The unbroken Ground State of General Relativity

and

DARK MATTER

WORK IN PROGRESS

Max Bañados (PUC). Talk at ESO, 01/2007

-- What is Dark Matter?

We don't know...

-- What is the Dark Matter Problem?

There is no problem! We just need to find what dark matter is made of...

-- There is something out there, that's for sure.

SUSY particles? Is this the only answer?

We shall suggest here a different type of ‘particle’ which may make up for dark matter.

These **“particles” are of topological** nature and are somehow **inherent to general relativity.**

They provide the right phenomenology, at least in some simple examples examined so far.

MENU

Incorporate the state $g_{\mu\nu} = 0$ as a solution of Einstein equations.

Find:

- The gravitational potential outside a spherical source becomes

$$\Phi \sim -\frac{MG}{r} + C_1\sqrt{\lambda}\log(r)$$

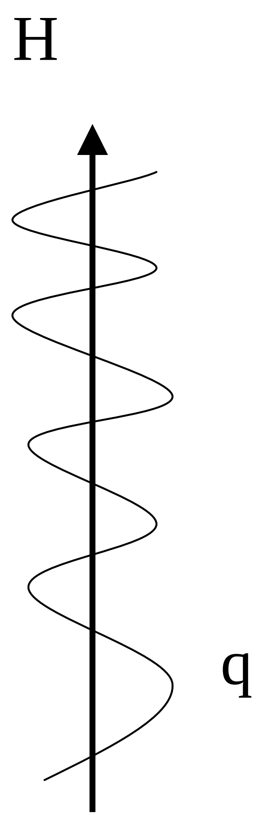
- The Friedman equation for an expanding Universe becomes

$$\frac{\dot{a}^2}{a^2} + \frac{k(1-2\lambda)}{a^2} = \frac{8\pi G}{3}\rho + \frac{C_2}{a^3}$$

λ is a coupling constant. C_1 and C_2 are adjustable constants.

An Analogy:

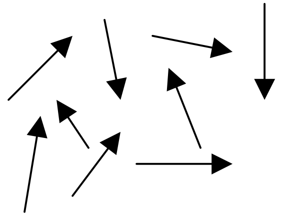
A charged particle near an external magnetic field



Observing the particle’s orbit we could infer the value of the field H

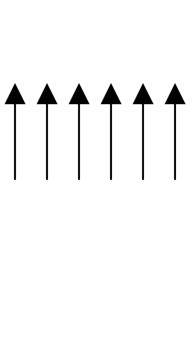
If H=0 the particle follows as straight line

Consider now a system of spins:



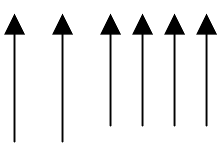
$$\langle \vec{S} \rangle_T = 0$$

Add an external field H. The spins align in the direction of the field



$$\langle \vec{S} \rangle_T \neq 0$$

Remove the external field, at a low temperature



$$\langle \vec{S} \rangle_T \neq 0$$

Without H, the particle still feels a magnetic field, a **“dark” magnetic field,** provided by the internal spin structure, not by currents

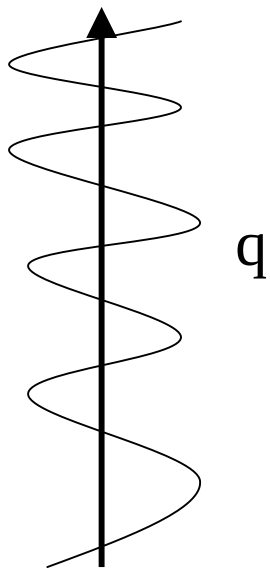
$$\vec{H}' = \langle \vec{S} \rangle_T$$

Keeping H, the total magnetic field is stronger…

$$\vec{H}_T = \vec{H} + \langle \vec{S} \rangle$$

visible field

"Dark field"



If we didn't know about spins, this experiment would make us think that there are more currents than the one generating H.

Is there a similar internal structure in Gravity?

Could the gravitational force be stronger than the visible mass provided?

Is there curvature not associated to matter?

MOND theories modify Newton's equations,

$$\mu(a/a_0) \quad m\, a = F$$

and achieve right phenomenology without dark matter.

The internal structure we have in mind shows up by studying Einstein equations in the limit of zero metric…

$$g_{\mu\nu} = 0$$

Topological Theories: A Universe with no metric

-- distances, times, shapes and forms are not defined

-- A football ball and a rugby ball are the same thing…

-- charges, spin, and other physical properties ***can*** be defined

-- Physicist E.Witten won the Fields Medal for his work on topological Universes and the theoretical 'physics' behind them

Is this a purely mathematical and Academic idea with no possible observational effect?

