Abundances in Globular Cluster Dwarfs

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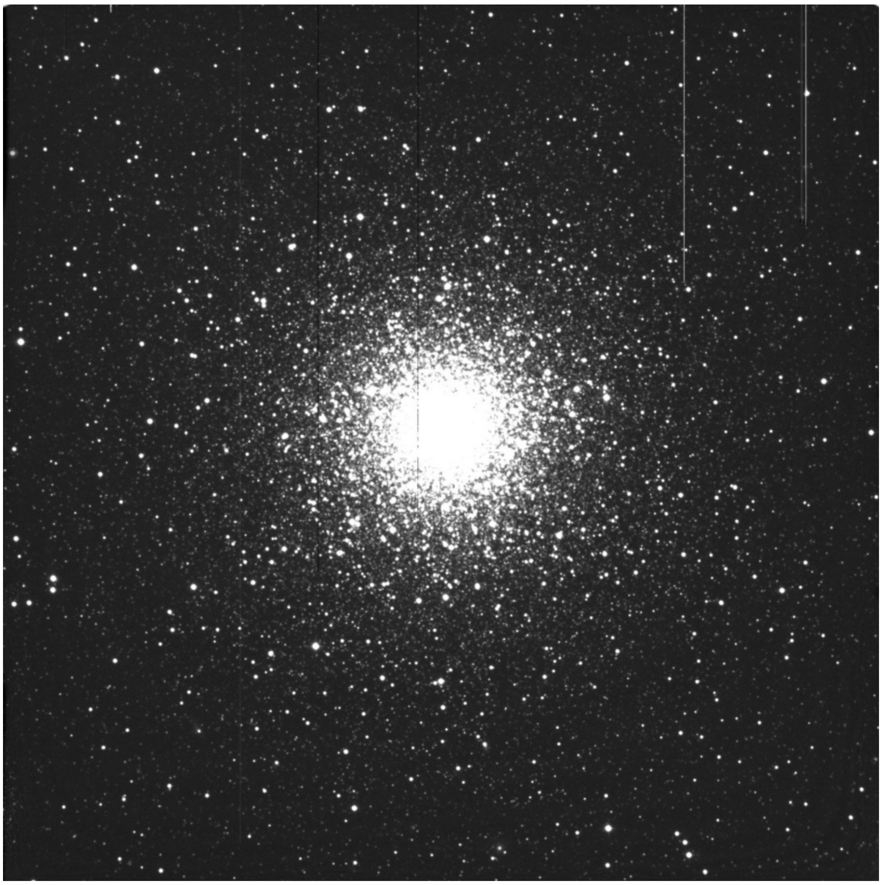


Figure 1: Image of a typical Globular Cluster (NGC 5024)

Globular Clusters are huge, very compact aggregates of hundreds of thousands of stars (see Figure 1). There are about 150 such systems in our Galaxy, the closest being about 10,000 light years from us; some of them are visible with the naked eye (ω Centauri, 47 Tucanae), and many others are spectacular objects visible with small telescopes.

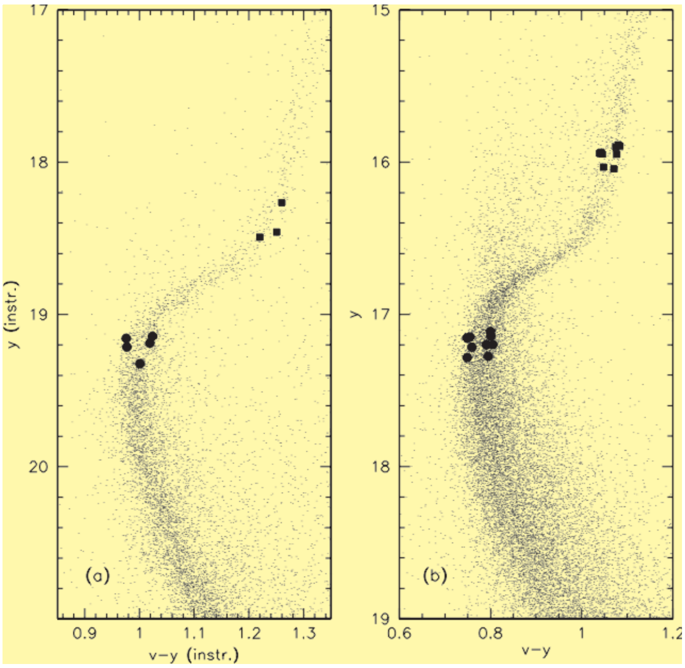
Globular Clusters play an important role in modern astrophysics mainly because they are the oldest objects in our Galaxy and in the whole Universe that we can accurately date. Provided that their distances are known, ages of Globular Clusters can in fact be determined quite precisely from the luminosity of the turn-off stars, that is the stars that are exhausting Hydrogen at their centre. The oldest Globular Clusters are so old that they formed when the Universe was very young, and very different from what it appears now. Accurately dating them is then basic to constraining the early epochs of formation of our own Galaxy, and even the age of the Universe. This last is important for cosmology: combined with estimates of the Hubble constant, it may tell us about the presence and nature of the mysterious dark energy, whose presence is suggested by the apparent decline of the luminosity of type Ia supernovae at high redshifts (Perlmutter et al. 1999), and by the characteristics of the X-ray emission of galaxy clusters (see the review by Rosati et al. 2002).

