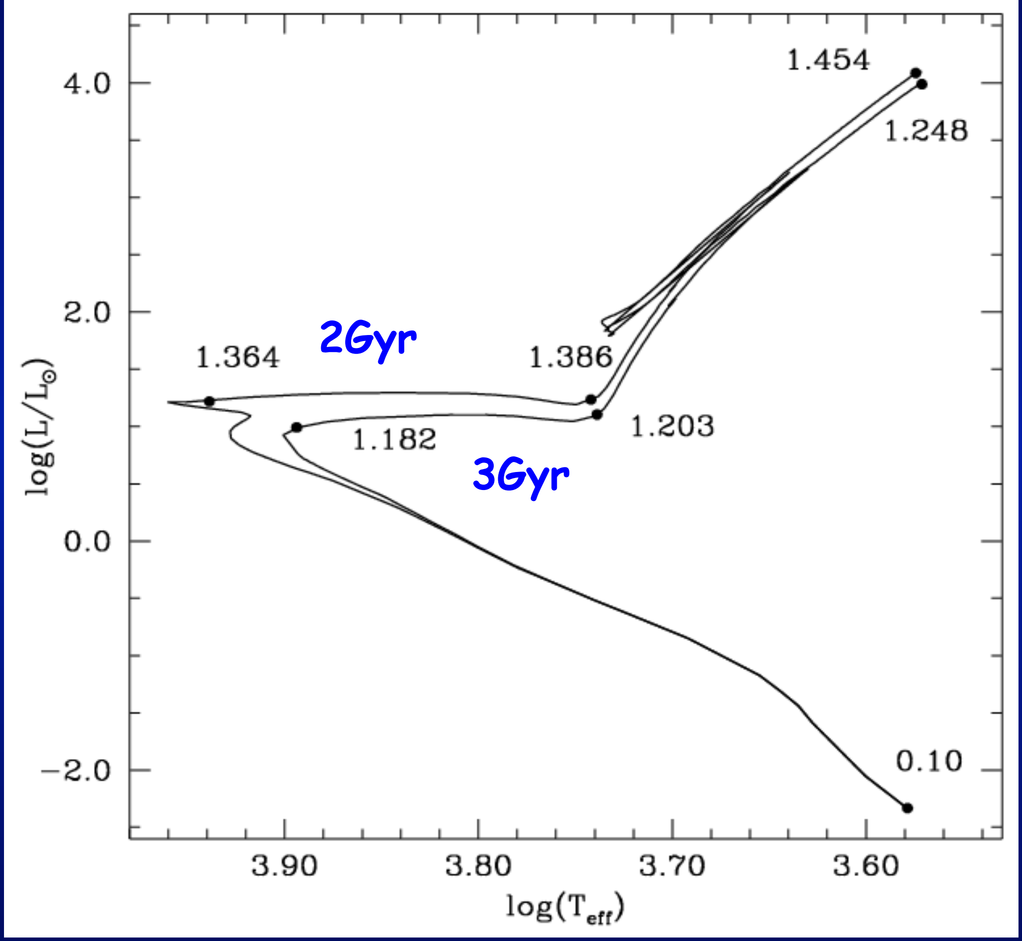
The theoretical CMD (or HRD) of an SSP is called **isochrone**, from the greek word meaning 'same age'.

Consider a set of evolutionary tracks of stars with the same initial chemical composition and various initial masses; different points along an individual track correspond to different values of the time t' and the same initial mass. An isochrone of age t is simply the line in the HRD that connects the points belonging to the various tracks (one point per track) where $t' = t$. This means that when we move along an isochrone, time is constant whereas the value of the initial mass of the stars populating the isochrone at each point is changing.

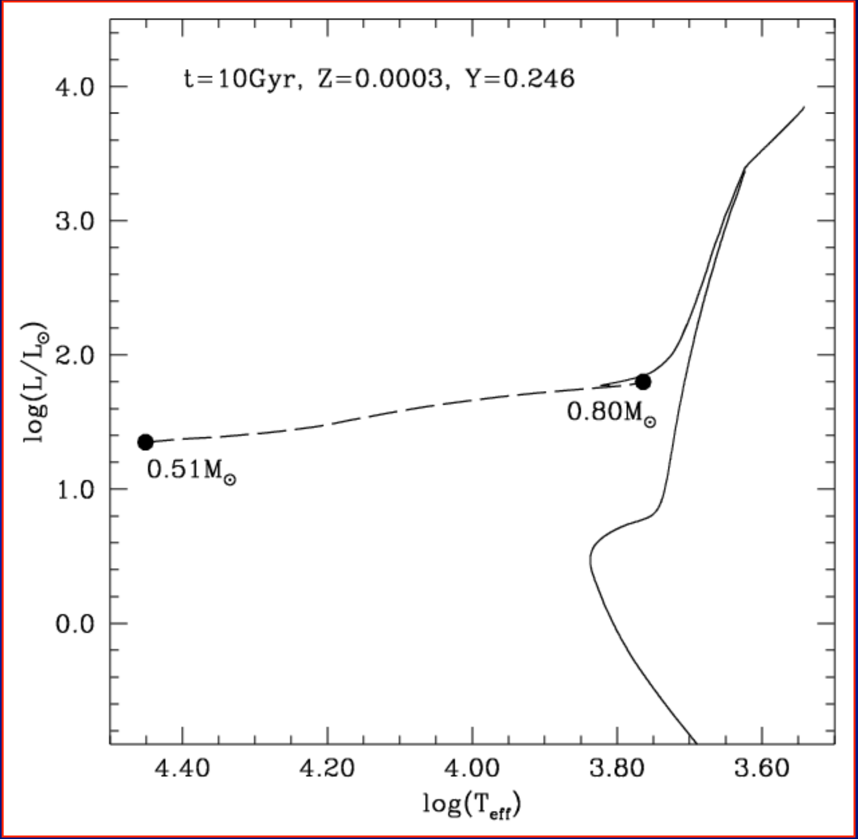
A point along an isochrone of age t is therefore determined by 3 quantities: bolometric luminosity, effective temperature and value of the evolving mass. Once an isochrone of a given age and initial chemical composition is computed from stellar evolution tracks, it can be transferred to an observational CMD by applying to each point a set of appropriate bolometric corrections.



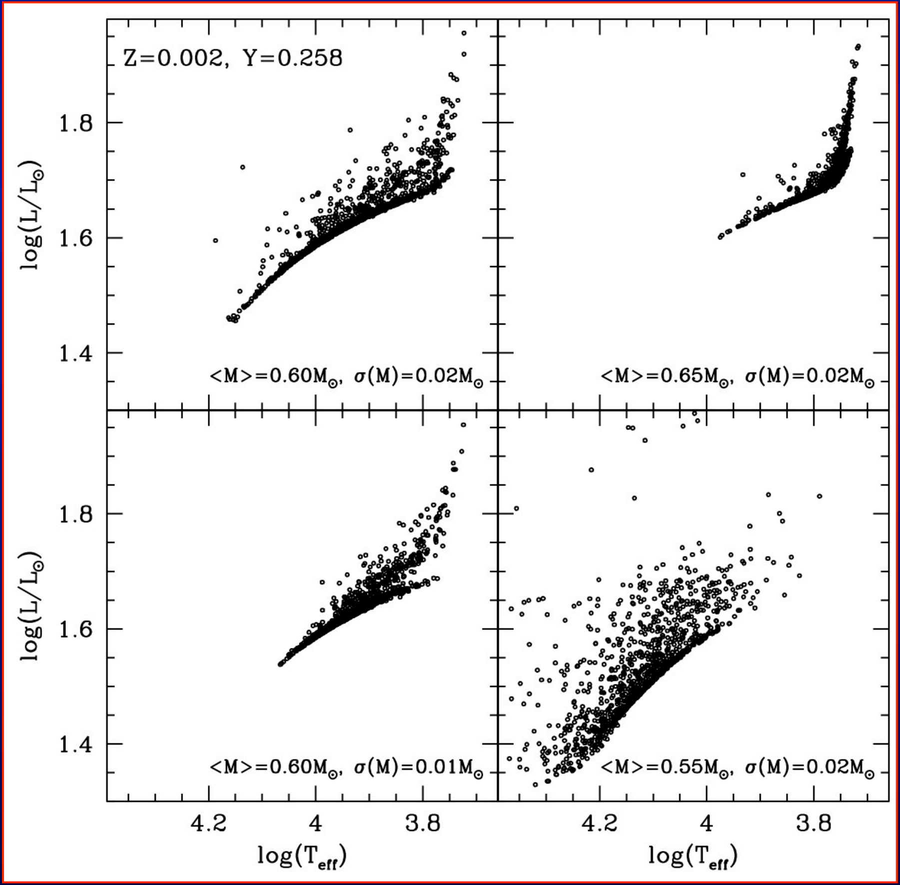
Isochrones: an interesting property



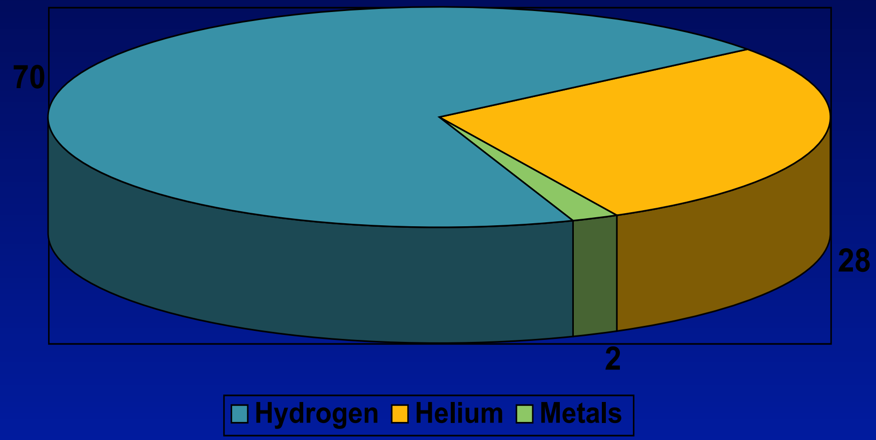
A problem with the HB: the T_{eff} (mass) distribution



The ZAHB and HB colour of an old isochrone depend on the ACTUAL mass loss along the RGB



The chemical composition of the stars



It is a common practice to use the symbols X, Y and Z to indicate the mass fractions of Hydrogen, Helium and all other elements heavier than Helium (called "metals") respectively

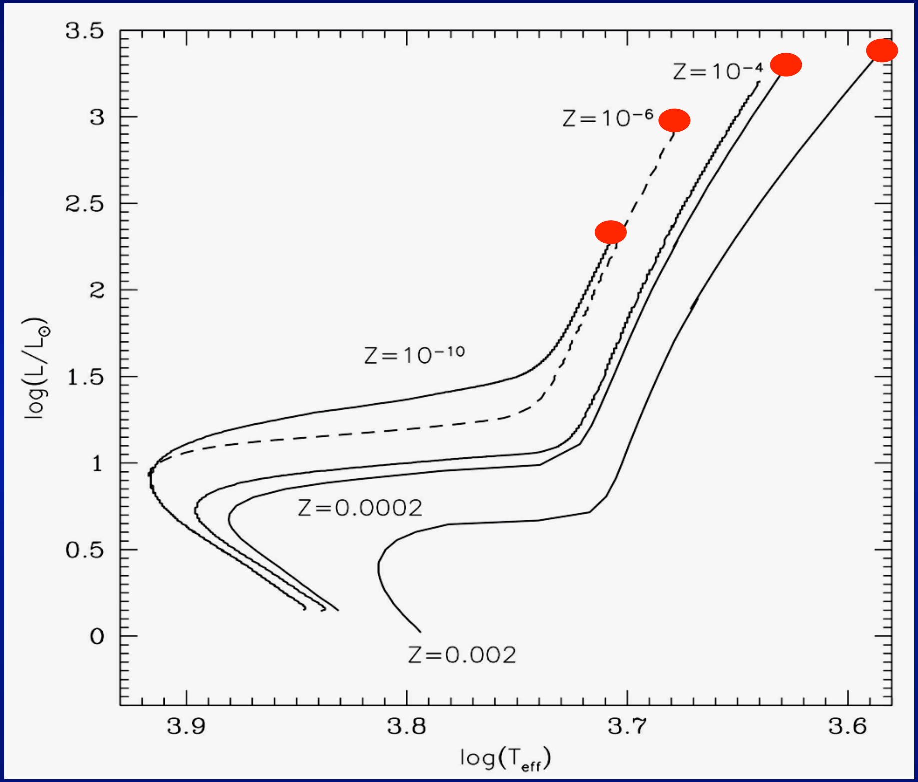
$$\boxed{X + Y + Z = 1}$$

For the metals, the distribution of the individual fractional abundances has to be specified too

The "standard" metal mixture is the Solar one. It is made up of ≈ 48% (in mass fraction) of Oxygen, ≈ 5% of Nytrogen and ≈ 17% of Carbon…

