HART: An Efficient Modeling Framework for Simulated Solar Imaging

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Haystack & AOSS Ray Tracer (HART)

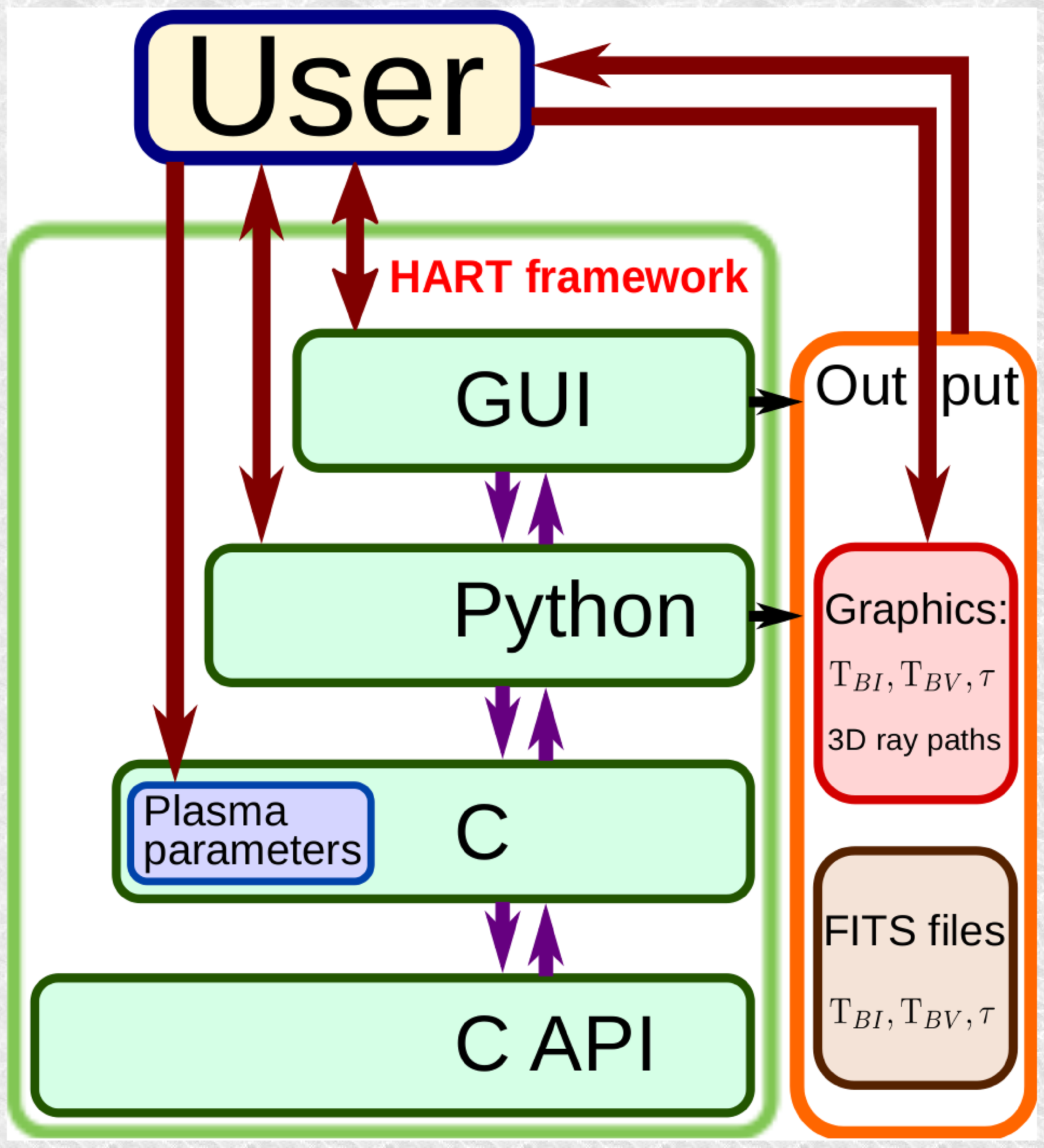
Why do we need HART? To provide the simulation and modeling support for interpreting high quality solar images from modern low frequency (LF) radio interferometers like the Murchison Widefield Array (MWA), Low Frequency Array (LOFAR), Long Wavelength Array (LWA), and others.

How does HART solve this problem? For every pixel of a simulated image HART computes the corresponding ray trajectories in the corona using the specified plasma properties. The brightness temperatures and other parameters of the pixels are found through simultaneous integration along the ray paths.