Observations of external galaxies indicate that Globular Clusters form during epochs of strong dynamical interactions. The lack of young Globular Clusters in our Galaxy can then be connected to the presence of the thin disc: in fact the thin disc would have been destroyed by strong dynamical interactions. The oldest components of the thin disc formed about 10 Gyr ago. While a substantial fraction of the Galactic Globular Clusters seems to be coeval and extremely old, there is a group of clusters that appears to be slightly younger (Rosenberg et al. 1999), although still older than the oldest components of the thin disc. These "younger" Globular Clusters differ systematically from the oldest ones: they are more metal rich, and more concentrated toward the centre of the Galaxy. This second group of clusters is probably related to the thick disc, and perhaps to the bulge. Comparison between the chemical composition of the thick and thin disc stars suggests that there was an interval of low star formation between these two phases of the history of our Galaxy (Gratton et al. 1996). This hiatus in the star formation might be due to the accretion of a gas rich satellite, which may have caused the heating of the pre-existing disc, a burst of star formation, and possibly a galactic wind that temporarily cleaned our Galaxy of gas from which stars could form. This strong dynamical interaction may be the site for the formation of the group of "younger" Globular Clusters. Precisely dating them can help to fix the scenario of the early evolution of the Galaxy.

Globular Cluster distances

Dating Globular Clusters requires knowledge of their distances. Globular Clusters are too far for direct distance estimates using trigonometric parallaxes with the presently available instrumentation, although in the next decade accurate distances will be likely obtained with GAIA. However, distances

Figure 2: Location in the colour magnitude of the stars observed in our programme in the Globular Clusters NGC6397 and NGC6752 (from Gratton et al. 2001)



to Globular Clusters can be obtained with various more indirect techniques. In the next few years, high precision distances will probably be obtained by comparing internal proper motions measured by HST with extensive radial velocity measurements obtained with the stellar multi-object spectrograph GIRAFFE fed by FLAMES at KUEYEN (VLT Unit Telescope 2). In the meantime, the best known method is the so-called Main Sequence Fitting. In this method, local subdwarfs whose parallaxes have been accurately measured by the ESA HIPPARCOS astrometric satellite are used as standard candles. Assuming that these stars are identical to main sequence stars in Globular Clusters, distances may be derived by the difference in the apparent magnitude. Since the luminosity of main sequence stars depends on temperature and metallicity, we need to know these quantities for both field and cluster stars. Note that these quantities must be derived differentially: what is important is that temperature and metallicities for field and cluster stars are on the same scale; absolute values have a much smaller impact. Up to now, all data about cluster abundances were based on giant stars; these values might not be consistent with those derived from dwarfs. Furthermore, temperatures are derived from colours; such a derivation is sensitive to the value adopted for interstellar reddening, but it is not demonstrated whether the reddening scale used for Globular Clusters is the same as for local subdwarfs, because the local subdwarfs lie within the dust absorbing layer, while Globular Clusters are much farther. This leads to about 6% uncertainty in the adopted distances. While this may appear as a small value, it translates into uncertainties of almost 2 Gyr in the ages.

On the other side, temperatures and metallicities can be derived, independently of concerns related to reddening, from high resolution spectra. In this case, temperatures may be derived e.g. from the strength of the Balmer lines (good temperature indicators for stars warmer than about 5000 K), and metal abundances from weak metallic lines. In principle the method requires extraction of these parameters for the same field and cluster stars used to derive distances, that is, unevolved stars that are still on the main sequence. However, these stars are very faint even in the closest Globular Clusters. Slightly evolved stars near the turn-off can still be used: these are accessible in the case of the closest Globular Clusters using UVES at KUEYEN. If this technique is used, uncertainties in the distances are reduced to about 3%, allowing determinations of ages with errors of only about 1 Gyr. Such an accurate determination allows much more critical tests for both cosmology and Galaxy evolution.

