Cosmology with Supernovae

Bruno Leibundgut

ESO Knowledge Exchange Series – 24 September 2018

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Cosmology with Supernovae

- Fundamentals
 - Observables
 - Theory
- Supernovae as distance indicators
 - Tests of General Relativity
 - Time dilation
 - Distance duality
 - Hubble Constant H_0
 - Mapping the expansion history $(q_0, \Omega_M, \Omega_\Lambda)$

Cosmic Distances

Separate the observed distances r(t) into the expansion factor a(t) and the fixed part x (called *comoving* distance) r(t) = a(t)x



Calculating Distances

Simple example of distances in flat space:

- Coordinates x and y
 - Distance: $dl^2 = dx^2 + dy^2$ (Cartesian)
- Coordinates r and θ
 - Distance: $dl^2 = dr^2 + r^2 d\theta^2$
- In general:

$$\mathrm{d}l^2 = \sum_{i,j=1,2} g_{ij} dx^i \, dx^j$$



Cosmic Distances

- In 4 dimensions
 - (time as the 0th coordinate) this becomes $ds^{2} = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} dx^{\mu} dx^{\nu} = g_{\mu\nu} dx^{\mu} dx^{\nu}$

using the (Einstein summation) convention where repeated indices are summed

or explicitly:

– Minkowski (flat) space

$$ds^{2} = (cdt \ dx \ dy \ dz) \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix}$$

Calculating Distances Expanding universe with scale parameter a(t) $ds^{2} = (cdt \ dx \ dy \ dz) \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^{2}(t) & 0 & 0 \\ 0 & 0 & a^{2}(t) & 0 \\ 0 & 0 & 0 & a^{2}(t) \end{pmatrix} \begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix}$

→Lemaître-Friedmann-Robertson-Walker (FRW) metric for an isotropic and homogeneous universe

 ds^2 is proper space and the metric $g_{\mu\nu}$ is the conversion from the coordinates dx^{μ}



Geodesic and Coordinate Transformation

Simple case:

- Minkowski space (flat):
- movement of a force-free particle (geodesic):



 $\frac{d^2x^i}{dt^2} = 0$, with $x^i = (x, y)$ (Cartesian coordinates)

- How does this look like in polar coordinates?

$$x'^{i} = (r, \Theta) \implies \frac{d^{2}x^{i}}{dt^{2}} \neq 0 \parallel \parallel$$

An everyday $r\theta z$ system



Geodesic

General equation of a freely moving particle $\frac{d^2 x'^l}{dt^2} + \Gamma_{jk}^l \frac{dx'^k}{dt} \frac{dx'^j}{dt} = 0$

• Note the affine connection (Christoffel Symbol) $\Gamma_{jk}^{l} = 0$ for Cartesian coordinates



Recap Einstein Equations

- Gravity is the dominant force in the universe
 → General Relativity
- Need the most general form of the metric → transformations between coordinate systems
 - find 'invariant' parameters
- Equation of motion for a force-free particle $(\ddot{x} = 0)$ in GR leads to affine connections \rightarrow Christoffel symbols
- Putting this together with the geometry and the energy content → Einstein Equations

Lemaître Robertson Walker Metric Assumes homogeneity and isotropy Line element in polar metric has angular and radial components

$$ds^{2} = -c^{2}dt^{2} + a^{2}(t)\left[\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2}\right]$$

$$g_{00} = -1; \quad g_{rr} = \frac{a^2(t)}{1 - kr^2};$$

$$g_{\theta\theta} = a^2(t)r^2; \quad g_{\phi\phi} = a^2(t)r^2\sin^2\theta$$



Light ray coming towards us

No angular dependence, hence $cdt = \pm a(t) \frac{dx}{\sqrt{1 - kx^2}}$

and integrated

$$s = a \int_0^x \frac{dx}{\sqrt{1 - kx^2}} = aS(x)$$

with

$$S(x) = \begin{cases} \arcsin(x) & k = 1\\ x & k = 0\\ \operatorname{arcsinh}(x) & k = -1 \end{cases}$$

