Calibration of interferometric data acquired with a single-mode interferometer

Mini-Workshop on VLTI Calibrators

ESO Garching

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Most of this presentation is published in:

« *The calibration of interferometric visibilities obtained with single-mode optical interferometers. Computation of error bars and correlations* »

To appear in A&A (accepted December 19, 2002)

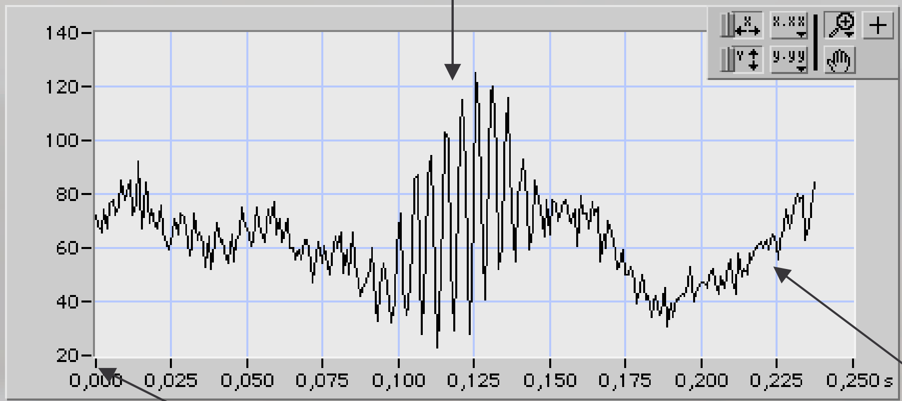
Available on astro-ph:

http://xxx.lpthe.jussieu.fr/PS_cache/astro-ph/pdf/0301/0301140.pdf

The fundamental question

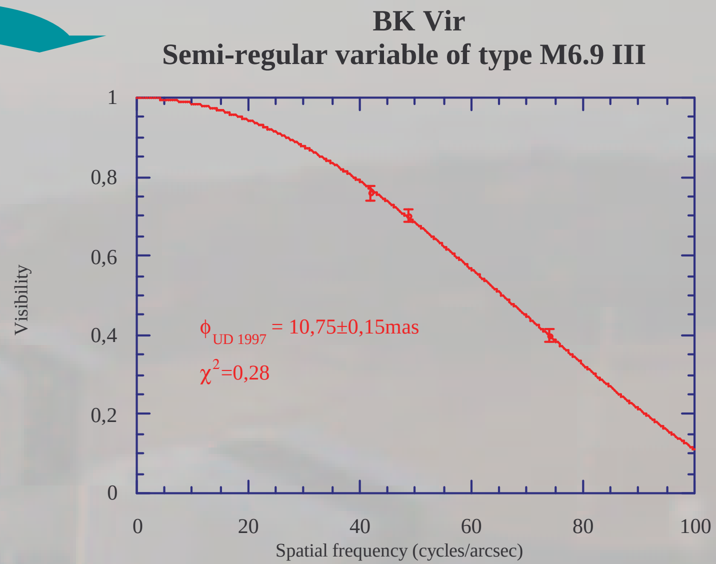
How can we get to the calibrated visibility?

The coherent signal fluctuates



The background may fluctuate

The incoherent signal fluctuates



Instantaneous contrast :

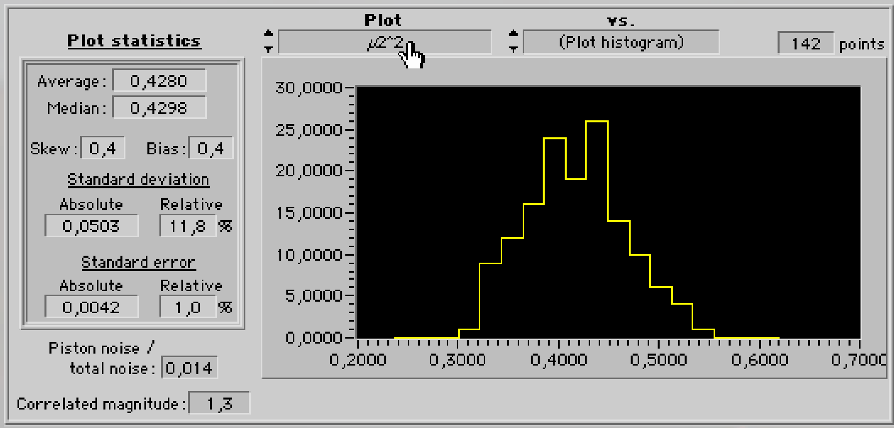
$$C(t) = \frac{2\sqrt{(P_1 + bg_1)(P_2 + bg_2)}}{(P_1 + bg_1) + (P_2 + bg_2)} \times T_{inst} \times T_{atm} \times V_{source}$$