UVES at the VLT is particularly suitable for a number of reasons: first, it is a very efficient spectrograph providing enough spectral resolution and S/N on faint sources (V~17) such as Turn-Off stars in Globular Clusters; second, the wide spectral coverage observable in a single exposure allows simultaneous observation of Hα and of a suitable number of metal lines in the blue portion of the spectra (the stars at the turn-off of metal-poor Globular Clusters have spectra with very few measurable lines). Third, the location of Paranal gives access to the closest Globular

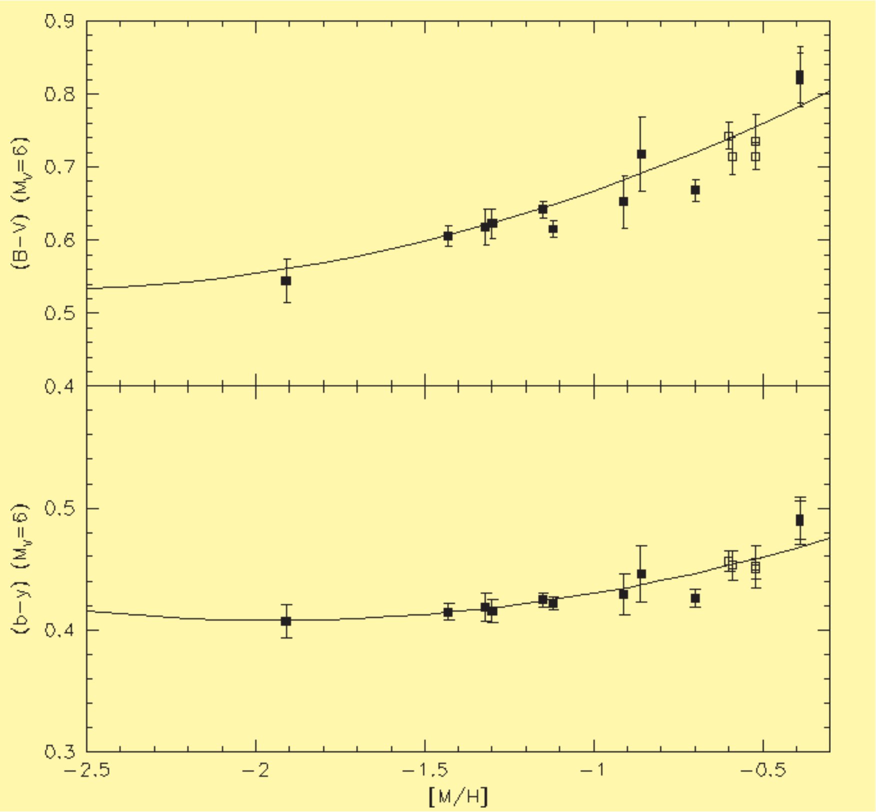


Figure 3: Observed colours of the Main Sequence at M_V = 6, as derived from local subdwarfs with accurate parallaxes from HIPPARCOS and metal abundances [M/H] determined in our programme. Filled symbols are stars actually used in the distance derivations; open symbols are control stars. The upper panel shows the run with metallicity for the Johnson B-V colour; the bottom panel is for the Strömgren b-y colour. Superimposed are the predictions by models by Straniero et al.

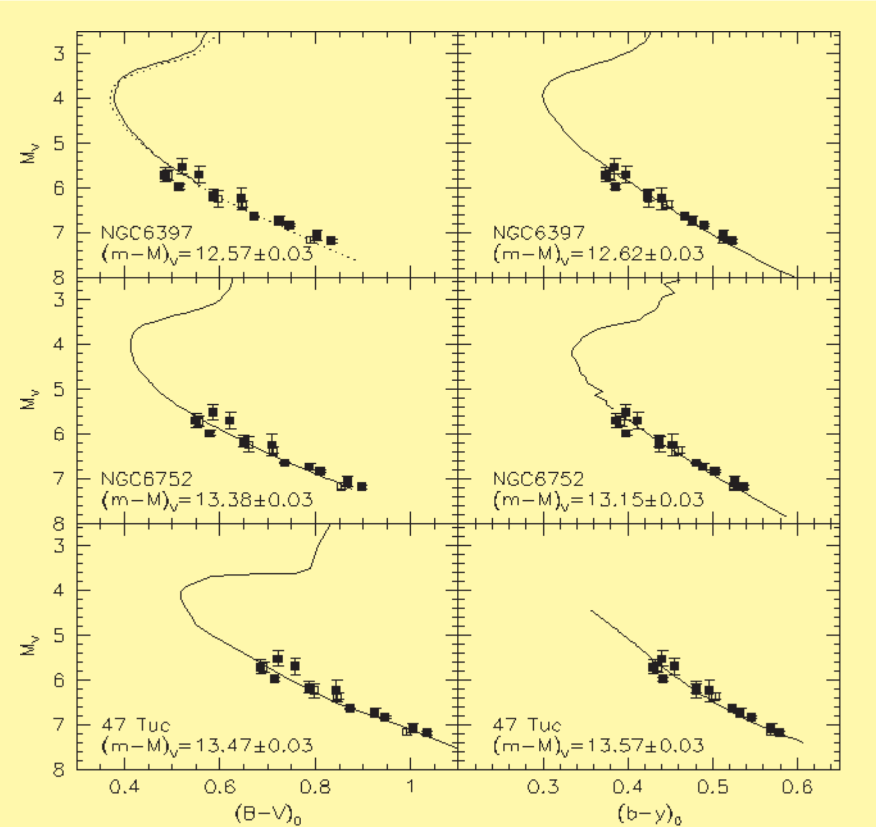


Figure 4: Fit of the Main Sequences of the program Globular Clusters NGC 6397, NGC 6752 and 47 Tucanae, with local subdwarfs. Left panels are for Johnson B-V colours; right panels are for Strömgren b-y colours

Clusters that are all located in the Southern hemisphere.

