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A study on the kinematics of the local system of young stars

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Abstract. Radial velocities of a sample of OB stars have been used to fit a two-dimensional model of the galactic field of velocities in the solar neighbourhood. Our model takes into account solar motion, galactic rotation, spiral structure and local kinematic effects. Analysis of the stellar velocity field is carried out via series of orthogonal functions.

Our results indicate that Gould's belt kinematics is clearly separated from that of the spiral arms, and is indeed a local feature. Concerning the rotation curve and the characteristics of the spiral structure in the solar neighbourhood, a good agreement is found with the results obtained by other authors who have in some cases used different approaches.

Key words: kinematics of OB stars – spiral structure – Gould's belt

1. Introduction

Young stars have been extensively used as probes of the galactic structure in the solar neighbourhood, in studies regarding both their spatial distribution (see e.g. Stothers & Frogel 1974; Schmidt-Kaler 1975) and their kinematics (Crézé & Mennessier 1973; Frogel & Stothers 1977; Byl & Ovenden 1978; Westin 1985). OB stars present several advantages over other bright kinematic tracers such as open clusters, cepheid variables, or red giants, among which we can point out the following ones:

— There is a relatively high number of OB stars whose distances and radial velocities are reliably determined.

— Their luminosity makes them visible over a large range of distances respective to other main-sequence stars.

— Their ages are small as compared to the dynamical evolution timescales of our Galaxy (Fuchs & Wielen 1987); so, their positions in the phase space are essentially the same at the present time as were at their birth.

— They have low velocity dispersions, and it is thus possible to detect systematic motions even from small groups of stars.

In this paper, we try to analyse the kinematic behaviour of an extended sample of 2,745 OB stars belonging to the catalog prepared by Figueras (1986). This catalogue makes use of the astrophysical information available for SAO stars compiled by Ochsenbein (1980), allowing the determination of individual spectroscopic distances. Known binary systems and clusters members have been removed from the considered sample.

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The distribution of the stars in heliocentric distance is shown in Table 1. Typical ages of these stars as estimated from their spectral types are of about 10^8 years although no individual age determination has been carried out.

We shall discuss the fit of a two-dimensional kinematic model of the solar neighbourhood to this observational material, comparing it to the results obtained by other authors from different methods.

2. A model of the galactic kinematics in the solar neighbourhood

As indicated above, we have proposed a two-dimensional model which provides the radial velocity of a star as a function of its position and a set of characteristic parameters. We have used only radial velocities, since proper motions are well below their probable errors for most of these stars. The effects taken into account are the following ones:

— The peculiar motion of the Sun, projected onto the galactic plane. It is thus characterized by the modulus of the velocity and the apex longitude, whose values we try to find out.

— The pure, axisymmetric differential rotation of our Galaxy. It is a well known fact that the rotational velocity around the galactic center is a slowly-varying function of the galactocentric distance at the Sun's position; this allow us to approximate it by a second-degree polynomial whose variable is the difference in star's and Sun's galactocentric distances. We have always taken as known values both the Sun's galactocentric distance and its velocity of rotation around the galactic center, according to the IAU recommended values 8.5 kpc and 220 km s^{-1} respectively (Kerr & Lynden-Bell 1986).

— The influence of spiral structure on the kinematics of such stars. While it is clear that OB stars have not had time enough to reach dynamical equilibrium with the spiral perturbation (Reddish 1978), it can be reasonably expected that they reproduce the kinematic conditions of the gas they were formed from. For several years, it has been thought that large-scale, density-wave induced shock fronts were main responsibles for star formation in spiral arms of galaxies (Roberts 1970); however, more recent studies on the interstellar medium components cast some doubts on this picture (Appenzeller et al. 1980; Combes & Gerin 1985). Then, it is a controversial matter whether the kinematics of star-forming regions results from either a large-scale shock front pattern or from a normal density wave pattern approximable by a linear version of Lin's theory (Lin & Shu 1964; Rohlfs 1977). In this work, we have chosen this last scenario, where systematic motions of OB stars due to spiral arms can be expressed via the

Table 1. Distribution in heliocentric distance of the stars in our sample

R (pc)	Number of stars
0–200	651
200–400	635
400–600	446
600–800	233
800–1000	158
1000–1200	87
1200–1400	74
> 1400	461

equations deduced for a gaseous disk in their linear approximation, without any shock front playing a dominant role.