Dependence of the evolutionary properties on chemical abundances

The metallicity



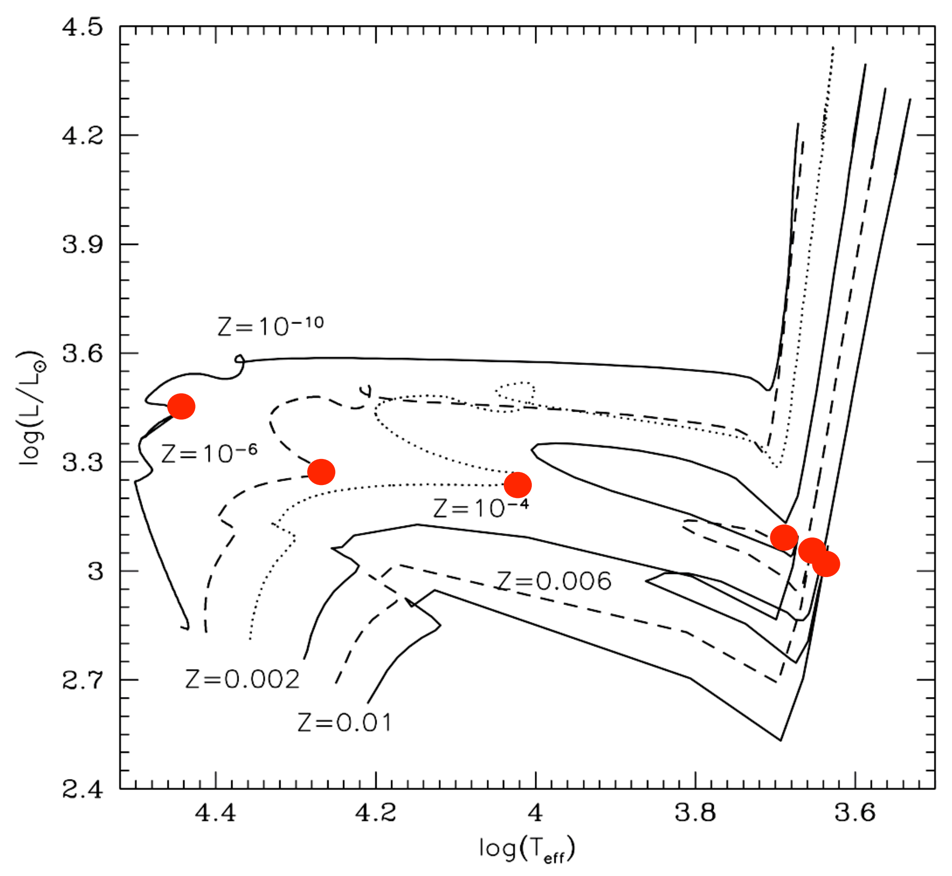
✓The evolutionary tracks are brighter;

✓They are located at larger effective Temperatures. In particular note the strong dependence of the RGB location…

✓The evolutionary lifetime of the core H-burning stage is shorter;

✓The brightness of the Red Giant Branch Tip is lower;

	$Z=10^{-10}$	$Z=10^{-6}$	$Z=10^{-4}$	$Z=2\ 10^{-4}$	$Z=2\ 10^{-3}$
t_{Hc} (10^8yr)	66.6	65.0	61.1	61.5	70.1
$t_{H\text{-}shell}$ (10^8yr)	4.04	5.07	8.06	9.24	21.8

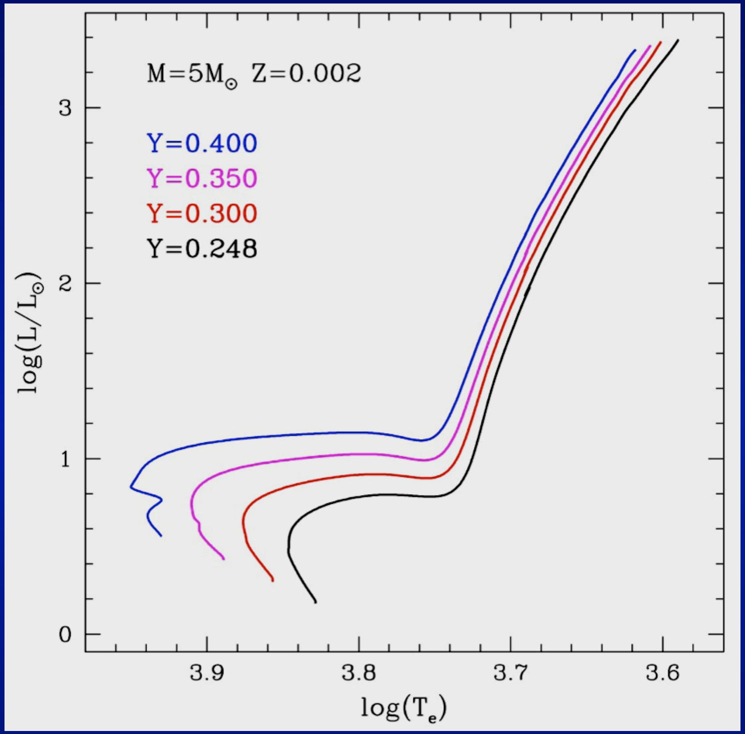


- Note the different morphology of the core H-burning locus;

- The blue loop corresponding to the core He-burning has a maximum extension and then it vanishes…;

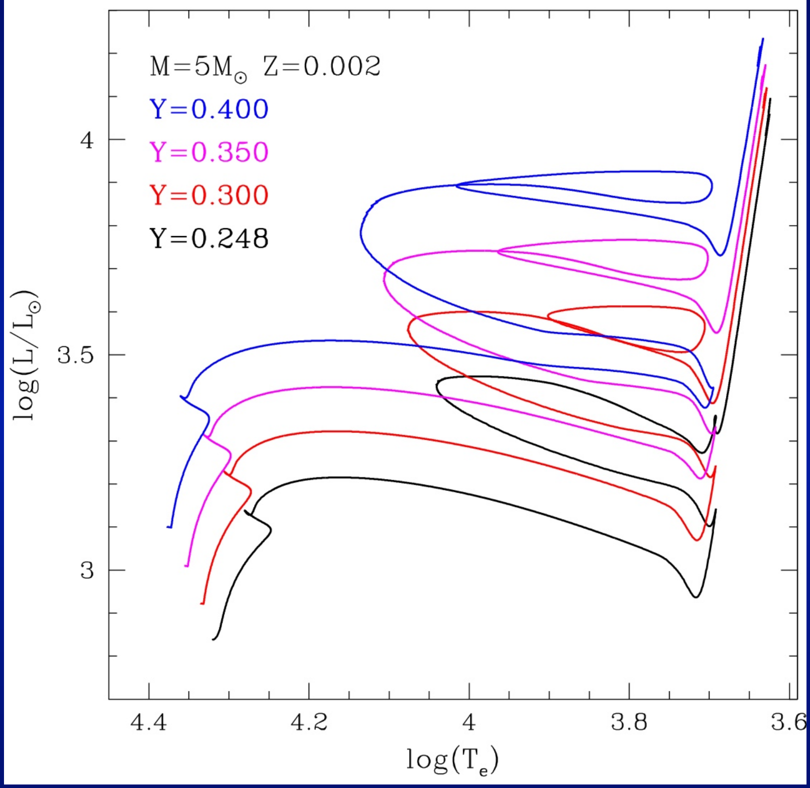
- For low – enough – metallicity, stars fail to reach the Hayashi track, igniting He-burning at large T_{eff};

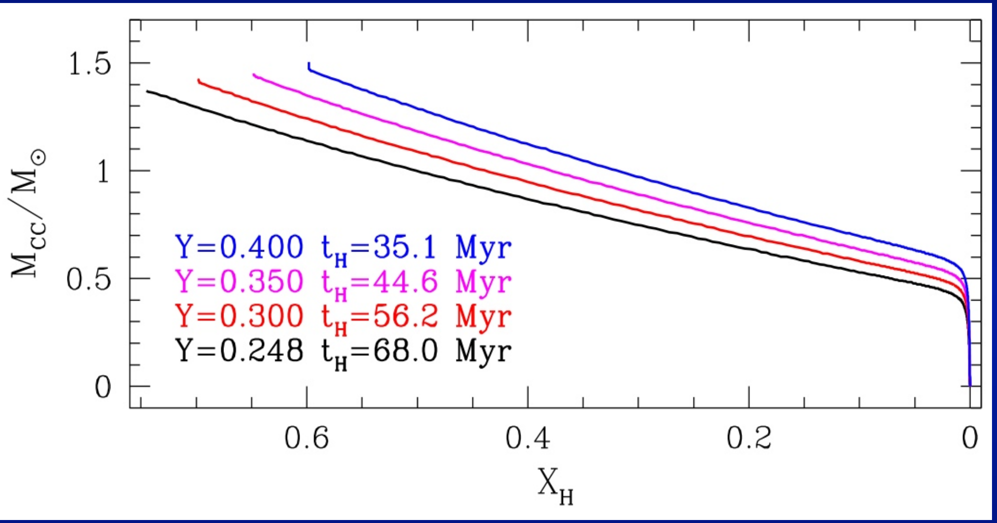
The initial Helium content



Y	t_H(Gyr)	t_{Tip}(Gyr)	$\log(L_{tip}/L_\odot)$	$M_{cHe}/M_\odot$
0.248	5.73	6.34	3.387	0.484
0.300	4.00	4.47	3.374	0.472
0.350	2.83	3.19	3.353	0.461
0.400	1.97	2.25	3.329	0.450

The initial Helium content





From the H-R diagram to the observational CMDs

Evolutionary stellar models provide:

- Evolutionary lifetimes ⇒ Star counts
- Bolometric luminosity
- Effective temperature
- Mass – different than the initial one

Gravity
T_{eff}

Bolometric corrections
+
color-T_{eff} relations
+
spectral library

•Magnitudes & Colors
•Spectral indices
•Integrated spectra

FROM THEORY TO OBSERVATIONS

Almost all information we gather from observations of stars come from the detection and analysis of the photons they emit; observations of the stellar radiation provide low- and high-resolution spectra and broad band photometry that have to be related to the properties predicted by stellar models.

$$M_{bol} = M_{bol,\odot} - 2.5\ \log(L/L_{\odot})$$
$$M_{bol,\odot} = 4.74$$

The (absolute) magnitude in a given photometric band A is defined in terms of M_{bol} as follows

$$M_A = M_{bol} - BC_A$$

where BC_A is the bolometric correction to the photometric band A.

The observed magnitude of a star, called apparent magnitude (m_A) is related to M_A as follows:

$$m_A = M_A + 5\ log(d) - 5 + A_A$$

where d is the distance in parsec and A_A is the effect of the interstellar extinction.
$(m - M)_0 \equiv m_A - M_A - A_A$ is called **distance modulus**
$(m - M)_A \equiv m_A - M_A$ is called **apparent distance modulus**

A distance modulus equal to 0 corresponds to a distance of 10 pc.

Spectral Energy Distribution

The light we receive from a star has been released by the photosphere, where the optical depth τ (the probability that a photon has one interaction with the stellar matter)

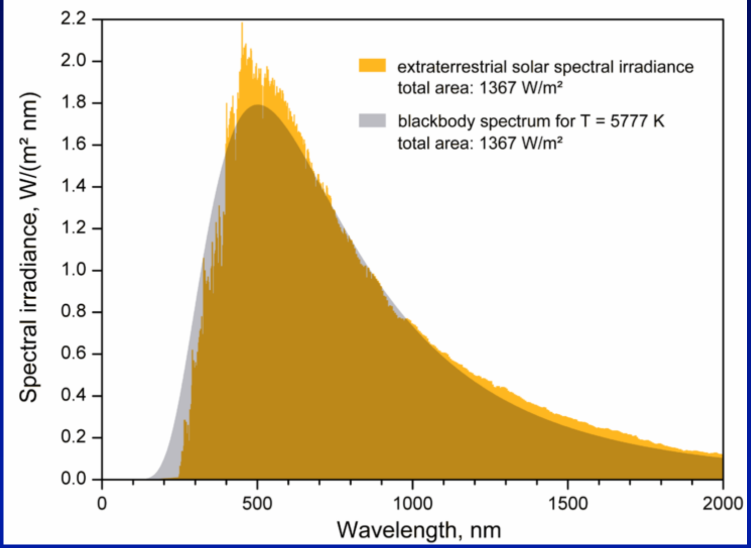
$$\tau = \int_r^\infty \kappa_{rad}\, \rho\, dr$$

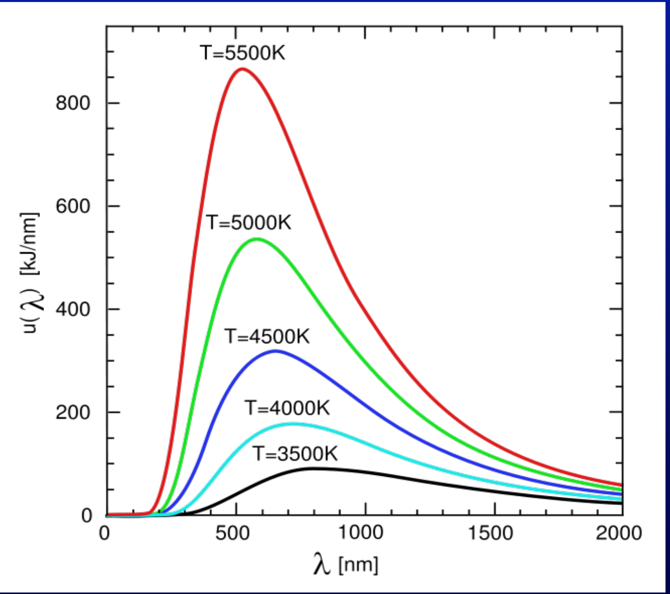
(r is the radial coordinate of a generic point within the star) is about 1. This light has a **blackbody spectrum**, whose energy distribution depends only on T_{eff}.

After being released from the photosphere, the photons have to cross the overlying stellar atmosphere, where $\tau < 1$. Here the spectrum of the star is modified and what is observed is no longer exactly equal to a blackbody.

