Plasma Physics and Controlled Fusion, Vol. 31, No. 13, pp. 2003 to 2010, 1989
Printed in Great Britain.

0741-3335/89 $3.00 + .00


# EMISSION AND ABSORPTION PROFILES IN ELECTRON CYCLOTRON RADIATION FOR NON-THERMAL PLASMAS

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(*Received* 17 *February* 1989; *and in revised form* 6 *June* 1989)

**Abstract**—The emission and absorption profiles in momentum space for electron cyclotron radiation for arbitrary distribution functions are investigated. It is shown that in general for non-Maxwellian distributions the predominant contributions to emission and absorption come from different groups of electrons, a circumstance to be related to the non-validity of Kirchhoff's law.

## 1. INTRODUCTION

THERE IS a great deal of interest in the problem of the relations between reverse processes for non-Maxwellian distributions, involving particle (SOBELMAN *et al.*, 1981) and radiative (OXENIUS, 1986) phenomena. A special case of relevance in plasma physics is the process of emission and absorption of electron cyclotron waves in magnetized plasmas. According to Kirchhoff's law, in a Maxwellian plasma the emission and absorption coefficients are proportional. The emission and absorption profiles in momentum space, describing the relative contributions of electrons at a given velocity, are also proportional and therefore the two competitive processes are determined by the same group of electrons. For an arbitrary electron momentum distribution the two profiles may differ considerably and the predominant contributions to the processes of emission and absorption come from different regions of phase space. This is understood from the fact that wave absorption is the difference of "true" absorption and stimulated emission. The two processes are in general described by large and nearly equal quantities and therefore the resulting global absorption may change from positive to zero or negative values for a relatively small variation of the parameters characterizing the wave and the electron momentum distribution. The region of the accessible phase space in which absorption takes place may then be different from that of spontaneous emission. In some cases, we obtain that finite emission occurs in the absence of absorption. The accessible momentum space is determined by the relativistic resonance condition. This relates $p_\perp$ and $p_\parallel$ and allows a great simplification in the analysis of the wave processes in the momentum space which can be carried out in one direction. The plan of the paper is as follows. In Section 2, we develop the theory of emission and absorption for arbitrary wave frequencies and direction of propagation and derive the phase space profile of the emission and absorption coefficients for an arbitrary distribution function. In Section

3, we apply the theory to the special case of a loss-cone distribution. In Section 4, we give the conclusion.

## 2. THEORY OF EMISSION AND ABSORPTION OF ELECTRON CYCLOTRON RADIATION

Emission and absorption of electron cyclotron waves at frequency $\omega$ are described by the emission $\beta(\omega)$ and absorption $\alpha(\omega)$ coefficients. For a Maxwellian velocity distribution function, the two coefficients are related by Kirchhoff's law

$$\beta(\omega) = B_0(\omega, T_e)\alpha(\omega) = (T_e\omega^2/8\pi^3c^2)\alpha(\omega), \tag{1}$$

where $T_e$ is the electron temperature and c is the speed of light. The global results of emission and absorption are the cumulative contributions of electrons in a given velocity range, thus,

$$\alpha(\omega) = \int \mathrm{d}\mathbf{p}\alpha(\mathbf{p}) = \sum_{n=1}^{\infty} \int \mathrm{d}\mathbf{p}\alpha_n(\mathbf{p}),$$

where $\mathbf{p}$ is the electron momentum. Now for cyclotron resonance processes $\alpha_n(\mathbf{p}) \propto \delta(\gamma - Y_n - N_\parallel p_\parallel/mc)$, where $\gamma = (1+p^2/m^2c^2)^{1/2}$, $Y_n = n\omega_c/\omega$, $N_\parallel$ is the parallel refractive index and $m$ is the electron rest mass. Using the $\delta$ function, we obtain

$$\alpha(\omega) = \sum_{n=1}^{\infty} \int_{v_-}^{v_+} \mathrm{d}v_\parallel W_n(v_\parallel),$$

where $\mathbf{v} = \mathbf{p}/mc$,

$$v_\pm = p_\pm/mc = [N_\parallel Y_n \pm (N_\parallel^2 - 1 + Y_n^2)^{1/2}]/(1-N_\parallel^2), \quad N_\parallel^2 - 1 + Y_n^2 \geqslant 0,$$

and we consider for simplicity the case of most interest, i.e. $|N_\parallel| < 1$. $W_n(p_\parallel)$ is proportional to the power absorption per unit interval in momentum space near a given value of $p_\parallel$. A similar quantity can be defined for the emitted power, namely

$$\beta(\omega) = \sum_{n=1}^{\infty} \int_{v_-}^{v_+} \mathrm{d}v_\parallel G_n(v_\parallel).$$