The free parameters for the spiral perturbation are the amplitudes in the antigalactocentric and tangential directions and the phase of the spiral pattern at the Sun's position. Models are proposed with different values of the pitch angle and the number of arms, but keeping the interarm spacing within the limits suggested by mapping of spiral structure tracers.

— The fourth considered effect arises from the fact that a substantial fraction of the used stars in our sample belong to the Gould's belt, a structure owing well known kinematic peculiarities (Lindblad 1974; Tsioumis & Fricke 1979). We have tried to adjust a model consisting of an expansion of the whole sample of stars from a central point, while the expansion rate is allowed to vary with distance to the center of expansion in a linear way. As we have made no distinction between stars belonging to the Gould's belt and the galactic belt, the variation in the expansion rate may not be a real effect, but the result of variations in the fraction of stars associated to each one of the belts at each point. Therefore, the expansion law represents an averaging between two different groups of stars, and its parameters are then related to the expansion velocity, spatial extent and the fraction of stars in our total sample which belong to the Gould's belt.

Quite probably, this is a very oversimplified picture of the Gould's belt, since there is some evidence that its kinematics cannot be described by a simple model (Eggen 1975; Frogel & Stothers 1977) and, moreover, the kinematics of an expanding group grows in complexity with its age (Lindblad 1980). It should, however provide acceptable estimates of the characteristics of this strucutre.

The expansion parameters are: the constant term in the expansion law, the variation rate with distance, and the heliocentric coordinates of the expansion center.

Equation (1) gives the systematic component of the radial velocity expected for a star at R kiloparsecs from the Sun in the direction of galactic longitude l when the quoted effects are considered:

$$
\begin{aligned}
v_p = & -(v_\odot \cos l_\odot - a_e \cos l_e)\cos l - (v_\odot \sin l_\odot - a_e \sin l_e)\sin l \\
& + \Theta(\varpi_\odot)(\sin(l+\theta) - \sin l) + a_r \Delta\varpi \sin(l+\theta) \\
& + b_r (\Delta\varpi)^2 \sin(l+\theta) \\
& - \Pi_1 \cos\psi_\odot \left(\cos m\left(\theta - \frac{\ln \frac{\varpi}{\varpi_\odot}}{\mathrm{tg}\, i}\right)\cos(l+\theta) - \cos l \right) \\
& - \Pi_1 \sin\psi_\odot \sin m\left(\theta - \frac{\ln \frac{\varpi}{\varpi_\odot}}{\mathrm{tg}\, i}\right)\cos(l+\theta) \\
& - \Theta_1 \sin\psi_\odot \left(\cos m\left(\theta - \frac{\ln \frac{\varpi}{\varpi_\odot}}{\mathrm{tg}\, i}\right)\sin(l+\theta) - \sin l \right) \\
& + \Theta_1 \cos\psi_\odot \sin m\left(\theta - \frac{\ln \frac{\varpi}{\varpi_\odot}}{\mathrm{tg}\, i}\right)\sin(l+\theta) \\
& + a_e \frac{R_{i'}}{d_i} a_e R_e \cos l_e \frac{\cos l_i}{d_i} - a_e R_e \sin l_e \frac{\sin l_i}{d_i} + b_e R_i .
\end{aligned}
\tag{1}
$$

$v_\odot$, $l_\odot$ are the modulus of the solar peculiar motion projected onto the galactic plane and the longitude of the apex; a_r, b_r are the coefficients of the polynomical development of the galactic differential rotation, $\Delta\varpi$ is the difference of galactocentric distances between the star and the Sun, and θ is the difference of their galactocentric longitudes. $\Theta(\varpi_\odot)$ is the velocity of rotation at the Sun's position (galactocentric distance $\varpi_\odot$), taken as a fixed quantity as quoted above; Π_1, Θ_1 are the amplitudes of the spiral perturbation in the antigalactocentric and tangential directions, $\psi_\odot$ is the spiral pattern phase at the Sun, m is the number of spiral arms, and i is the pitch angle of the spiral structure. The Gould's belt related parameters are: the constant term of expansion, a_e; the rate of variation in the expansive velocity, b_e; the heliocentric distance of its center, R_e; and its galactic longitude, l_e. The distance of a star to the center of expansion is represented by d_i.