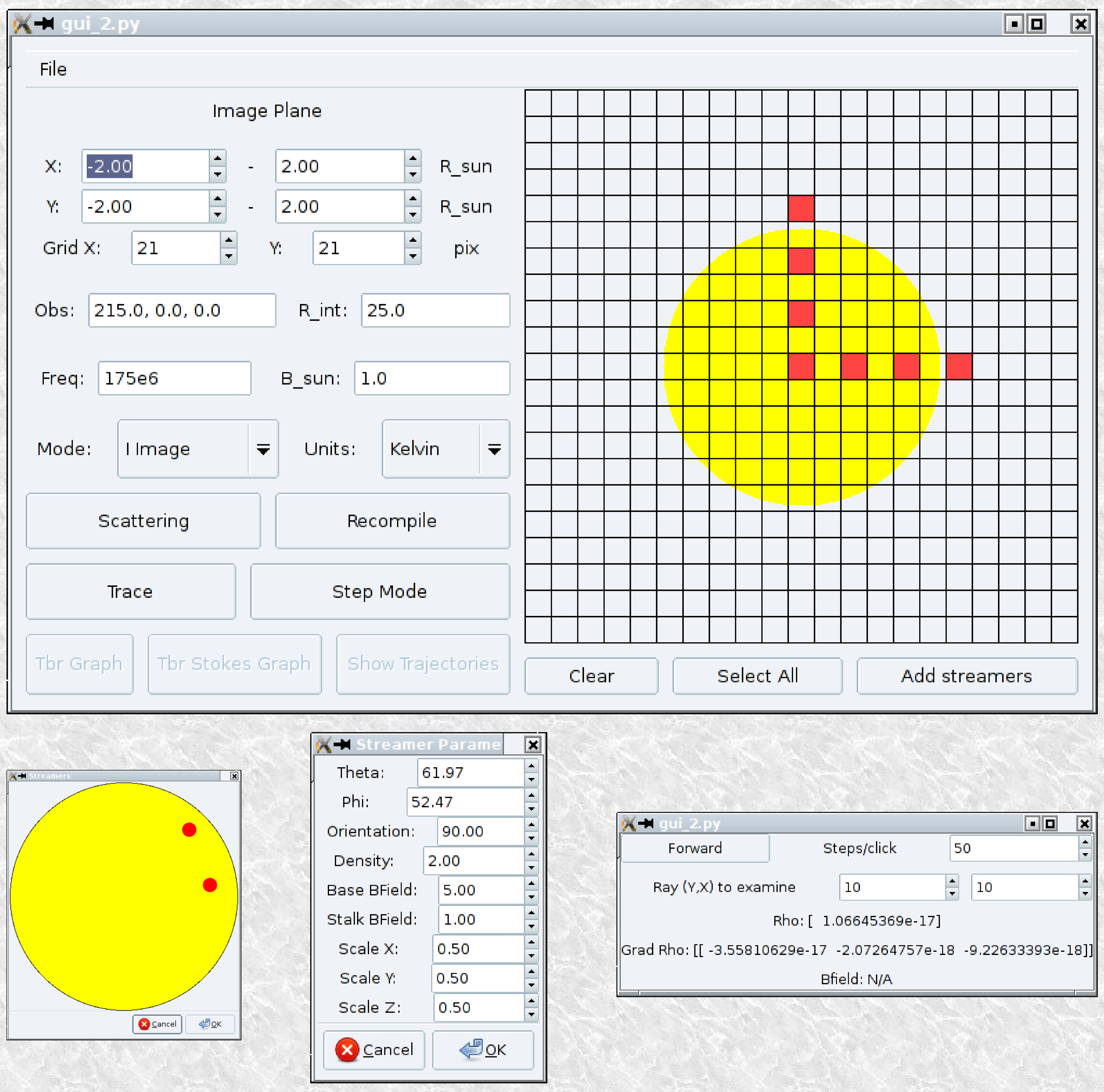
What does it do? The appearance of the LF sun images is modified very substantially due to effects like refraction, scattering, and the dichroic and birefringent nature of the coronal plasma. To correctly interpret the LF images these effects need to be correctly accounted for.