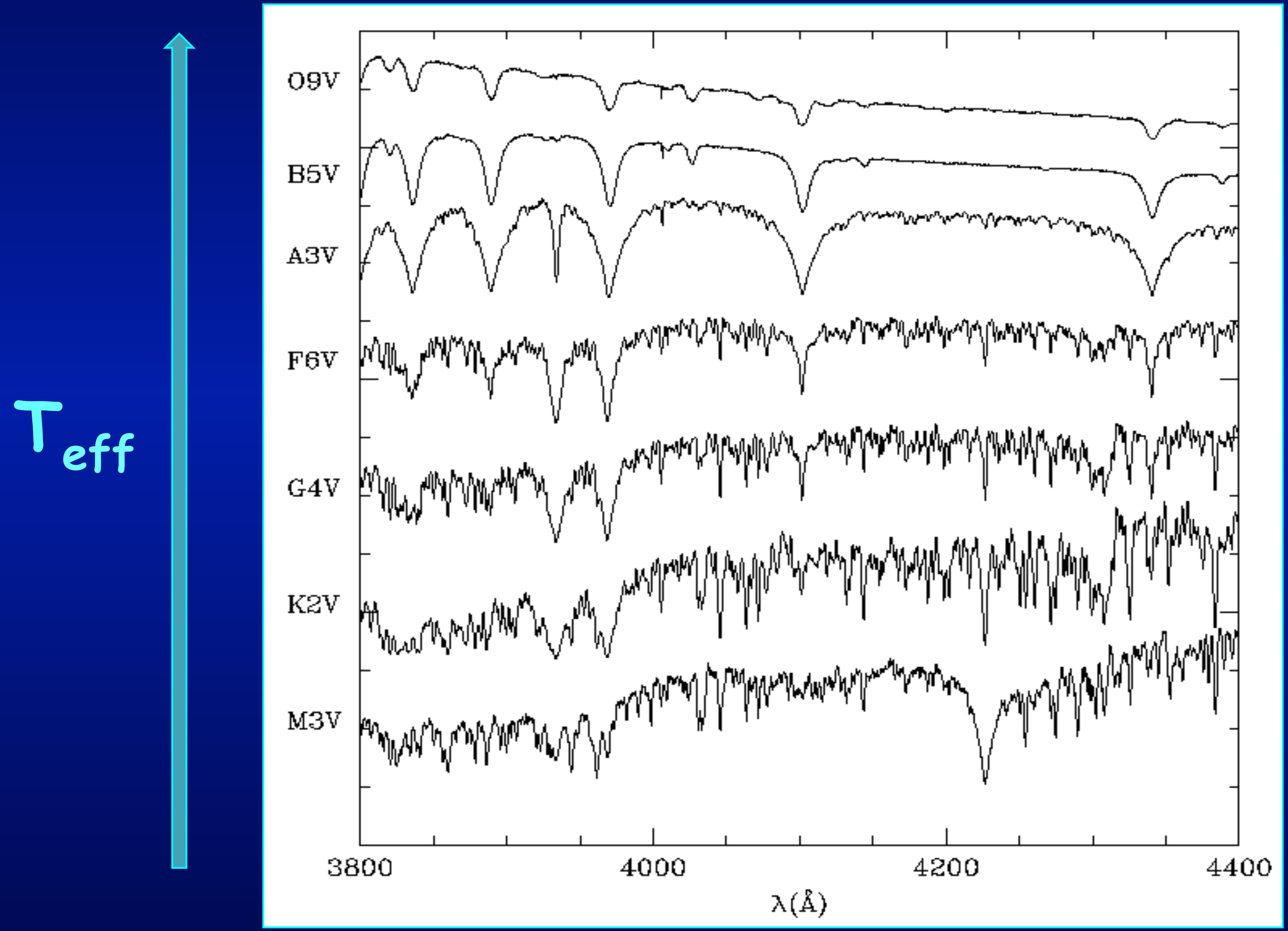
It is the crossing of the atmosphere that introduces a dependence of the spectral energy distribution on gravity and chemical composition, in addition to the T_{eff}.

Theoretical models of stellar atmospheres and predictions of the spectral energy distribution of the emerging photons are necessary to compute bolometric corrections to any given photometric band.





Examples of stellar spectra



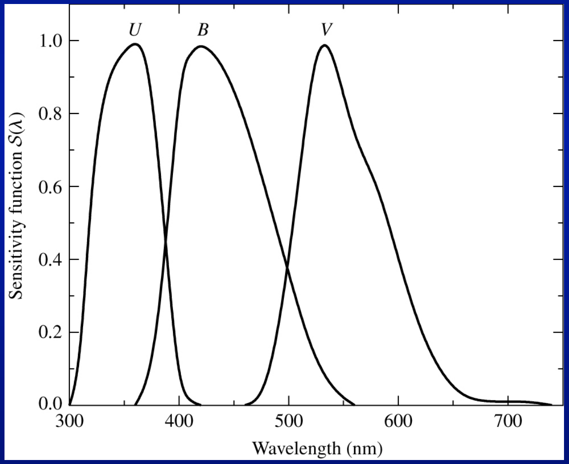
Bolometric corrections

Flux received at a distance d from a star of radius R: $f_\lambda = 10^{-0.4A_\lambda}(R/d)^2 F_\lambda$

Apparent magnitude (the definition)

$$m_{S_\lambda} = -2.5 \log\left(\frac{\int_{\lambda_1}^{\lambda_2} \lambda f_\lambda S_\lambda \mathrm{d}\lambda}{\int_{\lambda_1}^{\lambda_2} \lambda f_\lambda^0 S_\lambda \mathrm{d}\lambda}\right) + m_{S_\lambda}^0$$

$\lambda f_\lambda S_\lambda$ is proportional to the photon flux (i.e. number of photons by unit time, surface, and wavelength)



From stellar models we want to predict the absolute magnitude

$$M_{S_\lambda} = -2.5 \log\left[\left(\frac{R}{10\ \mathrm{pc}}\right)^2 \frac{\int_{\lambda_1}^{\lambda_2} \lambda F_\lambda 10^{-0.4A_\lambda} S_\lambda \mathrm{d}\lambda}{\int_{\lambda_1}^{\lambda_2} \lambda f_\lambda^0 S_\lambda \mathrm{d}\lambda}\right] + m_{S_\lambda}^0$$

We want to eliminate the dependence on R

Definition of bolometric correction:

$$BC_S = M_{Bol} - M_{S_\lambda}$$

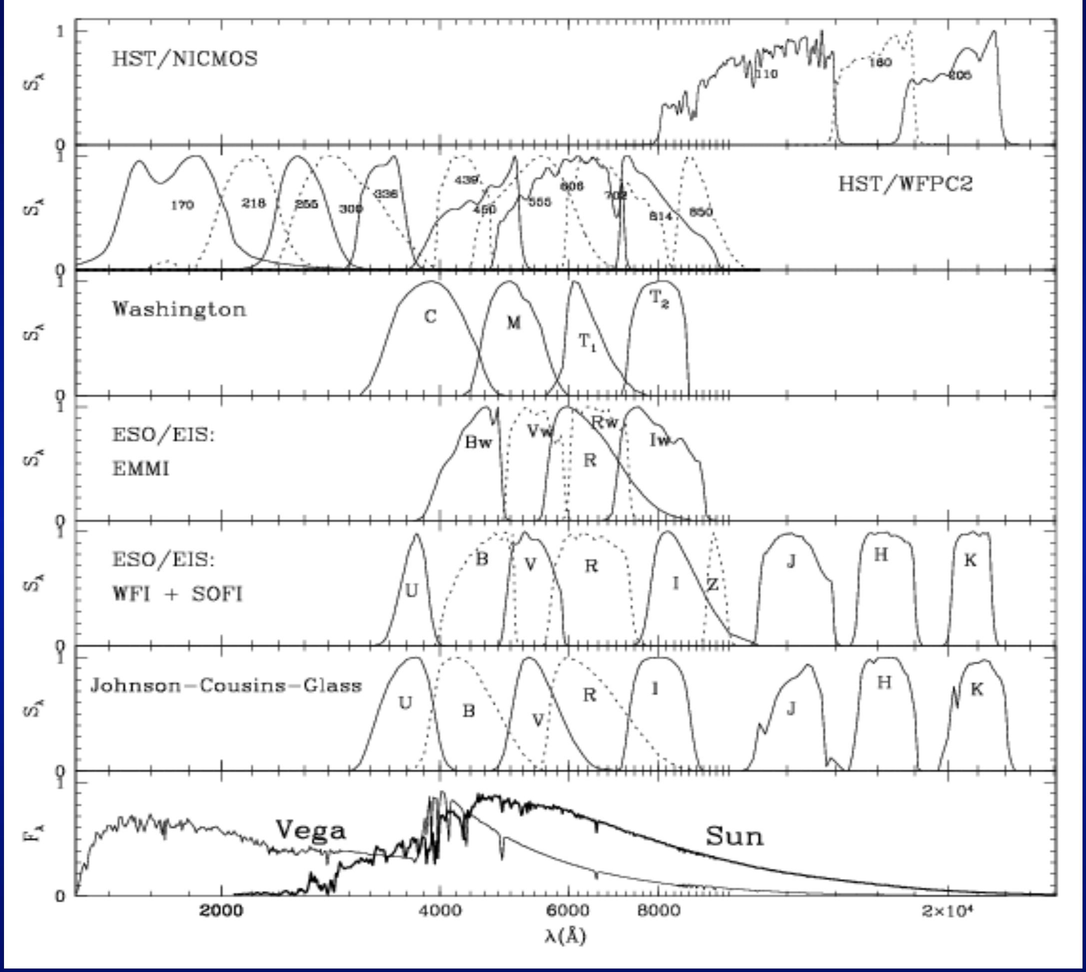
$$M_{bol} = M_{bol,0} - 2.5\log(L/L_0) = M_{bol,0} - 2.5\log(4\pi R^2 F_{bol}/L_0)$$

$$F_{bol} = \int_0^\infty F_\lambda \, d\lambda = \sigma T_{eff}^4$$

$$BC_{S_\lambda} = M_{\mathrm{bol},\odot} - 2.5\ \log\left[4\pi(10\,\mathrm{pc})^2 F_{\mathrm{bol}}/L_\odot\right] + 2.5\ \log\left(\frac{\int_{\lambda_1}^{\lambda_2} \lambda F_\lambda 10^{-0.4A_\lambda} S_\lambda \mathrm{d}\lambda}{\int_{\lambda_1}^{\lambda_2} \lambda f_\lambda^0 S_\lambda \mathrm{d}\lambda}\right) - m_{S_\lambda}^0$$

$$M_{bol,0} = 4.77 \longleftrightarrow L_0 = 3.844 \times 10^{33}\, ergs^{-1}$$

We have to fix the solar bolometric magnitude, the spectrum of the reference star (e.g. Vega) and its magnitudes in the various photometric filters



HST/NICMOS
110
160
205
HST/WFPC2
170
218
255
300
336
439
450
555
606
702
814
850
Washington
C
M
T_1
T_2
ESO/EIS:
EMMI
Bw
Vw
Rw
R
Iw
ESO/EIS:
WFI + SOFI
U
B
V
R
I
Z
J
H
K
Johnson-Cousins-Glass
U
B
V
R
I
J
H
K
Vega
Sun
S_λ
F_λ
2000
4000
6000
8000
2×10^4
λ(Å)

Color indices

A generic colour index $(A-B)$ is defined as

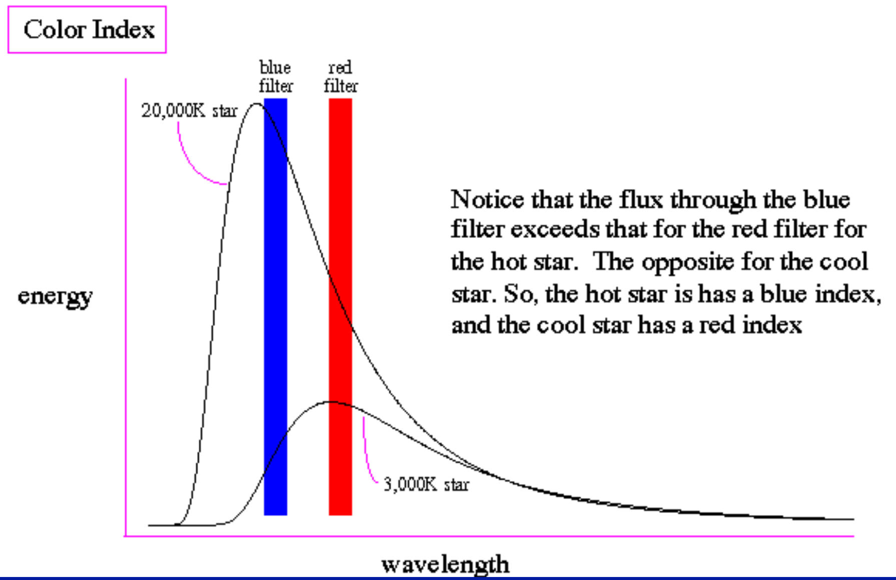
$$(A-B)_0 \equiv M_A - M_B$$

the difference of the magnitudes in two different photometric filters. It is clearly independent of the stellar distance, but it is affected by the extinction.
If we denote with $(A-B) \equiv m_A - m_B$ the observed value of the index $(A-B)_0$, we have that

$$(A-B)_0 = M_A - M_B = m_A - 5\,log(d) + 5 - A_A - m_B + 5\,log(d)$$
$$-5 = (A-B) - (A_A - A_B) = (A-B) - E(A-B)$$

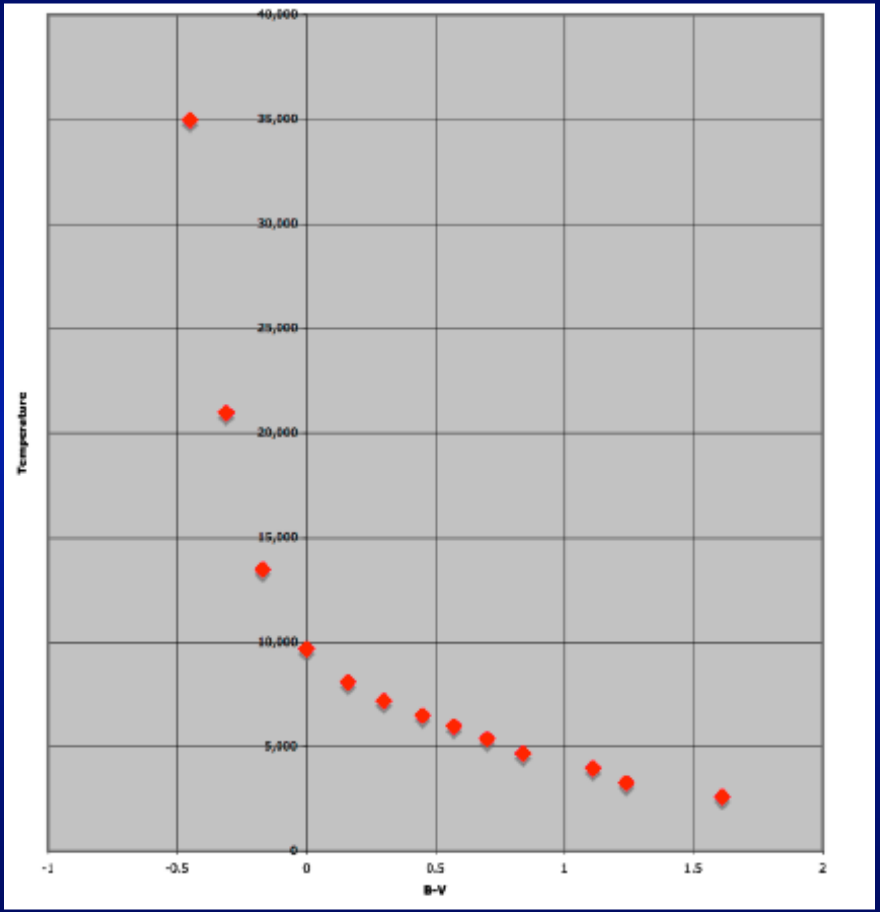
where we have defined $E(A-B) \equiv A_A - A_B$ (colour excess or reddening).

