Feasibility Study
- Interim Report -

Calibration System
based on a Laser Frequency Comb

Interim Report
in response to the
ESO Technical Specification and Statement of Work
Doc. No. OWL-SPE-ESO-00000-0185 Iss. 2.0

Authors: Andreas Sizmann, Marc Fischer and Ronald Holzwarth

Copyright: Menlo Systems GmbH, 2006

Version: 0.1
# Table of Contents

## 1 SUMMARY

## 2 INTRODUCTION AND OVERVIEW

2.1 STUDY MOTIVATION, PRESENTATION AND OUTLINE

2.2 A BRIEF HISTORY OF FREQUENCY COMBS

2.2.1 HARMONIC CHAINS

2.2.2 MEASUREMENT TOOLBOX FOR OPTICAL FREQUENCY DIFFERENCES

2.2.3 FIRST EXPERIMENTS WITH PICOSECOND LASERS IN THE 70s

2.2.4 Ti:SAPPHIRE FREQUENCY COMBS

2.2.5 ABSOLUTE FREQUENCY MEASUREMENTS WITH Ti:SAPPHIRE LASERS

2.2.6 PHOTONIC CRYSTAL FIBER COMBS

2.2.7 FIBER BASED FREQUENCY COMBS

2.2.8 UV AND IR COMBS

## 3 STATE-OF-THE-ART FREQUENCY COMBS

3.1 PRINCIPLE OF OPTICAL FREQUENCY COMB GENERATION

3.2 FEMTOSECOND LASERS

3.2.1 HISTORY

3.2.2 GROUP-VELOCITY DISPERSION

3.2.3 SELF-PHASE MODULATION

3.2.4 COMBINED EFFECT OF GROUP-VELOCITY DISPERSION AND SELF-PHASE MODULATION

3.3 Ti:SAPPHIRE AND OTHER BULK ALL-SOLID-STATE LASERS FOR FREQUENCY COMBS

3.3.1 BASIC CONSIDERATIONS

3.3.2 PASSIVE MODELOCKING

3.3.3 PERFORMANCE OF BULK SOLID STATE FREQUENCY COMB GENERATORS

3.4 FIBER LASERS FOR FREQUENCY COMBS

3.4.1 BASIC CONSIDERATIONS

3.4.2 PASSIVE MODELOCKING

3.4.3 PERFORMANCE OF FIBER LASER FREQUENCY COMB GENERATORS

3.5 OTHER OPTICAL COMB GENERATORS

3.6 NONLINEAR SPECTRAL BROADENING OF THE COMB

3.7 OFFSET-FREQUENCY DETECTION AND COMB STABILIZATION

3.7.1 OFFSET-FREQUENCY DETECTION

3.7.2 COMB STABILIZATION

## 4 ENABLING TECHNOLOGY FOR HIGH-REPETITION-RATE COMBS

4.1 BASIC CONSIDERATIONS

4.2 HIGH-REPETITION-RATE (HRR) FEMTOSECOND LASERS

4.2.1 FUNDAMENTALLY MODE-LOCKED HRR LASER SOURCES

4.2.2 HARMONICALLY MODE-LOCKED HRR LASER SOURCES

4.3 MODE FILTER TECHNIQUES

4.4 NONLINEAR PROCESSES

4.5 PROMISING TECHNICAL SOLUTIONS FOR HRR COMBS
5 PROPOSED SOLUTION

5.1 ASSESSMENT OF SPECIFICATIONS
5.1.1 PRELIMINARY SPECIFICATIONS
5.1.2 WAVELENGTH RANGE
5.1.3 RELATIVE INTENSITY BETWEEN LINES
5.1.4 ATTENUATION OF “SUPERMODES”

5.2 ASSESSMENT OF COMPETING TECHNICAL SOLUTIONS

5.3 SYSTEM DESIGN PROPOSAL

5.4 DETAILED TECHNICAL DESCRIPTION OF THE PROPOSED SOLUTION

6 SCHEDULE, COST AND RISK

6.1 DEVELOPMENT AND LABORATORY TESTING PLAN
6.2 REALIZATION AND INITIAL VERIFICATION
6.3 ON-SITE SYSTEM LINE-UP AND TESTING (SLAT), VERIFICATION
6.4 COST AND TIME ESTIMATION
6.5 RISK ANALYSIS

7 REFERENCES

7.1 BOOKS AND BOOK CHAPTERS ON FEMTOSECOND LASERS AND OPTICAL FREQUENCY COMBS
7.2 JOURNAL PUBLICATIONS ON HIGH-REPETITION-RATE LASER TECHNOLOGY
7.3 JOURNAL PUBLICATIONS ON FEMTOSECOND OPTICAL FREQUENCY COMB SCIENCE AND TECHNOLOGY
1 Summary

The interim report presents a technology review and identifies promising solutions for a calibration system based on a laser frequency comb. The system is intended to provide a means of unprecedented accuracy of calibration of an ultra-stable array of high-resolution spectrographs. The spectrographs are designed for the first-time direct measurement of cosmic dynamics in the Cosmic Dynamics Experiment (CODEX) instrument, which leads to specific requirements for the calibration unit as outlined in the ESO Technical Specification and Statement of Work Doc. No. OWL-SPE-ESO-00000-0185 Iss. 2.0 of 27.02.2006.

The calibration system is required to provide a long-term stable (> 10 years) and accurate (to $10^{-11}$) comb of equidistant frequencies in the 400 – 680 nm region (minimum range), with possible extension to 350 – 800 nm. The optimum comb line separation shall be 15 GHz.

The 15-GHz frequency comb requires a femtosecond pulse source with 15 GHz repetition rate and sufficient power to achieve nonlinear spectral broadening of the comb over the desired wavelength range. This is a combination of requirements that so far no laser crystal or laser glass is able to fulfil. To our best knowledge, the shortest pulses obtained from lasers with 15 GHz repetition rate are in the picosecond regime, and the highest repetition rate achieved with (sub-) 100-fs lasers is 4 GHz (see Fig.1.1). Surprisingly, new types of optically pumped high-repetition-rate semiconductor lasers are closest to the femtosecond regime, with one demonstration of sub-500-fs pulses from a 10-GHz system [Hoogland2005].

Figure 1.1: State-of-the-art of ultrafast laser performance shows that very high repetition rates (>10 GHz) are achieved with picosecond pulses, whereas (sub-) 100-fs pulses are generated only to up to 4 GHz. The desired frequency comb parameters lie in the quadrant of >10-GHz femtosecond operation, for which no published demonstration has been found. (see also section 4)
Sub-100-fs laser sources have been demonstrated to operate up to 4 GHz, with 20-fs pulses from a 3.5-GHz Ti:sapphire laser system [Bartels2005] and 80-fs pulses from a 4-GHz Cr:YAG system [Leburn2004]. Turn-key femtosecond fiber laser systems have been demonstrated to achieve up to 250 MHz repetition frequency.

At this point of the study, we believe that a 15-GHz comb is best realized as follows (Fig.1.2): at repetition frequencies near 2 GHz the pulse energy can still be amplified enough for nonlinear frequency conversion. A femtosecond source with 1 – 4 GHz repetition rate or alternatively a 250-MHz system with a first filter stage to achieve a &gt;1 GHz pulse train can be used, and the subsequent amplification stage provides the power for the nonlinear processes. After frequency conversion and broadening, an external mode filter cavity is added to enhance the repetition rate from a few GHz to 15 GHz.

**Figure 1.2:** System layout based on a 250-MHz erbium-doped fiber laser emitting at a wavelength of 1560 nm (see also section 4).

If a frequency-tuning approach is compatible with the calibration procedure, a tunable two-port scheme will be considered.

