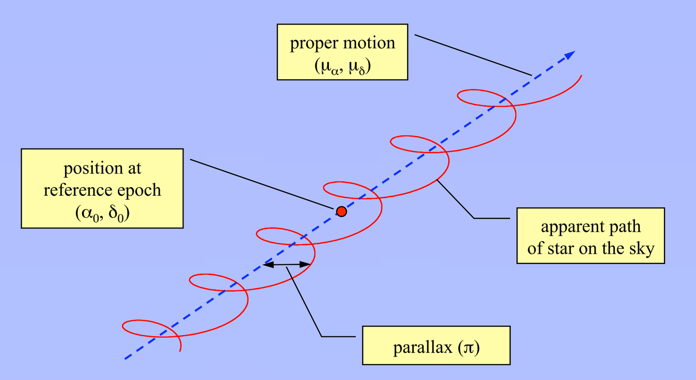
Combining and comparing astrometric data from different epochs

A case study with Hipparcos and Nano-JASMINE



Introduction

Astrometry is the accurate determination of positions, distances (through parallaxes) and proper motions of stars. The data reduction of space based astrometry missions is done using a least-square solution to determine the stellar parameters from the observation data. In the future we will have access to several independent astrometric catalogues, produced by different space projects. Improved proper motions can be computed by comparing the positions in catalogues at different epochs.

Daniel Michalik[1], Lennart Lindegren[1], David Hobbs[1], Uwe Lammers[2] and Yoshiyuki Yamada[3]

[1] Lund Observatory, Lund University
[2] European Space Agency/ESAC
[3] Department of Physics, Kyoto University

E-mail: Daniel.Michalik@astro.lu.se

Theory

A natural extension of the least-squares approach is to make a **joint solution** of the data from two or more missions. This allows to obtain good results even when each data set alone is insufficient for an accurate reduction. We developed a method to combine information from different sets of astrometric data in a statistically optimal way by making a joint astrometric solution. This is now part of the Astrometric Global Iterative Solution which will be used for the data reduction of two upcoming space missions, Gaia and Nano-JASMINE. Solving for astrometric parameters is done by forming a set of normal equations from observation equations to which we can now add the normal equations of the older mission (e.g. from Hipparcos) before solving for the unknowns. This requires that both sets of data use the same reference epoch.

Conventional catalogue combination

In the conventional approach, catalogues are reduced independently and the combination is done a posteriori.

$$\left.\begin{array}{l} \boldsymbol{N}_1\boldsymbol{x} = \boldsymbol{b}_1 \quad \rightarrow \quad \boldsymbol{x}_1 \\ \boldsymbol{N}_2\boldsymbol{x} = \boldsymbol{b}_2 \quad \rightarrow \quad \boldsymbol{x}_2 \end{array}\right\} \Rightarrow \hat{\boldsymbol{x}}$$

Positions and parallaxes: $\sigma^{-2} = \sigma_1^{-2} + \sigma_2^{-2}$

Proper motions: $\sigma = \dfrac{\sqrt{\sigma^2_{position1} + \sigma^2_{position2}}}{\Delta T}$