Strange Consequences

- k=1
 - closed universe
 - distances increase and then decrease again with increasing x
- k=0
 - 'critical' universe
 - expands forever
- k=-1
 - open universe
 - expands forever

The Energy-Momentum Tensor

Use the form for the 'perfect fluid'

$$T^{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$



Einstein's Field Equation

The (time) evolution of the scale factor depends only on the time-time component of the Einstein equation:

$$R_{00} - \frac{1}{2}g_{00}R = \frac{8\pi G}{c^4}T_{00}$$
$$- T_{00} = \rho c^2 \text{ (energy density)}$$
$$- \text{ time part } R_{00} - \frac{1}{2}g_{00}R = \frac{3}{c^2}\left(\frac{\dot{a}}{a}\right)^2$$

Friedmann Equation

Time evolution of the scale factor is described through the time part of the Einstein equations

Assume a metric for a homogeneous and isotropic universe (metric is diagonal in polar coordinates) and a perfect fluid

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho(t)$$

Friedmann Equation

Put the various densities into the Friedmann equation

$$\frac{\dot{a}^2}{a^2} = H^2 = \frac{8\pi G}{3}\rho(t) - \frac{k}{a^2} = \frac{8\pi G}{3}\left(\rho_M + \rho_\gamma + \rho_{vac}\right) - \frac{k}{a^2}$$
Use the critical density $\rho_{crit} = \frac{3H_0^2}{8\pi G} \approx 2 \cdot 10^{-29} \ g \ cm^{-3}$ (flat

universe), define the ratio to the critical density $\Omega = \frac{\rho}{\rho_{crit}}$

Most compact form of Friedmann equation

$$1 = \Omega_M + \Omega_\gamma + \Omega_{vac} + \Omega_k$$

with $\Omega_k = -\frac{k}{a^2 H^2}$

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Dependence on Scale Parameter

For the different contents there were different dependencies for the scale parameter

$$\rho_M \propto a^{-3}$$
 $\rho_\gamma \propto a^{-4}$ $\rho_{vac} = const$

Combining this with the critical densities we can write the density as

$$\rho = \frac{3H_0^2}{8\pi G} \left[\Omega_M \left(\frac{a_0}{a}\right)^3 + \Omega_\gamma \left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_k \left(\frac{a_0}{a}\right)^2 \right]$$

and the Friedmann equation
$$H^2 = H_0^2 \left[\Omega_M (1+z)^3 + \Omega_\gamma (1+z)^4 + \Omega_\Lambda + \Omega_k (1+z)^2 \right]$$

Redshift

Redshift is directly related to the ratio of the scales between emission and absorption of a photon



This is remarkably simple as a measurement in a spectrum tells the scale changes

Cosmological Redshift

For two different times we get

$$\frac{dt_1}{a(t_1)} = \frac{dt_2}{a(t_2)}$$

- i.e. the time scales with the scale parameter If the time intervals dt are interpreted as oscillation periods, e.g. of a photon, then

$$\frac{dt_1}{dt_2} = \frac{\nu_2}{\nu_1} = \frac{a(t_1)}{a(t_2)} = \frac{1}{1+z}$$

with z as the redshift between the two times

Energy-Momentum Tensor

• The time (00) component of the Einstein equations is

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3p)$$

- As long as pressure and density are positive the universe decelerates $\ddot{a} < 0$.
- Acceleration requires $\rho c^2 + 3p < 0$ or $\omega < -\frac{1}{3}$.