Inputs and Outputs

Software Architecture



Graphical User Interface



Under the Hood

Ray Tracing Formulation

- **High efficiency:** The 2-stage scheme requires only two plasma density calculations unlike four for the classical Runge-Kutta method
- **Good precision:** Better than the second order, due to the property of direction vector magnitude conservation; adaptive step size
- **Correctness control:** Through monitoring the ray states and approximate parabolic and Snell reflections

Fermat principle → Ray Tracing Algorithm

$$\delta \int_{P_1}^{P_2} n\, ds = 0$$

↓

Six ray equations:

$$\begin{cases} \dfrac{d\mathbf{r}}{ds} = \mathbf{v} \\ \dfrac{d\mathbf{v}}{ds} = \mathbf{v}\times\left[\dfrac{\nabla n}{n}\times\mathbf{v}\right] \end{cases}$$

ds is path element
$\mathbf{r}$ is position
$\mathbf{v}$ is direction
n is index of refraction

→ Converts initial ray position and direction into the new ones along the ray path: $(\mathbf{r}_0,\mathbf{v}_0)\to(\mathbf{r}_1,\mathbf{v}_1)$:

$$\mathbf{r}_{1/2} = \mathbf{r}_0 + \mathbf{v}_0\frac{ds}{2}$$

$$\mathbf{\Omega}_0 = \left(\frac{\nabla n}{n}\right)_{\frac{1}{2}}\times\mathbf{v}_0\frac{ds}{2}$$

$$\mathbf{\Omega}_{1/2} = \left(\frac{\nabla n}{n}\right)_{\frac{1}{2}}\times(\mathbf{v}_0+\mathbf{v}_0\times\mathbf{\Omega}_0)\frac{ds}{2}$$

$$\mathbf{v}_1 = \mathbf{v}_0 + \frac{2}{1+|\mathbf{\Omega}_{1/2}|^2}\times(\mathbf{v}_0+\mathbf{v}_0\times\mathbf{\Omega}_{1/2})\times\mathbf{\Omega}_{1/2}$$

$$\mathbf{r}_1 = \mathbf{r}_{1/2} + \mathbf{v}_1\frac{ds}{2}$$

Plasma density

Method for Smooth Stitching Chromospheric and Coronal N_e Distributions

Chromosphere: $N_e = f(r), r\in[1, r_1]$; Corona: $N_e = g(r,\theta), r\in[r_2,\infty]$

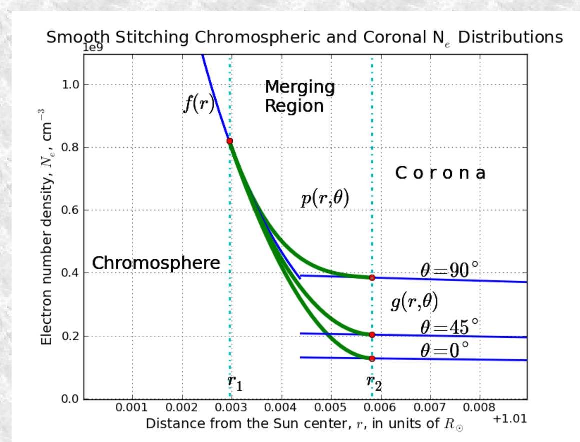
Link function: $p(r,\theta,\varphi) = a(r) + b(r)\,g(r_2,\theta)$, a, b – 4-polynomials

To find p, i.e. a and b coefficients, two linear systems with the same matrix **P** and RHSs c_a and c_b are solved for the coefficient 4-vectors $\mathbf{a}$ and $\mathbf{b}$:

$$\begin{cases} a(r_1) = f(r_1) \\ a(r_2) = 0 \\ a_r(r_1) = 0 \\ a_r(r_2) = 0 \end{cases} ; \quad \begin{cases} b(r_1) = 0 \\ b(r_2) = 1 \\ b_r(r_1) = f_r(r_1)/g(r_2,\theta) \\ b_r(r_2) = g_r(r_2,\theta)/g(r_2,\theta) \end{cases} ; \quad \mathbf{P} = \begin{vmatrix} 1 & r_1 & r_1^2 & r_1^3 \\ 1 & r_2 & r_2^2 & r_2^3 \\ 0 & 1 & 2r_1 & 3r_1^2 \\ 0 & 1 & 2r_2 & 3r_2^2 \end{vmatrix}$$

$$\mathbf{P}\,\mathbf{a} = \mathbf{c}_a \;;\; \mathbf{P}\,\mathbf{b} = \mathbf{c}_b \,.$$

