High Performance Graphical Data Trending in a Distributed System


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ABSTRACT
Trending near real-time data is a complex task, specially in distributed environments. This problem was typically tackled in financial and transaction systems, but it now applies to its utmost in other contexts, such as hardware monitoring in large-scale projects. Data handling requires subscription to specific data feeds that need to be implemented avoiding replication, and rate of transmission has to be assured. On the side of the graphical client, rendering needs to be fast enough so it may be perceived as real-time processing and display.

ALMA Common Software (ACS) provides a software infrastructure for distributed projects which may require trending large volumes of data. For theses requirements ACS offers a Sampling System, which allows sampling selected data feeds at different frequencies. Along with this, it provides a graphical tool to plot the collected information, which needs to perform as well as possible.

Currently there are many graphical libraries available for data trending. This imposes a problem when trying to choose one: It is necessary to know which has the best performance, and which combination of programming language and library is the best decision. This document analyzes the performance of different graphical libraries and languages in order to present the optimal environment when writing or re-factoring an application using trending technologies in distributed systems. To properly address the complexity of the problem, a specific set of alternative was pre-selected, including libraries in Java and Python, languages which are part of ACS. A stress benchmark will be developed in a simulated distributed environment using ACS in order to test the trending libraries.

Keywords: Trending, Plotting, Benchmarking, Distributed Systems, ACS

1. INTRODUCTION
Atacama Large Millimeter/Sub-Millimeter Array (ALMA) is a joint project between astronomical organizations of Europe (ESO), North America (NRAO), and Japan (NAOJ). ALMA is a large radio-astronomical project, it will consist of at least 50 twelve meter antennas operating in the millimeter and sub-millimeter wavelength range with baselines up to 10 km. It will be located at an altitude above 5000 m on the Chajnantor plateau in the middle of the Chilean Atacama desert. The science commissioning of ALMA will start in 2012 when the array will be fully operational for astronomical observations.

ALMA Common Software (ACS) is an Object Oriented CORBA middleware framework (for science facilities) that handles communication between distributed objects. ACS was built to support the complex control requirements of ALMA radio telescopes, but can be used to support control and data flow for any system with similar performance requirements. Particularly, it features a Sampling System, which is used to continuously collect the values of an indicated set of properties around the system. This data can then be displayed by a
graphical client (the Sampling System GUI), in form of a dynamical plot. This client is written in Java, using the JChart2D library, and is currently being used at the ALMA observatory. Nevertheless, a question remains open: Which is the best library/language combination for such a task? One point to note is that ACS is open source (it is distributed under LGPL), so any base software used in it (like graphics libraries) must be compatible with this.

The work presented in this paper aims at evaluating different alternatives for high performance graphical data trending in distributed systems. It is important to distinguish trending (constructing graphs of data flowing in real-time) from plotting (where graphs, possibly very complex, are constructed from statically available data). Trending is inherently real-time, and is mostly concerned with showing trends for data that evolves in time.

First the most common data trending solutions were identified, and a list of alternatives is described. Performance tests are applied to the tool developed for the ALMA project in order to assess the performance of different graphical libraries. The problem of selecting a programming language and trending tool are described, then we discuss the state of the art in trending tools and graphical trending libraries. A methodology to test performance is then proposed, and the resulting benchmarks are discussed. Some real-case scenario tests were performed.

2. PROBLEM

When a development team is given the task of creating a software project, one of the first questions is the selection of the programming language. This is a very important decision, as performance is greatly affected by the paradigm and implementation of the available compilers for that programming language. The usual manner to solve this is to prototype, use previous experience and test the different wanted characteristics, such as modularity, performance, and scalability.

There are comparisons between different programming languages, such as, in which the author gives a walk through the origin of each language, and puts emphasis in the validation of a programming language comparison, because that depends on different characteristics, such as programmer capabilities, the different task and work conditions, the handling of misunderstood requirements, the paradigms employed (object oriented, imperative, generic programming). As is to be expected, such comparisons tend to concentrate only in one area.

Aside the selection of programming language, the team has to consider the selection of a graphical library. This is a nested decision, as very few libraries are available for several languages. Then, the question arises: “Of all the graphical libraries available, which is the best one for my project?” Strictly speaking, one should analyse a couple of libraries and then make a decision. In practice this is almost never done, as schedules are usually tight so there is no time available to evaluate every choice.