We addressed this issue in an ESO Large Programme. UVES high resolution spectra were obtained for about 15–20 stars in each of the Globular Clusters NGC6397, NGC6752 and 47 Tucanae. These three Globular Clusters were selected for observations because they are the closest to the Sun, except M4, which however has a variable foreground reddening, making it less suitable for precise dating. These three clusters cover a wide range in metal abundance, from very metal-poor ([Fe/H]~ –2.0: NGC6397) to rather metal-rich ([Fe/H]~ –0.7: 47 Tucanae). NGC 6397 and NGC6752 belong to the group of old clusters, while 47 Tucanae is likely slightly younger (Rosenberg et al. 1999). In each of these clusters, we selected for observations two groups of stars, one near the turn-off, and the second one at the base of the subgiant branch (see Figure 2). This choice allowed us to make further tests on the program stars, described in the next Sections. In order to ensure that the analysis of the stars in these clusters is strictly identical to that of field subdwarfs, we also acquired spectra of about thirty such stars, selected to have good Hipparcos parallaxes.

The analysis of the spectra of all these stars was done using the same procedures: effective temperatures were derived from the wings of Hα, using the same precepts for both field and cluster stars. From these analyses, we derived effective temperatures with errors of about 150 K for each star. Most of this error stems from uncertainties in the flat fielding procedure, so that the error is fairly independent of the actual S/N of the spectra. However, averaging results from all stars, we were able to constrain the temperatures for the stars in a cluster to within 30 K. This in turn translates into unprecedented accuracies of about 0.005 mag in the estimates of the interstellar reddening E(B-V), and of about 0.04 dex in the metal abundance [Fe/H]. Furthermore, we derive the abundances of important elements like O, Mg, Si, Ca, and Ti, so that appropriate values of the overall metal abundance could be obtained for each star.

Once reddening and metal abundances for both field stars and Globular Clusters were derived, distances could be obtained by fitting the main sequence of the Globular Clusters to the location occupied by the field subdwarfs. Only unevolved stars (that is stars with an absolute magnitude $M_V > 5.5$) were considered, in order to avoid possible concerns due to differences between the ages of field and cluster stars. However, before this fitting is done, the temperature (colours) of the field stars should be corrected to take into account the difference in metallicity between the field and the Globular Cluster stars. This was done using theoretical relations by Straniero et al. (1997): in Figure 3 we compare the prediction for the colour of the main sequence at $M_V = 6$ obtained with these models with the observed colours of the field subdwarfs used in the distance derivations. We considered two independent colours for each star: the broad band Johnson *B-V* and the intermediate band Strömgren *b-y* colours. The agreement between observations and theoretical predictions is excellent, with only a small offset for *B-V*. Note however that these relations are only used to correct colours, i.e. differentially, so that this small offset has no impact in our analysis.

Figure 4 shows the fits we obtained for the three clusters, in both the Johnson (*V, B-V*) and Strömgren (V, b-y) colour-magnitude diagrams. There are small differences in the results obtained with different colours that can be attributed to small errors in the colour transformations from observed to standard sequences in the used photometry. However, the agreement is on the whole very good: distances estimated with this procedure have errors as small as 3.5%, and they are the best estimates currently available for these three clusters.

Globular Cluster Absolute Ages and their impact

The ages of the three Globular Clusters that can be obtained from the luminosity of the turn-off point using these distances are 13.8 ± 1.1 Gyr for NGC6397, 13.7 ± 1.1 Gyr for NGC 6752, and 11.2 ± 1.1 Gyr for 47 Tuc. This last cluster turns out to be about 2.5 Gyr younger than the other two, in excellent agreement with the age difference obtained by Rosenberg et al. (1999) using relative dating methods.

Leaving aside 47 Tucanae, the age of the two other clusters (that are coeval to the oldest Globular Clusters, according to Rosenberg et al. 1999) is 13.7 ± 0.8 ± 0.6 Gyr, where the first error bar accounts for internal errors, and the second one for systematics, including uncertainties in the stellar models. This estimate for the age of Globular Clusters coincides with the age of the Universe determined by the WMAP group for a standard Λ_{CDM} model (Spergel et al. 2003). This indicates that, in the framework of a standard Λ_{CDM} model, the Galactic Globular Clusters began to form very early, within 1.4 Gyr from the Big Bang. Alternatively, this age estimate, combined with the estimate of the Hubble constant given by the HST Key Program (Freedman et al. 2001) and the WMAP experiment (Spergel et al. 2003) can be used to constrain the value of the matter density Ω_M in a flat Universe $\Omega_{tot} = \Omega_M + \Omega_\Lambda = 1$, as determined by the spectrum of perturbations of the microwave background (Spergel et al. 2003). This estimate is independent from results provided by type Ia SNe and clusters of galaxies. The results of this exercise are shown in Figure 5: Ω_M is constrained to be $\Omega_M < 0.57$ (and $\Omega_\Lambda > 0.43$) at the 95% level of confidence. This confirms the need for some form of dark energy ($\Omega_\Lambda \neq 0$) providing the observed acceleration in the expansion of the Universe.