- **High numerical efficiency** independent of the problem dimensions



- **Seamless** merging of the 1D plasma density distribution in chromosphere with 2D distribution in corona

Brightness Temperature

In total intensity:

$$T_{B_{i+1}} = T_{e_i} + e^{-\alpha_i\Delta s}\,(T_{B_i} - T_{e_i})$$

In Stokes parameters:

$$\begin{bmatrix} T_{BI_{i+1}} \\ T_{BQ_{i+1}} \\ T_{BU_{i+1}} \\ T_{BV_{i+1}} \end{bmatrix} = \begin{bmatrix} T_{e_i} \\ 0 \\ 0 \\ 0 \end{bmatrix} + e^{m\Delta s}\cdot \begin{bmatrix} T_{BI_i} - T_{e_i} \\ T_{BQ_i} \\ T_{BU_i} \\ T_{BV_i} \end{bmatrix}$$

Vector form:

$$\mathbf{T}_{B_{i+1}} = \mathbf{T}_{e_i} + e^{m_i\Delta s}\,(\mathbf{T}_{B_i} - \mathbf{T}_{e_i})$$

$e^{m\Delta s}$ - Mueller matrix

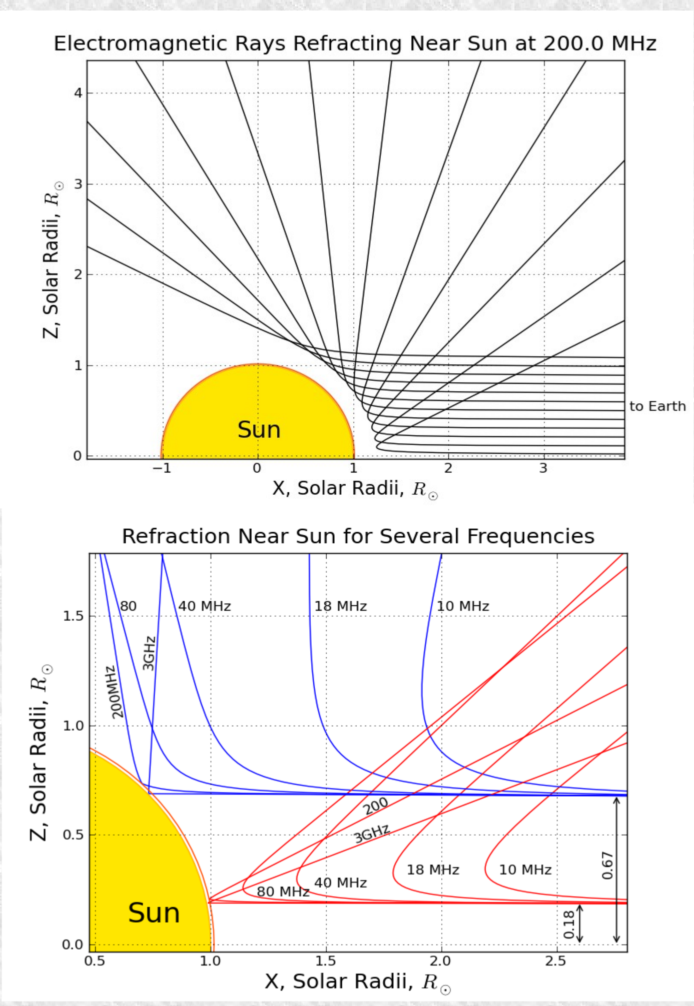
$$m = \begin{bmatrix} \alpha & \beta & \gamma & \delta \\ \beta & \alpha & \mu & \nu \\ \gamma & -\mu & \alpha & \eta \\ \delta & -\nu & -\eta & \alpha \end{bmatrix}$$ - differential Mueller matrix