Energy-Momentum Tensor

- A general form is an equation of state $p = \omega \rho c^2$. ω is the equation of state parameter.
- Inserting this into the conservation equation gives $\frac{\dot{\rho}}{\rho} = -3(1+\omega)\frac{\dot{a}}{a}$ which integrates to $\log(\rho) = -3(1+\omega)\log(a) + const.$
- Exponentiating yields $\rho \propto a^{-3(1+\omega)}$

Matter

• The pressure in matter is negligible compared to the mass content (think mc^2) and hence $\omega = 0$

• Thus
$$\rho_M \propto a^{-3}$$

• Inserting this in the Friedmann equation for a flat universe (k=0) provides the time dependence of the scale factor $a(t) \propto t^{2/3}$

Radiation

- Radiation decreases with the volume (i.e. number of photons), but has one additional factor due to the redshift $\omega = \frac{1}{3}$ and hence $\rho_{\gamma} \propto a^{-4}$
- The time dependence here is now $a(t) \propto \sqrt{t}$

Vacuum energy

- A special case is $\rho_{\Lambda} = const$.
- In this case the density is associated to the vacuum
- Now the scale factor grows exponentially $a(t) \propto e^{Ht}$

With the equation of state parameter $\boldsymbol{\omega}$

General luminosity distance

$$D_{L} = \frac{(1+z)c}{H_{0\sqrt{|\Omega_{k}|}}} S\left\{ \sqrt{|\Omega_{k}|} \int_{0}^{z} \left[\Omega_{k}(1+z')^{2} + \sum_{i} \Omega_{i}(1+z')^{3(1+\omega_{i})} \right]^{-\frac{1}{2}} dz' \right\}$$

- with $\Omega_{k} = 1 - \sum_{i} \Omega_{i}$ and $\omega_{i} = \frac{p_{i}}{\rho_{i}c^{2}}$

- $\omega_M = 0$ (matter)
- $\omega_{\gamma} = \frac{1}{3}$ (radiation)
- $\omega_{\Lambda} = -1$ (cosmological constant)

Synopsis

- Gravity \rightarrow Einstein Equations
 - 'Contents' \rightarrow Energy-momentum tensor
 - perfect fluid, density, pressure
 - dependence on different contents
 - matter, radiation, vacuum, curvature
 - FRW metric (isotropy, homogeneity) →
 different curvature models
- Time evolution → Friedmann Equation
 Hubble constant
- Redshift \rightarrow related to scale factor

Lookback Time

Consider

$$H = \frac{\dot{a}}{a} = \frac{\mathrm{da}}{\mathrm{dt}}\frac{1}{a} = \mathrm{dt}\ln\left(\frac{\mathrm{a}(\mathrm{t})}{\mathrm{a}_0}\right) = \frac{1}{\mathrm{dt}}\ln\left(\frac{1}{1+\mathrm{z}}\right) = -\frac{1}{1+\mathrm{z}}\frac{\mathrm{dz}}{\mathrm{dt}}$$

Inserting into the Friedmann equation we find the equation for the time interval

$$dt = \frac{-dz}{H_0(1+z)\sqrt{\Omega_M(1+z)^3 + \Omega_\gamma(1+z)^4 + \Omega_\Lambda + \Omega_k(1+z)^2}}$$
and integrating
$$t_0 - t_1 = \frac{1}{H_0} \int_0^{z_1} \frac{dz}{(1+z)\sqrt{\Omega_M(1+z)^3 + \Omega_\gamma(1+z)^4 + \Omega_\Lambda + \Omega_k(1+z)^2}}$$
Age in a matter dominated universe $(t_1 = 0, z = \infty)$

$$t_1 = \frac{1}{H_0} \int_0^{\infty} \frac{dz}{(1+z)\sqrt{\Omega_M(1+z)^3 + \Omega_\gamma(1+z)^4 + \Omega_\Lambda + \Omega_k(1+z)^2}}$$

$$t_{0,M} = \frac{1}{H_0} \int_0^\infty \frac{uz}{(1+z)^{5/2}} = \frac{z}{3H_0}$$
 and $t_{0,\gamma} = \frac{1}{2H_0}$