Some vocabulary

- ***Visibility:***
- ***Coherence factor (μ):*** uncalibrated contrast of the fringes
- ***Transfer function:*** ratio of expected visibility to measured coherence factor (no T_{atm} for single-mode instruments)

Estimate of coherence factors

- Squared coherence factors are computed for each scan (photometric and background corrections are performed)



- They define a statistics (histogram) from wich a standard deviation is derived

- Eventually one gets:

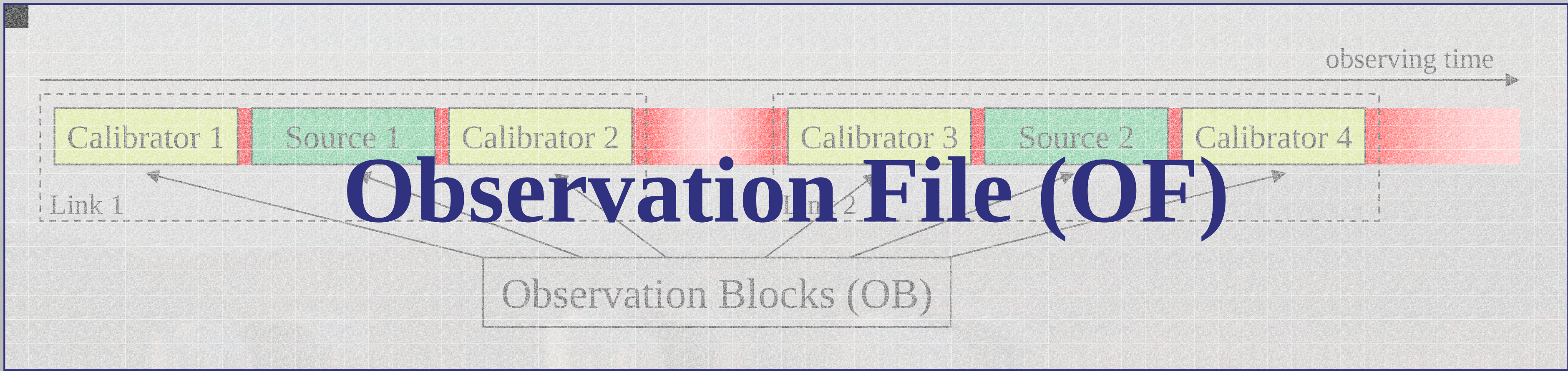
$$\mu_1^2 \pm \sigma\left(\mu_1^2\right)$$

$$\mu_2^2 \pm \sigma\left(\mu_2^2\right)$$

(Case of a 2 output beam-combiner, e.g.: MIDI and VINCI)

Then the calibration process starts !