Competing technical solutions will be assessed as part of the complete Feasibility Study. After a critical review of the components, sub-units and anticipated performance, a system design proposal will be formulated along with an assessment of timelines, cost and risk of its realization.
2 Introduction and overview

2.1 Study motivation, presentation and outline

Optical frequency combs are a novel technique for ultra-precise measurements of time and frequency. This technology has been successfully employed in experiments of fundamental tests of physics, such as the determination of the fine structure constant, Lamb-shift and Rydberg constant. The optical frequency comb is an enabling technology for a variety of other precision measurements. Here we study its application as the frequency calibration unit for the first-time direct measurement of cosmic dynamics in the Cosmic Dynamics Experiment (CODEX).

The unique advantage of an absolute wavelength calibration to unprecedented precision comes with some development risk as in every pioneering application. However, the basic technology and experience of Menlo Systems and the MPQ team, who have developed optical frequency combs into a reliable, robust commercial product, will minimize such risk. As a key subsystem of the CODEX instrument it is essential to guarantee long-term service-free operation in a remote location. Some technological risk originates from the development of a custom-tailored solution according to the ESO specifications for the calibration unit whereas the operational risk can be minimized by providing a laboratory-standard operating environment. The development challenge is mainly due to the requirements of the very large comb frequency spacing and the need for nonlinear frequency conversion and broadening with sufficient power.

The study outline is shown in Fig.2.0. The authors of the Interim Report are Dr. Andreas Sizmann, Dr. Marc Fischer and Dr. Ronald Holzwarth of Menlo Systems GmbH in Marinsried/Munich.

The final document is intended to propose the most promising realization of a calibration unit based on a laser frequency comb. The comb will be employed for the calibration of an ultra-stable array of high-resolution spectrographs. The spectrographs are designed for CODEX instrument. The specific requirements for the calibration unit are outlined in the ESO Technical Specification and Statement of Work, Doc. No. OWL-SPE-ESO-00000-0185 Iss. 2.0 of 27.02.2006

This document will be presented in three stages of the Feasibility Study.
1. The Interim Report, version V.0.x (Issue 0), will be presented (with the basic table of contents as listed in section 7.1.1 of the ESO TechSpec and SOW. ) at the intermediate progress meeting. The Interim Report covers the technology review material, identifies promising solutions and contains an assessment of the comb specifications (see Fig. 2.0).
2. The Draft Feasibility Study, version V.1.x (Issue 1), will be delivered prior to the study presentation and discussion at ESO.
3. The Final Report, version V.2.x (Issue 2) will include comments and clarification of issues raised before and at the presentation meeting.

The document is organized as follows:
The proposed solution and possible alternatives with trade-offs will be summarized in Section 1 of the completed feasibility study.
Section 2 gives a historical and technical overview of the frequency comb technique.
Section 3 is a state-of-the-art frequency comb technology overview. It presents the principles of optical frequency comb generation beginning with femtosecond lasers, followed by nonlinear spectral broadening and stabilization of the comb. Section 4 is a review of enabling technology for the high-repetition rate comb used for the CODEX instrument calibration. It discusses the components and sub-systems needed for the large mode spacing and identifies promising technical solutions. Here, the Interim Report covers some concepts that will be refined and subsequently selected. Section 5 discusses the requirements, which is still part of the Interim Report. However, the Interim Report does not cover the assessment of competing solutions (with their technological maturity or development uncertainties). In this section, the Final Report will present the detailed technical description of the proposed solution. Section 6 is related to the implementation of the selected system design: it addresses the work parts and schedule of the development, testing&verification and system-line-up for operation as a calibration unit for CODEX. The related time lines, costs and risks involved are also estimated and discussed. As this will be developed from the refined concept of a calibration system, it will be contained in the Final Report but not in the Interim Report. Section 7 provides a list of relevant references and information resources.

Figure 2.0: Outline of the feasibility study. Identifying the most promising solution follows a comprehensive technology review which covers both the state-of-the-art combs and high-repetition rate technology. The final system system design proposal is developed through an assessment of competing solutions and the assessment of the comb specifications, followed by a critical review of the availability and specifications of components, subsystems and of the anticipated comb performance.
2.2 A brief history of frequency combs

2.2.1 Harmonic chains

The observation of sharp optical resonances by nonlinear laser spectroscopy with a resolution much beyond the measurement limits of wavelength interferometry had long created a strong need for methods to measure the frequency rather than the wavelength of light. The quest for an optical frequency counter is almost as old as the laser itself. Ali Javan, the co-inventor of the helium-neon laser, was the first to superimpose the beams from two different lasers with a beam splitter on a photo detector to observe a beat note, similar to the interference of the sound waves from two tuning forks [Javan1962]. This was an extraordinary result, because it proved that laser waves can behave like classical radio waves. A coherent laser wave can have a well defined phase and amplitude, so that it must be possible to count the wiggles of such a light wave. However, at a frequency near 500 000 billion oscillations per second, there are no electronic detectors and circuits fast enough to build an optical frequency counter.

At MIT in the early 1960s, Ali Javan started a research project, aimed at extending microwave frequency counting techniques into the optical spectral region. He experimented with whisker-like metal-insulator-metal point contacts as antennas, detectors and mixers for infrared laser waves. Such elements were later used by John Hall and Ken Evenson at NBS (now NIST) in Boulder to realize the first harmonic laser frequency chain, that was used to determine the speed of light by measuring both the wavelength and the frequency of a methane-stabilized 3.39-µm helium-neon gas laser [Evenson1972]. Harmonic laser frequency chains were highly complex systems, engineered to measure just one particular optical frequency, and only a handful of these chains have ever been constructed at a number of well-equipped national metrology laboratories. In the early 1980s, a chain at NBS in Boulder had been perfected so that it could measure the frequencies of some iodine-stabilized visible helium-neon lasers to 10 decimal digits. This demonstration led the Conférence Générale des Poids et Mesures in 1983 to redefine the meter by defining the speed of light in vacuum $c$ as exactly 299 792 458 meters per second. One meter is then the distance travelled by light during the time of $1/299\,792\,458$ seconds. From now on, one could determine the precise wavelength of a laser in vacuum, $\lambda$, by simply measuring the frequency $f$, since $c = f \cdot \lambda$.

Unfortunately, the complex NBS frequency chain had to be abandoned soon after this definition was in the books, and for the next decade there was not a single laboratory in the U.S. that could have followed this prescription. A number of European laboratories followed this path successfully, notably the Observatoire de Paris (now BNM SYRTE) and the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig. In an article published in early 1996 [Schnatz1996], a team from the PTB laid claim to the first phase coherent frequency measurement of visible radiation. An elaborate frequency chain filling three large laboratories spread over two separate buildings was assembled to compare the frequency of the red intercombination line of atomic calcium with the microwave frequency of a cesium atomic clock. To reach sufficient phase stability, the clock frequency was first reduced to the 100 MHz of a stable quartz oscillator. From here, the chain traversed the entire electromagnetic spectrum in discrete steps, always generating some harmonic frequency in a suitable nonlinear element and producing enough power for the next step with a phase-locked transfer oscillator. A tricky puzzle had to be solved to reach the desired final frequency with the help of several auxiliary oscillators.
2.2.2 Measurement toolbox for optical frequency differences

A small frequency difference or gap between two laser frequencies can be measured rather easily by superimposing the two laser beams on a photo detector and monitoring a beat signal. The first experiments of this kind date back to the advent of cw He-Ne-lasers in the early
1960s. Modern commercial fast photodiodes and microwave frequency counters make it possible to directly count frequency differences up to the order of 100 GHz.

Since the gap between the endpoint of a traditional harmonic laser frequency chain and an unknown optical frequency to be measured can easily amount to tens or hundreds of THz, there has long been a strong interest in methods for measuring much larger optical frequency differences.