In our case, the main problem is to find the best choice in the data trending area, where performance of the graphical representation are critical. There are several ways to measure the performance and quality of graphical libraries, some examples are:

- Number of chart types handled.
- Trace options.
- Plot functionality.
- Frames Per Second (FPS).
- Data volume vs. performance

In the present work the most important metrics are FPS and the data volume vs. performance. Large amounts of data need to be quickly processed and displayed, together with additional information.

The development team has still two decisions open, both of them pending from a proper comparison. But there is yet another problem. Many systems in real world applications are not single-node systems, so that performance tested on a single computer may give a different result than when deployed on a distributed system.
This is the case of ALMA and ALMA Common Software (ACS) distributed applications. Distributed systems are complex, and communications is an important part of the data pre-processing.

Given this, a third question remains open: “How does a distributed system affect my trending application’s performance?”

### 3. STATE OF ART

Currently, a huge range of plotting solutions exists in the market. As time passes, more and more libraries are developed, or existing ones are improved, extended or updated. This makes it difficult both choosing one of such libraries, and to compare them in a distributed environment. In this section we aim to present some of the most used plotting and data trending solutions. This list is constrained to the set of languages that are used in ALMA Common Software (i.e., Java, C++ and Python). They are of help in understanding which variables are important to measure when considering the election of one solution.

#### 3.1 Graphical Libraries

The most significant part of the work when plotting data collected over a distributed system is the plotting itself. In the following section we present some of the most used plotting libraries, giving an overview of their implementations.

##### 3.1.1 Java

**JFreeChart**\(^9\) is a very popular chart library, since it allows to create complex charts easily. One of the most important characteristics, from the programmer’s point of view, is its well-documented API. Currently, **JFreeChart** supports different chart types, such as X-Y charts (line, spline, and scatter), pie charts, Gantt charts, bar charts (either horizontal, vertical and stacked and independent), and finally a single value graph (like for the case of a thermometer, compass, or speedometer). The flexible design of this library allows to extend the application in both the server and client sides. It also supports many output formats, including Swing components, image files, and vector graphics. Finally, **JFreeChart** is open-source software, distributed under the terms of the GNU Lesser General Public Licence (LGPL), which allows to use it both in open-source and proprietary applications.

**JChart2D**\(^10\) is a charting library designed for displaying multiple traces, which in turn consist of trace-points. Its main advantage is that it provides the programmer a minimalistic way to work with charts. It is important to note that **JChart2D** is centered around a Swing widget (Chart2D), so knowledge of Java AWT and Swing technologies is very useful. Some of its features are the renderization of the traces via lines, discs, dots, or filled polygons, multiple axes on top, bottom, left and right side, zoomable charts, multiples traces with different behavior in a simple chart, a toolbox of UI controls for charts via pop-up menus, automatic choice of units, optional display of grids, labels, and more. **JChart2D** is intended for engineering tasks, since its speciality is the dynamic precise display of the data in a minimalistic way, without much configuration. **JChart2D** is also published under LGPL.

**JCCKit**\(^11\) is a very small chart library, which features a very flexible framework for creating scientific charts and plots with the necessary elements. **JCCKit** is suitable for scientific applets, because it is written for the JDK 1.1.8 platform. The purpose of this library is to provide a flexible kit for programming applets and application for the visualization of some data. Some important features of **JCCKit** are that it is highly configurable, its extensibility and customizability, the automatic update of changed data, dynamic charts and plots, automatic rescaling, good support for logarithmic axes, line styles, symbols, fonts, error bars, and the compatibility with AWT graphic, SVG, and more.