For a Maxwellian distribution, $G_n(v_\parallel) = B_0 W_n(v_\parallel)$, we obtain that the electrons giving the predominant contribution to emission and absorption lie in the same range of velocities. For non-Maxwellian momentum distributions, $G_n(v_\parallel)$ is not in general proportion to $W_n(v_\parallel)$ and emission and absorption depend on electrons in different regions of momentum space. In order to derive $G_n(v_\parallel)$ and $W_n(v_\parallel)$ we first consider the case of a system in which plasma polarization effects can be neglected. In this case (BEKEFI, 1966),

$$\beta(\omega) = \int \mathrm{d}\mathbf{p}\beta_E(\mathbf{p})f(\mathbf{p}) = \sum_{n=1}^{\infty} \int \mathrm{d}\mathbf{p}\beta_{En}(\mathbf{p})f(\mathbf{p}),$$

$$\alpha(\omega) = \int \mathrm{d}\mathbf{p}[\alpha_A(\mathbf{p}) - \alpha_S(\mathbf{p})]f(\mathbf{p}),$$

where $f(\mathbf{p})$ is the electron momentum distribution and $\alpha_A(\mathbf{p})$ and $\alpha_S(\mathbf{p})$ describe "true" absorption and stimulated emission, respectively. For $f(\mathbf{p}) = f(p_\perp, p_\parallel)$, we obtain (BEKEFI, 1966)

$$\alpha(\omega) = -(8\pi^3c^2/\omega^2) \sum_{n=1}^{\infty} \int \mathrm{d}\mathbf{p}\beta_{En}(\mathbf{p})(m\gamma/p_\perp)L_n f,$$

where

$$\gamma L_n = Y_n(\partial/\partial p_\perp) + (N_\parallel p_\perp/mc)(\partial/\partial p_\parallel),$$

and we use the relation $\beta_{En} \propto \delta(\gamma - Y_n - N_\parallel p_\parallel/mc)$. It appears that the emission coefficient $\beta(\omega)$ is obtained from the absorption coefficient $\alpha(\omega)$ using the transformation

$$L_n f \rightarrow -(\omega^2/8\pi^3c^2)p_\perp f/m\gamma. \tag{2}$$

At first glance, the previous formulation is only applicable to a tenuous plasma, for which the ray refractive index (BEKEFI, 1966) $N_r = 1$. We now show that the adopted procedure is valid for arbitrary plasma densities. As shown by equation (1.139) of BEKEFI (1966), the radiation intensity emerging outside the plasma is given by

$$I_\omega = \int_0^{\tau_0} S_\omega \exp(-\tau)\,\mathrm{d}\tau, \quad \tau_0 = \int_0^{l_0} \alpha(\omega)\,\mathrm{d}l,$$

where $l_0$ is the length of the ray path in the plasma and $S_\omega = \beta(\omega)/N_r^2\alpha(\omega)$. We define $\alpha$ from the electromagnetic theory of wave propagation, namely, from the appropriate dispersion relation or from the Poynting theorem. Now, one of Kirchhoff's laws states that in a thermal plasma the source function $S_\omega$ is independent of the plasma density and is, therefore, a universal function of the temperature, i.e. $S_\omega = B_0(\omega, T_e)$. For a non-Maxwellian distribution, $S_\omega$ is still independent of the plasma density (BEKEFI, 1966). Thus, we may define the source term without the ray refractive index, i.e. $S_\omega = \beta/\alpha$, provided we compute $\alpha$ and $\beta$ consistently using equation (2). This procedure is supported by the full electromagnetic theory of radiation which uses Maxwell's equations and the fluctuation–dissipation theorem (FREUND *et al.*, 1978; FIDONE *et al.*, 1980). Note that $\beta$ and $\alpha$ may depend on the plasma density, but $\beta/\alpha$ does not (except in some pathological cases of strong plasma polarization effects). However, here we are mainly interested in the case of waves and densities for which refractive effects are small ($N_r \simeq 1$). In fact, for arbitrary direction of wave propagation, polarization and plasma density, we are faced with the problem of the effect of non-reciprocity of a magnetized plasma, which requires a full-wave solution of the radiation problem. As shown by FIDONE and GRANATA (1979), in a relatively tenuous plasma or for special directions of propagation the effects of non-reciprocity can be neglected.