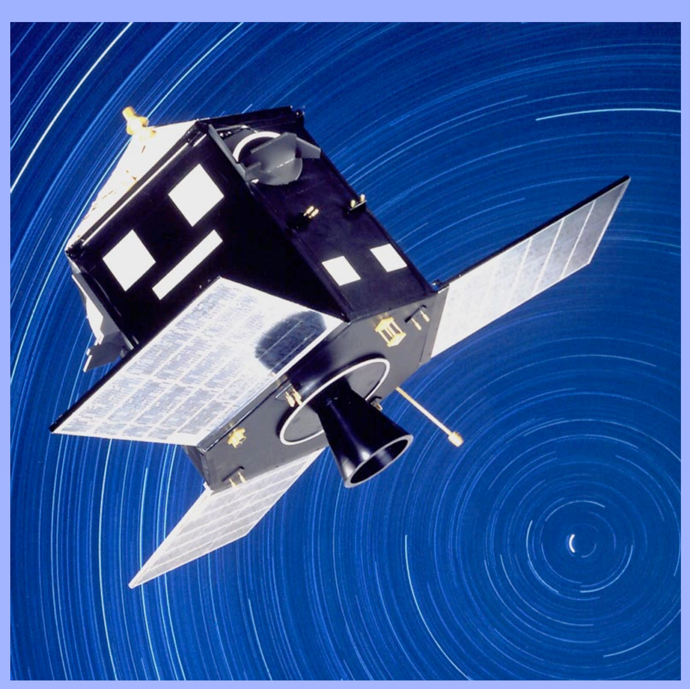
Joint least-squares solution

In our proposed approach the normal equations from the two missions are combined before the solution:

$$(\boldsymbol{N}_1 + \boldsymbol{N}_2)\boldsymbol{x} = \boldsymbol{b}_1 + \boldsymbol{b}_2$$
$$\Rightarrow \hat{\boldsymbol{x}}_{\text{joint}}$$

Performs better by taking correlations between the parameters into account, see Results!

Hipparcos and Nano-JASMINE

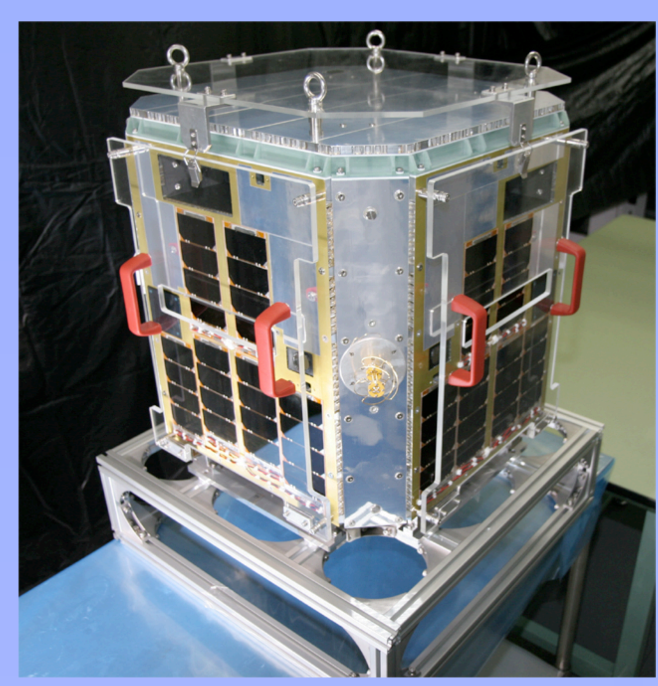

Picture courtesy ESA

The Hipparcos mission was launched by ESA in 1989 as the first space astrometry mission. It surveyed ~118,000 bright stars with ~1 mas accuracy. The catalogue is still the main source of fundamental data for stars in our neighbourhood. The accuracies (Robust Scatter Estimate) of the astrometric parameters of its fifteen thousand brightest stars are

$$\sigma_{position} \sim 0.6 \text{ mas}$$
$$\sigma_{parallax} \sim 0.8 \text{ mas}$$
$$\sigma_{proper\ motion} \sim 0.7 \text{ mas/yr}$$

Nano-JASMINE is the first space astrometry satellite mission in JAPAN and a technical demonstrator for the future JASMINE mission. It is scheduled for launch in November 2013. Simulations are based on the original version of its scanning law. For the fifteen thousand brightest Hipparcos stars we expect accuracies (RSE) of

$$\sigma_{position} \sim 2.5 \text{ mas}$$
$$\sigma_{parallax} \sim 3 \text{ mas}$$
$$\sigma_{proper\ motion} \sim 4.5 \text{ mas/yr}$$