![Figure 2.2: A typical situation in frequency metrology, a well known reference frequency and an unknown frequency tens or hundreds of THz apart.](image)

Motivated by such problems in precision spectroscopy of atomic hydrogen, a general solution for the measurement of large optical frequency gaps was introduced in 1988 by T.W. Hänsch and D. McIntyre with the invention of the optical frequency interval divider (OFID) which can divide an arbitrarily large frequency difference by a factor of precisely two [McIntyre1988]. An OFID receives two input laser frequencies $f_1$ and $f_2$. The sum frequency $f_1 + f_2$ and the second harmonic of a third laser $2f_3$ are created in nonlinear

![Figure 2.3: Principle of the divider stage.](image)
crystals. The radio frequency beat signal between them at \(2f_1 - (f_1 + f_2)\) is used to phase-lock the third laser at the midpoint \(f_3 = (f_1 + f_2)/2\). Phase-locking of two optical frequencies is achieved electronically by locking the phase of their beat signal to zero or, to reduce \(1/f\) noise, to a given offset radio frequency, provided by a local oscillator. Techniques of conventional radio frequency phase-locked loops can be applied. With a divider chain of \(n\) cascaded OFIDs, the original frequency gap can be divided by a factor \(2^n\).

Another more compact setup to measure frequency gaps on the order of a few THz is an optical frequency comb generator (OFCG) based on the very efficient creation of side bands in a large index electro optic phase modulator. To enhance the efficiency an electro optic modulator is placed inside a Fabry-Perot cavity. To further reduce losses a monolithic resonator can be formed by placing high reflectivity coatings on the end facets of the electro optic crystal. The cavity can be locked to the carrier wave by adjusting temperature and a dc offset applied to the crystal. If the modulation frequency matches the free spectral range of this optical resonator, the side bands are again in resonance and create further side bands. This technique has been pioneered by M. Kourogi in the group of M. Ohtsu (Tokyo, Japan) [Kourogi 1994, Kourogi1995, Kourogi1996]. Some details including different locking schemes and further references can be found in the PhD thesis of Th. Udem [Udem1997]. The width of such a comb is limited by dispersion. The refractive index in the crystal depends on the wavelength and therefore the modes of the resonator do not match the modulation side bands far away from the carrier. To measure optical frequency differences beat signals can be observed between cw lasers and sidebands on different sides of the carrier and frequency gaps on the order of 8 THz can be bridged in this way. To measure larger gaps, a chain of OFIDs can be followed by an OFCG. These OFCGs have been used in the “prior-femtosecond” time quite extensively as witnessed by several publications [Udem1997, Ye1997, Huber1998 vonZanthier1999]. Furthermore, the output of such an OFCG corresponds to a short pulse in the time domain and can be spectrally broadened in a nonlinear fiber [Imai1998]. Typical parameters for such an OFCG around 800 nm that we used for optical frequency metrology in our lab were: a free spectral range (FSR) of 3.16 GHz which corresponds to a 21 mm long LiNbO\(_3\) crystal, a dc offset voltage up to 1000 V can be applied and a rf power of 2 W at 6.32 GHz (= \(2 \times \text{FSR}\)) is coupled into the rf resonator for modulation. Another variant uses an phase modulation inside an parametric oscillator [Diddams1999].

![Figure 2.4: Principle of an optical frequency comb generator (OFCG). Side bands are created efficiently in an electro-optic crystal.](image)
As the most powerful addition to this optical frequency difference measurement toolbox let us
now turn our view to the frequency combs actively generated by mode-locked short pulse
lasers. First let us look back to the early days of mode-locked lasers.

2.2.3 First experiments with picosecond lasers in the 70s

It has been known since the early experiments with multi-mode helium-neon lasers
[Hargrove1964] that the longitudinal modes of a laser are well defined and their phases can be
coupled so as to produce a short light pulse circulating inside the cavity [Yariv1965,
McDuff1967]. Much shorter pulses were produced in the mid 70s with broadband dye lasers by
locking their axial modes with the help of a saturable absorber or by synchronous pumping
with a modulated argon laser [Shank1990]. With such lasers at hand resonant excitation with
separated light pulses was explored at the time by M. Salour at MIT [Salour1977] and by V.
Chebotaev at Novosibirsk [Baklanov1977b]. Also T.W. Hänsch at Stanford used Ramsey-like
excitation with a coherent train of multiple light pulses for high resolution spectroscopy of
atomic resonance lines [Teets1977].

T.W. Hänsch and co-workers demonstrated that a synchronously pumped mode-locked
picosecond dye laser could produce a stable phase coherent pulse train which they used for
Doppler-free two-photon excitation of atomic sodium [Eckstein1978]. The comb lines served
as a frequency ruler to measure some atomic fine structure intervals.
First, they replaced the original radio frequency driver for the modulator of the argon pump
laser with a high quality frequency synthesizer. The performance of the dye laser improved so
much that they were the first to generate sub-picosecond pulses directly from a synchronously
pumped dye laser [Ferguson1978].

They went on to investigate Doppler-free polarization spectroscopy with this frequency
comb [Ferguson1979], as well as two-photon spectroscopy with the frequency comb of an FM
mode-locked laser where the phases of the modes adjust so that the intensity remains constant
but the frequency sweeps back and forth periodically [Hänsch1980].

At that time there was no way to know the absolute positions of the comb lines because the
dispersion inside the laser resonator would lead to unknown phase slips of the carrier wave
relative to the pulse envelope. Such phase slips shift the entire comb spectrum by an unknown
amount $f_{CE}$, as worked out in considerable detail already in the 1978 Stanford Ph.D. thesis of J.
Eckstein [Eckstein1978]. With a comb spectrum spanning only 800 GHz, there was no means
to observe and measure the offset frequency $f_{CE}$. Therefore, it was not known how to measure
absolute optical frequencies with this laser frequency combs in the late seventies.

In 1990 another interesting idea in this direction was published by T.W. Hänsch, a proposal for
a synthesizer of sub-femtosecond pulses that would superimpose a wide comb of frequencies
from separate phase-locked continuous-wave laser oscillators [Hänsch1990].

2.2.4 Ti:sapphire frequency combs

In the early 1990s, the technology of ultrafast lasers advanced dramatically with the discovery
of Kerr-lens mode locking by W. Sibbett at the University of St. Andrews [Spencer1991].
Soon, commercial Ti:sapphire femtosecond lasers became available that made the generation of ultrashort light pulses much easier, and amplifiers have been developed that produce pulses at a reduced repetition frequency but with increased pulse energy.

In amplified Ti:sapphire systems it is sufficient to focus the laser beam into a glass slide to produce a white light continuum which can be dispersed by a prism into a rainbow of colors. Such white light pulses are produced by a combination of self-focusing, self phase modulation, and other nonlinear processes. A striking feature is the laser-like speckle pattern in the rainbow colors which indicate a high degree of spatial coherence.

Such observations inspired T.W. Hänsch to start a series of discussions and experiments. A discussion with F. Krausz at the Technical University of Vienna resulted in the first observation of such pulse-to-pulse phase slips in an interferometric correlation experiment in 1996 [Xu1996].

In early 1997 T.W. Hänsch and M. Bellini at the European Laboratory for Nonlinear Spectroscopy, LENS, in Florence, Italy produced a white light continuum by focusing part of the laser beam into a thin plate of CaF$_2$. They used an amplified Ti:sapphire femtosecond laser producing pulses of 1 mJ energy at a rate of 1 kHz, a common tool in many ultrafast laboratories for pump-probe experiments.

They split the laser beam in two parts and focused these beams at two spatially separate spots. Would the two white light pulses interfere? With a somewhat misaligned Michelson interferometer they created two beams that would escape in two slightly different directions. By adjusting the length of one arm they could make sure that the two focused pulses arrived on the CaF$_2$ plate at precisely the same time. As a result, they observed stable interference fringes of high contrast for all the colors of the white light continuum [Bellini2000].

In an earlier joint experiment at the Lund Laser Center, the same problem for the generation of high harmonic radiation in a gas jet had been investigated [Zerne1997].

The lesson to be learned from these experiments is that no matter how complicated the process of white light continuum generation might be, the process was perfectly reproducible. If such pulses were separated in time rather than in space, they would interfere in the spectrum to produce a very broad frequency comb.