Look now at the spectral energy distribution of stars with different T_{eff} and see why colour indices are related to the stellar T_{eff} (in the same way as magnitudes are related to the stellar luminosity). You should also realize that some colour indices are more sensitive to T_{eff} than others.



Spectral Type, Color and Effective Temperature for Main-Sequence Stars

Spectral Type	B-V	T_e(K)
O5	-0.45	35,000
B0	-0.31	21,000
B5	-0.17	13,500
A0	0.00	9,700
A5	0.16	8,100
F0	0.30	7,200
F5	0.45	6,500
G0	0.57	6,000
G5	0.70	5,400
K0	0.84	4,700
K5	1.11	4,000
M0	1.24	3,300
M5	1.61	2,600



Spectral Type, Color and Effective Temperature & BC_V for Main-Sequence Stars

Main-Sequence Stars (Luminosity Class V)

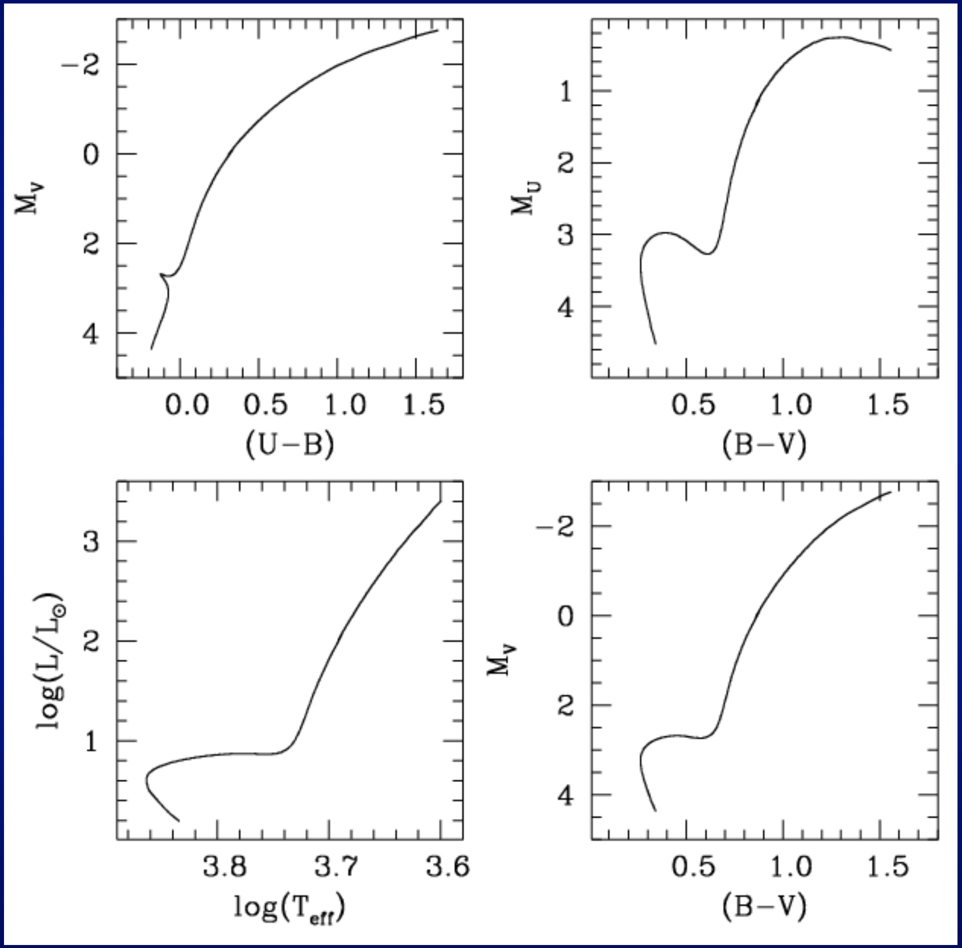
Sp. Type	T_e (K)	$L/L_\odot$	$R/R_\odot$	$M/M_\odot$	M_{bol}	BC	M_V	$U-B$	$B-V$
O5	42000	499000	13.4	60	−9.51	−4.40	−5.1	−1.19	−0.33
O6	39500	324000	12.2	37	−9.04	−3.93	−5.1	−1.17	−0.33
O7	37500	216000	11.0	—	−8.60	−3.68	−4.9	−1.15	−0.32
O8	35800	147000	10.0	23	−8.18	−3.54	−4.6	−1.14	−0.32
B0	30000	32500	6.7	17.5	−6.54	−3.16	−3.4	−1.08	−0.30
B1	25400	9950	5.2	—	−5.26	−2.70	−2.6	−0.95	−0.26
B2	20900	2920	4.1	—	−3.92	−2.35	−1.6	−0.84	−0.24
B3	18800	1580	3.8	7.6	−3.26	−1.94	−1.3	−0.71	−0.20
B5	15200	480	3.2	5.9	−1.96	−1.46	−0.5	−0.58	−0.17
B6	13700	272	2.9	—	−1.35	−1.21	−0.1	−0.50	−0.15
B7	12500	160	2.7	—	−0.77	−1.02	+0.3	−0.43	−0.13
B8	11400	96.7	2.5	3.8	−0.22	−0.80	+0.6	−0.34	−0.11
B9	10500	60.7	2.3	—	+0.28	−0.51	+0.8	−0.20	−0.07
A0	9800	39.4	2.2	2.9	+0.75	−0.30	+1.1	−0.02	−0.02
A1	9400	30.3	2.1	—	+1.04	−0.23	+1.3	+0.02	+0.01
A2	9020	23.6	2.0	—	+1.31	−0.20	+1.5	+0.05	+0.05
A5	8190	12.3	1.8	2.0	+2.02	−0.15	+2.2	+0.10	+0.15
A8	7600	7.13	1.5	—	+2.61	−0.10	+2.7	+0.09	+0.25
F0	7300	5.21	1.4	1.6	+2.95	−0.09	+3.0	+0.03	+0.30
F2	7050	3.89	1.3	—	+3.27	−0.11	+3.4	+0.00	+0.35
F5	6650	2.56	1.2	1.4	+3.72	−0.14	+3.9	−0.02	+0.44
F8	6250	1.68	1.1	—	+4.18	−0.16	+4.3	+0.02	+0.52

Spectral Type, Color and Effective Temperature for Main-Sequence Stars (continued)

Main-Sequence Stars (Luminosity Class V)

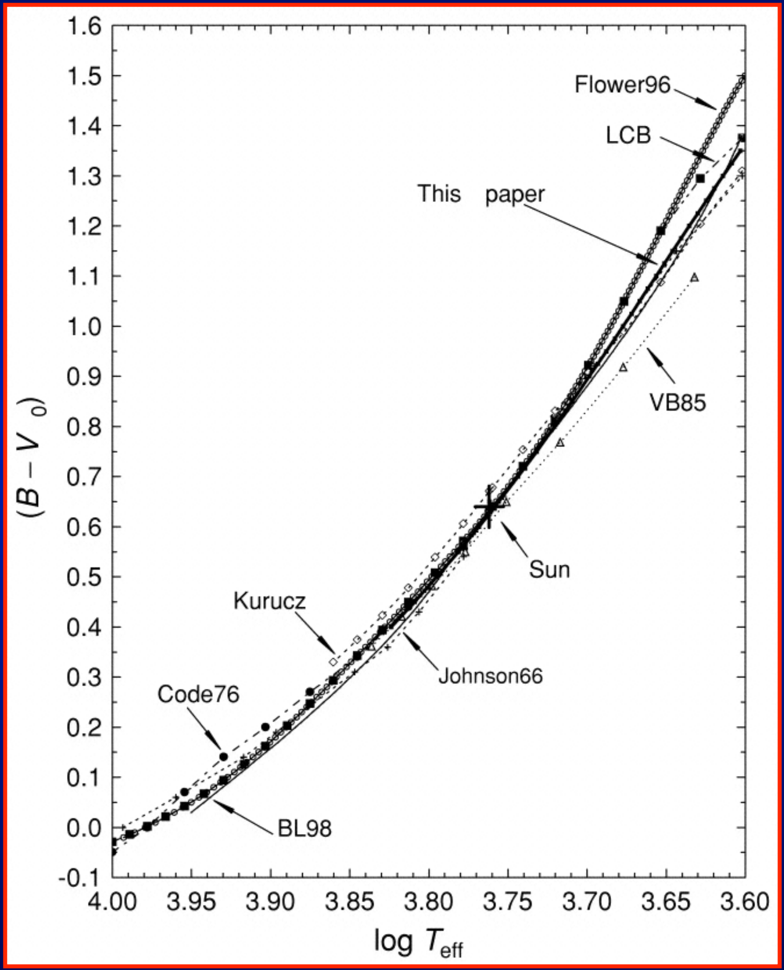
Sp. Type	T_e (K)	$L/L_\odot$	$R/R_\odot$	$M/M_\odot$	M_{bol}	BC	M_V	$U-B$	$B-V$
G0	5940	1.25	1.06	1.05	+4.50	−0.18	+4.7	+0.06	+0.58
G2	5790	1.07	1.03	—	+4.66	−0.20	+4.9	+0.12	+0.63
Sun[a]	5777	1.00	1.00	1.00	+4.74	−0.08	+4.82	+0.195	+0.650
G8	5310	0.656	0.96	—	+5.20	−0.40	+5.6	+0.30	+0.74
K0	5150	0.552	0.93	0.79	+5.39	−0.31	+5.7	+0.45	+0.81
K1	4990	0.461	0.91	—	+5.58	−0.37	+6.0	+0.54	+0.86
K3	4690	0.318	0.86	—	+5.98	−0.50	+6.5	+0.80	+0.96
K4	4540	0.263	0.83	—	+6.19	−0.55	+6.7	—	+1.05
K5	4410	0.216	0.80	0.67	+6.40	−0.72	+7.1	+0.98	+1.15
K7	4150	0.145	0.74	—	+6.84	−1.01	+7.8	+1.21	+1.33
M0	3840	0.077	0.63	0.51	+7.52	−1.38	+8.9	+1.22	+1.40
M1	3660	0.050	0.56	—	+7.99	−1.62	+9.6	+1.21	+1.46
M2	3520	0.032	0.48	0.40	+8.47	−1.89	+10.4	+1.18	+1.49
M3	3400	0.020	0.41	—	+8.97	−2.15	+11.1	+1.16	+1.51
M4	3290	0.013	0.35	—	+9.49	−2.38	+11.9	+1.15	+1.54
M5	3170	0.0076	0.29	0.21	+10.1	−2.73	+12.8	+1.24	+1.64
M6	3030	0.0044	0.24	—	+10.6	−3.21	+13.8	+1.32	+1.73
M7	2860	0.0025	0.20	—	+11.3	−3.46	+14.7	+1.40	+1.80