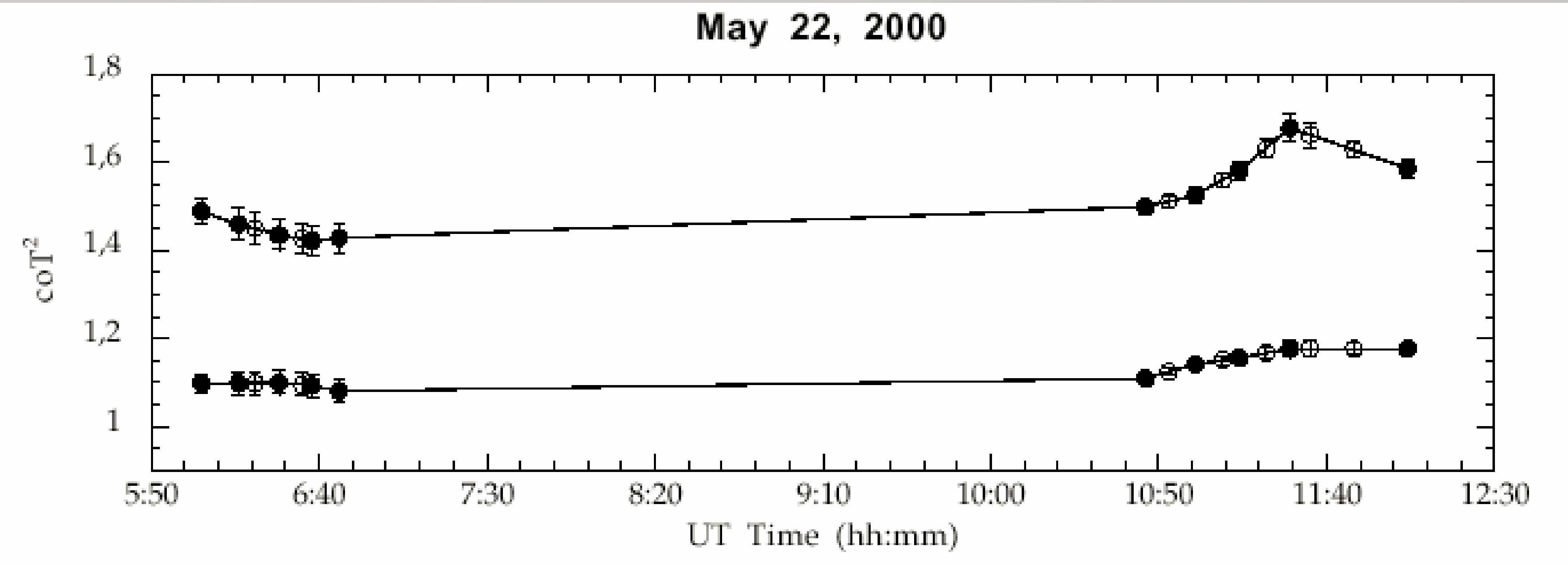
Principles of the calibration



- 1 OF = 1 set-up
  - same night
  - same detector parameters (frame rate, number of frames, ...)
  - same filter...

- Principle : follow slow coherence loss fluctuations

Example of transfer function



It varies slowly enough that the linear interpolation is valid

Steps

1. Derive calibrator's expected visibility: usually a uniform disk diameter is used to predict visibility at the spatial frequency S

$$V_{\exp}(S) = \left| \frac{2J_1(\pi\theta_{UD}S)}{\pi\theta_{UD}S} \right|$$

2. Derive instantaneous transfer function for each channel

$$T_i^2(t_1) = \frac{\mu_i^2}{V_{\exp}^2(S)}$$

3. Interpolate transfer function at time τ when source was observed

$$T_i^2(\tau) = \left(\frac{t_2 - \tau}{t_2 - t_1}\right) T_i^2(t_1) + \left(\frac{\tau - t_1}{t_2 - t_1}\right) T_i^2(t_2)$$

4. Calibrate single channel visibility for science target

$$V_{ST,i}^2 = \frac{\mu_{ST,i}^2}{T_i^2(\tau)}$$

5. Derive a final visibility estimate from the two single channel visibilities

Error bars and model fitting: the issue of correlations

- In order to perform a meaningful estimate of error bars it is necessary to evaluate correlations between quantities:
  - correlated noise between the two outputs of the interferometer (not necessary in the case of AMBER as there is a single output per baseline). This correlated noise is partly due to turbulence and partly to the correlated noise of the detector.
  - correlated error due to the use of the same calibrator diameter estimates in the two channels

- Correlations for 3T, 4T, etc … simultaneously recorded visibilities

Error bars and model fitting: the issue of correlations

- Correlations between visibilities recorded at different times, with different baseline,..., have to be taken into account for model fitting. The (possible) correlation is due to the use of common calibrators (the expected visibilities are then correlated).

Correlation between channels for a co-axial beamcombiner

The correlation ρ_{12} is computed (Monte-Carlo) from the measured correlations of the μ^2 measurements (source and calibrators) and from the errors on μ^2 and the calibrator diameters uncertainties

$$r = \frac{\langle \mu_1^2 - \overline{\mu_1^2} \rangle \langle \mu_2^2 - \overline{\mu_2^2} \rangle}{\sqrt{Var(\mu_1^2) Var(\mu_2^2)}},$$

The final visibility estimator is the least-square best average value:

$$V^2 = \frac{V_1^2(\sigma_2^2 - \rho_{12}\sigma_1\sigma_2) + V_1^2(\sigma_1^2 - \rho_{12}\sigma_1\sigma_2)}{\sigma_1^2\sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

An error bar is derived from the fit as well as a quality factor. Data with too large a χ^2 are ***a priori*** eliminated

$$\sigma_{V^2}^2 = \frac{(1 - \rho_{12}^2)\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

$$\chi^2 = \frac{(V_1^2 - V_2^2)^2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

This method can be generalized to the case of more than two-way beamcombiners and a correlation matrix has to be saved with the data ...