In an abbreviated form, Eq. (1) can be expressed as

$$
v_p(R, l) = \sum_{j=1}^{13} a_j f_j(R, l) \tag{2}
$$

where the a_j's contain combinations of the kinematic parameters whose values we want to determine.

3. Mathematical treatment

Instead of directly using formula (1) to determine the kinematic parameters via, say, maximum likelihood, we have introduced several modifications. Here we give a brief outline, but the process has been extensively described elsewhere (Comerón 1989; Comerón & Torra 1990).

Firstly, we have averaged radial velocities of the stars in such a way that the effect of cosmic dispersion is reduced. This is done by means of a network of fictitious stars, regularly distributed. Each of the real stars contributes to the radial velocity assigned to the fictitious star with a weight that decreases with distance between the real and the fictitious star; in turn, this weight function depends on the density of catalogued stars at each point. This averaging is done with observed radial velocities and also with the $f_i(R, l)$ functions appearing in the right member of Eq. (2). Thus, we obtain the averaged quantities $\bar{v}_p(R, l)$, $\bar{f}_i(R, l)$, which hold the relationship:

$$
\bar{v}_p(R, l) = \sum_{j=1}^{13} a_j \bar{f}_j(R, l). \tag{3}
$$

Next step consists of developing $\bar{v}_p(R, l)$, $\bar{f}_j(R, l)$ as a series of orthogonal functions. We have chosen the product of Jacobi polynomials $J_n^{0,1}(x)$ and Fourier series terms for such a development, so that the averaged radial velocity $\bar{v}_\rho(R, l)$ takes the form:

$$\bar{v}_\rho(R, l) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{q=0}^{1} c_{nmq} J_n^{0,1}\left(\frac{2R}{R_0} - 1\right) \sin\left(ml + q\frac{\pi}{2}\right). \tag{4}$$

An analogous formula is obtained for each of the $\bar{f}_j(R, l)$, with coefficients $k_{nmq}^j \cdot R_0$ is an arbitrary parameter, and it corresponds to the radius of the network of fictitious stars. The coefficients are found by multiplying both members by the product $J_n^{0,1}((2R/R_0) - 1)\ \sin(ml + q(\pi/2))$ and numerically integrating over all the network of fictitious stars.

This procedure results in low correlations among coefficients, and allows an easy manipulation of the kinematical effects to be taken into account in a given model. It is also important that discrepances between theory and observation do not appear as residuals for each individual star, but as residuals on the coefficients calculated from the whole sample, thus allowing an easier interpretation. In addition, the terms showing the highest residuals tell us about how the unconsidered effects in a given model affect the field of radial velocities as a function of position, thus giving clues about the nature of unconsidered effects themselves.

Series of orthogonal functions as a tool for data analysis have been extensively discussed by Brosche (1966), and applied by him and other authors to a wide variety of astronomical problems (see e.g. Brosche & Schwan 1981; Brosche et al. 1989).

Once the coefficients of the series development have been determined, we can propose a system of as many equations as the number of retained terms of the series development:

$$c_{nmq} = \sum_{j=1}^{13} a_j\, k_{nmq}^j. \tag{5}$$

We are able to assign a probable error to c_{nmq} by considering the cosmic dispersion of our stars, observational errors and uncertainties in the knowledge of their distances. Errors in c_{nmq} are much large than those in the k_{nmq} (due only to distance uncertainties), so these later can be ignored.

The final step in our procedure is solving Eqs. (5) via the maximum likelihood criterion and a iterative schema, obtaining in this way the kinematic parameters sought for.

4. Results and discussion

Our best results are those obtained for a two-armed spiral Galaxy with a pitch angle of $i = -8°$, thus agreeing with other studies (see e.g. Burton & Bania (1974), from a kinematic study of young, distant objects). Good agreement is obtained too for three or four arms, provided that we keep the interarm spacing at 2–3 kpc. Numerical values for the free parameters in Eq. (1) are shown in Table 2. The residuals in the terms of the series development are generally below or close to the probable error, except for the term with $n = 1, m = 2, q = 0$, whose interpretation we shall discuss later.

