AS2001
Nucleosynthesis and the Chemical Evolution of the Universe
Tutorial 4 – Answers

Question 1

$$f_N(8M_\odot) = \frac{\int_8^{20} M^{-\frac{7}{3}}\,\mathrm{d}M}{\int_{0.1}^{20} M^{-\frac{7}{3}}\,\mathrm{d}M} = \frac{20^{-\frac{4}{3}} - 8^{-\frac{4}{3}}}{20^{-\frac{4}{3}} - 0.1^{-\frac{4}{3}}} = 0.002 = 0.2\% \tag{1}$$

$$f_M(8M_\odot) = \frac{\int_8^{20} M \times M^{-\frac{7}{3}}\,\mathrm{d}M}{\int_{0.1}^{20} M \times M^{-\frac{7}{3}}\,\mathrm{d}M} = \frac{20^{-\frac{1}{3}} - 8^{-\frac{1}{3}}}{20^{-\frac{1}{3}} - 0.1^{-\frac{1}{3}}} = 0.074 = 7.4\% \tag{2}$$

There are several ways to answer this question. For example, one can multiply the 20 $M_\odot$ / year by the above 7.4 % to get the amount of mass / year cycled through SN. If this number is divided by some sort of average mass of a SN progenitor (i.e. a star with $M > 8M_\odot$), you have the answer. However, I like the following better: use the star formation rate to get the constant of proportionality in the IMF, $\Phi(M) = KM^{-\frac{7}{3}}$:

$$20 = \int_{0.1}^{20} M \times KM^{-\frac{7}{3}}\,\mathrm{d}M, \tag{3}$$

hence

$$K = \frac{20}{3(0.1^{-\frac{1}{3}} - 20^{-\frac{1}{3}})} \tag{4}$$

Then one can get the total number of stars forming / year

$$N = \int_{0.1}^{20} KM^{-\frac{7}{3}}\,\mathrm{d}M = 5\frac{(0.1^{-\frac{4}{3}} - 20^{-\frac{4}{3}})}{(0.1^{-\frac{1}{3}} - 20^{-\frac{1}{3}})} = 60.3 \tag{5}$$

Therefore we would expect $60.3 \times 0.002 = 0.12$ SN / year, or roughly 1 every 8 years.

Question 2

Irregular galaxies undergo short bursts of star formation with quiescent periods inbetween. We have $\mu = 1 - \alpha n \psi t$ where μ is the gas mass fraction, ψ is the star formation rate / total mass during a burst, n is the fraction of time spent bursting and α is the lock-up fraction.

$$\mu = \frac{7 \times 10^{10} M_\odot}{10^{11} M_\odot} = 1 - 0.9 \times \frac{0.1\ \text{Gyr}}{\text{Period of bursts}} \times \frac{100 M_\odot/\text{yr}}{10^{11} M_\odot} \times 10\ \text{Gyr} \tag{6}$$

$$\text{Period} = \frac{0.9}{0.3} \times 10^{-1+2-11+10}\ \text{Gyr} = 3\ \text{Gyr} \tag{7}$$

$$t_{95} = \frac{1-\mu}{\alpha n \psi} = \frac{0.95 \times 3 \times 10^{11}}{0.9 \times 0.1 \times 100} \text{ yr} = 32 \text{ Gyr} \tag{8}$$

So the galaxy can continue forming stars for another 20 odd Gyr!