Correlation between non-simultaneous visibilities

Visibilities are measured at 2 different baselines S_1 and S_2 and calibrated by A, B, C and D. Visibilities are linearly linked to the expected visibilities of the calibrators:

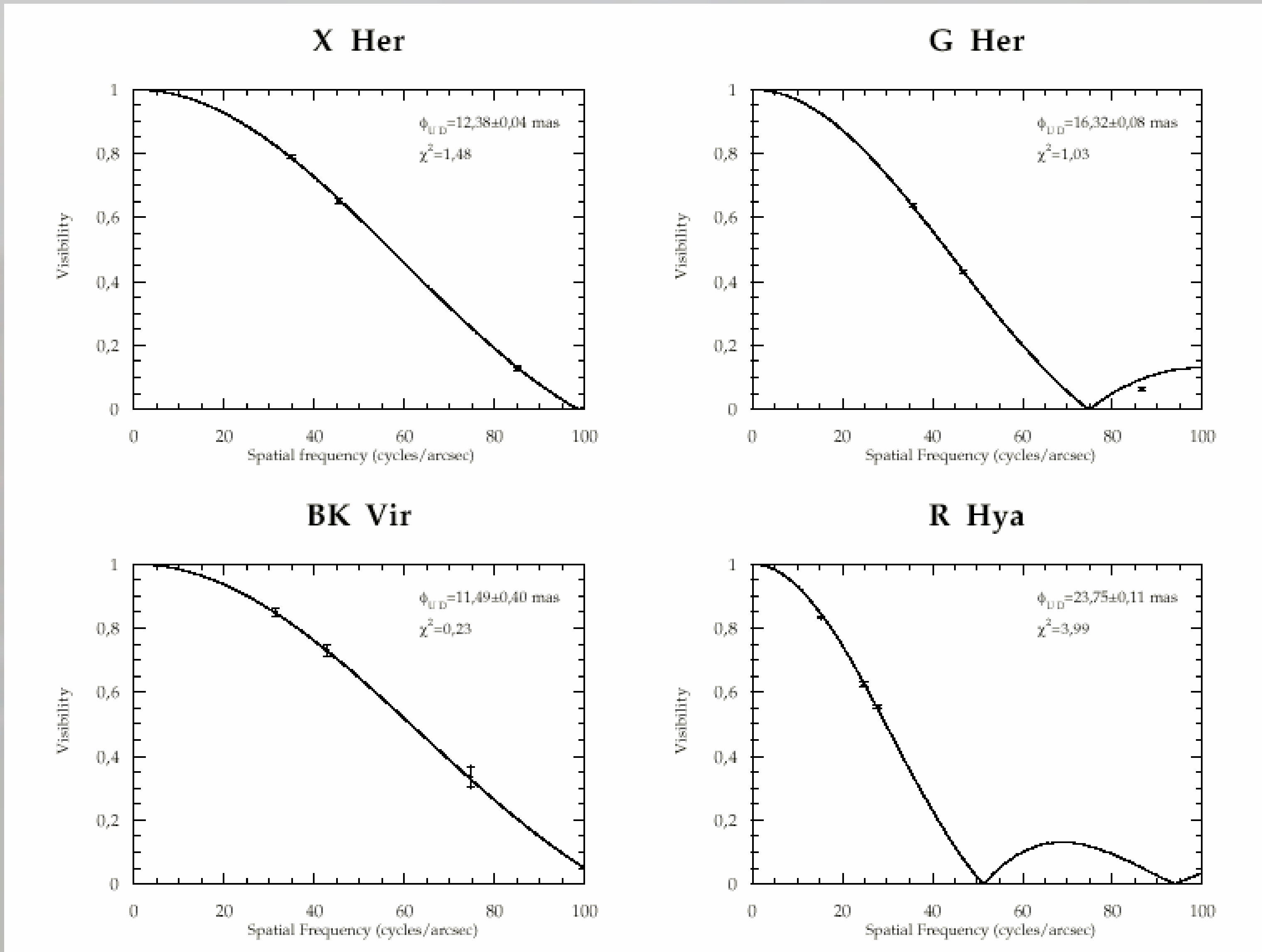
$$\begin{cases} V^2(S_1) = \alpha_1 V^{a^2}(S_1) + \beta_1 V^{b^2}(S_1) \\ V^2(S_2) = \alpha_2 V^{c^2}(S_2) + \beta_2 V^{d^2}(S_2) \end{cases}$$