We should mention that we have tried a simplified model without the expansion of the local system, in order to see if, as suggested by some authors (see e.g. Byl & Ovenden 1978; Pellegatti Franco 1983), the Gould's belt kinematics may be explained as a local appearance of the galactic-wide spiral structure. Our results seem to be conclusive in that only a very small absolute value of the pitch angle ($i \cong -2°$) and a spiral phase angle of $\psi \cong 270°$ may reproduce the effect of such an expansion; and, even in this case, the required amplitude on the tangential component of the velocity due to the spiral perturbation would have a sign opposite to that predicted by the density wave theory. Thus, we are enforced to accept the Gould's belt kinematics as a local, peculiar feature.

Table 2. Results for the general model

Solar velocity	$v_\odot = 16.8 \pm 1.0$ km s^{-1}
	$l_\odot = 68°3 \pm 3°0$
Rotation curve	$a_r = 6.7 \pm 2.1$ km s^{-1}
	$b_r = -6.1 \pm 2.1$ km s^{-1}
Spiral arms	$\Pi_1 = -5.6 \pm 1.7$ km s^{-1}
	$\Theta_1 = 5.7 \pm 2.1$ km s^{-1}
	$\psi_o = 135° \pm 18°$
Expansion	$R_e = 80 \pm 40$ pc
	$l_e = 146° \pm 32°$
	$a_e = 3.5 \pm 0.7$ km s^{-1}
	$b_e = -3.9 \pm 1.1$ km s^{-1} kpc^{-1}

These values are obtained by taking $p_o = 8.5$ kpc, $\Theta(p_o) = 220$ km s^{-1}, $i = -8°$

Concerning the results of our general model, we shall divide the discussion in four parts, according to the effects taken into account.

4.1. Rotation curve

The velocity of rotation around the galactic center, when the maximum slightly outside the solar circle, at $\varpi = 9.1 \pm 0.3$ kpc. The probable error would be larger had we not considered $\varpi_\odot$, $\Theta(\varpi_\odot)$ as exactly known quantities. Figueras (1986) also found a maximum when applying the Ogorodnikov-Milne model up to second degree, but studies on the large-scale rotation curve show that this maximum, if existing, is not a conspicuous one; Rohlfs et al. (1986) found a maximum at $\varpi \cong 6$ kpc, and the mass-model by Haud et al. (1985) reproduces it at $\varpi \cong 7.5$ kpc. Anyway, it seems to be clear that, outside the solar circle, and maybe beyond the optical limits of our Galaxy, the velocity of rotation keeps constant or even increases with distance (Chini 1985).

Due to the initial values we have taken as known data, Oort's constant A depends only on the derivative $\partial\Theta/\partial\varpi$. We find $A = 9.6 \pm 1.0$ km s^{-1} kpc^{-1}, a rather low value as compared to that found by other authors (e.g. Westin 1985; Hron 1987). We shall see later that this fact may be partially explained by the spiral kinematic pattern.

4.2. Spiral structure

The phase of the spiral pattern at the Sun places it in the interarm region, near the inner edge of the Orion-Cygnus arm. According to this, the center of the arm (as defined by the spiral potential minimum) lies at $\varpi \cong 9$ kpc, but the typical spiral arm population objects are somewhat closer to the Sun, as expected from the fact that most young objects are formed before reaching the potential minimum (Tilanus & Allen 1989; Kaufman et al. 1989). It is remarkable this agreement between mapping of the spiral structure tracers (Schmidt-Kaler 1975; Elmegreen 1985) and our kinematic determination of the spiral pattern phase at the Sun.

Amplitudes of the spiral perturbation in the antigalactocentric and tangential directions are about 5–6 $km\,s^{-1}$. Both amplitudes have signs compatible with those expected from the density wave theory, and the sign of Π_1 places the corotation circle well away from the Sun. The angular velocity of the spiral pattern, Ω_p, may be estimated from these values, although the uncertainties are big. We obtain $\Omega_p = 10 \pm 5\,km\,s^{-1}\,kpc^{-1}$, a value compatible with $\Omega_p = 13\,km\,s^{-1}\,kpc^{-1}$ used by Lin et al. (1969) and other authors.