Let us recall the main formulae of General Relativity

$$
\begin{aligned}
G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2} g^{\alpha\beta} g_{\mu\nu} R_{\alpha\beta} \\
R^{\mu}{}_{\nu\alpha\beta} &= \partial_\alpha \Gamma^{\mu}{}_{\nu\beta} + \Gamma^{\mu}{}_{\sigma\alpha}\Gamma^{\sigma}{}_{\nu\beta} - \partial_\beta \Gamma^{\mu}{}_{\nu\alpha} - \Gamma^{\mu}{}_{\sigma\beta}\Gamma^{\sigma}{}_{\nu\alpha} \\
\Gamma^{\mu}{}_{\alpha\beta} &= \frac{1}{2} g^{\mu\sigma}(g_{\sigma\alpha,\beta} + g_{\sigma\beta,\alpha} - g_{\alpha\beta,\sigma})
\end{aligned}
$$

The metric inverse appears in two places and this is why $g_{\mu\nu} = 0$ has never been considered as part of general relativity.

HOWEVER.... Note that the metric inverse always appears in company with a metric. Thus, when $g_{\mu\nu} = 0$ the above formulae have the structure

$$\frac{\mathbf{0}}{\mathbf{0}} = ?$$

THIS IS OUR KEY OBJECT

Let us write the Einstein tensor in the form:

$$G_{\mu\nu}(\Gamma, g) = R_{\mu\nu}(\Gamma) - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} R_{\alpha\beta}(\Gamma), \tag{1}$$

The relationship between $\Gamma^{\mu}{}_{\alpha\beta}$ and $g_{\mu\nu}$ is represented by

$$g_{\mu\nu;\rho} = g_{\mu\nu,\rho} - \Gamma^{\sigma}{}_{\mu\rho} g_{\sigma\nu} - \Gamma^{\sigma}{}_{\nu\rho} g_{\mu\sigma} = 0. \tag{2}$$

- If $g_{\mu\nu}$ is invertible then (2) implies $\Gamma^{\mu}{}_{\alpha\beta} = \frac{1}{2} g^{\mu\sigma}(g_{\sigma\alpha,\beta} + g_{\sigma\beta,\alpha} - g_{\alpha\beta,\sigma})$, and we recover the usual Einstein tensor.

- If $g_{\mu\nu} = 0$, then Eq. (2) is an identity and Γ is fully decoupled from $g_{\mu\nu}$.

The curvature $R_{\mu\nu}(\Gamma)$ can take arbitrary values!

But, is the Einstein tensor really well defined at $g_{\mu\nu} = 0$...?

- The question is, what is the value of

$$\frac{1}{2} g_{\mu\nu} g^{\alpha\beta} R_{\alpha\beta}(\Gamma) = ?????$$

  when $g_{\alpha\beta} \to 0$.

- At $g_{\mu\nu} = 0$ the only tensor structure present is the curvature $R_{\mu\nu}(\Gamma)$.

- Thus, if the above limit exist, it must be proportional to $R_{\mu\nu}(\Gamma)$ itself

- We thus write

$$\lim_{g \to 0} \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} R_{\alpha\beta} = \lambda' \, R_{\mu\nu}$$

  where λ' is a dimensionless constant

The curvature and Einstein tensor are different from zero at the 'unbroken' ground state: ***Spontaneous "Geometrization".***

To incorporate $g_{\mu\nu} = 0$ as a solution to Einstein equations we write

$$G_{\mu\nu} = \lambda R_{\mu\nu}(\Gamma) + T_{\mu\nu}$$

'DARK MATTER' **VISIBLE MATTER**

General Relativity is now coupled to a topological theory, which contributes to the energy momentum tensor.

These topological 'particles' may account for Dark Matter

The topological connection Γ is not an independent 'outside' field

The key point is to realize that the Cristoffel symbol approaches the topological connection in the limit $g_{\mu\nu} = 0$,

$$\frac{1}{2} g^{\mu\sigma}(g_{\sigma\alpha,\beta} + g_{\sigma\beta,\alpha} - g_{\alpha\beta,\sigma}) \quad \longrightarrow \quad \Gamma^{\mu}{}_{\alpha\beta}, \qquad (g \to 0)$$

Now, this limit is tricky. This is a limit on a space of functions!

For example, let $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$. What is the value of

$$\frac{f'(x)}{f(x)}$$

as $a_0, a_1, a_2, \ldots \to 0$?

Let $f(x) = a + bx + cx^2$ and take the limit $a, b, c \to 0$ in three different ways.