From the Poynting theorem $\alpha(\omega) = W/S$, where $W$ is the power absorption per unit volume and $S$ is the magnitude of the Poynting vector, and

$$W = n_e \int \mathrm{d}\mathbf{p} mc^2\gamma(\partial f/\partial t)_{cy}, \tag{3}$$

where $n_e$ is the electron density,

$$\left(\frac{\partial f}{\partial t}\right)_{cy} = 4\pi e^2 \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}N_{\parallel}(\gamma/p_{\perp})L_n D_n \delta(\gamma - Y_n - N_{\parallel}p_{\parallel}/mc)L_n f, \tag{4}$$

$$D_n = (p_{\perp}/8\omega)|\mathbf{E}\cdot\mathbf{\Pi}_n|^2,$$

$\mathbf{E}$ is the wave electric field, $\Pi_{1n} = nJ_n(\rho)/\rho$, $\Pi_{2n} = -iJ_n'(\rho)$, $\Pi_{3n} = (p_{\parallel}/p_{\perp})J_n(\rho)$, $\rho = k_{\perp}p_{\perp}/m\omega_c$, $-e$ is the electron charge, and $\mathbf{k}$ is the wave vector. Using equation (3), we obtain

$$W_n(v_{\parallel}) = -2\pi(mc)^3\omega_p^2[\gamma(D_n/S)L_n f]_{v_{\perp}=v_{\perp R}}, \tag{5}$$

where $v_{\perp R}^2 = \gamma_n^2 - \gamma_{\parallel}^2$, $\gamma_n = Y_n + N_{\parallel}v_{\parallel}$, $\gamma_{\parallel} = (1+v_{\parallel}^2)^{1/2}$ and $\omega_p$ is the plasma frequency. Similarly,

$$G_n(v_{\parallel}) = (\omega^2/8\pi^3c^2)2\pi(mc)^3\omega_p^2 c[(D_n/S)v_{\perp}f]_{v_{\perp}=v_{\perp R}}. \tag{6}$$

It is of interest to note that $v_{\pm}$ are the solutions of the equation $v_{\perp R}^2 = \gamma_n^2 - \gamma_{\parallel}^2 = 0$; thus $W_n(v_{\pm}) = G_n(v_{\pm}) = 0$. Note also that $D_n/S$ is independent of the wave electric field strength. Equations (4) and (5) can be studied for arbitrary values of $v_{\perp}$ but here we only consider the case of moderate values of $v_{\perp}$, i.e. for $\rho/n = N_{\perp}(\omega/n\omega_c)v_{\perp} < 1$. In this case, the polarization factor $D_n$ can be simplified and we obtain (ABRAMOVITZ and STEGUN, 1965)

$$|\mathbf{E}\cdot\mathbf{\Pi}_n|^2 \simeq (Y_n/N_{\perp})^2|E_x - iE_y + (N_{\perp}/Y_n)v_{\parallel}E_z|^2 J_n^2(\rho)/v_{\perp}^2, \tag{7}$$

where $N_{\perp} = (c/\omega)k_{\perp}$. Equation (6) is valid for arbitrary values of $N_{\parallel}$, except for the extraordinary mode at $n = 1$ near perpendicular propagation (BORNATICI et al., 1979; MAZZUCATO et al., 1987). It is found that for $N_{\parallel} \neq 0$ the $v_{\parallel}$ dependence in the polarization factor generally plays a minor role in the determination of the emission and absorption profiles and therefore it is neglected. Dropping the common factor in equations (4) and (5), we then define the two profiles from the relations

$$W_n(v_{\parallel}) = -[\gamma(J_n^2/v_{\perp})L_n f]_{v_{\perp}=v_{\perp R}}, \tag{8}$$

$$G_n(v_\parallel) = (\omega^2/8\pi^3c^2)c[J_n^2 f]_{v_\perp = v_{\perp R}}. \tag{9}$$