According to our results, the combined rotation curve (pure, axisymmetric differential rotation plus spiral arms perturbation) is nearly flat in the solar neighbourhood, and this rises the estimated value of A, when both effects are not separately considered, up to 13 $km\,s^{-1}\,kpc^{-1}$.

We may also find an estimation for the arm/interarm surface density contrast. Our value is $\sigma_{arm}/\sigma_{interarm} = 1.14 \pm 0.09$, consistent with the simplificative hypothesis of the linear theory of density wave according to which the perturbations in both density and potential are small as compared to the unperturbed values.

The unperturbed surface density of the galactic disk is also estimated from the self-consistency conditions on the density wave. Our value is $\sigma_0 \cong 53\,M_\odot\,pc^{-2}$. In order to compare it to the volume density in the galactic plane, we need some assumptions on the density law of gravitating material as a function of height on the galactic plane. We have approached it by an exponential law, with a scale height of 475 pc for the typical stellar population of the galactic disk (Robin & Crézé 1986), and the resulting value is then $\rho_0 \cong 0.06\,M_\odot\,pc^{-3}$. This value reproduces the density of visible material in the solar neighbourhood (Mihalas & Binney 1981), but it is clearly below the estimates of 0.15 $M_\odot\,pc^{-3}$ for the total gravitating mass. This discrepancy may however be due to the large uncertainties accumulated along our procedure.

4.3. *Gould's belt*

Our kinematic determination of the center of the Gould's belt places it at 80 ± 40 pc from the Sun, in the direction $l = 146° \pm 32°$. This center is in a good agreement with the geometric center of the plot of stars assigned to the Gould's belt from a positional criterion by Westin (1985) and with the center of the local distribution of neutral hydrogen and H_2CO as mapped by Lindblad et al. (1973) from a kinematic model of an expanding ring. In order to determine the local extent and velocity of expansion, we have reduced our observational horizon to 1.4 kpc, because of the strong bias introduced by stars with overestimated distances. From the values in Table 2, we are able to estimate the mean distance of Gould's belt stars to its center to 250 pc. If we suppose a uniform distribution of stars within the belt, its diameter results to be about 750 pc, in agreement with the results by Stothers & Frogel (1974), Tsioumis & Fricke (1979) and Westin (1985).

Our model allows us to estimate the product of the expansion velocity, considered as a constant, by the fraction of stars in our sample belonging to the Gould's belt. This product is about 2 $km\,s^{-1}$. If we accept the expansion velocity given by Tsioumis & Fricke (1979) as 4–5 $km\,s^{-1}$, this would imply that 35–50% stars of our sample up to 1.4 kpc belong to the Gould's belt. The radius-to-velocity ratio gives an estimation of the expansion age of the belt of $t_{exp} \cong 7\;10^7$ years.

Assuming an initial circular symmetry for the expansion of the Gould's belt, it is expected to show a clear distortion as a consequence of the shearing motions induced by differential rotation, as noted by Lindblad (1974). The resulting form would be an ellipse, whose major axis is directed towards a point lying between the galactic center and the direction of rotation, and whose eccentricity grows with distance as a consequence of the greater expansion age of the stars lying farther away from the center. This would show in our results as a high residual in the terms depending linearly with heliocentric distance [$n=1$ in Eq. (4)], and having a doubly sinusoidal dependence with galactic longitude [$m=2$, $q=0$]. As mentioned at the beginning of this discussion, this is just our highest residual, which is qualitatively explained in this way.

4.4. *Solar velocity*

The values $v_\odot = 16.8 \pm 1.0\,km\,s^{-1}$, $l_\odot = 68°3 \pm 3°0$ quoted in Table 2 are those referred to a centroid which moves as a typical OB star sharing both the expansive motion of the Gould's belt and the streaming motion related to the spiral arms. Other centroid definitions are also possible, and the resulting values of $v_\odot$ and $l_\odot$ are changed accordingly. However, the differences are much smaller than those found by Jaschek & Valbousquet (1989) among values of the solar velocity as defined from stars of different spectral types.

5. Conclusions

We have proposed a kinematical model of the solar neighbourhood which successfully fits the observed field of radial velocities of an extended sample of OB stars. The method used for analysing the observations presents several interesting features, mainly an easy and unambiguous geometrical interpretation of residuals.