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Cosmic Distances We can now also express the luminosity distance $D_L = a_0 x_1 (1 + z)$ in these terms - from the metric for a light ray coming towards us we have $\frac{dr}{cdt} = \frac{\sqrt{1-kx^2}}{a(t)}$ which turns into $\frac{a_0}{c}\frac{dx}{\sqrt{1-kx^2}} = (1+z)dt$ - after integration we have (using dt from above) $\frac{a_0}{c} \int_0^{x_1} \frac{dx}{\sqrt{1 - kx^2}} = \int_0^{z_1} \frac{dz}{H_0 \sqrt{\Omega_{matter}(1 + z)^3 + \Omega_{rad}(1 + z)^4 + \Omega_A + \Omega_k(1 + z)^2}}$ - solutions of the left side are $\frac{a_0}{c} \times \begin{cases} \frac{\arcsin(x_1\sqrt{k})}{\sqrt{k}} \\ x_1 \\ \frac{\arcsin(x_1\sqrt{-k})}{\sqrt{k}} \end{cases}$ k > 0k = 0k < 0

Luminosity Distance

Putting this together with the appropriate trigonometric functions gives

$$D_{L} = a_{0}x_{1}(1+z) = \frac{c(1+z)}{H_{0}\sqrt{|\Omega_{k}|}}S\left(\sqrt{|\Omega_{k}|}\int_{0}^{z} \frac{dz'}{\sqrt{\Omega_{M}(1+z')^{3} + \Omega_{\gamma}(1+z')^{4} + \Omega_{\Lambda} + \Omega_{k}(1+z')^{2}}}\right)$$

with $S(y) = \begin{cases} \sin(y) & k > 0\\ y & k = 0\\ \sinh(y) & k < 0 \end{cases}$

Luminosity distance as a function of today's measurements (H_0 , Ω 's) and the redshift z

Luminosity Distance

- The rate of the photon arrivals is reduced by

a factor $\frac{a(t_1)}{a(t_0)} = \frac{1}{1+z}$ and the energy of the photons ($E = h\nu$) is also reduced by a factor (1 + z) (remember luminosity L is energy per time)

$$l = \frac{L}{4\pi x_1^2 a^2 (t_0)(1+z)^2}$$

- Set $D_L = x_1 a(t_0)(1 + z)$ and we recover the

equation for the luminosity distance $l = \frac{L}{4\pi D_l^2}$

Angular size distance

- A different method is to measure the angle of a distant object of known size $D_A = \frac{l}{\theta}$ (here *l* is the size of the object; θ the observed angle)
- Inspection of the metric (here we only need the $g_{\theta\theta}$ part), which gives $l = x_1 a(t_1)\theta$ and inserting this in the equation above

yields
$$D_A = x_1 a(t_1)$$
 and with $\frac{a(t_1)}{a(t_0)} = \frac{1}{1+z}$
we find $\frac{D_L}{D_A} = (1+z)^2$.

Distance Duality

- This is quite remarkable for high redshifts
 - the physical distances differ for the same redshift!
 - an object for which we could measure the angular size distance and the luminosity distance would give a different number of Mpc!
 - a direct consequence of general relativity $\frac{D_L}{D_A} = (1+z)$

<u>c</u>	z	$\frac{D_L}{D_A}$
	0.1	1.21
	0.15	1.32
	0.2	1.44
	0.25	1.56
2	0.3	1.69
	0.35	1.82

Distance Duality

- Now measured in several systems
 - galaxy clusters
 - Sunyaev-Zeldovich effect
 - gravitational lenses
- Type II Supernovae
 - use two different methods to the same object
 - Expanding Photosphere Method
 - equates luminosity distance with angular size distance
 - Standardizable Candle Method
 - pure luminosity distance

Expansion of the Universe

Luminosity distance in an isotropic, homogeneous universe as a Taylor expansion

$$D_{L} = \frac{cz}{H_{0}} \left\{ 1 + \frac{1}{2} (1 - q_{0})z - \frac{1}{6} \left[1 - q_{0} - 3q_{0}^{2} + j_{0} \pm \frac{c^{2}}{H_{0}^{2}R^{2}} \right] z^{2} + O(z^{3}) \right\}$$