Our distance estimates can also be used to derive the luminosity of the horizontal branch (a benchmark for distance scales, as well as for theoretical models). When coupled with estimates of the apparent magnitudes of RR Lyrae stars in the LMC (e.g. using the derivation of Clementini et al. 2003 based on photometric data acquired with the Danish 1.5 m telescope and metallicities derived from FORS spectra), they can be used to derive the distance to the closest satellite to our Galaxy, the first step in the extragalactic distance scale. The value we obtain is 50 ± 4 Kpc, the same value adopted in the HST Key Project to derive the Hubble constant.

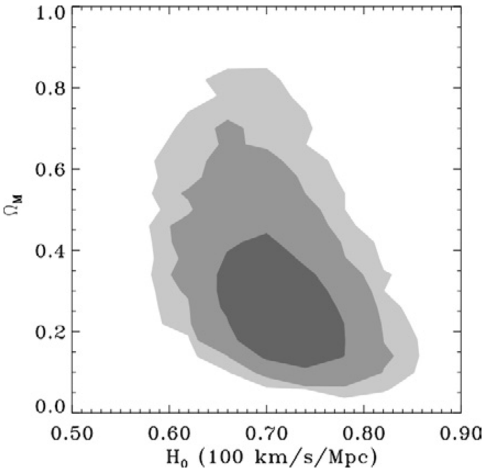


Figure 5: Allowed values of Ω_M as a function of H_0 derived using our estimate for the age of the oldest Globular Clusters (13.7 ± 1.4 billion years), for a flat Universe ($\Omega_{tot} = 1$)

No evidence for element sedimentation in Globular Cluster stars

The most important theoretical uncertainty in the evolution of solar type stars concerns the impact of element sedimentation due to microscopic diffusion. Microscopic diffusion is a basic physical mechanism; it needs to be included in the solar models in order to predict correctly the very accurate and detailed run of the sound velocity within the interior of the Sun provided by helioseismology. Microscopic diffusion is a slow process and its effects may take billion of years to show up. Element sedimentation causes two important effects: first, Globular Cluster ages computed from models that include the effect of microscopic diffusion are about 1 billion years smaller than those obtained neglecting this effect. Second, the abundances of heavy elements for metal-poor stars near the turn-off should be quite different from those obtained for stars at the base of the subgiant branch, where the inward deep penetration of the outer convective envelope should have cancelled the sedimentation effects due to microscopic diffusion. This may have important consequences, e.g. on the interpretation of the observed abundances of Lithium (see below). Detailed predictions that takes into account partial ionisation and the effects of radiation pressure have been presented by Richard et al. (2002): these authors found that Fe is expected to be overabundant (and Li underabundant) by quite a large factor in turn-off stars with an initial value of [Fe/H]=–2 and an age of 12–14 Gyr.

Our observations of turn-off and subgiants of NGC6397 (a metal poor cluster with [Fe/H] = –2.0) may be used to test the effects of sedimentation. We found that there is no appreciable difference between the abundances of Fe and several other elements (Gratton et al. 2001). This severely constrains the impact of diffusion. The most reasonable explanation for the lack of evidence of sedimentation in stars of NGC6397 is that there is a region at the base of the outer convective envelope mixed up by turbulence that cancels the effects of sedimentation. Richard et al. (2002) showed that such a mixing effectively reduces the impact of microscopic diffusion on both the ages of Globular Clusters and on the depletion of the primordial Lithium abundances. This makes both ages and the interpretation of the Lithium abundances much sounder.

A second generation of stars in Globular Clusters?

Precise dating is not the only reason why Globular Clusters are interesting. They are also very dense stellar environments and this may cause systematic differences with respect to stars in the general (low density) field. A very intriguing difference concerns the anticorrelation between abundances of elements like O and Na that the Lick-Texas group (Kraft, Sneden and co-workers: see Kraft 1994) found among the stars they observed in Globular Clusters (close to the tip of the red giant branch). Figure 6 illustrates this anticorrelation: within Globular Clusters, stars that are rich in Oxygen are poor in Sodium, and vice versa. Such an anticorrelation is not present among stars in the general field and thus seems a peculiarity of Globular Clusters (Gratton et al. 2000).