For a Maxwellian distribution, $f \propto \exp(-\mu\gamma)$ and we obtain

$$G_n(v_\parallel) = B_0(\omega, T_e) W_n(v_\parallel). \tag{10}$$

The absorption spectrum for a Maxwellian distribution was discussed by FIDONE *et al.* (1988). It is found that $W_n(v_\parallel)$ has a maximum in the interval $(v_-, v_+)$ which is rather sharp for $N_\parallel \neq 0$. In this case, the predominant contribution to the absorption comes from the electrons with parallel velocity near the maximum. From equation (9), we obtain that the predominant contribution to the emission comes from the same group of electrons. For a non-Maxwellian distribution this is not generally true, since the ratio $G_n(v_\parallel)/W_n(v_\parallel)$ in general is not constant.

## 3. NUMERICAL RESULTS

In order to illustrate the deviation between $G_n(v_\parallel)$ and $W_n(v_\parallel)$, we consider the loss-cone momentum distribution

$$f = \frac{\mu_\parallel^{1/2}\mu_\perp^{1+l/2}}{(2\pi)^{3/2}(mc)^3}\frac{1}{2^{l/2}\Gamma(1+l/2)}v_\perp^l \exp(-\mu_\perp v_\perp^2/2 - \mu_\parallel v_\parallel^2/2)$$

where $\mu_\perp = mc^2/T_\perp$, $\mu_\parallel = mc^2/T_\parallel$ and $l$ is a given number representing the mirror effect. For simplicity, we consider a tenuous plasma, for which $N_\perp^2 = 1 - N_\parallel^2$. According to equations (8), this implies that $W_n$ and $G_n$ vanish for $N_\parallel \to 1$. For $l = 0$ and $T_\parallel = T_\perp$, we obtain the weakly relativistic Maxwellian distribution, for which equation (9) holds to the order $(p/mc)^2$. This is shown in Fig. 1, where $G_n(v_\parallel)$ and

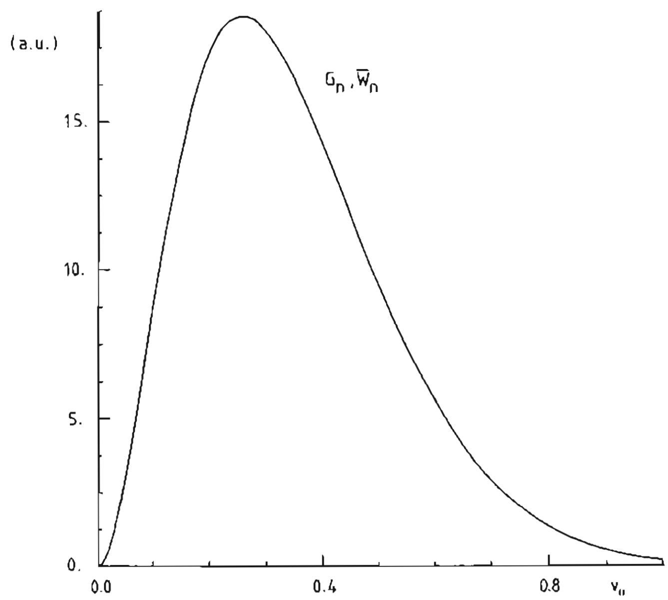


FIG. 1.—$G_n$ and $\bar{W}_n$ in arbitrary units versus $v_\parallel$ for an isotropic Maxwellian distribution and $n = 2$, $\omega = 2\omega_c$, $N_\parallel = 0.5$ and $T_e = 50$ keV.

$\bar{W}_n = W_n(v_\parallel)B_0(\omega, T_\perp)$ are presented for $n = 2$, $\omega = 2\omega_c$, $N_\parallel = 0.5$, and $T_\perp = T_\parallel = 50$ keV. As expected, the two curves coincide, which shows that for the Maxwellian distribution the Kirchhoff's law has a counterpart in momentum space.