At the MPQ in Garching, a serious experimental effort towards optical frequency measurements with femtosecond laser frequency combs was started in 1997. By that time, hundreds of Ti:sapphire femtosecond lasers were in use in laboratories around the world, but they were mostly used to study ultrafast phenomena. Nobody had ever looked for any comb lines.

The experiments at the MPQ started with a commercially available “Mira” femtosecond laser (Coherent Inc.). With a repetition frequency of 76 MHz, the comb spectrum of this femtosecond laser was so densely spaced that no standard lab spectrometer could resolve the comb lines. Therefore, the method of choice was heterodyne detection, employing a cw diode laser as a local oscillator. The diode laser beam and the pulse train were superimposed with a beam splitter, and a beat signal was detected with an avalanche photodiode after some spectral filtering. The first steps towards using the femtosecond frequency comb were the observation of stable comb lines. As a next step the spacing of these comb lines was investigated.

Two diode lasers were phase-locked to two arbitrarily chosen comb lines and used an optical interval divider stage to produce a new frequency precisely at the center. A beat note with the nearest comb line confirmed that the comb lines were perfectly evenly spaced, way out into the wings of the emission spectrum, within a few parts in 10$^{17}$ [Udem1999a].
2.2.5 Absolute frequency measurements with Ti:sapphire lasers

It was now certain that the frequency comb of such a mode-locked femtosecond laser did not suffer from “coherence collapse” and could serve as a ruler in frequency space to measure large optical frequency intervals. The first demonstration of an optical frequency measurement with a femtosecond laser comb was the determination of the frequency interval between the cesium D1 resonance line and the fourth harmonic of a transportable CH₄-stabilized 3.39 µm He-Ne-laser, which had been calibrated with a harmonic laser frequency chain at the PTB Braunschweig [Udem1999b]. The optical cesium frequency was needed for a determination of the fine structure constant α from the atomic recoil energy as measured by atom interferometry in the group of Steve Chu at Stanford.

These experiments demonstrated to the optical frequency metrology community that femtosecond laser frequency combs could be powerful tools to measure the frequency of light.

The next goal was the measurement of an absolute optical frequency. The art of measuring frequency intervals is easily transferred to an absolute measurement by measuring the interval defined by different harmonics of the same laser. In the most simple case this can be done by comparing an optical frequency f with its second harmonic 2f. Unfortunately this requires an octave spanning frequency comb and such a device was not available at that time.

What was available at the MPQ was the remainder of a harmonic frequency chain that linked a 3.39-µm methane-stabilized helium-neon laser to the Hydrogen 1S – 2S transition [Udem1997]. It required only minor modifications to produce two different multiples, 4/7 and 1/2, of the dye laser frequency at 486 nm, which could be bridged with a relatively narrow femtosecond laser comb which had a span of only 44 THz [Reichert2000, Niering2000]. The CH₄-stabilized He-Ne laser served now as part of an interval divider stage and no longer as an intermediate reference standard. The primary reference for this first absolute frequency measurement was a commercial HP cesium atomic beam clock which was used to determine the pulse repetition rate [Reichert2000].

Once the optical frequency gap was measured, the absolute frequency of the dye laser as well as the absolute frequencies of all the comb lines were known at the same time. To control the position of the comb lines, the MPQ team learned how to change the carrier-envelope offset frequency of the MIRA Ti:sapphire laser by tilting an end mirror where the spectrum is slightly dispersed by a prism pair inside the cavity (for details see section 3.7). In this way, the first optical frequency measurement was at the same time also the demonstration of femtosecond laser pulses with controlled slips of the carrier-envelope phase [Reichert1999].

The news of this novel development spread rapidly. In the fall of 1998 J. Hall visited the MPQ and soon became an ardent evangelist for “this goofy technique, that makes everything obsolete that we have worked on for so long.” He started to assemble a powerful professional team in Boulder to advance research on femtosecond laser frequency combs, and he persuaded his colleague Steve Cundiff at JILA, an expert on femtosecond lasers from Lucent (Bell Laboratories), to visit the MPQ in the spring of 1999. An increasingly heated competition did much to accelerate the development of the new tools in the following months and to ignite a firework of novel applications in the years to follow. In March 1999, just before this started the MPQ group submitted patent applications on the frequency comb technology.

In June 1999, the MPQ group could directly compare the hydrogen frequency with a highly accurate transportable cesium fountain clock (PHARAO), built at the LPTF (now BNM
SYRTE) in Paris [Niering2000]. This measurement yielded a new value of the hydrogen 1S – 2S frequency accurate to $1.8 \times 10^{-14}$, surpassing all earlier optical frequency measurements by more than an order of magnitude. By now, the compelling advantages of laser frequency combs had been clearly demonstrated. A number of different possible approaches for carrier-envelope offset phase control were soon proposed [Telle1999].

2.2.6 Photonic crystal fiber combs

At the CLEO conference in Baltimore, MD, held in May 1999, researchers from Lucent (Bell Laboratories) had reported on a novel micro-structured “rainbow fiber” that could broaden the spectrum of pulses of a Ti:sapphire femtosecond laser oscillator without further amplification to a rainbow of colors [Ranka2000]. This looked like the missing piece to drastically simplify the setup, provided that this magic fiber would preserve the phase coherence of successive pulses.

At this point a run to get a piece of the magic fiber started. John Hall’s team in Boulder was successful to get a piece of the Lucent fiber and in October 1999, they could demonstrate the first octave-spanning self-referencing laser frequency comb [Diddams2000, Jones2000]. The MPQ team realized a similar comb system a few weeks later [Holzwarth2000], after having received some “photonic crystal fiber” from the group of Philip Russell at the University of Bath in the UK. This British researcher had actually pioneered micro-structured silica fibers some years earlier [Birks1995]. Both laboratories submitted their first short publications on octave-spanning frequency combs on the same day (Nov. 12, 1999) to the CLEO 2000 conference.

The MPQ experiment used a small commercial Ti:sapphire ring laser for their first octave-spanning frequency comb, producing pulses of about 25 fs duration at a repetition frequency of 625 MHz. Launching about 170 mW into a 30 cm length of photonic crystal fiber, we immediately produced a frequency comb spanning more than an octave. The spectrum showed a complicated structure, with valleys and peaks, but it offered useable comb lines everywhere. Together with a nonlinear interferometer for control of the offset frequency $f_{CE}$, the entire optical setup did easily fit on a single breadboard. While the traditional harmonic frequency chains with their factory halls full of lasers could measure just one single optical frequency, our new system was ready to measure any frequency throughout the visible and near infrared. The Boulder experiment was very similar in nature, except that the Ti:sapphire laser was a home-built system with a repetition frequency of 100 MHz.

Since then, Ti:sapphire femtosecond lasers have been developed that produce an octave-spanning spectrum directly from the oscillator, without any need for external spectral broadening [Matos2004].

In a first stringent test, the MPQ group compared an octave spanning frequency comb synthesizer with the more complex frequency synthesizer used in the 1999 hydrogen frequency measurement [Holzwarth2000]. By starting with a common 10-MHz radio-frequency reference and comparing comb lines near 350 THz, they could verify agreement within a few parts in $10^{16}$, limited by Doppler shifts due to air pressure changes or thermal expansion of the optical tables. In 2002, a group at the PTB in Braunschweig demonstrated how a femtosecond laser frequency comb generator can be used as a transfer oscillator to precisely measure optical frequency ratios [Stenger2002]. As a test case, they measured the frequency ratio between the...
second harmonic of a Nd:YAG laser and the fundamental frequency, verifying the expected value of 2 with an uncertainty of 7 parts in $10^{19}$. More recently, the MPQ group has pushed a related experiment to an uncertainty of 6 parts in $10^{21}$ [Zimmermann2004]. In 2004, researchers in Boulder compared four different frequency combs from different laboratories, finding agreement between neighboring comb lines at an uncertainty level of $10^{-19}$ [Ma2004]. So far, no systematic error has been identified which would limit the potential accuracy of future precision spectroscopy or optical atomic clocks which will use frequency combs to deliver superior stable optical transitions to the radio-frequency domain.