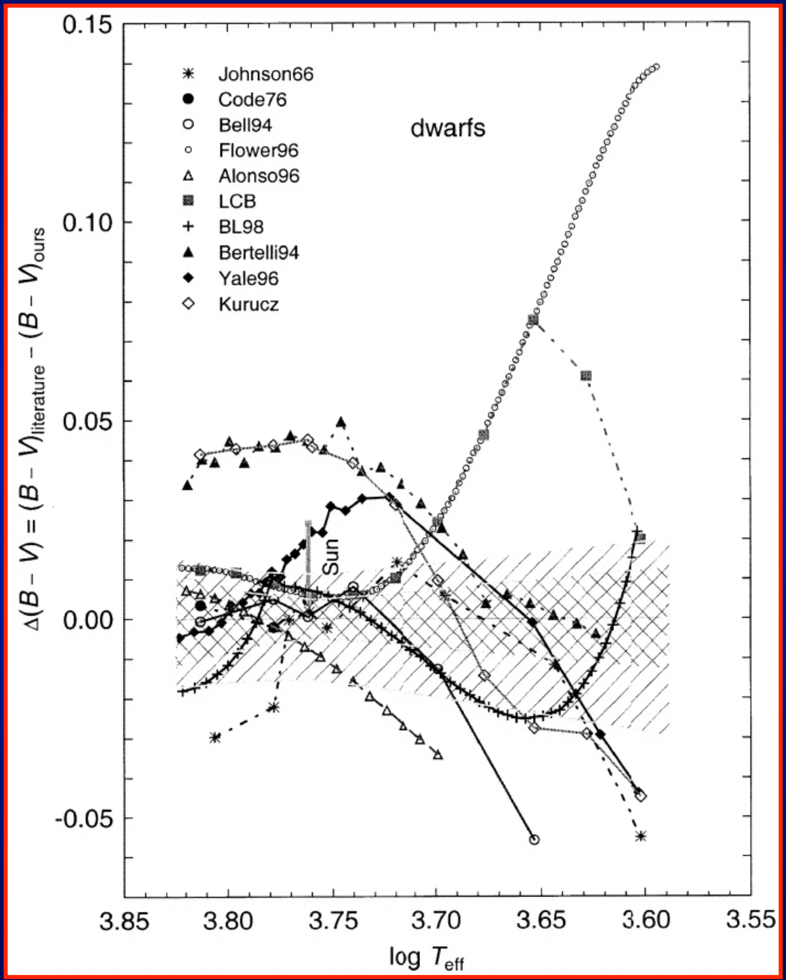
Impact on model predictions



The uncertainty in the color-T_{eff} relations and BC scale

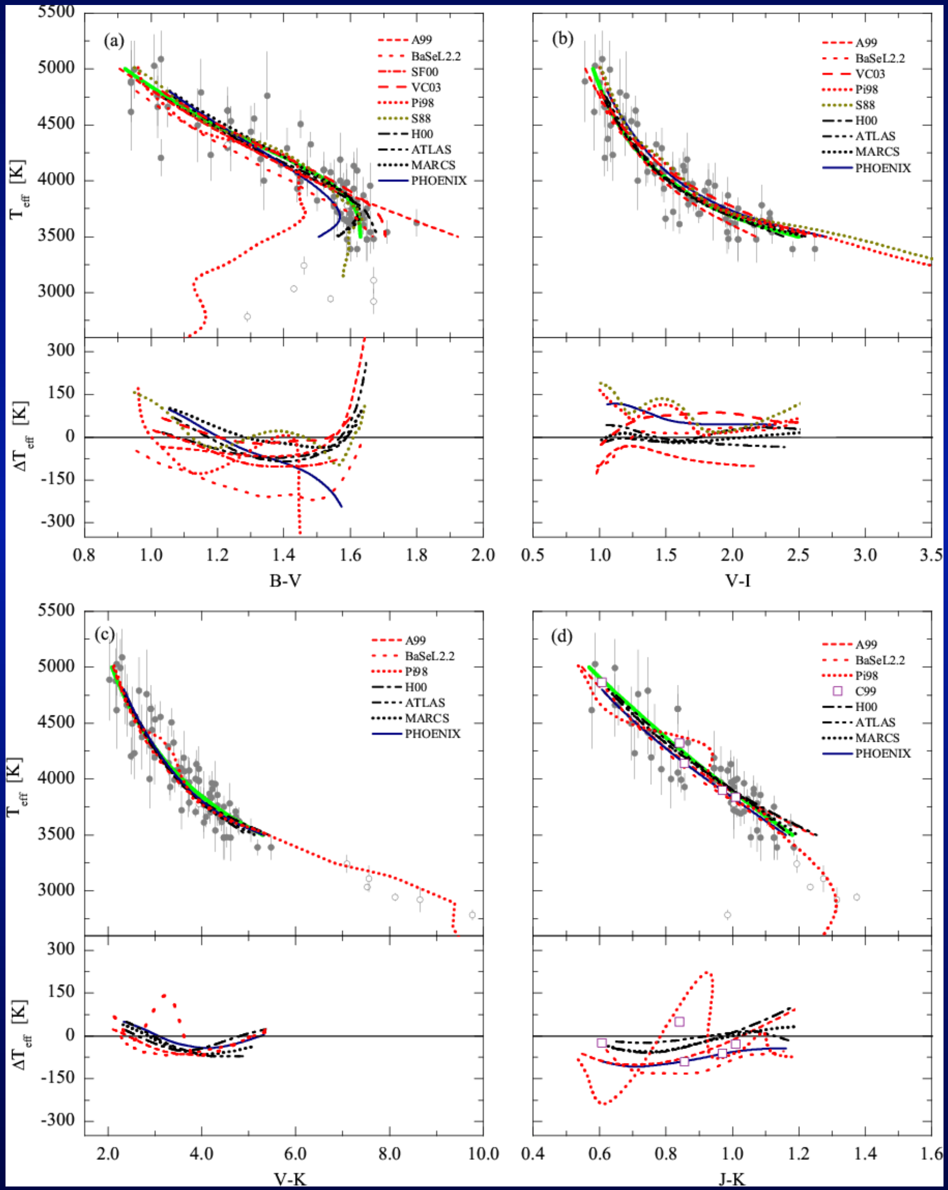
Dwarfs





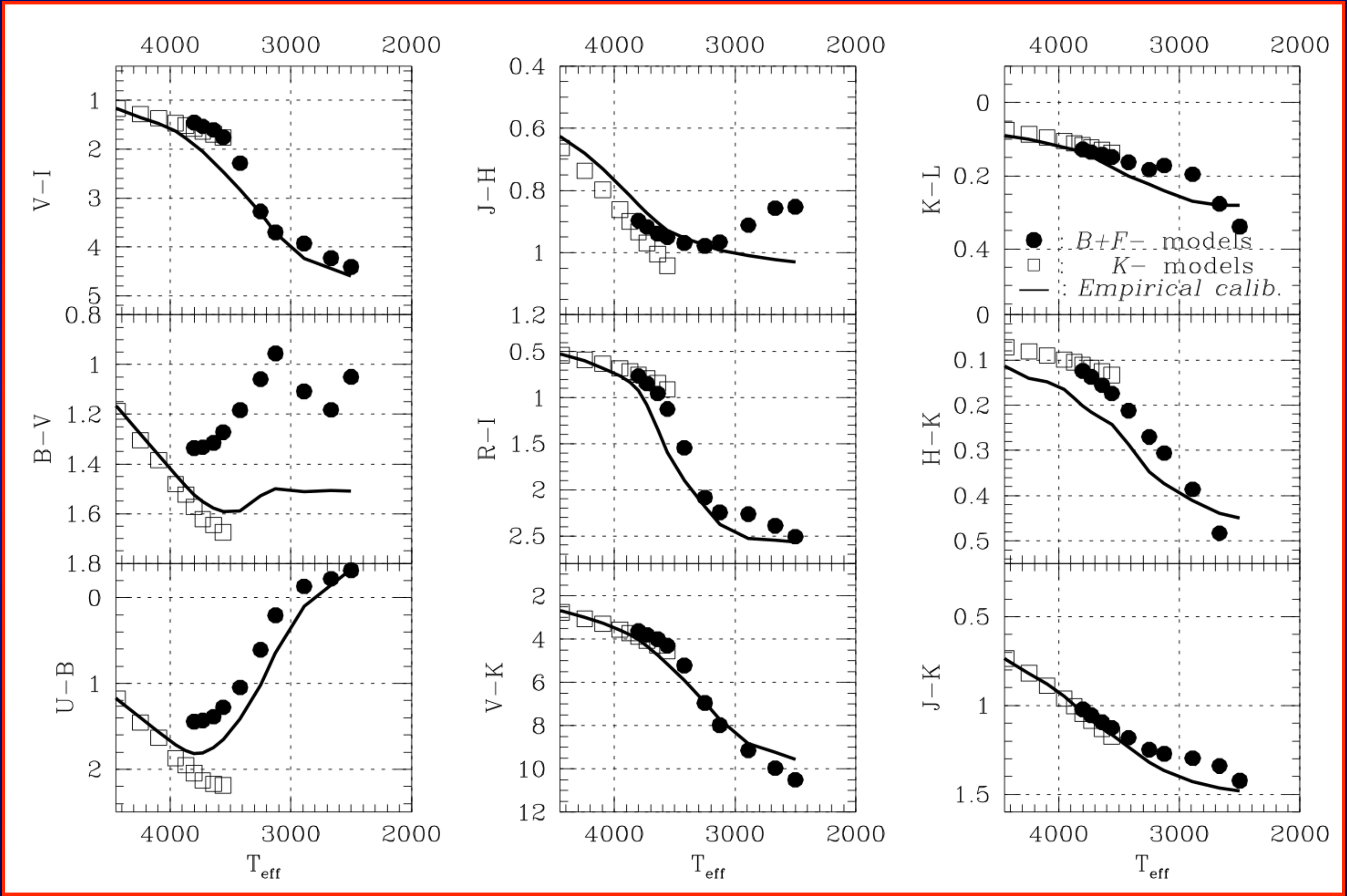
The uncertainty in the color-T_{eff} relations and BC scale

Late type Giants



The uncertainty in the color-T_{eff} relations and BC scale

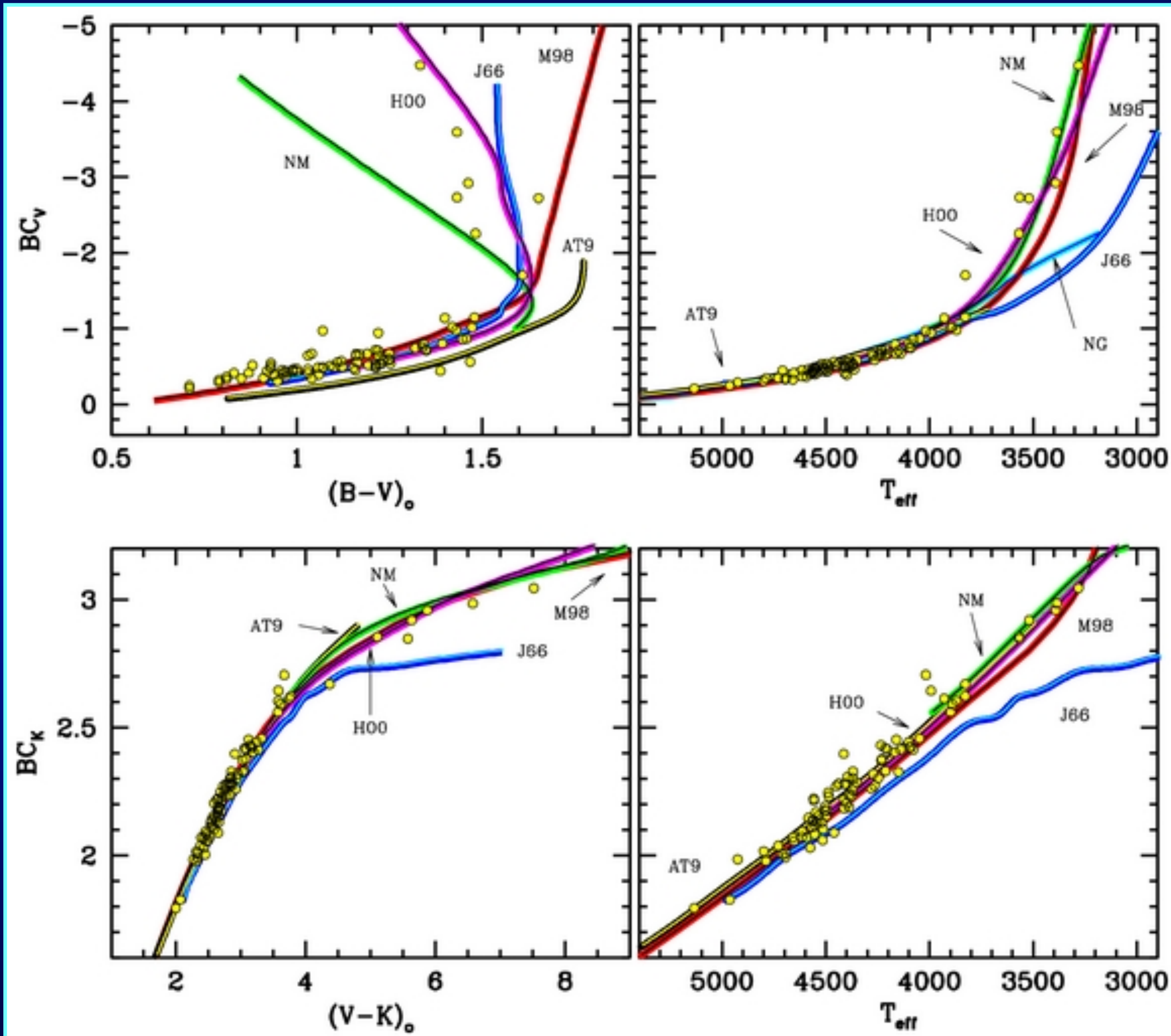
Cool Giants



K = Kurucz; B = Bessell et al.; F= Fluks et al.

The uncertainty in the color-T_{eff} relations and BC scale

Cool Giants

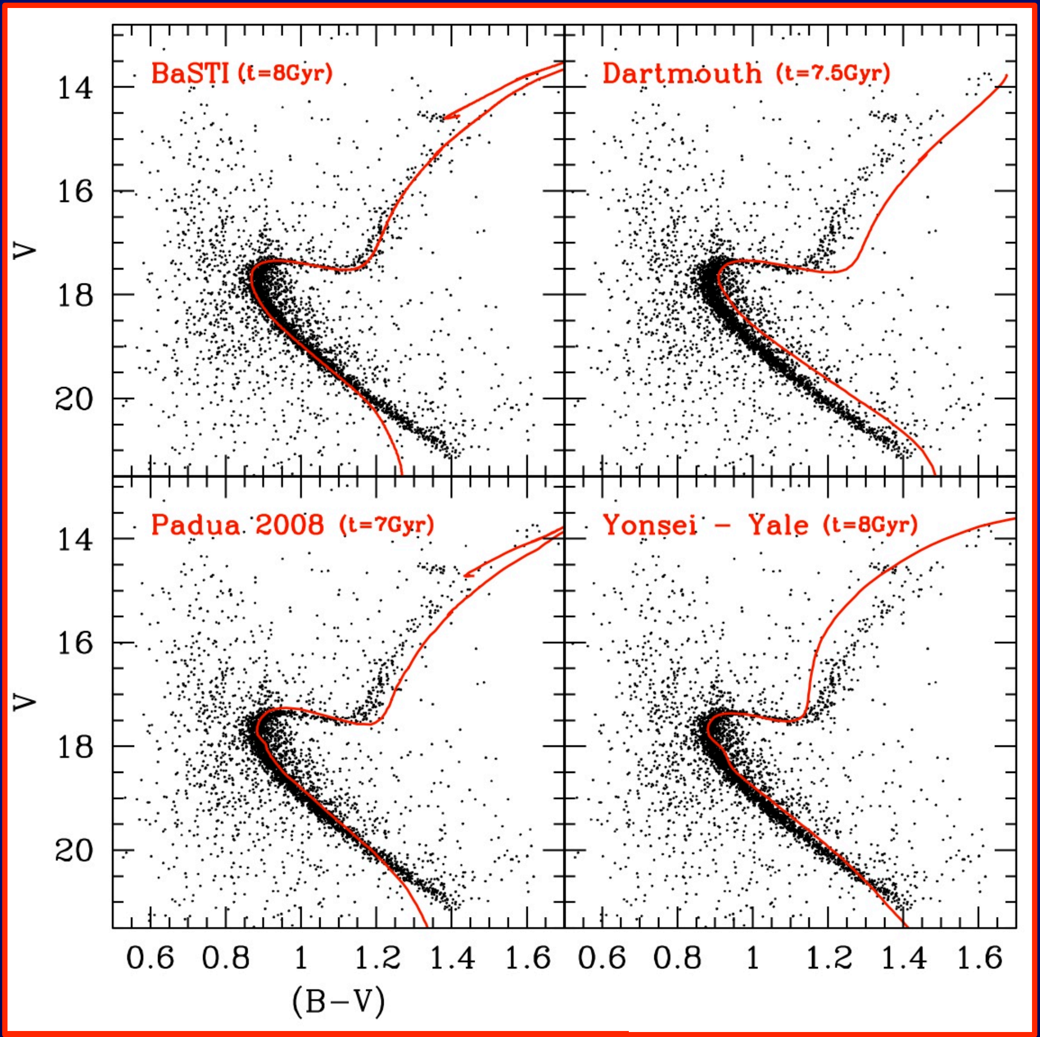


J66= Johnson;
AT9 = Kurucz et al.;
NG= Hauschildt et al.;
M98 = Montegriffo et al;
H00 = Houdashelt et al.;
NM = Bessell et al.