**QN Plot**\(^12\) is a chart implementation that provides a way to program graphs (of one or more functions) as Swing components. Its design makes it possible to render large amounts of real-time data, which is an advantage when it comes to its usage in distributed systems. Some features of **QN Plot** are the coordination of different kinds of big decimals having arbitrary precision, its high performance with large amounts of data, all the classes in its implementation are thread-safe, the schemes of the axes have been specially written to choose step sizes for the index automatically. Finally, **QN Plot** is free software, released under the 2-clause BSD license.
3.1.2 Python

The PyQwt module is a set of Python bindings for the Qwt C++ class library which extends some features of the Qt framework to be able to build widgets for scientific and engineering applications. This library provides widgets to plot 2-dimensional data and work with the control of bounded or unbounded floating point values, and very large integer values, with various widgets displaying them. The main idea of PyQwt is to merge some Python modules, so as to have a complete framework to work with data. It mixes the GUI library PyQt, the plotting library Qwt, and NumPy and SciPy to cover the mathematical data manipulation because those libraries have a lot of computational methods that help manipulating the data easily, because the combination of NumPy and SciPy offers an environment very similar to Matlab.

Matplotlib is a very powerful plotting library developed for the Python programming language, which produces high quality figures in different formats. One of the main goals of this library is to make it easy to plot and manipulate data, because in a few lines one can generate histograms, bar charts, scatter-plots, simple plots, etc. Besides the Python language, Matplotlib uses NumPy, a numerical mathematics extension for Python, to do all the heavy mathematical processing. Matplotlib is very similar to MatLab, because it offers the PyLab interface to simplify learning, principally to the MatLab user. This makes it a good choice for numerical mathematics and signal processing tasks. Finally, Matplotlib is distributed under a BSD-style license.

Looking at other Python libraries, we found Biggles, which provides a lot of useful tools to create and manipulate scientific plots, having too as main idea the complete customization of the plots, so for Biggles the plots are a set of very simple objects. The idea of the Biggles objects is a good classification, taking two categories of objects, the containers and the components; so when we have a container we may use a lot of components for that container. Finally, we have the concept of Container for all the plots, tables, etc and the idea of the Components that need a Container to work, and can’t be visualized on their own. Biggles is a new graphical library, so it has far to go. This library is distributed under the terms of the GNU General Public License.

PyQtGraph is a very good library, because it uses two very popular Python modules and combines them in a nice way, we are talking of PyQt and NumPy, so the idea of PyQtGraph is to combine all the features of NumPy with the widgets provided by the PyQt wrapper. The objectives of this library are to be a good library to work with mathematics, scientific, and engineering applications. PyQtGraph is a very fast library, because it is written purely in Python and uses all the numerical capacities of NumPy and the fast display of Qt applications. Beside all the widgets that PyQtGraph provides, there are two important features, the highly feature-rich plotting systems and an image display system with region-of-interest widgets.

3.1.3 C++

Previously, we mentioned the PyQwt library, and we said that it is a wrapper for Qwt, while Qwt is an extension to the Qt library that contains useful GUI Components and utilities. Its main feature is the 2D plot widget, but Qwt provides several widgets/components that facilitate programming. Some of these components are scales, sliders, dials, compasses, thermometers, wheels and knobs to control or display values, arrays, or ranges of type double. Qwt is distributed under the terms of the Qt License, a variation of the GNU LESSER GENERAL PUBLIC LICENSE (LGPL) with some exceptions.

Koolplot is a very simple-to-use library that allows to create and manipulate 2D graphs. It is very small and basic, so it is not recommended for use in complex graph situations. A feature of Koolplot is the compatibility with the MinGW compiler, so it can be used on Linux and Microsoft Windows platforms. Finally, Koolplot is in the public domain.

wxMathPlot is a properly built library to add 2D plot scientific functionality into wxWidget, a cross-platform C++ library to create applications for Microsoft Windows, OS X and Linux. As it can be inserted into wxWidgets, it allows to embed inside every wxWidgets application a window for plotting different types of data. Some of the most important features of wxMathPlot are the completely mouse-driven view control (pan, zoom, scroll, etc), the different output formats of the screenshots (BMP, PNG and JPEG), the flexible axis positioning, the several layers to plot data from vectors, movable objects, bitmaps, etc. Finally, wxMathPlot is distributed under the thetherms of the wxWindows Licence, that is essentially LGPL with some exceptions.
Carnac Chart Library is an extension to the Qt library that adds powerful visualization. The main idea is to allow programming complex charts with minimum effort. Some features of Carnac are the large number of different chart types supported, and its flexibility in setting up axes and labels.