For $T_\parallel \neq T_\perp$ and $l = 0$, i.e. for the anisotropic Maxwellian distribution, we find that $G_n/\bar{W}_n$ is not constant in momentum space and Kirchhoff's law is not valid, as appears in Fig. 2, for the same parameters of Fig. 1, but $T_\parallel = 2$ keV. However, Fig. 2 shows that the two processes are determined by electrons in the same range of $v_\parallel$ and $v_\perp$.

For a loss-cone distribution, i.e. $l \neq 0$, equation (10) becomes

$$G_n/\bar{W}_n = \mu_\perp[Y_n(\mu_\perp - l/v_{\perp R}^2) + N_\parallel \mu_\parallel v_\parallel]^{-1}. \tag{11}$$

This equation shows that now the sign of $G_n/\bar{W}_n$ can change. In particular, for small $v_{\perp R}$, $W_n$ can become negative, i.e. stimulated emission is dominant compared to "true" absorption. This is illustrated in Fig. 3, for the parameters of Fig. 2 and $l = 2$. It appears that for the loss-cone distribution the emission and absorption profiles in momentum space are quite different from each other and that the absorption profile is radically changed from both the isotropic and the anisotropic Maxwellian case. Two distinct groups of electrons are mainly responsible for stimulated emission and "true" absorption, for $v_\parallel < 0.1$ and $v_\parallel > 0.1$, respectively. Note that, for $v_\parallel \simeq 0.1$, the spontaneous emission is maximum whereas the global absorption is practically zero. This illustrates the microscopic behaviour of a system of electrons for which Kirchhoff's law is not applicable. Because of the presence of regions of positive and negative absorption in momentum space, the integrated absorption for loss-cone

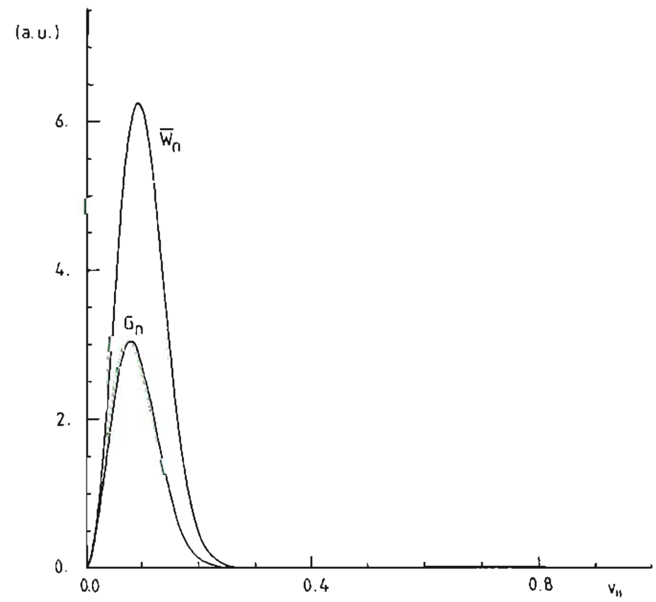


FIG. 2.—As in Fig. 1 for an anisotropic Maxwellian distribution with $T_\perp = 50$ keV, $T_\parallel = 2$ keV.

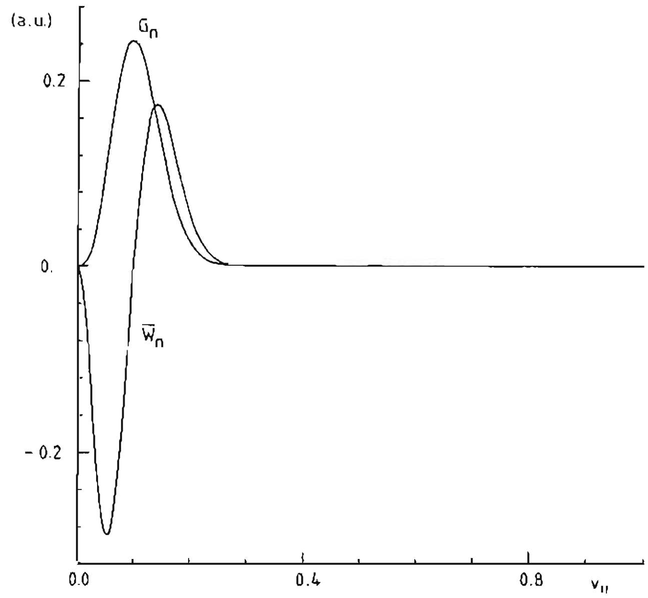


FIG. 3.—As in Fig. 2 for a loss-cone distribution with $l = 2$.