- In the order c, b, a

$$\lim \frac{f'}{f} = \lim \frac{b + 2cx}{a + bx + cx^2} = \lim \frac{b}{a + bx} = \lim \frac{0}{a} = 0$$

- In the order a, b, c

$$\lim \frac{f'}{f} = \lim \frac{b + 2cx}{a + bx + cx^2} = \lim \frac{b + 2cx}{bx + cx^2} = \lim \frac{2cx}{cx^2} = \frac{2}{x}$$

- First $a \to 0$ and then $c = b$ together

$$\lim \frac{f'}{f} = \lim \frac{b + 2cx}{a + bx + cx^2} = \lim \frac{b + 2cx}{bx + cx^2} = \lim \frac{b(1 + 2x)}{b(x + x^2)} = \frac{(1 + 2x)}{(x + x^2)}$$

In conclusion, $\lim \frac{f'}{f} = q(x)$ is an arbitrary function, ${\Gamma^{\mu}}_{\alpha\beta}$ (!), hopefully fixed by the theory.

BIANCHI IDENTITIES

The other type of quotient involves $f(x)$ and $g(x)$, two different components of the metric tensor. What is the value of

$$\lim \frac{f(x)}{g(x)}$$

as $f, g \to 0$??

- f and g transform differently under coordinate changes. If we set, e.g., $\lim \frac{f}{g} = 1$ this would not be preserved by a coordinate change.

- If we set, $\lim \frac{f}{g} = q(r)$, an arbitrary **finite** function, it would imply a fine tunned limit. Not natural because f and g are different components of the metric.

- The most natural result for the limit which preserves coordinate invariance is

$$\text{either:} \quad \frac{f(x)}{g(x)} = 0, \quad \text{or} \quad \frac{f(x)}{g(x)} = \infty$$

Hopefully the theory will guide us on the correct choice.

Applying the limit

For the following metric we consider the limit $A, B, N \to 0$

$$ds^2 = -A^2(r)dt^2 + N^2(r)dr^2 + B^2(r)(d\theta^2 + \sin^2\theta d\phi^2)$$

R(Γ)

The curvature becomes

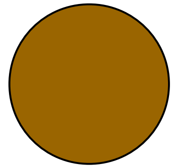
$$\begin{aligned}
R_{tt} &= \frac{A}{N}\left(-\frac{A'}{N}\frac{N'}{N} + \frac{A''}{N} + \frac{2A'B'}{NB}\right) && \longrightarrow \quad 0 \\
R_{rr} &= -\frac{A''}{A} + \frac{A'N'}{AN} - \frac{2N'B'}{NB} && \longrightarrow \quad q(r) \\
R_{\theta\theta} &= \frac{B}{N}\left(-\frac{B'A'}{NA} + \frac{B'N'}{N^2} - \frac{B''}{N}\right) - \frac{B'^2}{N^2} + 1 && \longrightarrow \quad 1
\end{aligned}$$

We choose here

$$\lim \frac{A}{N} = 0, \qquad \lim \frac{B}{N} = 0,$$

because it gives a finite result.

Spherical source:



Solve the vacuum Einstein equations assuming spherical symmetry

$$ds^2 = -f(r)\,h(r)dt^2 + \frac{dr^2}{h(r)} + r^2 d\Omega^2$$

$$\begin{aligned} h &= 1 - \frac{2M}{r} & (1) \\ f &\simeq 1 + C\sqrt{\lambda}\,\log(r) & (2) \end{aligned}$$

- C is an adjustable integration constant.
- $C\log(r)$ is characteristic of flat rotation curves
- The exact solution for $f(r)$ is an hypergeometric function.

FRW cosmology

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

The new equations leds to a modified Friedman equation,

$$\frac{\dot{a}^2}{a^2} + \frac{k(1 - 2\lambda)}{a^2} = \frac{8\pi G}{3}\rho + \frac{C_2}{a^3}$$

- If $\lambda = 1/2$ the spatial sections could be curved
- C_2 is an adjustable integration constant contributing to the matter sector $\sim 1/a^3$

Conclusions

We have modified Einstein equations to incorporate the state with zero metric as a solution

The new energy-momentum tensor contain two contributions ‘visible’ and ‘dark’

For a spherical source the gravitational potential develops a logarithmic branch characteristic of flat rotation curves

For FRW cosmology, there is an adjustable contribution in the matter section.

Future work:

-- Give precise definitions to the limits on space of functions

-- Solve realistic galactic models and fit to observational data

-- Explore the coupling of topological theories on other scales

-- Bianchi identities as ‘equation of motion’ of the topological theory