GNUplot++ is a wrapper of GNUplot through C++. GNUplot is a very old and properly build command-line program that can generate different types of plots, frequently used for publication quality graphics, and is multi-platform (Linux, Microsoft Windows, Mac OS X, etc) GNUplot++ mixes the powerful GNUplot tools with many features of standard C++, for example templates class. It uses many features of standard C++, like integration to the standard template library (STL) and its iterators. GNUplot++ is distributed under GPL.

4. METHODOLOGY PROPOSAL

The graphical library benchmark was performed in the same conditions for each library, the idea was to test the Frames Per Second (FPS) to compare their performance. Each program has a similar code structure, with the following conditions:

- One thread in charge of feeding the plot with random data.
- A main thread to execute the program.
- A program widget without details, only the dynamic plotted data (axis, labels, etc)

The next step is a comparison between graphical libraries and study the behavior with different latency times (data actualization rate), and finally extract a conclusion about the best performing graphical library. Finally, with the graphical library chosen, the task is to compare language performance.

On the other hand, the real benchmark was performed on an existing application, the Sampling System GUI. The ACS Sampling System is a collection of objects designed to easily sample an ACS Components Property value over time. The Sampling System GUI (SSG) is a client to the Sampling System written in Java, using the JChart2D library. SSG communicates with several Sampling Managers to create Sampling Objects and group them as needed, in order to sample properties, and finally plot the values in user-time (i.e., as they arrive through the ACS Notification Channel). SSG allows easy, quick visualization of system behavior during a period of time, or under certain circumstances, and gives the possibility of visually correlating the values of different properties of the system.

5. GRAPHICAL LIBRARY BENCHMARK

This section presents the results of the different performance tests applied to a selection of graphical libraries. The technical characteristics of the computer are:

- Intel(R) Core(TM)2 Duo CPU 2.66 [GHz]
- 4 Gigabyte RAM
- Operating System Fedora 12 for i686

Each benchmark consisted in taking two simple examples of dynamic data plotting for each library, in which the data was a random value to simulate a real environment. The tests were repeated for periods between data updates of 100 [ms], 10 [ms] and 1 [ms] in each case, comparing frames per second (FPS) and their variances. The 100 [ms] test is slow updates, 10 [ms] is rather fast and 1 [ms] is a real stress test. The scripts running the tests measure the FPS each 200 data objects, each measurement is one data point in the graphs given below. Note that raw FPS can be misleading, for smooth trending graphs 20 FPS is adequate, while 50 FPS is absolutely perfect. Perhaps a better measure would be to see how many data points can be updated at a given rate while still reaching 20 FPS (or 50, as the case may be). Other considerations are the impact on performance if there is a clear trend (new data are just added near the end of the graph) or appear scattered. The Data from each plot, come from a method that generates random values, and the plot refresh is performed using each library’s own methods. The difference between the FPS and the arrived data is that the FPS is the amount of refresh in the plot, considering the data that have been sent.

The data coming in from a method that
5.1 Java

Here we compare two Java libraries, JFreeChart and JChart2D. The resulting FPS of each library for 100 [ms] are shown in figure 1(a). At first look, JFreeChart shows a inconspicuous lower performance graphing data with 100 [ms] of latency time. There is little difference between the averages of each library, of the order of 0.024. JChart2D has a standard deviation of 0.055 and a average of 9.975, which means that the difference between the FPS are very similar, and has a minimal variation. JFreeChart has a standard deviation of 0.111 and a average of 9.952, which means that the difference between the FPS at each point has a minimal variation, but is higher compared to JChart2D. An important fact is that this latency is slow, so taking it as reference the scientific data visualization topic we need receiving the data with a higher frequency.

The second test was performed with an interval of 10 [ms], giving the results in figure 1(b). Decreasing the interval ten times we see that the difference between the averages of the libraries increases a little more, we are talking about 2.22 frames more for JChart2D. JChart2D has a standard deviation of 6.861 and a average of 96.974, which means that the difference between the FPS are now higher than in the previous test, ant it is a large for this same task. JFreeChart has a standard deviation of 10.717 and a average of 94.754, which means that the difference between the FPS at each point has a noticeable variation, and is higher than the one exhibited by JChart2D. Also, JChart2D has a regular performance varying a little at each point, but JFreeChart does not have a regular performance, the three and four points showed a substantial fall. This fact makes this library unsuitable, because visualizing scientific data requires a regular expected behavior.