Buzzoni et al. (2010)

A crucial issue: the reliability of models…

Eclipsing binaries can represent an important benchmark for model libraries

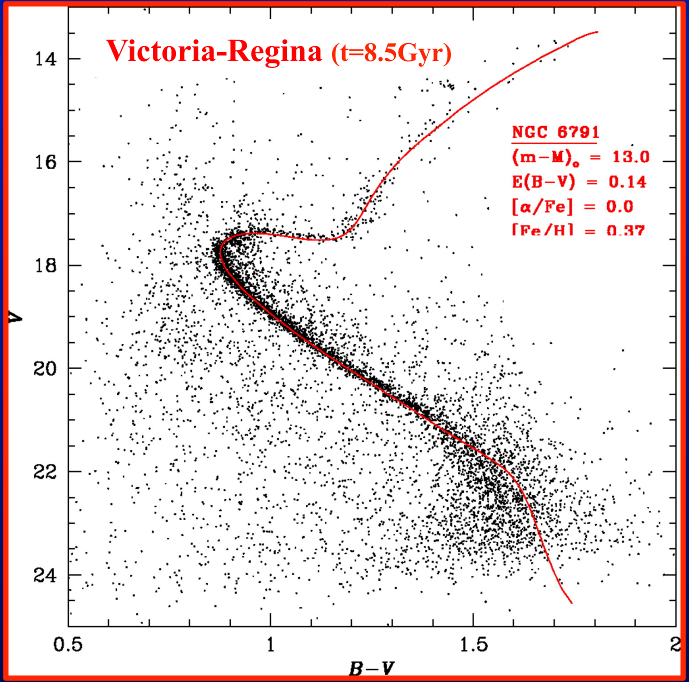


Photometry by Stetson et al. (2003)

The case of V20 in the Galactic Open Cluster NGC6791
(Grundahl et al. 2008)

$(m-M)_V = 13.46 \pm 0.10$

$E(B-V) = 0.15 \pm 0.02$



The effect of the presence of unresolved binaries in the CMD

Binaries that are able to survive in the dense environment of star cluster (in particular GCs) are so close that even the *HST* is not able to resolve the single components…

So, light coming from each star will combine, and the binary system will appear as a single point-like source…

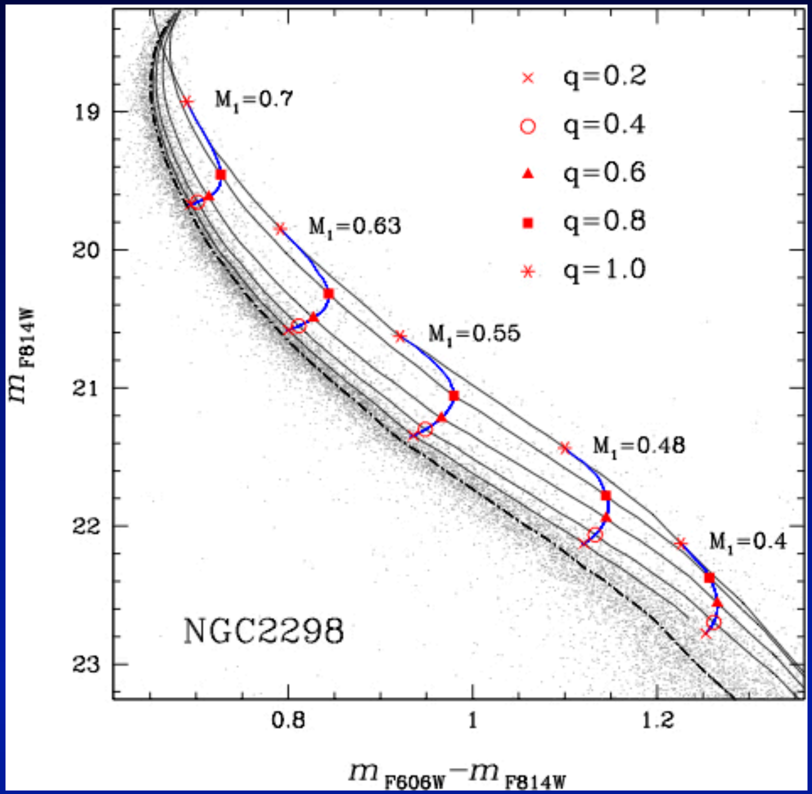
when indicating with M_1, M_2, F_1 and F_2 the mass and fluxes of the two components of the systems, the binary will appear as a single object with a magnitude:

$$m_{\rm bin} = m_1 - 2.5\ \log\left(1 + \frac{F_2}{F_1}\right)$$

In the case of a binary formed by two MS stars (MS-MS binary) the fluxes are related to the two stellar masses and its luminosity depends on the mass ratio $q=M_2/M_1$

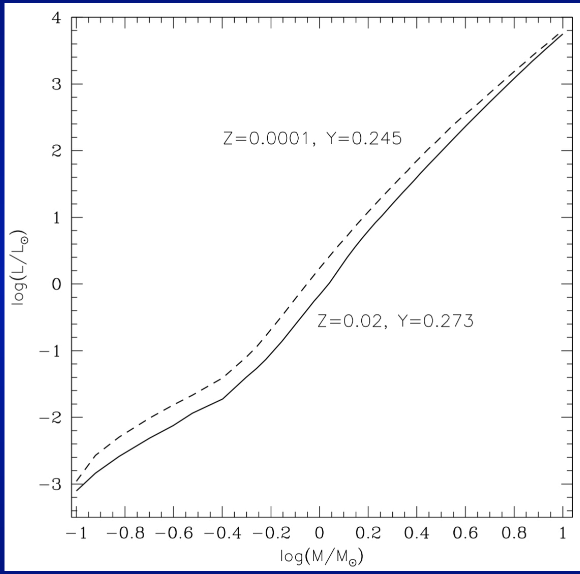
- ✓ The equal-mass binaries form a sequence running almost parallel to the MS, and ~0.75mag brighter;
- ✓ When the masses of the two components are different, the binary will appear redder and brighter than the primary and populate a CMD region on the red side of the MS ridge line (MSRL) but below the equal-mass binary line;

Model MS-MS binary sequences with different mass ratios



Ingredients:

- mass – luminosity relation; $L \propto M^3$



- effective temperature from evolutionary tracks;
- bolometric corrections to move from the H-R diagram to the observational CMDs;

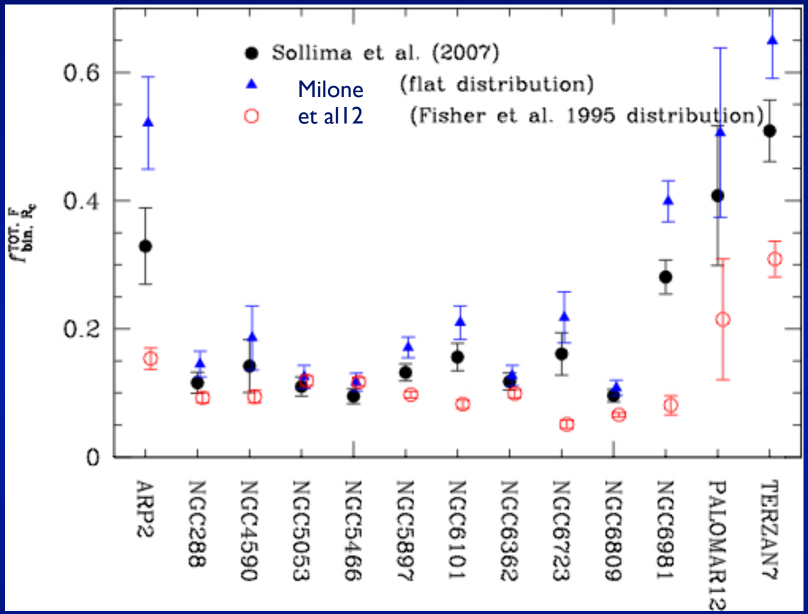
Warning:

for low- and very low-mass MS stars ($M<0.8M_\odot$) the evolutionary lifetime is so long that we can neglect any change in their location in the H-R diagram due to evolution;

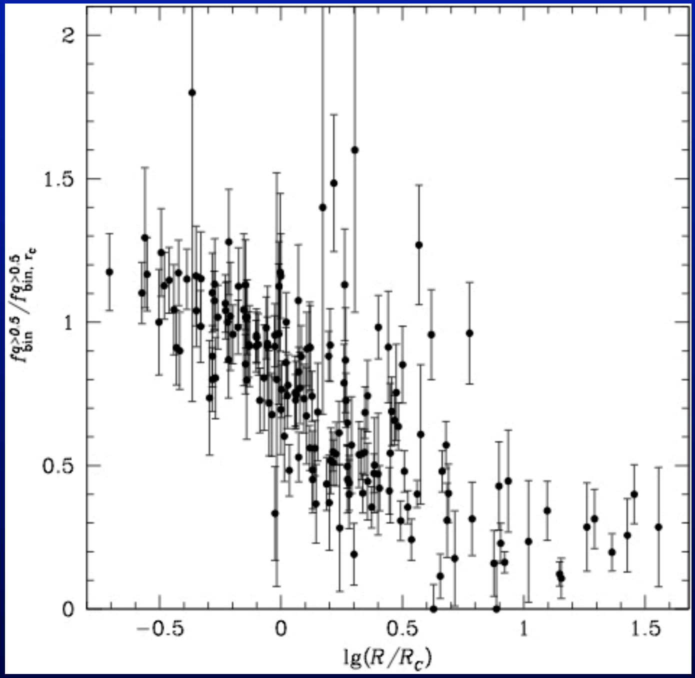
this is no longer true for more massive stars; … time has to be taken into account..

what about the true fraction of unresolved binaries in various environment?

The binary fraction in the core of Galactic GCs

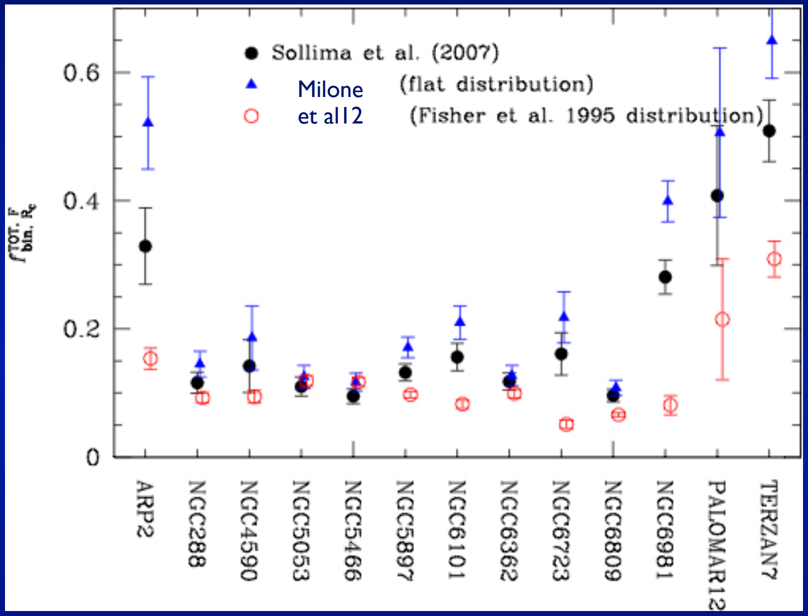


Dynamical effects hugely affect binary evolution. So, we can expect a huge change in the binary fraction in the various cluster regions…



what about the true fraction of unresolved binaries in various environment?