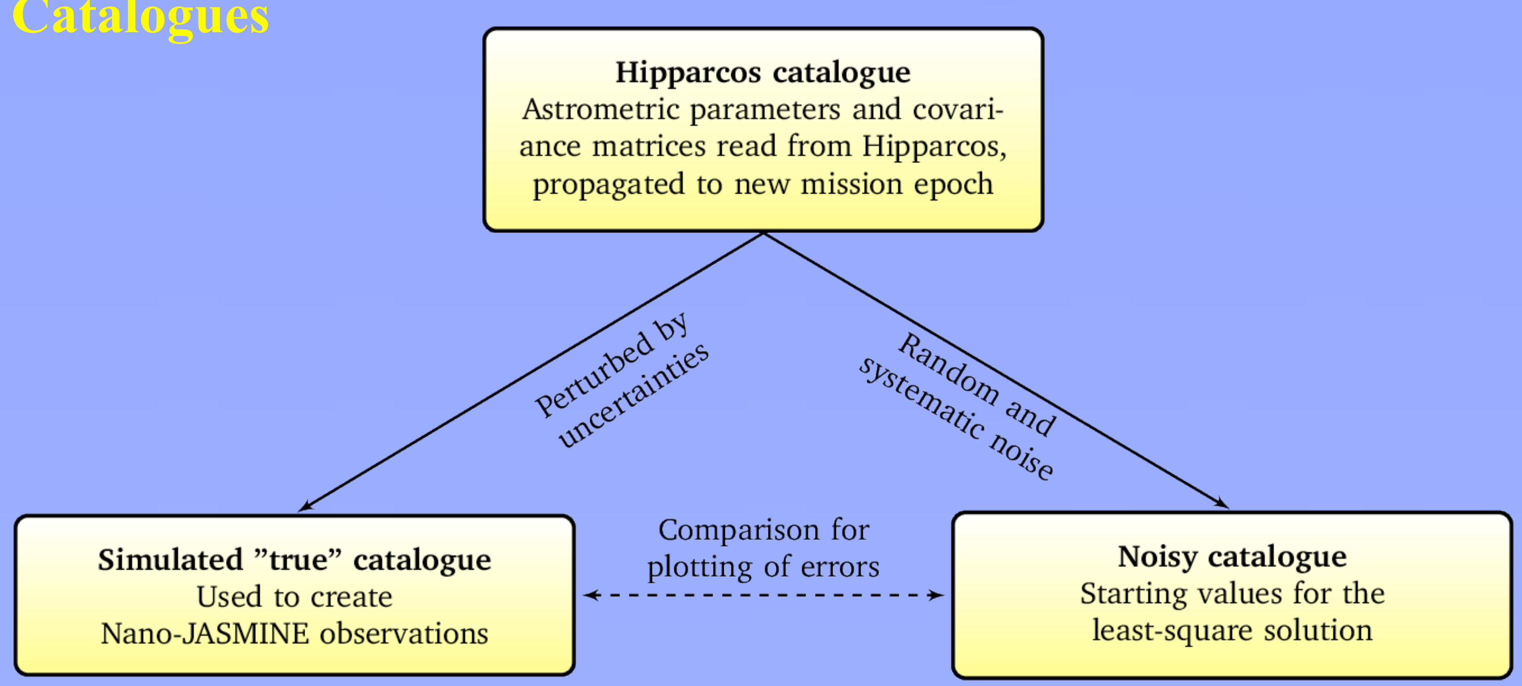

Picture courtesy Nano-JASMINE team

The epoch difference between Nano-JASMINE and Hipparcos is ΔT = 23.75 years. Combining the two data sets gives a big improvement of the proper motions and a significant improvement in position and parallaxes.

Simulations

Simulations are carried out using AGISLab, a software package aiding the development of algorithms for the data reduction of Gaia, developed at Lund Observatory.

