AS2001
Nucleosynthesis and the Chemical Evolution of the Universe
Tutorial 2

Question 1

Given the Fermi-Dirac/Bose-Einstein statistic

$$f(\mathbf{p},t)\mathrm{d}^3p = \frac{g}{(2\pi\hbar)^3}\frac{\mathrm{d}^3p}{\exp\left(\frac{E}{kT(t)}\right)\pm 1} \tag{1}$$

(where $E=\sqrt{p^2c^2+m^2c^4}$), derive the energy density distribution of photons

$$\epsilon_\nu \mathrm{d}\nu = \frac{8\pi h}{c^3}\frac{\nu^3\mathrm{d}\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}, \tag{2}$$

where ν is the photon frequency. Hints: photons are bosons; $g_\gamma = 2$; $m_\gamma = 0$;

Hence calculate the total energy density $\epsilon = \int_0^\infty \epsilon_\nu\,\mathrm{d}\nu$. Hint: $\int_0^\infty \frac{x^3\,\mathrm{d}x}{\exp(x)-1} = \frac{\pi^4}{15}$.

Now find the number density of photons, n. Hint: $\int_0^\infty \frac{x^2\,\mathrm{d}x}{\exp(x)-1} = 2\zeta(3)$, where $\zeta(3)\approx 1.202$ (Riemann's zeta function of order 3).

What then is the average photon energy $\langle E\rangle$?

Given that the temperature of the photons filling the universe today is $T_0 = 2.725$ K, express η (the ratio of the number density of baryons to the number density of photons) in terms of $\Omega_B h^2$.

The binding energy of the deuterium nucleus is ≈ 2.2 MeV. When the temperature has fallen so that $2.7\,kT < 2.2$ MeV then the average photon does not have enough energy to dissociate a deuterium nucleus. Yet deuterium does not form until much later, $kT\approx 0.06$ MeV. Given equation (2) and your result for η, can you see why (qualitatively)?