distributions can be positive, zero or negative depending on the parameters, as is known. In particular, in the case of Fig. 3 the integrated absorption vanishes, whereas the integrated emission coefficient is non-zero. Thus, this specific electron system can emit waves for which it is transparent. Furthermore, the analysis of the angular emission and absorption spectra shows that the angle for which the absorption vanishes is in the region of maximum emission. This is illustrated in Fig. 4, where the integrated emission coefficient $\beta_n$ and the normalized absorption coefficient $\bar{\alpha}_n = \alpha_n B_0(\omega, T_\perp)$ are shown versus $N_\parallel$. This situation is quite general for loss-cone distributions and similar features can be found in frequency spectra.

## 4. CONCLUSION

We have investigated the relation between the emission and absorption profiles in momentum space for Maxwellian and non-Maxwellian electron momentum distributions. We have found that the absorption profile is very sensitive to the wave parameters and the shape of the electron distribution, in contrast with the emission profile. The peculiar behaviour of the absorption profile results from the fact that the two competitive mechanisms of "true" absorption and stimulated emission are of comparable and large magnitudes. The ratio between emission and absorption profiles then in general depends on the electron velocity. This is an alternative statement of the non-validity of Kirchhoff's law for the emission and absorption coefficients for a non-Maxwellian distribution. We have applied the general theory to the special case of a loss-cone distribution. The analysis has revealed that for given values of $\omega$, $N_\parallel$ and $l$, the predominant contributions to the emission and absorption coefficients come from different groups of electrons. We have also found that finite emission may occur in the absence of wave absorption. These facts must be considered when emission and

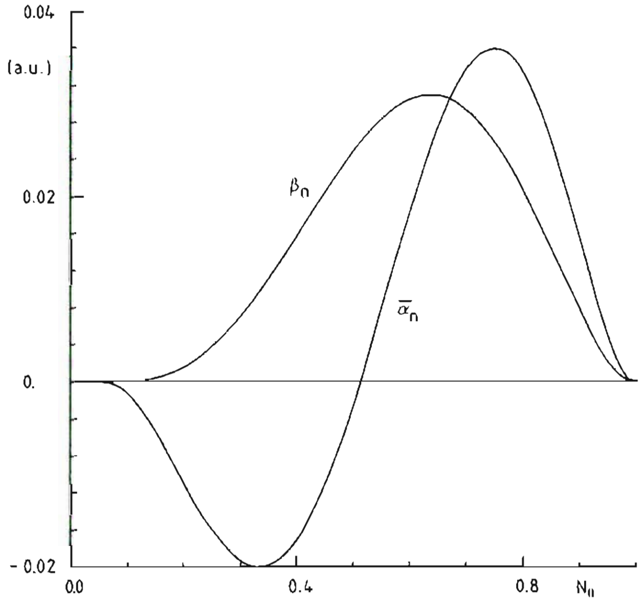


FIG. 4.—$\beta_n$ and $\bar{\alpha}_n = B_0(\omega, T_\perp)\alpha_n$ versus $N_{\|}$, for the conditions of Fig. 3. Here $\beta_n$ and $\alpha_n$ are the integrals over $v_{\|}$ of the quantities $G_n$ and $W_n$ defined in equations (8).

absorption are used to obtain information on the shape of a non-Maxwellian distribution. In fact, in a low density plasma the optical thickness is generally much less than unity and the emitted intensity is proportional to the emission coefficient $\beta$, from which we can obtain information on the predominant group of electrons. A similar procedure is applied to transmission measurements. As shown in the case of a loss-cone distribution with $l = 2$, the two measured quantities are related to different groups of electrons, in contrast with the case of the Maxwellian distribution.

In conclusion, it is worth mentioning that the problem discussed in this paper for the electron cyclotron radiation, namely, the relation between reverse processes for non-Maxwellian distributions, is also of relevance in other radiation and/or particle processes and, therefore, the approach outlined here could have wider applications in the field of non-thermal plasmas.

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