The fit of the proposed model to the observed kinematics of our sample of stars allows several conclusions on the considered kinematical effects. Our results tend to confirm a rather low value of Oort's constant A of 9.6 $km\,s^{-1}\,kpc^{-1}$, rising up to 13 $km\,s^{-1}\,kpc^{-1}$ when spiral arms contribution is not subtracted. The spiral structure as deduced from kinematics of young stars is essentially coincident with that deduced from other methods, and it seems that kinematics of young stars is well described by the hydrodynamic equations for a gaseous disk.

Gould's belt kinematic parameters are confirmed, and the kinematic determination of its center is in agreement with that determined from mapping of the spatial distribution of gas or stars. The Gould's belt kinematics cannot be satisfactorily explained by a global spiral structure.

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References

Appenzeller I., Lequeux J., Silk J., 1981, Star Formation, SAAS-FEE Course 1980, Geneva Observatory
Brosche P., 1966, Veröff. des Astronomisches Rechen-Instituts no. 17, Heidelberg
Brosche P., Schwan H., 1981, A&A 99, 311

Brosche P., Wildermann E., Geffert M., 1989, A&A 211, 239
Burton W.B., Bania T.M., 1974, A&A 33, 425
Byl J., Ovenden M.W., 1978, ApJ 225, 496
Combes F., Gerin M., 1985, A&A 150, 327
Comerón F., 1989, Tesis de Licenciatura, Univ. of Barcelona
Comerón F., Torra J., 1990, in: Proc. of the Coll. "Errors, Bias and Uncertainties in Astronomy", Strasbourg, 1989, Cambridge Univ. Press, Cambridge (in press)
Crézé M., Mennessier M.O., 1973, A&A 27, 281
Chini R., 1985, in: The Milky Way Galaxy, eds. H. van Wörden et al., IAU
Eggen O.J., 1975, PASP 87, 37
Elmegreen D.M., 1985, in: The Milky Way Galaxy, eds. H. van Wörden et al., IAU Coll. 106
Figueras F., 1986, Tesi Doctoral, Univ. of Barcelona
Frogel J.A., Stothers R., 1977, AJ 81, 890
Fuchs B., Wielen R., 1987, in: The Galaxy, eds. G. Gilmore, B. Carswell, Reidel, Dordrecht
Haud U., Jôeveer M., Einasto J., 1985, in: The Milky Way Galaxy, eds. H. van Wörden et al., IAU Coll. 106
Hron J., 1987, A&A 176, 34
Jaschek C., Valbousquet A., 1989 (preprint)
Kaufman M., Bash F.N., Hine B., Rots A.H., Elmegreen D.M., Hodge P.W., 1989, ApJ 345, 674
Kerr F.J., Lynden-Bell D., 1986, MNRAS 221, 1023
Lin C.C., Shu F.H., 1964, ApJ 140, 646
Lin C.C., Yuan C., Shu F.H., 1969, ApJ 155, 721
Lindblad P.O., Grape K., Sandqvist Aa., Schober J., 1973, A&A 24, 309
Lindblad P.O., 1974, in: Stars and the Mily Way System, ed. L.N. Mavridis, Springer, Berlin Heidelberg New York
Lindblad P.O., 1980, Mitt. astr. Ges. 48, 151
Mihalas D., Binney J., 1981, Galactic Astronomy, Freeman, San Francisco
Ochsenbein F., 1980, Bull. Inform. CDS no. 19, 74
Pellegatti Franco G.A., 1983, Ap&SS 96, 195
Reddish V.C., 1978, Stellar Formation, Pergamon, Oxford
Roberts W.W., 1970, in: The Spiral Structure of Our Galaxy, eds. W. Becker, G. Contopoulos, IAU Coll. 38
Robin A., Crézé M., 1986, A&A 157, 71
Rohlfs K., 1977, Lectures in Density Waves Theory, Springer, Berlin Heidelberg New York
Rohlfs K., Chini R., Wink J.E., Böhme R., 1986, A&A 158, 181
Schmidt-Kaler T., 1975, Vistas Astron. 19, 69
Stothers R., Frogel J.A., 1974, AJ 79, 1974
Tilanus R.P.J., Allen R.J., 1989, ApJ 339, L57
Tsioumis A., Fricke W., 1979, A&A 75, 1
Westin T.N.G., 1985, A&AS 60, 99