The Oxygen-Sodium anticorrelation is a sign of the presence of material processed throughout the complete CNO cycle. In fact, at high temperatures, Hydrogen is burnt into Helium through a chain that includes as intermediate steps the participation of Carbon, Nitrogen, and Oxygen atoms. When the temperature is low (a few million degrees), as in the central regions of main sequence stars like the Sun, only part of this cycle is active, due to the large Coulomb barrier of Oxygen nuclei: practically, only Carbon and Nitrogen nuclei participate, and most of the original Carbon is transformed into Nitrogen because Nitrogen has a much smaller cross section for proton capture and then tends to accumulate. However, at the higher temperatures of a few tens of millions of degrees K that may be reached in the H-shell burning of Red Giants, Oxygen nuclei also participate in the cycle, and they are effectively transformed into Nitrogen nuclei too, because the cross section for proton capture is much larger than that for proton capture on Nitrogen. Hence, material coming from this region is depleted in Oxygen. However, at the same temperature, proton capture on Neon nuclei effectively produces Sodium; hence this material will be rich in Sodium.

It is not easy to bring material processed through the complete CNO cycle to the surface of small mass red giants like those in Globular Clusters. The structure of the star in fact prevents such a phenomenon, unless some deep mixing not predicted by standard models (i.e. normal non rotating stars) occurs. The reason is the large jump in entropy due to the variation of the molecular weight left over in the star by the maximum extension of the central convective region when the stars left the main sequence. Only when this molecular weight barrier is cancelled by the outward shift of the H-burning shell of the star evolving along the red giant branch is deep mixing allowed. This result is fully confirmed by observations of the field stars (see Figure 7). Why then do stars in Globular Clusters behave differently?

The critical observation is that of dwarfs in Globular Clusters. In fact, the central temperature of these stars is still

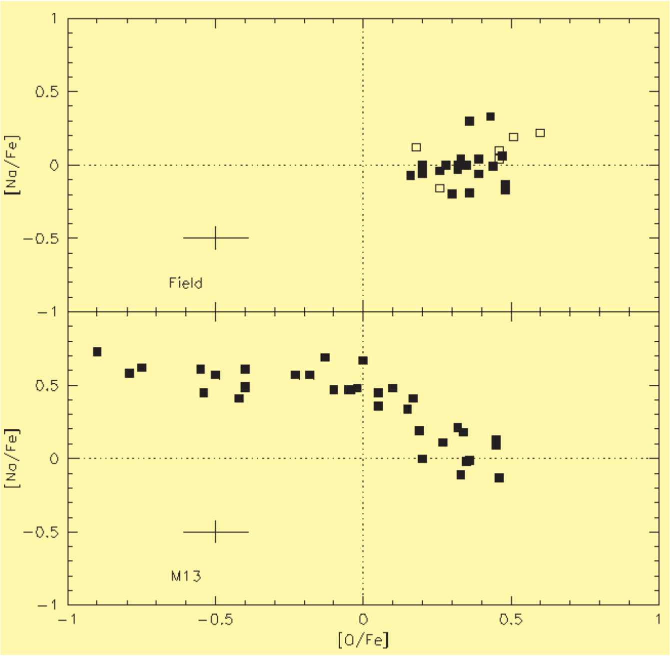


Figure 6: Run of the ratio between the abundances of Na and Fe, against the ratio between the abundances of O and Fe, for stars in the Globular Cluster M13 (lower panel), and in the field (upper panel). Note that an extended O-Na anticorrelation is present only among Globular Cluster stars (from Gratton et al. 2000).

too low for complete CNO burning. If then the O-Na anticorrelation is observed also in these stars, the deep mixing hypothesis is untenable. We performed this test using the turn-off stars we observed in NGC6752, a cluster that shows a clear O-Na anticorrelation among its giants. We found (see Figure 8) that the O-Na anticorrelation is present also among dwarf stars, where it is actually very similar to what is observed in giants (Gratton et al. 2001). It is then clear that the O-Na anticorrelation is not due to deep mixing.

What is then the source of the CNO processed material we see in a large fraction of the Globular Cluster stars? The most probable sources are now extinct massive AGB stars (stars with masses of 4–5 solar masses), where large amounts of material processed through the complete CNO cycle at the base of the outer convective are brought to the surface, and then lost at low velocity by the slow wind blowing from these stars. The escape velocity from Globular Clusters is typically a few tens of km/s, so that this material may be retained by these massive, concentrated objects, explaining the difference between cluster and field stars. Detailed computations show that massive metal-poor AGB stars may do the job. In some way, this material should arrive where it is observed, that is in much smaller unevolved stars. This requires a transport mechanism: the most likely is that the O-poor, Na-rich stars belong to a second generation, born from the ejecta of these massive AGB stars. These stars should be a bit younger than the others: however, the age difference that corresponds to the lifetime of 4–5 solar masses stars is tiny (only 100 or 200 million years), compared with the age of the clusters (about 13 billion years). Such a small age difference would go undetected as far as the magnitude and colour of the turn-off in the colour-magnitude diagram are concerned.