The third test was performed with a latency time of 1 [ms]. The resulting FPS of each library are shown in figure 1(c). Finally, the difference between FPS for each library is more noticeable, we are talking about 225.98 FPS, and JChart2D outperforms JFreeChart. JChart2D has a standard deviation of 207.715 and a average of 858.307, which means that the difference between the FPS are now higher related to the two previous tests, we also note that as the FPS at each point are large values in comparison, so that implies the big difference between the values. JFreeChart has a standard deviation of 171.445 and a average of 632.322, which means that this library has a lower standard deviation related to the JChart2D, but the value still being a large value for a standard deviation; in another hand, the average has a lower value related the JChart2D average. Like the previous test, we can look the irregularities of the JFreeChart performance, at points 3, 6, 8 and 10, so this reaffirms the previous fact that the performance of JChart2D is most recommendable.

5.2 Python

We also compared two Python libraries, PyQwt and Matplotlib. First, the programs start with an interval of data actualization of 100 [ms], and the resulting FPS of each library are shown in figure 2(a). In this test, as we know in the case of Java, is a kind of non realistic test, because the data was actualized slowly. PyQwt has a more stable performance, showing a standard deviation of 0.0004 that is very close to zero, keeping the FPS near to 10, but the average of 9.999 is lower in relation to Matplotlib. In the side of Matplotlib, the performance has higher and lower points, but at point number 4 it stabilizes, having an average close to 10.029. At first look,
Matplotlib is a better choice because it has a better average, but a higher standard variation, 0.010; but if the programmer is looking for a library with stable performance, choosing Matplotlib means taking some risks.

Second, the programs start with an interval of data actualization of 10 [ms], and the resulting FPS of each library are shown in figure 2(b). In this test, a notorious difference comes out, because the difference between the averages is around 55.90 FPS, so PyQwt library is the best choice with a average of 98.500. Anyway, Matplotlib tries to gain performance from point 5 on forward, but still having a lower average of 42.598. The standard variation of these libraries are very similar, being 3.932 and 3.887 relatively. On the other hand, the PyQwt has a more constant performance, which tell us the stability of the library.

Third, the programs start with an interval of data actualization of 1 [ms], and the resulting FPS of each library are as given in figure 2(c). In this stressed situation the notorious difference between the performance of PyQwt and Matplotlib is finally visible, showing averages of 697.050 and 42.463, respectively. Aside of the performance average, the standard deviation of Matplotlib of 2.809 is much lower compared to the PyQwt standard deviation of 300.325, but this has direct relation with the obtained values at each point. As final words it is necessary to say that Matplotlib is not designed purely to create dynamic plots, its main goal is to create a lot of graph types in a easy way. On the other hand, PyQwt has the direct binding from the Qwt library that was designed to obtain high performance in data trending.

5.3 C++

Finally, on the C++ side we present the results of the comparison between Qwt and wxMathPlot. The thread handling in C++ is more efficient, as it isn’t hindered by portability layers. As a result, the FPS values obtained are much larger than the ones in previous tests. FPS in this case are almost equivalent to the data generation rate.

First, the programs start with an interval of data actualization of 100 [ms] (latency time), and the resulting Frames Per Second (FPS) of each library are shown in figure 3(a). The Qwt library shows better performance
than wxMathPlot, which is reflected in the average of each, 51916.5 and 14589.5, respectively. On the other hand, the standard deviation tells us about the stability of each library, in this case wxMathPlot has a decreasing stability, with a standard deviation of 3605.235, which is acceptable because the quantity of data is large enough. Also the Qwt library, has a more unstable behavior with higher and lower points, which is defined by their standard deviation of 51916.5 that is greater that the previous by one order of magnitude.

Continuing with the test, now with a latency rate of 10 [ms], the results in FPS are in figure 3(b). In this case the Qwt performance increases a little, but at point 2 for example, it is surpassed by wxMathPlot. But in general, Qwt has better performance with a average of 47830.2 in comparison to the 26988.1 of wxMathPlot. Respect to the standard deviation wxMathPlot remains the decreasing stability showing a value of 16242.181, that is very high in relation with the standard deviation of Qwt, 5949.169. Also Qwt still shows an irregularity in its performance, due to thread behavior.