The binary fraction in the core of Galactic GCs



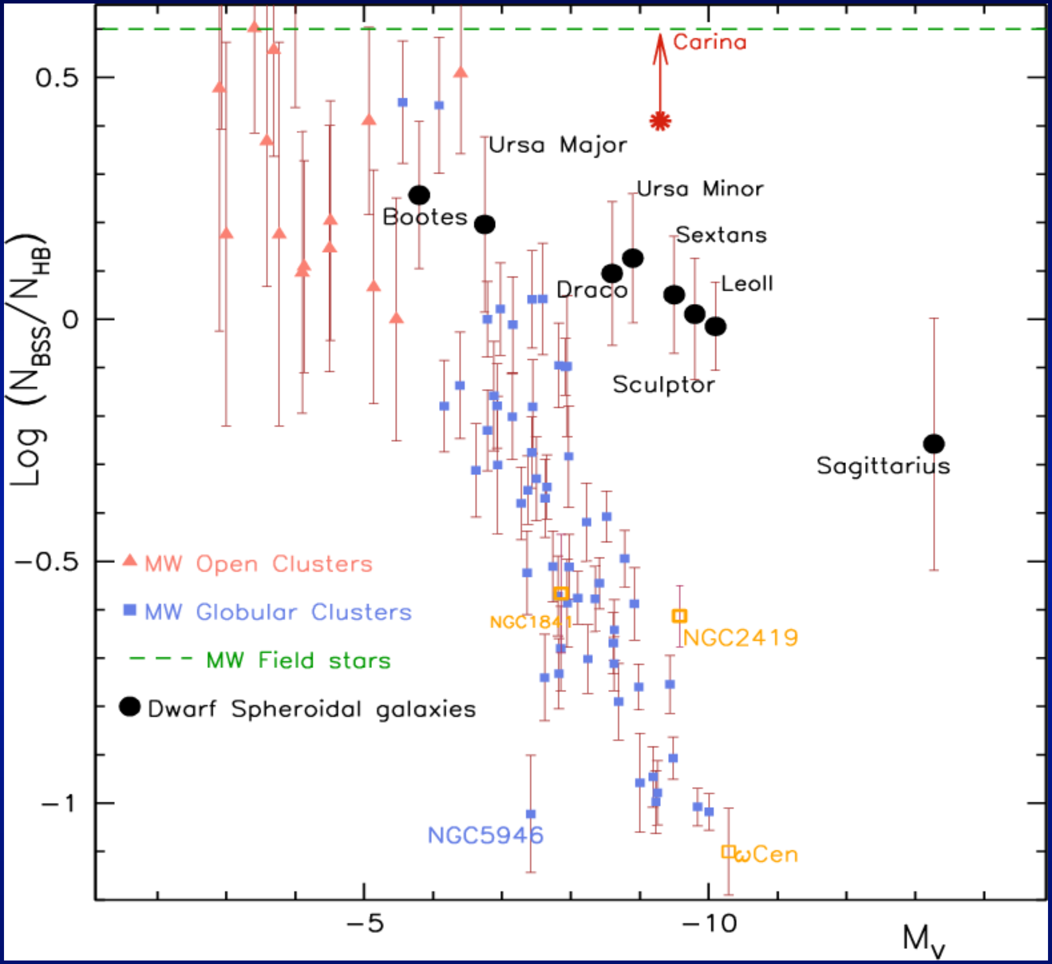
As a consequence of dynamical effects, we can also expect a huge change in the binary fraction when considering different stellar environments…

Local Group dwarfs:

- ✓ no clues… but…

Galactic field:

- ✓ field G-dwarfs (Duquennoy & Mayor 91) show a binary fraction of about 0.6;
- ✓ Binaries appear to be far more common in stars more massive than $1M_{\odot}$ than below, with a multiplicity fraction approaching 100% above a few $M_{\odot}$ (Goodwin 2010);
- ✓ Blue Stragglers frequency…



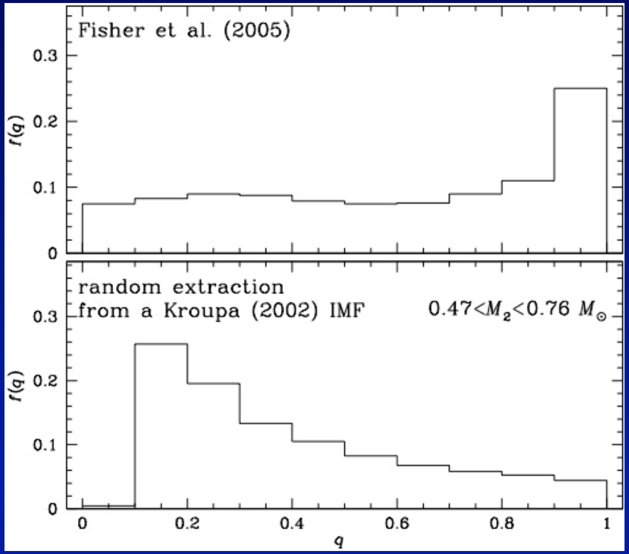
Momany et al. (2007)

mass-ratio (q) distribution

Up to now, there are few observational constraints on the overall mass-ratio distribution of the binary population.

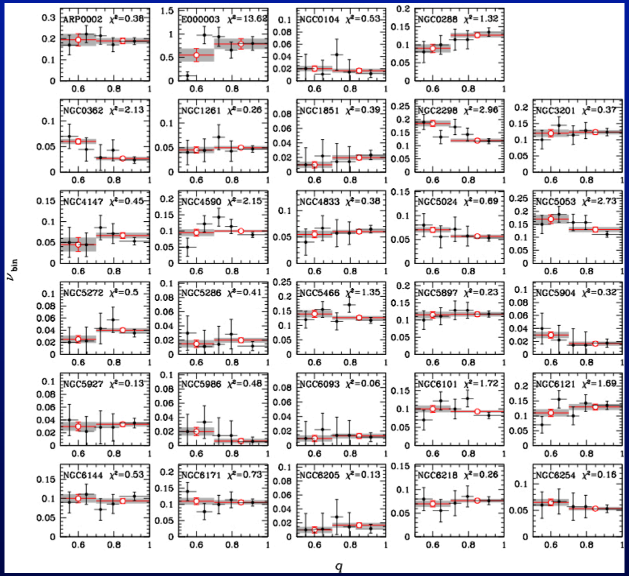
One of the few measures of $f(q)$ for binary systems, comes from Fisher et al. (2005) who estimated the q distribution function from spectroscopic observations of field binaries within 100 parsecs from the Sun.

The binary systems located in the local field has been also studied by Tout (1991), who suggested that the f(q) function can be derived by randomly extracting secondary stars from the "adopted" (observed) initial mass function;



what about in Galactic GCs?

these results are consistent with a flat distribution



The Initial Mass Function: IMF

The IMF provides the information about the "initial" number of star (ΔN) in a given mass range (M – M+ΔM)

The IMF is a fundamental parameter for understanding galaxy properties at all redshifts, including mass/light ratios, star formation rate, metal enrichment, luminosity enrichment;

The main open question is whether the IMF is Universal, or rather it depends on local conditions, including star formation density, cosmic time, etc.;

Theretical derivation of IMF is complex, and far from satisfactory;

Usually, the IMF is represented as a power law, or lognormal functions;

The definition of Initial Mass Function

The *Initial Mass Function* (IMF) is defined as the number of stars per unit volume per unit *logarithmic* mass.

$dN = \xi(\log m_*)\, d\log m_* \sim m_*^{-\Gamma}$ in the case of a power law

This implies:

$dN/dm \sim m^{-\alpha}$ with $\alpha = \Gamma + 1$

where $N(m,m+dm) = \Phi(m)\, dm$, and $\Phi(m)$ represents the number of stars in the mass intervall $[m, m+dm]$.

Salpeter 1955
Bastian et al. 2010

Where has the IMF been measured?
Where can it be measured best?

Edwin E. Salpeter 1955: Measurement of the stellar mass function in the field star population of the Milky Way.

$\xi(\log m_*) \sim (m_*/M_{sun})^{-1.35}$ "Salpeter IMF"

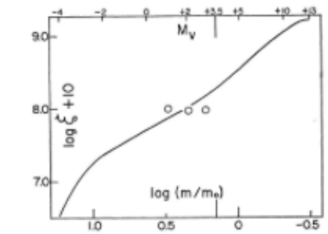
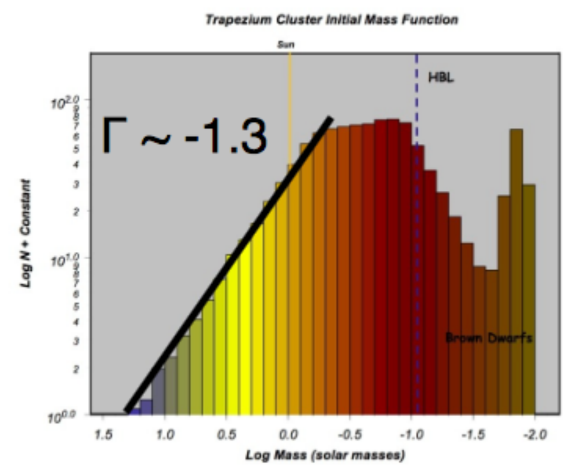


FIG. 2.—The logarithm of the "original mass function," ξ, plotted against the mass, 𝔐, in solar units.

= a power-law IMF with a steep increase towards low-mass stars.

Initial means that the IMF is the mass distribution of stars *at the time of their formation*. In a star cluster, this would be the mass spectrum of the cluster right *after all stars have formed*.



Measuring the initial mass function in the field star population has a dramatic disadvantage: The *star-formation history* of the Milky Way (or the extragalactic system where the IMF shall be deduced) must be known. To convert from the observed distribution of low-mass stars to the complete initial mass spectrum, it must be known *how many stars were formed at any given time in the galaxy*.

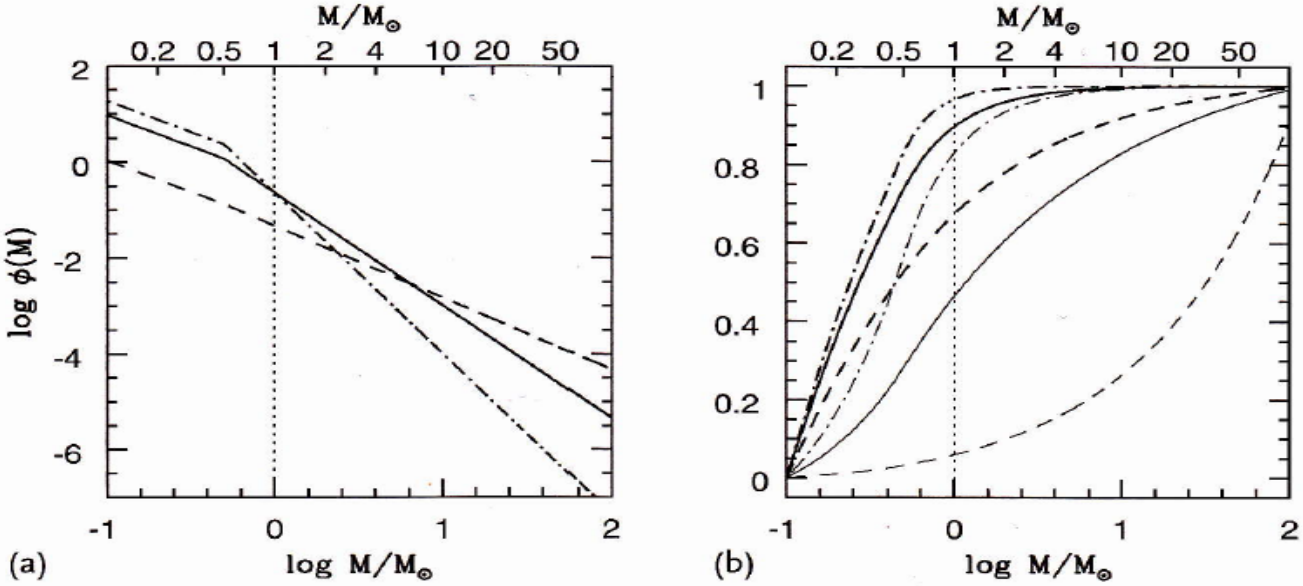


Figure 8.1 (a) Three different IMFs normalized to have the same total stellar mass of 1 $M_\odot$. Below 0.5 $M_\odot$ all three IMFs have the same slope $s = 1.3$ and above it $s = 1.5$, 2.35 (Salpeter) and 3.35, shown as dashed, solid, and dot-dashed lines, respectively. (b) Cumulative distributions of the number (thick lines) and one of the stellar mass (thin lines) for the three IMFs with the same line encoding as in (a).

IMF has a drastic effect on stellar demography.

Most of the nucleosynthesis comes from $M>10M_\odot$

Most of the light in a >10Gyr old stellar population comes from $M\sim1M_\odot$ stars.

For the same amount of gas, different IMFs imply different proportions of low mass to high mass stars.

In a Salpeter IMF, 0.6% of the stars are more massive than $8M_\odot$, while for s=3.35 and 1.5, these fractions are 0.03% and 9%, respectively.

The mass in stars heavier than $8M_\odot$ is 20% for a Salpeter IMF, it drops to 1% for s=3.35, and boost to 77% for a top-heavy s=1.5 IMF.

Is the IMF universal?

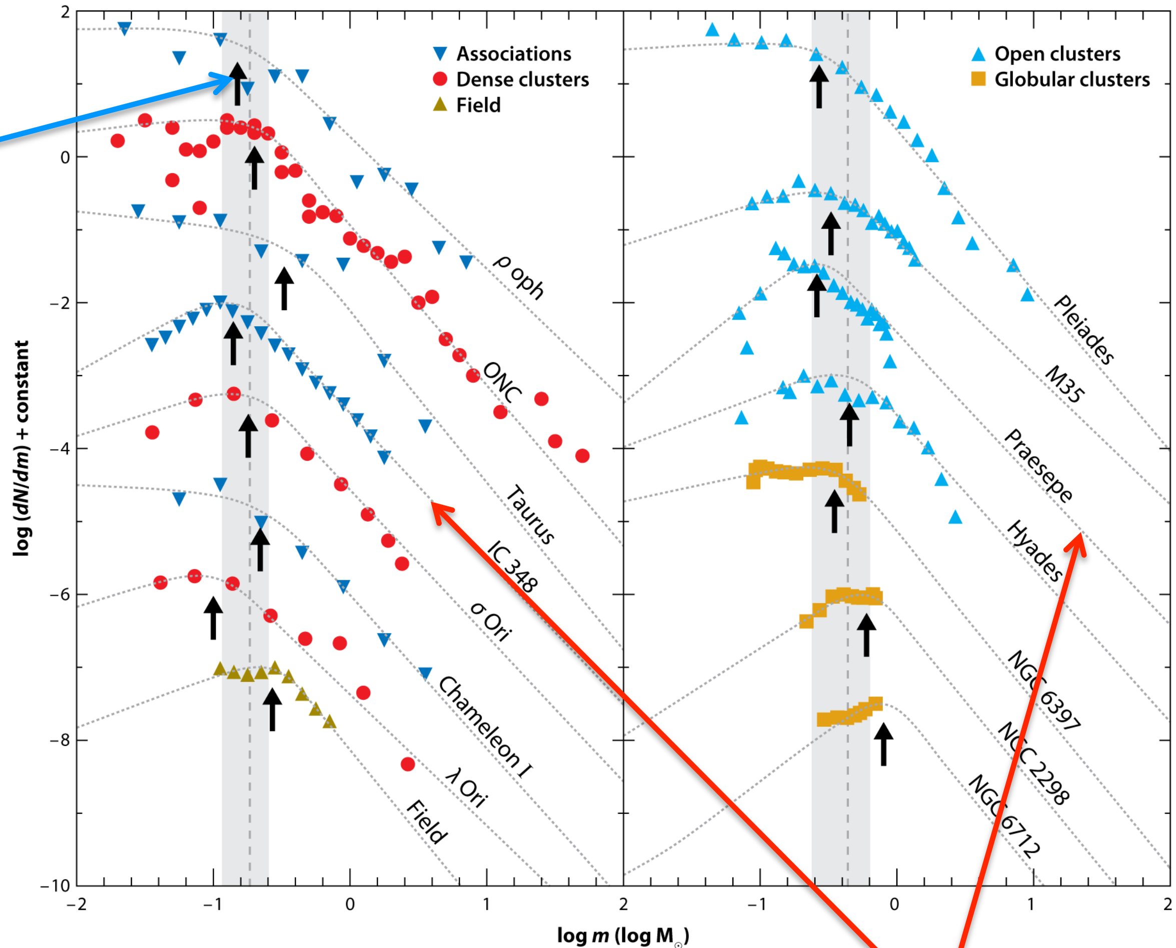
characteristic mass (m_p)
(the most frequent mass in the stellar population)

tapered power-law

$$\chi(m) = \frac{dN}{dm} \propto m^{-\alpha}\left[1 - e^{(-m/m_p)^{\beta}}\right]$$