2.2.7 Fiber based frequency combs

As an alternative to the well analyzed Ti:sapphire ultrafast technology erbium ($\text{Er}^{3+}$) doped fiber laser are currently under investigation by several groups [Tauser2003, Washburn2004, Schibli2004].

These lasers have many advantages. They can be built with reliable and certified components from the telecom industry, assuring uninterrupted use for years. Also the intrinsic stability of the fiber laser designs allow for continuous operation. This is one basic requirement needed to operate a frequency comb as the clockwork for a future "optical clock". At the same time power consumption and maintenance are much lower than for Ti:sapphire lasers that require a large-frame green pump laser.

Other advantages of using frequency combs based on erbium fiber lasers are cost-effectiveness and a compact setup, as well as their wavelength around 1.55 µm. Matching the telecommunication bands, a time or frequency standard as realized by an optical clock may be broadcasted via existing terrestrial fiber transmission networks.

Disadvantages are the relatively low repetition rate of up to roughly 100 MHz for the fundamental frequency (i.e. one pulse circulating in the laser cavity) and the relatively high-frequency phase noise, a topic that is still under investigation. Also the output power is usually lower and pulse length longer than for Ti:sapphire systems.

The current system design of choice makes use of a seed-laser - amplifier system. Without elaborate chirped pulse amplification schemes the achievable output power is limited by nonlinear effects in the fibers. By carefully adjusting the dispersion regime for the amplifier 300 mW of average power in sub 100 fs pulses at a mean wavelength of 1560 nm and at a repetition frequency of 100 MHz can be achieved.

The output pulses of the amplifier are launched into a highly nonlinear fiber, creating a frequency comb spanning from 1.0 µm to 2.3 µm. To access the visible part of the spectrum different approaches have been realized, including second-harmonic generation and subsequent spectral broadening in photonic crystal fibers.

Current research puts its focus on the noise properties of the fiber based frequency combs and especially on the noise properties of continuum generation.

2.2.8 UV and IR combs

For many applications it would be useful to extend the range of frequency combs into previously inaccessible regions. The mid-infrared (mid-IR) region between 2 and 5 µm can be
reached by difference frequency generation. This has the further advantage that the frequency combs created in that manner no longer exhibit an offset frequency. At the same time such difference frequency combs might give a good seed pulse for seeding mid-IR optical parametric amplifier systems.

Another interesting range is the UV part of the spectrum. This would allow tests of new limits in UV spectroscopy. One candidate is the hydrogen-like helium ion with a $1S - 2S$ two-photon transition near 60 nm.

Since there is hardly any gain medium available that supports lasing in the XUV spectral range and it is almost impossible to create strong feedback on such a gain medium due to the lack of high-reflectance mirrors in that wavelength range, it is difficult to set up a laser oscillator directly. A more promising approach to create laser-like radiation in the XUV and soft X-Ray spectral region is to up-convert available highly coherent visible and near-infrared (NIR) laser radiation using nonlinear frequency conversion processes. There the nonlinear response of dielectric media is utilized, to create harmonics and/or sum and difference frequencies of the laser’s frequency.

High harmonic generation is a well-known and often-used technique for low repetition frequency amplified pulses. For spectroscopic applications it is much preferred to have a higher repetition frequency on the order of the fundamental laser repetition frequency.

A way to generate high harmonics at a high repetition frequency is to feed the pulses into an enhancement resonator. High harmonic radiation down to wavelengths of 60 nm at a repetition frequency of 112 MHz has recently been generated by the MPQ group [Gohle2005b]. To this end, the pulses from a mode-locked Ti:sapphire laser oscillator were stacked in a dispersion-compensated passive build-up cavity and a xenon gas jet was placed at an intra-cavity focus. The high harmonic radiation is coupled out by external reflection from a thin sapphire Brewster plate, that has a refractive index smaller than 1 in the extreme ultraviolet. Similar experiments have also been reported by Jun Ye in Boulder [Jones2005b].
3 State-of-the-art frequency combs

3.1 Principle of optical frequency comb generation

To understand the mode structure of a femtosecond frequency comb and the techniques applied for its stabilization one can look at the idealized case of a pulse circulating in a laser cavity with length \( L \) as a carrier wave at \( f_c \) that is subject to strong amplitude modulation described by an envelope function \( A(t) \). This function defines the pulse repetition time \( T = f_r^{-1} \) by demanding \( A(t) = A(t - T) \) where \( T \) is calculated from the cavity mean group velocity:

\[
T = 2L/v_g.
\]

The pulses however are not necessarily identical. This is because the pulse envelope \( A(t) \) propagates with \( v_g \) while the carrier wave travels with its phase velocity. As a result the carrier shifts with respect to the pulse envelope after each round trip by a phase angle \( \Delta \phi \) as shown in Fig. 3.1. Unlike the envelope function, which provides us with a more rigorous definition of the pulse repetition time \( T = f_r^{-1} \), the electric field is, in general, not expected to be periodic in time. Because of the periodicity of the envelope function the electric field at a given place (e.g. at the output coupler) can be written as

\[
E(t) = A(t) e^{-2\pi if_c t} + \text{c.c.} = \sum_q A_q e^{-2\pi i(f_c + qf_r) t} + \text{c.c.}.
\] (3.1)

As the envelope function \( A(t) \) is strictly periodic it has been written as a Fourier series

\[
A(t) = \sum_q A_q e^{-2\pi if_q t},
\] (3.2)

where \( A_q \) are Fourier components of \( A(t) \). Equation 3.1 shows that, under the assumption of a periodic pulse envelope, the resulting spectrum represents a comb of laser frequencies separated by the pulse repetition frequency \( f_r \). Since \( f_c \) is not necessarily an integer multiple of \( f_r \) the modes are shifted from being exact harmonics of the pulse repetition frequency by an offset \( f_o < f_r \):

\[
f_o = nf_r + f_o.
\] (3.3)

with a large \( (\geq 10^6) \) integer \( n \). This equation maps two radio frequencies \( f_r \) and \( f_o \) onto the optical frequencies \( f_o \). While \( f_r \) is readily measurable and usually lies between a few 10 MHz and a few GHz depending on the length of the laser resonator, \( f_o \) is not easy to access unless the frequency comb contains more than an optical octave as discussed later. The intuitive picture given here can even cope with a frequency chirp, i.e. a carrier frequency that varies across the pulse. In this case the envelope function becomes complex in value and the comb structure derived above stays valid provided the chirp is the same for all the pulses. Under this assumption, which is reasonable for a stationary pulse train, \( A(t) \) remains a periodic function. In the time domain the frequency offset is obvious because the group velocity differs from the phase velocity inside the cavity and therefore the carrier wave does not repeat itself after one round trip but appears phase shifted by \( \Delta \phi \) as shown in Fig. 3.1. The offset frequency is then calculated from \( f_o = \Delta \phi / (2\pi T) \) [Ferguson1979, Baklanov1977a, Wineland1989].
Figure 3.1: Top: Consecutive pulses of the pulse train emitted by a mode locked laser and the corresponding spectrum. As the carrier wave at $w_c$ moves with the phase velocity while the envelope moves with a different group velocity the carrier wave (blue) shifts by $Dj$ after each round trip with respect to the pulse envelope (red). Bottom: This continuous shift results in a frequency offset $2\pi\Delta\omega = \omega_\phi / T$ of the comb from being exact harmonics of the pulse repetition frequency $\nu$ [Ramsey1995, Quinn1999, Reichert2000].