The correlation factor between the 2 visibilities is in the general case:

$$\begin{aligned} \rho(V^2(S_1), V^2(S_2)) = & \frac{\alpha_1\alpha_2\sqrt{Var(A)Var(B)}\rho(A,C)}{\sqrt{Var(V^2(S_1))Var(V^2(S_2))}} \\ & + \frac{\alpha_1\beta_2\sqrt{Var(A)Var(D)}\rho(A,D)}{\sqrt{Var(V^2(S_1))Var(V^2(S_2))}} \\ & + \frac{\beta_1\alpha_2\sqrt{Var(B)Var(C)}\rho(B,C)}{\sqrt{Var(V^2(S_1))Var(V^2(S_2))}} \\ & + \frac{\beta_1\beta_2\sqrt{Var(B)Var(D)}\rho(B,D)}{\sqrt{Var(V^2(S_1))Var(V^2(S_2))}} \end{aligned}$$

It is therefore necessary that the pipeline outputs α and β to compute the correlation a posteriori.

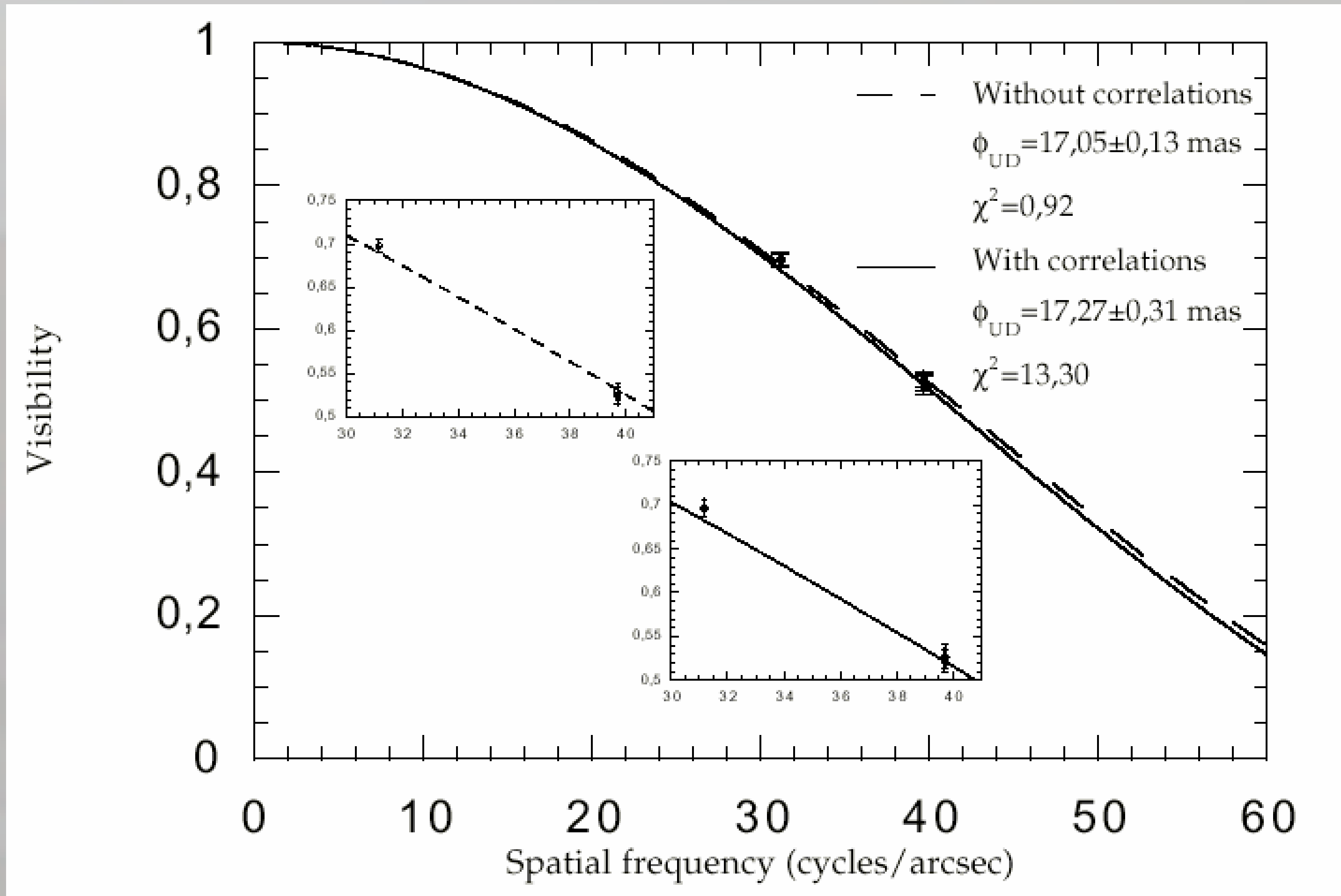
Examples of calibrated data and fits



Another example with SW Vir

Visibilities are almost fully correlated as a single calibrator has been used and most noises are negligible except the error on the calibrator's diameter which dominates:

$$C = \begin{bmatrix} 1 & 0.96 & 0.96 \\ 0.96 & 1 & 0.97 \\ 0.96 & 0.97 & 1 \end{bmatrix}$$



With correlations the fit is not of excellent quality. Either the calibration was bad or the model is not correct at the level of the uncorrelated noise.

Generalization of the SW Vir example: *relative calibration*

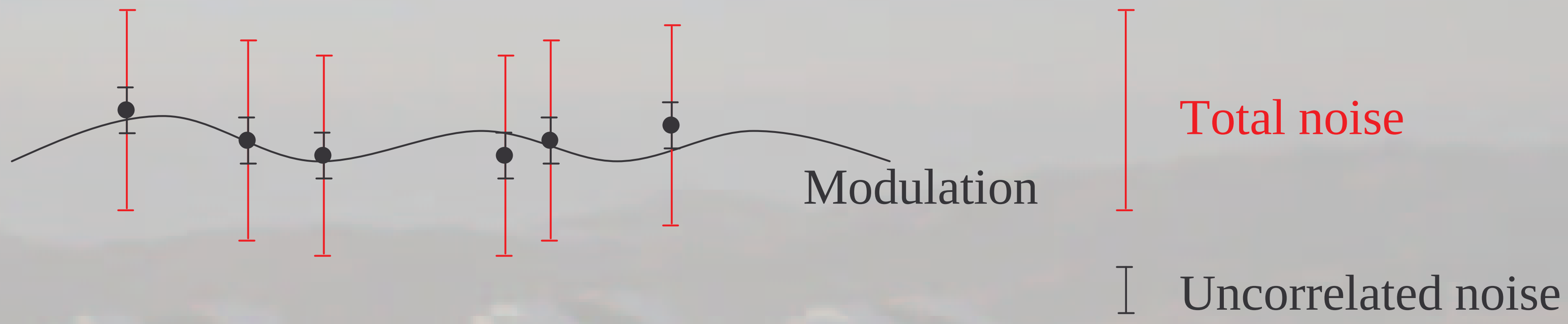
If a unique calibrator is used then the method allows to distinguish in the error bars the share of noise due to the uncertainty on the calibrator and the share of noise due to uncorrelated processes.

An application of this is the detection of faint V features despite « large » error bars.

Suppose calibrated visibilities have been measured with 1% error bars but are almost fully correlated as in SW Vir at the 97% level. Then the share of uncorrelated noise is only 0.03%.

Applying the χ^2 test, features far below the noise at the 0.03% level can therefore be detected.

This can be a very powerful method to detect exoplanets and relaxes the issue of the perfect knowledge of the calibrators.



If correlations are considered then the modulation is detectable otherwise it is not statistically significant.

Conclusions

- Calibration is an important issue but it has been demonstrated to work well on single-mode interferometers which means that meaningful error bars can be evaluated
- It is necessary to take all correlations into account to compute the best error bars
- Keeping track of correlations is necessary even after processing the data to obtain valuable fits
- Strategies taking into account correlations can be very powerful to measure faint signals such as exoplanets