α is the power law index in the upper end of the mass function

β is the tapering index to describe the lower end of the mass function

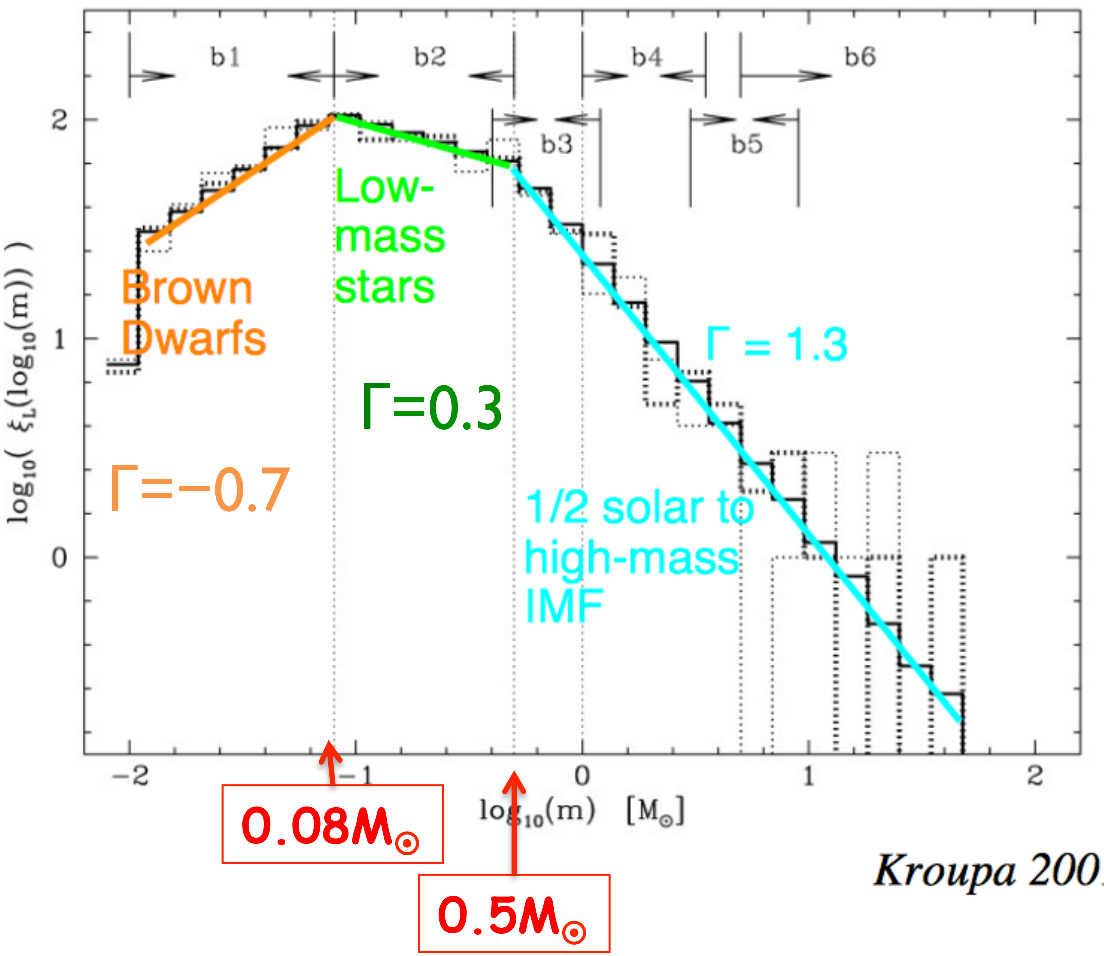


Bastian N, et al. 2010.
Annu. Rev. Astron. Astrophys. 48:339–89

mass slope of the IMF Salpeter

The "Kroupa" IMF

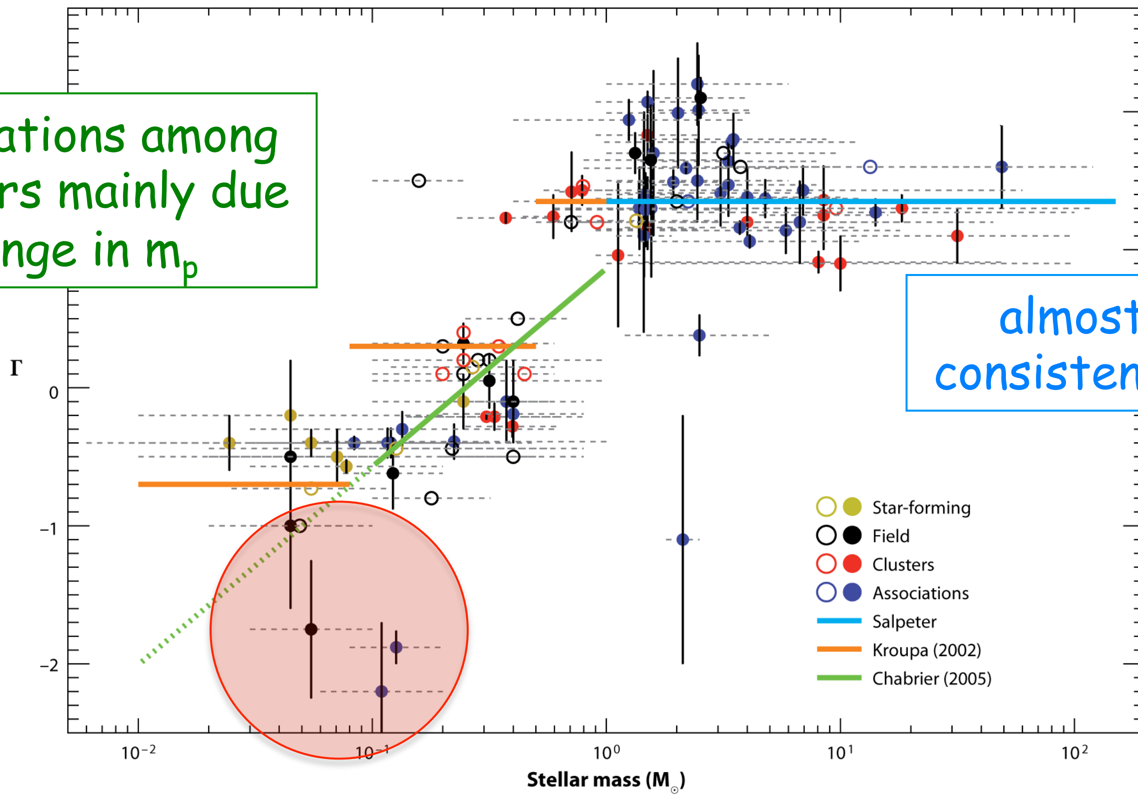
From an observational dataset obtained for young star cluster, Kroupa constructed a sort of "universal IMF"



Kroupa 2001

The "alpha" plot

some variations among the clusters mainly due to change in m_p



almost everywhere consistent with Salpeter

data in the very low-mass regime seem to be inconsistent with a universal IMF

The Star Formation History (SFH)

A Complex Stellar Population (CSP) is a collection of stars formed at different times and with different initial chemical compositions;

The fundamental information that characterizes a CSP is its **Star Formation History (SFH)**, that is the evolution with time of:

1. the amount (i.e. total mass) of stars formed (the **Star Formation Rate – SFR**)

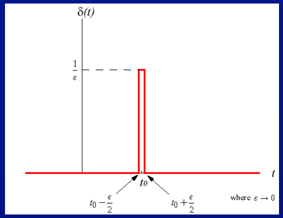
2. their initial chemical composition (the **Age-Metallicity Relation – AMR**)

It is possible formally to denote the SFH as the function $\Psi(t)\Phi(t)$ where $\Psi(t)$ is the SFR and $\Phi(t)$ is the AMR.

The SFR and AMR are not independent, given that each generation of stars will inject in the interstellar medium – through SNe and mass-loss processes along the AGB and RGB stages – large quantities of gas chemically enriched by nuclear processes.

The Star Formation Rate (SFR)

In case if a SSP, i.e. a stellar population of a common age, one has to assume an *instantaneous burst of star formation*. Then the SFR function is represented by a Dirac delta function;



In a CSP such as a galaxy, star formation takes place over a finite period of time; so it can be a complex function of the time…

On generic ground, one can expect that the star-formation rate decreases over time because more and more matter is bound in stars and thus no longer available to form new stars. But… this assumption relies on the fact that the system is isolated…

Since the star-formation history of a galaxy is a priori unknown, it needs to be parametrized in a suitable manner….

A "standard model" based on an exponentially decreasing star-formation rate was established for this:

$$\psi(t) = \tau^{-1} \exp\left[-(t - t_f)/\tau\right] \, \mathrm{H}(t - t_f)$$

where τ is the characteristic duration and t_f the onset of star formation; the Heaviside step function, $H(x) = 1$ for $x \geq 0$, $H(x) = 0$ for $x < 0$, accounts for the fact that $\psi(t) = 0$ for $t < t_f$.

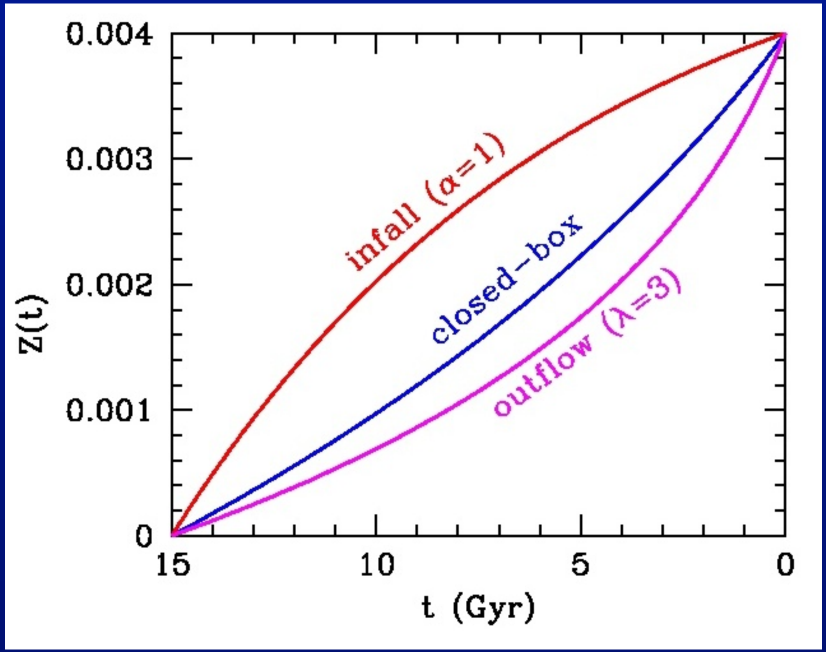
The Age-Metallicity relation

We expect the metallicity Z has to increase with star formation rate, integrated over the lifetime of the galaxy.

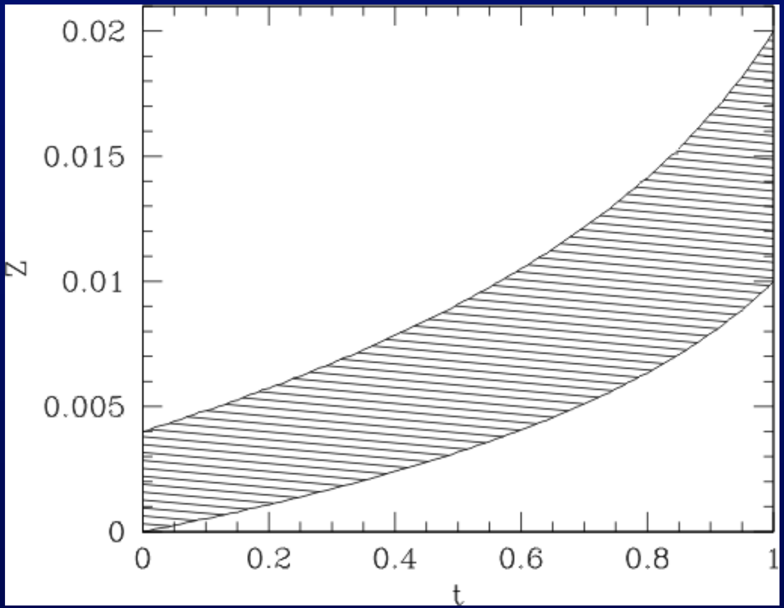
Simple chemical evolution model:

- We assume that at the formation epoch of the stellar population of a galaxy, at time $t = 0$, no metals were present; hence $Z(0) = 0$.
- The galaxy did not contain any stars at the time of its birth, so that all baryonic matter was in the form of gas.
- In addition, we consider the galaxy as a closed system out of which no matter can escape or be added later on by processes of accretion or merger.
- we assume that the timescales of the stellar evolution processes that lead to the metal enrichment of the galaxy are small compared to the evolutionary time-scale of the galaxy.

Under these assumptions, we can now derive a relation between the metallicity and the gas content of a galaxy.



0.004
0.003
0.002
0.001
0
Z(t)
infall (α=1)
closed-box
outflow (λ=3)
15
10
5
0
t (Gyr)



0.02
0.015
0.01
0.005
0
Z
0
0.2
0.4
0.6
0.8
1
t