Catalogues



To simulate observations we need to know the true values of the astrometric parameters, which are of course not known. We therefore simulate a "true" catalogue by perturbing the Hipparcos catalogue with an error distribution $\boldsymbol{e}$ consistent with Hipparcos uncertainties. We use 5 Gaussian random variates $\boldsymbol{g}$ scaled by the square root $\boldsymbol{L}$ of the covariance matrix $\boldsymbol{C}$ to introduce the correct correlations between the errors $\boldsymbol{e}$ in each parameter.

$$\boldsymbol{C} = \boldsymbol{L}\boldsymbol{L}^T \quad \text{(Cholesky)}$$
$$\boldsymbol{e} = \boldsymbol{L}\boldsymbol{g}$$

$$\alpha = \alpha_0 + e_{\alpha\star} \sec\delta_0$$
$$\delta = \delta_0 + e_\delta$$
$$\pi = \pi_0 + e_\pi$$
$$\mu_{\alpha\star} = \mu_{\alpha\star 0} + e_{\mu_{\alpha\star}}$$
$$\mu_\delta = \mu_{\delta 0} + e_{\mu_\delta}$$

Reconstruction of the Hipparcos normal equations

N is taken as the covariance inverse: $\boldsymbol{N}_{\text{HIP}} = \boldsymbol{C}^{-1}$

$\boldsymbol{b}_{\text{HIP}}$ is chosen so that solving for Hipparcos only the update $\boldsymbol{x_d}$ would recover the original source values. The vector $\boldsymbol{x_d}$ is the difference between the original Hipparcos and the current noisy values.

$$\boldsymbol{b}_{\text{HIP}} = \boldsymbol{N}_{\text{HIP}}\boldsymbol{x_d} = \boldsymbol{N}_{\text{HIP}} \begin{pmatrix} (\alpha_o - \alpha_c)\cos\delta_o \\ \delta_o - \delta_c \\ \pi_o - \pi_c \\ \mu_{\alpha\star o} - \mu_{\alpha\star c} \\ \mu_{\delta o} - \mu_{\delta c} \end{pmatrix}$$

Results

The tables below show first results of simulation runs. As expected the combination of Hipparcos and Nano-JASMINE gives a great improvement in proper motions. Additionally we show that our proposed joint solution performs significantly better than the conventional catalogue combination method. This can be understood as follows.The astrometric parameters in the Hipparcos (or Nano-JASMINE) catalogue are not uncorrelated. The huge improvement of the proper motions therefore brings some improvement also to the other parameters, provided that the correlations are properly taken into account. This is the case for the joint solution, but not for the conventional combination.

The position values from Hipparcos have been propagated to the Nano-JASMINE mid-mission epoch J2015. Simulations of Nano-JASMINE are based on a conservative observation performance model and an optimal scanning law.

		Position @J2015 [mas]		Parallax [mas]	Proper motions [mas/year]	
		α	δ	π	$\mu_{\alpha\star}$	μ_δ
mag < 7.5 ~15 000 stars	Hipparcos only (Hip)	18.19	14.84	0.77	0.63	0.80
	Nano-JASMINE only (NJ)	2.56	2.54	4.65	4.50	3.05
	Conventional combination Hip + NJ	2.54	2.51	0.111	0.110	0.77
	Joint solution Hip + NJ	**2.41**	**2.4**	**0.108**	**0.105**	**0.75**
	Improvement of joint solution	5.2%	4.4%	3.2%	4.4%	3.5%
mag < 11.5 ~117 000 stars	Hipparcos only (Hip)	27.06	22.35	1.18	1.14	0.94
	Nano-JASMINE only (NJ)	4.57	4.53	5.43	8.38	8.02
	Conventional combination Hip + NJ	4.51	4.44	1.15	0.197	0.194
	Joint solution Hip + NJ	**4.43**	**4.26**	**1.11**	**0.188**	**0.185**
	Improvement of joint solution	1.8%	3.9%	4.0%	4.5%	4.5%

Lund Observatory

Design consultant: Helene Jönsson