Hubble's Lawdeceleration jerk/equation of state

$$H_0 = \frac{\dot{a}}{a}$$
 $q_0 = -\frac{\ddot{a}}{a}H_0^{-2}$ $j_0 = \frac{\ddot{a}}{a}H_0^{-3}$

Local Universe ($z \ll 1$) Hubble Law $D = \frac{v}{H_0} = \frac{cz}{H_0}$

Luminosity distance

$$D_L = \sqrt{\frac{L}{4\pi F}}$$

Distance modulus

$$m - M = 5 \log(D_L) - 5$$

Distance in units of pc

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Hubble Constant

• Measure cosmic expansion velocity per unit scale length

$$H_0 = \frac{v}{D_L}$$
 (units: $km \ s^{-1} \ Mpc^{-1}$)

- Ignore higher-order cosmological effects

 de/acceleration
- Spectroscopy \rightarrow redshift \rightarrow velocity
- Photometry \rightarrow brightness \rightarrow distance



SN Classification

Time Dilation in SNe la

Uniform light curve shapes in a given filter

Distant supernovae should show a 'slower' light curve





Time dilation



Time Dilation

'Tired Light' can be excluded beyond doubt ($\Delta \chi^2 = 120$)



Distance to SN PS1-13bni (z = 0.335)

Use EPM and CSM to measure distance to same supernova

SN	Dilution factor	Filter	D _L Mpc	Averaged D _L Mpc	t_0^{\star} days*	Averaged t_0^{\star} days [*]	t_0^\diamond MJD	Estimate of t_0 via	
PS1-13bni	H01 – Hβ	B V I	1699 ± 451 1538 ± 1109 2078 ± 1082	1772±538	7.3 ± 12.4 5.4 ± 8.3 11.7 ± 9.7	8.1±5.9	56 401.3 ± 7.9		
	D05 – Hβ	B V I	2019 ± 542 1823 ± 1349 2488 ± 1336	2110±658	8.6 ± 13.6 6.6 ± 8.8 13.4 ± 10.5	9.5±6.4	56 400.0 ± 8.6	EPM	





Gall et al. 2018

SN	Estimate of t_0 via	t_0^\diamond mjd	V_{50}^* mag	I_{50}^* mag	$\frac{v_{50}}{\rm kms^{-1}}$	Estimate of velocity via	μ mag	D_L Mpc
PS1-13bni	EPM – H01 EPM – D05	56401.3 ± 7.9 56400.0 ± 8.6	23.39 ± 0.26 23.39 ± 0.26	23.18 ± 0.20 23.19 ± 0.20	5814 ± 1175 5913 ± 1237	Hβ	$\begin{array}{c} 40.65 \pm 0.76 \\ 40.68 \pm 0.77 \end{array}$	1348 ± 470 1368 ± 487

Distance Duality

First attempts inconclusive



SN la Hubble diagram





- Excellent distance indicators
- Experimentally verified
- Work of several decades
- Best determination of the Hubble constant

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Hubble Constant



Hubble Constant

Calibration of M(SN Ia @ max)

• Distance ladder





Distance ladder



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Cepheid Stars



Riess et al. 2011

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Hubble Constant

Caveats

- local calibrators still uncertain
 - Large Magellanic Cloud
 - Maser in NGC 4258
 - in the future geometric distances (parallaxes) to nearby Cepheids
- extinction
 - absorption of light by dust in the Milky Way and in the host galaxy
 - corrections not always certain
- peculiar velocities of galaxies
 - typically around 300 km/s



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Foundation Survey



Hubble Constant(s)