$\alpha\Delta s = \Delta\tau$ - optical depth from absorption coefficient

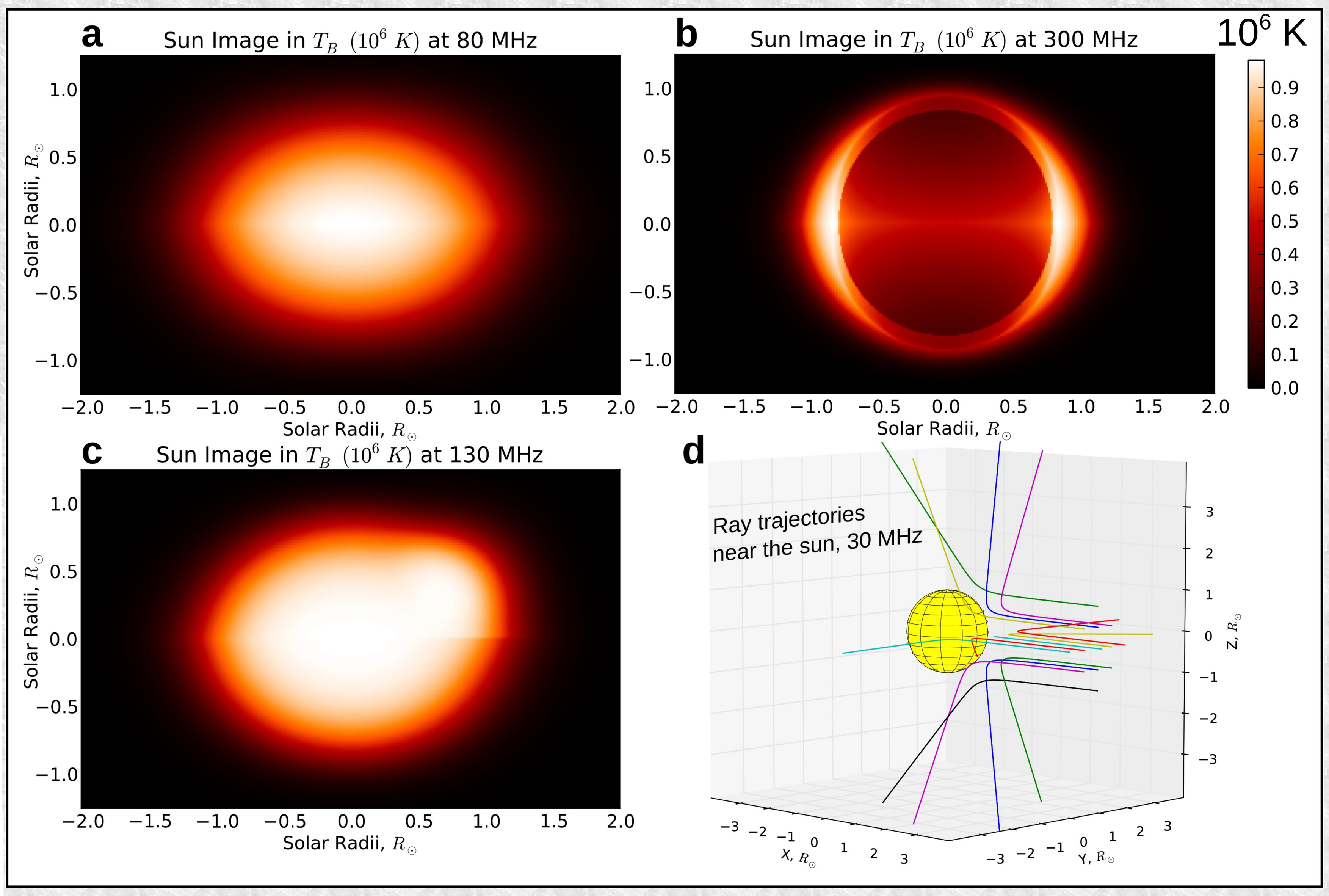
- **Conventional measure** of brightness related to the black body temperature by Rayleigh–Jeans law
- **The algorithm** computes redistribution of energy between the polarization states

2D trajectories

- **Refraction** in SGI XZ-plane
- **Upper panel**: a zone of avoidance at 200 MHz is close to the solar surface
- **Lower panel**: spectral decomposition in the corona. Wide-band observations probe its different layers



Simulation Results



- **Saito**'s (1970) model of the coronal density used
- **No** scattering
- **a, b, c:** simulated 300x186-pixel T_{BI} for a solar minimum
- **A coronal streamer** (30N, 60E) has been included in **c**
- **3D ray trajectories** computed at 30 MHz are shown in **d**
- **Note:**
- **Change** in the radio Sun appearance across the80-300 MHz range
- **Impact** of the streamer
- **Limb brightening** effect at 300 MHz

Conclusion and Future Work

The HART framework implements the propagation and radiative transfer modeling required for extracting science from modern low frequency arrays (e.g. MWA, LWA, and LOFAR).

Current active development efforts are directed at understanding and implementing:

- Polarization transport
- Anisotropic scattering
- Porting to GPU