AS2001
Nucleosynthesis and the Chemical Evolution of the Universe

Tutorial 1

Question 1

Given the Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho \tag{1}$$

and

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = -\frac{8\pi G}{c^2}p \tag{2}$$

derive the following differential equation:

$$\frac{\mathrm{d}\rho}{\mathrm{d}a} = -\frac{3}{a}(\rho + \frac{p}{c^2}). \tag{3}$$

Assuming that $p = wc^2\rho$, convince yourself that

$$\rho \propto a^{-3(1+w)} \tag{4}$$

is a solution to (3). The so-called 'vacuum energy' has $w = -1$, i.e. it has *negative* pressure. How does its density, ρ_{vac}, evolve with time? What is the ultimate fate of a universe that has $\rho_{\text{vac}} \neq 0$? In other words, find $a(t)$ for $t \to \infty$. Comment on the apparent gravitational effect of vacuum energy.

Question 2

What is approximately the size of the observable universe? Observations have shown that geometrical distortions are not blindingly obvious in the nearby universe. What does this immediately suggest about the total value of Ω_0? (Hint: What is the dimension of the right hand side of equation (1)/c^2?)

Question 3

$c = 2.998 \times 10^8$ m s^{-1}	(Speed of light)
$G = 6.672 \times 10^{-11}$ N m^2 kg^{-2}	(Newton's gravitational constant)
$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ J s	(Planck's constant)
$M_\odot = 1.989 \times 10^{30}$ kg	(Solar mass)

We know that General Relativity is incomplete because it does not incorporate the Uncertainty Principle (i.e. $\hbar$ appears nowhere in GR). Thus there must be situations/conditions where GR breaks down and where we need quantum gravity (which is yet to be invented) instead. We can derive characteristic quantum gravity scales by the following arguments:

The escape velocity of an object of mass m and radius R is given by $v = \sqrt{\frac{2Gm}{R}}$. What is therefore the radius of a black hole of mass m? What is the size of a solar mass black hole? Do we need to consider quantum mechanical effects in this situation?

In quantum mechanics particles can be described as 'waves' with a characteristic wavelength (called de Broglie wavelength) $\lambda = 2\pi\hbar/(mc)$. In general, quantum effects need to be taken into account when considering length scales of the order of the de Broglie wavelength of the particles involved.

Use this information to derive a characteristic mass (m_{Pl}) and hence a characteristic length (l_{Pl}) and time (t_{Pl}) for quantum gravity (ignore factors of 2 or 2π).

With the help of your tutor, can you derive the same result from the opposite direction (i.e. starting with quantum mechanics rather the GR)?