One might argue that no laser has line width zero and that one should treat the carrier not as a ideal single frequency wave $f_c$ but as a source with general line width function $C(t)$. Even if no technical noise would be present, there would still be some sort of fundamental Schawlow-Townes limit connected with the line width of each mode. As long as we still have the periodicity of $A(t)$ Eqn. 3.1 reads then as

$$E(t) = A(t) C(t) + c.c.$$

Fourier transforming $E(t)$ brings us into the frequency domain and back ($\omega = 2\pi f$):

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt, \quad E(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(\omega) e^{-i\omega t} d\omega$$

With the help of the convolution theorem

$$\sqrt{2\pi} A(t) C(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (A(t) \otimes C(t)) e^{-i\omega t} d\omega$$

we get

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \left( A(\omega) \otimes C(\omega) \right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(\omega') C(\omega - \omega') d\omega' + c.c.$$
The Fourier transforms of $A(t)$ and $C(t)$ are given by

$$A(\omega) = \sqrt{2\pi} \sum_{n=-\infty}^{+\infty} A_n \delta(\omega - n\omega)$$

$$C(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} C(t) e^{i\omega t} dt,$$

and therefore

$$E(\omega) = \sum_{n=-\infty}^{+\infty} A_n C(\omega - n\omega) + c.c.$$ (3.10)

This sum represents a periodic spectrum in frequency space with periodicity $f_r = 1/T$. The mode shape function is duplicated by the strong amplitude modulation induced by Kerr lens mode locking. Assuming the simplified case of a carrier wave $C(t) = e^{-2\pi f r t}$ brings us back to Eqn. 3.1. A chirp of the pulse may be hidden in the complex Fourier components $A_n$. Note that the only assumption necessary to create a precisely equidistant comb is the periodicity of the envelope function.

In the time domain, the output of a mode-locked femtosecond laser may be considered as a continuous carrier wave that is strongly amplitude modulated by a periodic pulse envelope function. If such a pulse train and the light from a cw laser are combined on a photo detector, the beat note between the carrier wave and the cw oscillator is, in fact, observed in a stroboscopic sampling scheme. The detector signal will thus reveal a slow modulation at the beat frequency modulo the sampling rate or pulse repetition frequency. A similar idea based on the stroboscopic sampling scheme has been reported previously by Chebotayev et al. [Chebotayev1990].

The important fact to learn from this section is that such a femtosecond frequency comb has two degrees of freedom which are the repetition frequency $f_r$ and the offset frequency $f_o < f_r$. Depending on the application one or both degrees of freedom have to be stabilized (see section 3.7). Furthermore the fast amplitude modulation of the Kerr lens keeps the inter-mode spacing constant even across a vast spectrum of modes. As the spectral width of these pulsed lasers scales inversely with the pulse duration the advent of femtosecond lasers has opened the possibility to directly access THz frequency gaps.

The outline of an optical frequency comb (OFC) system is shown in Figure 3.2 below. The following sections discuss the optical frequency comb generator (OFCG), which is a femtosecond laser with built-in comb control capabilities, followed by the nonlinear broadening and comb stabilization techniques.
Figure 3.2: OFC system with subsystems and external hardware (Radio-frequency reference, continuous-wave laser. The OFC contains the optical frequency comb generator (OFCG), followed by an optical amplifier to yield enough power for nonlinear spectral broadening. The broadenend comb is then used to detect the CEO in a nonlinear interferometer. In a beat detection unit, the broad spectrum can be overlapped with an external continuous-wave laser, and the respective beat signal can be detected. The optical interfaces between sub-units are labelled A through E. In the CODEX instrument application, the stabilized comb output at interface C is fed into the echelle spectrograph. In the lower half of the OFC system the electronic layer is shown as part of each subsystem and as a separate subsystem (control and measurement unit).

3.2 Femtosecond lasers

3.2.1 History

The first flash lamp pumped Nd:glass and Nd:YAG mode–locked laser appeared in the mid sixties with less than 100 ps in duration and demonstrated one of the most powerful interference phenomena in nature [Diels1996, Brabec2000]. The cw operation of dye lasers with broad bandwidth triggered the second generation of mode–locked lasers. Optical pulses shorter than 1 ps could be produced and improvements in the cavity design allowed breaking of the 100-fs barrier. Intracavity dispersion control by means of Brewster angled prism pairs was the next major breakthrough in 1984 [Fork1984]. This early work culminated in the production of 27 fs pulses from a Rhodamine 6G dye laser emitting around 620 nm [Valdmanis1986]. The development of new solid state laser materials led to the emergence of third generation laser sources with the discovery of self-mode locking in Ti:sapphire lasers [Spence1991], its explanation as being due to Kerr-lens mode-locking [Krausz1992], and development of the design to produce 10-fs pulses [Asaki1993]. Recently pulses shorter than 6 fs have been created directly from a Ti:sapphire laser oscillator [Morgner1999, Sutter2000] with the help of special dispersion-compensating mirrors. Ti:sapphire lasers nowadays represent convenient laboratory work horses and are commercially available in a variety of pulse lengths, repetition rates and peak powers and will be discussed in more detail below.
The shaping and propagation of femtosecond pulses in a dielectric medium is the subject of various textbooks. The following brief reviews follows closely the excellent books of Agrawal [Agrawal2001a,b], Rulliere [Rulliere2005] and Diels [Diels1996] as well as the PhD theses of A. Kasper [Kasper1997] and J. Reichert [Reichert2000].

As optical pulses travel in a transparent medium we can observe linear dispersive and nonlinear effects. The wavelength dependence of the light propagation factor

$$k(\omega) = \omega \frac{n(\omega)}{c}$$ (3.11)

leads to dispersive broadening of the pulses in the time domain while the nonlinear power dependent refractive index

$$n = n_0(\omega) + n_2 I(r,t)$$ (3.12)

changes the spectral and spatial properties of the pulse via self-phase modulation and the Kerr lens effect respectively.

### 3.2.2 Group-velocity dispersion

To gain some insight into the dispersive properties of short pulses in a medium we apply a Taylor expansion to $k(\omega)$ around $\omega_0$

$$k(\omega) = k(\omega_0) + \frac{\partial k}{\partial \omega} |_{\omega_0} (\omega - \omega_0) + \frac{1}{2} \frac{\partial^2 k}{\partial \omega^2} |_{\omega_0} (\omega - \omega_0)^2 + \cdots$$ (3.13)

The linear term does not change the envelope function of the pulse (i.e. the pulse length), linear dispersion just translates the pulse in time. All higher terms change the pulse duration, led by the quadratic term, the group-velocity dispersion (GVD) $k^* = \frac{\partial^2 k(\omega_0)/\partial \omega^2}{\partial k}$ . This measures (in first order) the spreading of a pulse as it travels with the group velocity

$$v_g = \frac{\partial \omega}{\partial k} = k'(\omega_0)^{-1} = \frac{c}{n + \omega \frac{\partial n}{\partial \omega}}$$ (3.14)

i.e. the GVD represents the wavelength dependence of the group velocity:

$$k'' = \frac{\partial^2 k}{\partial \omega^2} = \frac{\partial}{\partial \omega} (v_g^{-1})$$ (3.15)

The GVD is usually stated in fs$^2$ per cm. In the literature the GVD is sometimes denoted by $\beta_2 = k''$ or a dispersion parameter $D$ is introduced with $D = \frac{\hat{\beta}}{c^2} (v_g^{-1}) = -\frac{2\pi}{\lambda} k''$ in units of ps/(km
nm) [Agrawal2001]. For optical elements where the light travels a path length \( z \) the integrated contribution \( D_2 = k^* z \) is sometimes stated as GVD in fs\(^2\).

Higher order terms (third, fourth ..., order dispersion) are becoming relevant for very short pulses where a wide spectral bandwidth is covered.