This fascinating scenario for the evolution of clusters may help to understand one of the mysteries of Globular Clusters, the so-called second parameter. This concerns the horizontal branch of Globular Clusters, the phase where Globular Cluster stars burn helium at their centres. Theory predicts that the colour of stars along the horizontal branch should be essentially determined by their metal content. In the sixties, Sandage, van den Bergh and others noticed that there are pairs of Globular Clusters with apparently the same metal content, but very different colours of stars on the horizontal branch: the most famous pair includes M3 and M13. This anomaly indicates that there is a second parameter (other than metallicity) that determines the colour of the horizontal branch. This mystery has gone unsolved for over 35 years. The differences in colours are

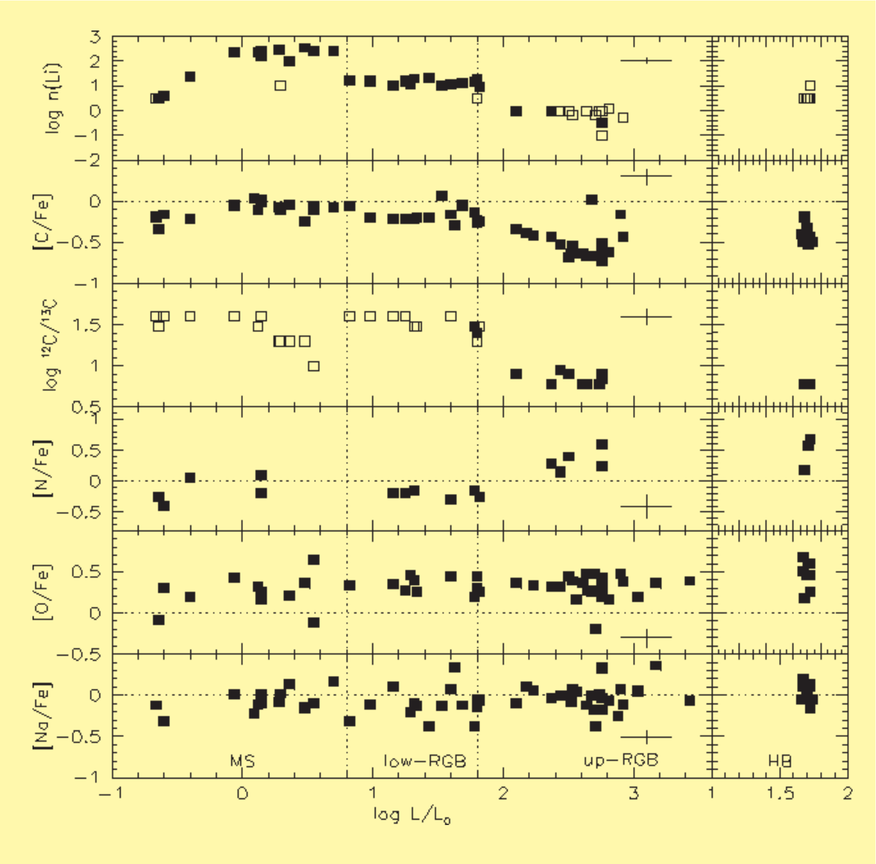


Figure 7: Overabundances of various elements with respect to Fe vs. stellar luminosity in field metal-poor stars. Stars evolve increasing their luminosity, that is, from left to right in these diagrams. The elements shown are Li, C, N, O, and Na, as well as the $^{12}C/^{13}C$ isotopic ratios. Two mixing episodes occur in these stars: 1) the first dredge-up at the base of the giant branch is due to the inward expansion of the outer convective envelope, in zone where incomplete CN H-burning has occurred during the main sequence. It only causes variations in the abundances of C and N (and their isotopes), and a decrease in the Li abundance. 2) A second episode occurs later, when the H-shell burning reaches the point of maximum penetration of the convective envelope (RGB bump); again, it only changes the abundances of C, N, and Li. Surface Na and O abundances are not modified during the evolution of small mass stars (from Gratton et al. 2000).