Finally we have the last test, that consist in stressing out C++ data trending, with a latency time of 1 [ms]. The results are in the figure 3(c). In this test, both libraries show very strange behavior, due to the data Thread actualization with a very low value as is 1 [ms]. Qwt still wins the match with wxMathPlot, but the irregularity is still present, because the standard deviation raises to 6271.181, but in the case of wxMathPlot that value decreases to 8805.198, which means that in stability wxMathPlot has the first place, but related to performance, Qwt wins with a average of 43771.6 over the wxMathPlot average of 21316.9.

### 5.4 General comments

The reasons of the lower performance of Python and Java with respect to C++ are very simple to explain. The execution of every Java program depends on the Java Virtual Machine. With respect to Python, PyQwt has simple bindings to the Qwt/C++ library, so they have a longer path to walk to interact with the final plot.

Another important issue is the threads implementation of each language. The C++ Standard Library has less features (functionality) and a limited scope related to the Java Standard Library, but includes native threads libraries. On the other hand, Python provides low-level primitives for working with multiple threads, but in difference to C++, Python has POSIX threads and non-POSIX threads. Software developed in Python is generally slower than Java, but development time is less with Python, since Python has dynamic typing and offers built-in high-level data types. Comparing Java with C++ brings us to similar conclusions.

### 6. REAL BENCHMARK

This section presents the result of testing the Sampling System GUI in a real case scenario at the ALMA Observatory. The Sampling System GUI is used in the same deployment as the operations software (ALMASW). Figure 4 roughly shows the distributed environment where ALMASW and the Sampling System GUI are deployed.

ACS and the sampling system run on the ACS server (figure 4). The sampling system GUI is run from the operations console. This setup was selected to monitor the behavior of the FrontEnd receiver during its locking routine. The monitoring setup considered two charts, each with:

- Five properties being sampled.
- Sampling frequency of 20 [Hz].
- Store a window of 15 minutes of plots.
- Storing all of the data to disk.

20 [Hz] is the maximum monitoring frequency available in ALMA since it is limited by its 48 [ms] period.

The test that was run on the FrontEnd receiver consisted in locking the band in 0.5 [GHz] steps over the range of the four available bands. The test took approximately 4 hours, time during which the sampling system GUI was continuously plotting and saving data to disk. At the time of the test only two antennas were available,
this test should be run again when there are six available antennas for such purpose. This would demonstrate how the overall system behaves when increasing the number of distributed nodes.

The overall result of the Sampling System is positive, since it met all the engineering requirements for monitoring during the specified test. It could cope with the amount of properties and the sampling rates specified by the engineering division. Data was properly stored to disk, and the plots provided a quick look for the data before it was more thoroughly analyzed.

7. CONCLUSIONS

Better ways of measuring performance, like the number of data streams that can be displayed at a given FPS or the impact of smoothly evolving data or scattered points, should be considered.

Looking backwards, the evolution of the different libraries give the programmers a lot of “path to follow” when starting a new project. The performance of the actual data trending libraries in different languages is very important because all of them try to offer “simple programming” in this application area. Comparing the features of the libraries is not so important, because most of them are rather robust, and those with more features just allow creating more elaborate plots. Without going further, there are other characteristics to consider, like “perdurability,” “modularity,” and “scalability” of software. For a long-lived project like ALMA (and its supporting software ACS) these characteristics are crucial. How to estimate the stability and longer range development and maintenance of open source software on which something like ACS is based is still very much an open question.

In section 5 we saw benchmarks of a couple of graphic libraries in the three languages that ACS uses, C++, Java, and Python. The benchmark shows the behavior of these libraries in three levels of stressed environments, giving us data to discriminate among them. Anyway, this is not a final decision, because one benchmark is not enough to test the real performance of a graphical library, but it is a good start to help discriminating them with the previous basic functionality (plotting random data). Also, other important characteristics haven’t been considered in detail. For a long-lived project like ALMA (and its supporting ACS package) using stable, well-maintained base software is crucial.

Finally, in section 6 we can see the performance of the existing scientific data visualization tool, developed by the Computer Systems Research Group (CSRG), and giving us the chance to analyze a real application in a real distributed system such as the ALMA project.
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