### 3.2.3 Self-phase modulation

In a dispersion and absorption free environment the intensity dependent refractive index \( n(t) = n_0 + n_2 I(t) \), where \( n_2 \) originates from \( \chi^{(3)} \) via \( n_2 = \frac{3}{4\pi\epsilon_0} \chi^{(3)} \) [Agrawal2001], leads to a self-induced phase shift after the pulse has propagated length \( l \) along the fiber:

\[
\phi_{NL}(l,t) = -n_2 I(t) \omega/c l/c \quad \text{with} \quad I(t) = |A(t)|^2
\]

(3.16)

The nonlinear phase shift has its maximum at the pulse center \( I(t_0) = I_0 \) and increases with the propagated distance \( l \). The maximal phase shift amounts to

\[
\phi_{\max} = I_0 n_2 \omega_0 c l/c = \frac{l}{L_{NL}}
\]

(3.17)

where the nonlinear length \( L_{NL} = 1/(n_2 \frac{\omega}{\omega_0} I_0) \) has been introduced as the effective propagation distance at which \( \phi_{\max} = 1 \).

This time dependent phase shift leads to a frequency modulation that is proportional to the time derivative of the self-induced phase shift \( \phi_{NL}(t) \). For fused silica with its positive Kerr coefficient \( n_2 \approx 3 \times 10^{-16} \text{cm}^2/\text{W} \) [Agrawal2001] the leading edges of the pulses create frequencies shifted to the red (\( \dot{\phi}_{NL}(t) < 0 \)) while the trailing edges cause blue-shifted frequencies to emerge. Self-phase modulation modifies the envelope function according to

\[
A(t) \rightarrow A(t) e^{i\phi_{NL}(t)}.
\]

(3.18)

Self-phase modulation thus produces a chirp without changing the shape or width of the pulse. This means that additional frequency components are created and the pulse can in principle be compressed afterwards. Because \( \phi_{NL}(t) \) has the same periodicity as \( A(t) \) the comb structure of the spectrum is not affected.

### 3.2.4 Combined effect of group-velocity dispersion and self-phase modulation

In many practical situations, however, there is normal group-velocity dispersion \( k^* > 0 \) in the wavelength-range of interest, whereas the anomalous-dispersion regime \( k'' < 0 \) applies to pulse propagation in standard telecommunication silica glass at wavelengths larger than 1.3 \( \mu \text{m} \). Consequently, the GVD broadens the pulses as they travel along the fiber. Typical length scales
for linear and nonlinear propagation effects provide a useful measure of the dominant pulse-shaping effect. The shorter the length scale, the more dominant is the related effect in the initial pulse propagation.

Effective self-phase modulation takes place when the so-called dispersion length
\[ L_D = T_0^2 / |k^*(\omega_c)| \] (where \( T_0 \) is the initial pulse duration) is much larger than the nonlinear length
\[ L_{NL} = 1 / (n_2 \frac{\omega}{c} I_0) \]. A useful parameter for nonlinear and dispersive pulse propagation, the ratio \( R \) of the length scales is given by [Agrawal2001]

\[ R = \frac{L_D}{L_{NL}} = \frac{n_2 \omega I_0 T_0^2}{c |k^*(\omega_c)|} \] (3.19)

In the dispersion dominated regime, \( R << 1 \), the pulses will disperse before any significant nonlinear interaction can take place while for \( R >> 1 \) dispersion can be neglected as an inhibitor of self-phase modulation and will affect the pulse shaping as soon as appreciable spectral broadening occurred. In the anomalous-dispersion regime dispersive and nonlinear pulse shaping effects can balance each other if the two length scales are comparable, i.e. if \( R \approx 1 \). Ultrashort pulses in this regime propagate with stable temporal and spectral shape and are called solitons because of their particle-like stability.

For stable mode–locked operation of a short–pulse laser two conditions must be fulfilled. The pulse must be able to circulate in the laser cavity without being dispersed and there must be a mechanism to form the pulses i.e. to mode–lock many continuous-wave modes. These mechanisms will be discussed in sections 3.3 and 3.4.

In the frequency domain, the axial modes of a “cold” laser cavity are not equally spaced due to GVD. The GVD is partially compensated by an intracavity dispersive delay line or specially designed mirrors [Szipöcs1997], as discussed in section 3.3.1. It is therefore worth pointing out, that pulse-stabilizing effects in the “hot” laser cavity, such as the balance of residual GVD and higher order terms with SPM in the soliton propagation regime of fiber or bulk lasers with residual anomalous dispersion, pull the modes on a perfectly regular frequency grid. The achievable pulse length is determined by the total number of modes that can contribute to the pulse. The broader the frequency comb the shorter is the shortest possible pulse length, ideally reaching the so-called Fourier limit. In fact, the spectral width is usually limited by the width over which the GVD and higher order terms can be compensated for by mode pulling [Morgner1999, Sutter2000].
3.3 Ti:sapphire and other bulk all-solid-state lasers for frequency combs

3.3.1 Basic considerations

Gain medium

Most commonly, a continuous-wave (cw) passive modelocking technique is used. A large number of axial continuous-wave laser modes is phase-synchronized and locked through a passive self-amplitude modulation mechanism inside the laser cavity thus producing an ultrashort pulse in the time domain, supported by a broadband laser gain medium that is continuously pumped. Self-amplitude modulation is typically achieved with an ultrafast mechanism of saturable absorption, thus providing more net gain for pulsed operation and forcing the laser to operate in this mode. Two essential conditions are that the gain bandwidth is large enough and that self-amplitude modulation is fast enough to favour the shaping of ultrashort pulses.

On the practical side, the gain medium should satisfy several additional factors, e.g. to achieve good power efficiency, to exhibit low thermal lensing and to withstand thermal stress without fracture. For remotely permanently operated long-lifetime systems, it will be essential to build a particularly power-efficient diode-pumped all-solid-state system for lifetime service-free operation. Most ultrafast lasers have either favourable properties for diode-pumped efficient cw-modelocked operation but have a relatively small gain bandwidth, too small for femtosecond pulses, such as Nd:YAG and Nd:YVO₄, or are less suitable for direct diode pumping but have a large gain bandwidth for femtosecond operation however with a smaller cross section and unfavourable thermal properties. Ti:sapphire is an exception due to its large cross section, good thermal properties and large gain bandwidth, yet requires a pump wavelength too short for direct diode pumping.

Ti:sapphire laser medium was the first laser medium that was able to support ultrashort pulses without cryogenic cooling [Moulton1986] and stimulated the field of all-solid-state ultrafast laser technology. Ultrashort pulses from Ti:sapphire lasers of less than 6 fs duration [Morgner1999, Sutter1999] contain less than two cycles of the optical field. Also, high average powers allow one to efficiently use cascades of nonlinear processes and to access new frequency domains. Therefore, it has also been a prime candidate for optical frequency comb generation with high power output for nonlinear frequency conversion and octave-spanning spectral width.

Today, a variety of all-solid-state optical frequency comb sources have been demonstrated [Ye2005], the mature Ti:sapphire laser still being the most common ultrafast laser medium, followed by the erbium-doped fiber laser [Tauser2004, Hong2003c, Hundertmark2004] (see below) for its advantage of efficiency, compact size and hands-off operation. Other solid-state laser materials such as Cr:forsterite [Corwin2004, Bartels2004] and Cr:LiSAF build compact diode-pumped bulk laser systems. With Cr:LiSAF efficient frequency conversion towards the blue end of the spectrum [Agate2002] and white-light comb generation [Holzwarth2001] have been demonstrated. Just like the mature erbium fiber systems, the Cr:LiSAF, Cr:LiSGAF or Cr:LiSCAF systems are particularly compact as they can be pumped at longer wavelength, thus eliminating the power-hungry and more complex pump laser frequency conversion scheme for the Ti:sapphire laser. Similarly, a Cr:forsterite comb generator can be efficiently pumped e.g.
with a diode-pumped Yb-fiber laser without the need for frequency doubling the pump [Corwin2004].

**Dispersion compensation**

The cavity is usually designed to meet the simultaneous needs for dispersion compensation, which requires some intra-cavity dispersive elements to over-compensate the normal group-delay dispersion (mainly due to the gain medium), and compactness (i.e. short roundtrip length $L$), providing a high repetition rate $f_r = v_g / L$ and higher power per comb line of the laser output. The higher repetition rate reduces the pulse peak power for a given output power, which may be an issue for maintaining e.g. Kerr-lens modelocking inside the cavity and which can be a disadvantage for extra-cavity nonlinear frequency conversion. On the other hand, the lower peak power can be an advantage in some spectroscopic experiments where nonlinear processes are to be suppressed while maintaining a high average power [Bartels1999].