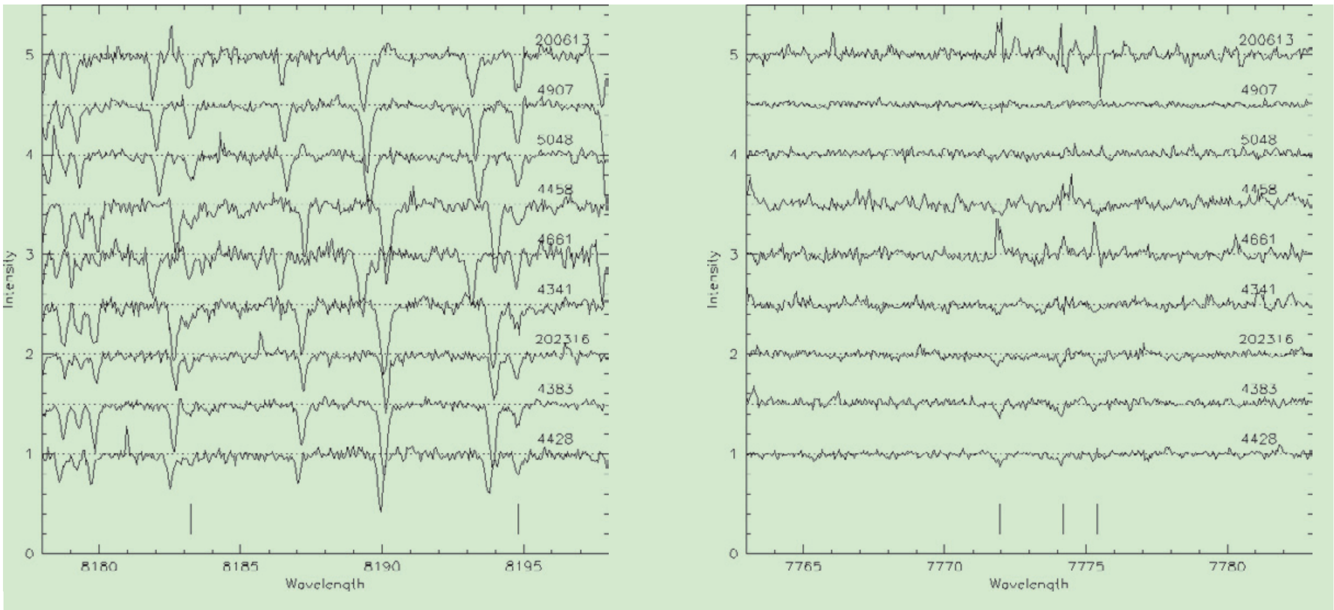


Figure 8: Plots of spectral regions including Na lines (panel a) and O lines (panel b) in dwarfs of the Globular Clusters NGC6752. The stars have virtually identical temperatures and chemical composition: the only difference is in the abundances of CNO elements and Na. Note that the strengths of the Na and O lines are anticorrelated each other: this trend is similar to that found in giants. Since the temperatures at the centres of these stars are not large enough for complete CNO cycle, the Na-rich, O-poor stars must contain material processed elsewhere (from Gratton et al. 2001).

essentially due to differences in masses of the stars on the horizontal branch: however, the reason for these different masses is not clear. In some cases, a difference in age may be the explanation. However, it has been shown by several authors that M3 and M13, for examples, have the same age.

A second generation born from the material expelled from massive AGB stars might explain the anomalous blue horizontal branch of clusters like M13, that has indeed a large population of O-poor, Na-rich stars. In fact this second generation of stars should also be enriched in Helium, produced by the H-burning. Stars richer in Helium evolve faster than normal stars: stars currently on the red giant branch would then be less massive, by a few hundredths of a solar mass. Not a large amount, but well enough to justify a very different colour when these stars are on the horizontal branch. Note however that M13 has virtually no star on the red side of the horizontal branch, so that this effect alone cannot solve the second parameter problem.

Of course many more observations are required to confirm this scenario. FLAMES, using both UVES and GIRAFFE spectrographs, is particularly well suited for such observations. ESO telescopes will likely play a basic role in future observations of Globular Clusters.

Globular Clusters and Ω_b

Besides Ω_M, Globular Clusters may be useful to determine the value of the baryonic component of the density of the Universe, Ω_b. In its first three minutes, the Universe was hot and dense enough to undergo nuclear reactions that formed ^{2}H, ^{3}He, ^{4}He and ^{7}Li. Production of heavier nuclei was not possible because of the rapid cooling of the Universe. According to Standard Big Bang Nucleosynthesis (SBBN), in the presence of three massless neutrinos, the primordial abundance of these light nuclei depends only on the baryon to photon ratio in the Universe, i.e. on the number of baryons, since the number of photons is known from the temperature of the cosmic microwave background (CMB). Therefore a determination of the primordial abundance of the light nuclei allows us, in principle, to determine Ω_b. Spite & Spite (1982) found that the warm metal-poor halo dwarf stars show the same lithium abundance independent of temperature or metallicity, the most straightforward explanation being that the lithium observed in these stars is the primordial lithium. This view may be challenged since the Li abundance in these stars might have been decreased by various stellar phenomena (stellar winds, convective and/or rotational mixing, diffusion, destruction in deep layers) and possibly increased by production through cosmic rays. Theories that predict Li depletion in metal poor stars imply the existence of a dispersion in Li abundances and the existence of a small number of highly depleted stars, as observed among halo field stars. In this respect a Globular Cluster is an ideal testing ground for such theories, since it allows us to observe a population of the same age and metallicity. However, the full power of the VLT is required to obtain high quality spectra of the faint TO stars even in nearby Globular Clusters. Our analysis of the Li abundance in the TO stars of NGC 6397 showed that they share the same Li abundance (within errors), and there is very little room for dispersion above the observational errors. Out of the 15 TO stars so far observed in this cluster none has been found to be strongly Li depleted. This result therefore supports the primordial nature of the Li observed in these stars. From this value we determined a value of the baryonic density that is consistent at 1.3 σ with the value determined from the WMAP experiment.

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