Planck satellite (CMB; 2016) measurement at $z \approx 1000$ $H_0 = (67.8 \pm 0.9) \ km \ s^{-1} Mpc^{-1}$ Riess et al. (local; 2016) $H_0 = (73.24 \pm 1.74) \ km \ s^{-1} \ Mpc^{-1}$ Dhawan et al. (local; NIR; 2018) same zero-point as Riess et al. (2016) $H_0 = (72.8 \pm 1.6(stat) \pm 2.7 (syst)) km s^{-1} Mpc^{-1}$ Riess et al. (local; 2018) $H_0 = (73.53 \pm 1.62) \ km \ s^{-1} \ Mpc^{-1}$

Hubble Constant

Latest values



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Gaia and H_0

- Calibrate Cepheid distances with parallaxes
 - long-period Cepheids so far not accessible
- Single step to the SNe la
- Helps to bypass intermediate steps and calibrators
 - reduced uncertainty on H_0
- Goal: uncertainty less than 1%



The SN Hubble Diagram





Supernova Cosmology



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Constant ω firmly established

N _{SN}	Ω _м (flat)	w (constant, flat)	Light curve fitter	Reference	
115	$0.263^{+0.042}_{-0.042}{}^{+0.032}_{-0.032}$	$-1.023^{+0.090}_{-0.090}{}^{+0.054}_{-0.090}$	SALT	Astier et al. 2006	
162	$0.267^{+0.028}_{-0.018}$	$-1.069^{+0.091}_{-0.083}{}^{+0.13}_{-0.13}$	MLCS2k2	Wood-Vasey et al. 2007	
178	$0.288^{+0.029}_{-0.019}$	$-0.958^{+0.088}_{-0.090}{}^{+0.13}_{-0.13}$	SALT2		
288	$0.307^{+0.019}_{-0.019}{}^{+0.023}_{-0.023}$	$-0.76^{+0.07}_{-0.07}{}^{+0.11}_{-0.11}$	MLCS2k2	Kasslar at al. 2000	
288	$0.265\substack{+0.016}_{-0.016}\substack{+0.025\\-0.016}_{-0.025}$	$-0.96^{+0.06}_{-0.06}{}^{+0.13}_{-0.13}$	SALT2		
557	$0.279^{+0.017}_{-0.016}$	$-0.997^{+0.050}_{-0.054}$	SALT2	Amanullah et al. 2010	
472		$-0.91^{+016}_{-0.20}{}^{\pm0.07}_{-0.14}$	SiFTO/SALT2	Conley et al. 2011	
472	0.269 ± 0.015	$-1.061^{+0.069}_{-0.068}$	SALT2	Sullivan et al. 2011	
580	0.271 ± 0.014	$-1.013^{+0.077}_{-0.073}$	SALT2	Suzuki et al. 2011	
740		-1.018±0.057 CMB	SALT2	Betoule et al. 2014	
740	0.293 <u>+</u> 0.034	-1.027±0.055 CMB+BAO	JAL12		
313	$0.277\substack{+0.010\\-0.012}$	$-1.186^{+0.076}_{-0.065}$	SALT2	Rest et al. 2014	
1049	0.306 ± 0.012	-1.031 ± 0.040	SALT2	Scolnic et al. 2018	
1369	0.324 ± 0.042	-0.986 ± 0.058	SALT2	Jones et al. 2018	

Status 2014



Status 2018



Scolnic et al. 2018

What next?

Already in hand

- ->1000 SNe la for cosmology
- constant ω determined to 5%
- accuracy dominated by systematic effects

Missing

- good data at z>1
 - light curves and spectra
- good infrared data at z>0.5
 - cover the restframe B and V filters
 - move towards longer wavelengths to reduce absorption effects

Cosmology – more?



Multi-parameter problem



Speculations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Einstein's cosmologal constant

No explanation in particle physics theories

Quintessence

Quantum mechanical particle field releasing energy into the universe

Signatures of high dimensions

Gravity is best described in theories with more than four dimensions

Phantom Energy

Dark Energy dominates and eventually the universe end in a (Big Rip)