Dispersion compensation into the slightly anomalous regime is desired to support soliton formation, which balances the net anomalous dispersion with the nonlinear effect of self-phase modulation in the Kerr medium (usually the gain medium). This balance forms $\text{Sech}^2$-shaped pulses with quasi-particle stability (solitons). The two most common techniques for dispersion compensation are illustrated in Fig. 3.3. below, the prism-pair delay-line [Fork1984] and the use of optimized-chirped [Jung1997] or double-chirped Bragg mirrors [Kärtner1997] with an optimized design to provide the desired group delay without the strong group-delay ripple which a simple straightforward chirp would introduce.

Obviously, the dispersive prism-delay-line approach requires a minimum cavity length as typically some 10 cm separation per prism pair is needed, which makes it difficult to raise the repetition rate beyond 200 MHz. On the other hand, the use of dispersion-compensating mirrors enables particularly compact cavity designs for femtosecond Ti:sapphire and Cr:forsterite ring lasers, so far demonstrated with a repetition rate of 300 MHz to up to 3.5 GHz [Bartels2005, Bartels1999].

![Figure 3.3: Two possibilities to introduce negative group velocity dispersion (“red pulses have a longer optical path than blue pulses”) into a laser cavity: a) through a prism pair and b) through so-called chirped mirrors.](image_url)

Figure 3.3: Two possibilities to introduce negative group velocity dispersion (“red pulses have a longer optical path than blue pulses”) into a laser cavity: a) through a prism pair and b) through so-called chirped mirrors.
Alternative methods for dispersion compensation are the use of grating pairs, providing high
dispersion in a compact setup at the expense of higher losses, and the Gires-Tournois-
interferometer (GTI) [Gires1964], which is also compact. The GTI is essentially a Fabry-Perot-
interferometer operated in reflection with the disadvantage of its limited dispersion-
compensating bandwidth.

Kerr nonlinearity

The high intracavity intensity of ultrafast lasers usually produces Kerr-nonlinear effects in the
gain medium, i.e. the refractive index of the medium is modified by the intensity \( I \) according to

\[
n(I) = n_0 + n_2 I(x, y, z, t) \tag{3.20}
\]

where \( n_2 \) is the material-dependent nonlinear coefficient, \( x \) and \( y \) are the transverse and \( z \) the
axial coordinate of the beam. Although \( n_2 \) is usually very small, the high peak power of
femtosecond pulses is strong enough to affect the laser modes two ways: in the transverse
direction through the Kerr-lensing effect as well as in the entire pulse phase and spectrum
through the fast self-induced phase modulation. In fact, both phenomena are important in cw-
modelocked lasers in the sense that they are effective means for ultrafast cw-modelocking
through the transverse Kerr-lensing effect (Kerr-lens modelocking, KLM, see section 3.3.2
below) and as self-phase modulation (SPM) provides a pulse-shaping effect, which cooperates
with the net-anomalous dispersion to form particularly stable and short and broadband pulses
(soliton regime, see section 3.2). Through the soliton effect the pulses can acquire additional
bandwidth outside the laser gain spectrum and can at the same time be compressed to yield the
shortest chirp-free, i.e. Fourier-limited, pulses.

3.3.2 Passive modelocking

General remarks

The shortest pulses directly produced from a laser are achieved by modelocking as mentioned
above. In order to lock the phase of a large number of axial modes, an amplitude modulation
mechanism produces side-band modes at the frequency offset of the repetition rate of the laser.
The phase-coherent side band modes then acts as injection locking signals for the adjacent axial
modes. In the time domain, the phase-coherent frequency comb of continuous-wave axial
modes form an ultrashort pulse with femtosecond duration, so-called continuous-wave (cw)
modelocked pulses.

For cw-modelocking operation of the laser the cavity roundtrip net gain must be larger for
ultrashort pulses than for long-pulse (or quasi-cw) operation. A cavity that favors short over
long pulses employs a fast active gain or loss modulation or a passive saturable absorber
mechanism.

Active modelocking through acousto- or electro-optic modulation, precisely tuned to the cavity
roundtrip time, is bandwidth-limited through the electronics. However, when pushing the limits
of active modelocking and subsequent nonlinear broadening, up to 45-THz optical frequency combs with a mode spacing of 6 GHz have been obtained [Imai1998] (see also section 3.3.3).

Passive loss modulation is commonly achieved with an all-optical mechanism such as a saturable absorber mechanism in the laser cavity which enables the self-amplitude modulation of the laser pulse once per round-trip in the cavity and which does not require electronic circuitry. Passive modelocking is therefore as fast as the kind of nonlinearity of the absorber, i.e. it is orders of magnitude faster than active modulation, and the modulation timing is inherently perfectly tuned to the cavity round trip frequency.

Saturable aborbers for femtosecond bulk solid state lasers can be realized in various ways, most commonly through the Kerr-lens-effect (Kerr lens modelocking, KLM), through semiconductor absorber materials (such as the semiconductor saturable absorber mirror, SESAM) and most recently with new materials such as carbon-nanotubes (CNT) [Rozhin2006]. Ultrafast far-off-resonant nonlinearities such as the Kerr-effect in optical crystals or glass fibers shape the shortest pulses to date. Somewhat slower near-resonant nonlinearities such as saturable absorption in semiconductor materials have also proven successful, as pulses much shorter than the picosecond saturation window can be obtained from this passive modelocking mechanism by combination with a fast Kerr-nonlinear soliton pulse shaping effect.

**Kerr-lens modelocking**

The Kerr effect in typical gain media acts on a time scale of a few femtoseconds. The transverse intensity profile of the TEM\(_{00}\) laser mode produces an index gradient which in turn acts as a self-induced Kerr-lens on the laser mode to change its intracavity transverse dimensions. The ultrafast Kerr lens in combination with an aperture that favours the transmission of the Kerr-focused beam becomes an effective ultrafast saturable absorber mechanism for generating femtosecond pulses at high output power.

The aperture may be realized in two ways. In “hard-aperture KLM”, a “hard” aperture such as a pinhole or a small mirror that leads to significant losses for long pulses or cw operation while offering much reduced losses for the Kerr-lensed laser mode of short pulses. In “soft-aperture KLM”, the short-pulse Kerr-lensed mode experiences a higher gain than the laser mode without Kerr lens because of its better spatial mode overlap with the coaxial pump mode in the gain medium. The pump mode therefore provides a “soft” gain aperture such that the laser favours short over long pulses.

**Semiconductor saturable absorbers**

A few nanometers thickness of semiconductor material, essentially a quantum well, is sufficient to absorb a few percent of laser light. A few percent absorption of the laser light suffices to deplete the initial state and to saturate the absorption loss in the material. As a resonant effect, involving actual excitation of carriers, it yields a slower response than the far-off-resonant Kerr effect. In fact, there are two time scales of carrier relaxation, the intra-band thermalization within 60-300fs and the recombination and trapping of carriers on a few-picosecond-to-nanosecond timescale. Both processes support modelocking, the slower process is useful for self-starting and the faster time scale supports sub-100-fs pulses in the absence of the KLM effect. Here the Kerr-nonlinearity balances off the residual anomalous group-velocity
dispersion and leads to soliton compression of the laser pulses to pulse widths below the absorber saturation window [Brovelli1995, Kärtner1996, Collings1996].

Such a saturable absorber, integrated into a Bragg mirror, is known as semiconductor saturable absorber mirror (SESAM) [Keller1992], and in a special configuration with the saturation layer on top of the mirror, is also known as saturable Bragg reflector (SBR) [Tsuda1995]. Depending on the laser wavelength, a suitable SESAM for the 0.8-µm region is composed of GaAs with Bragg mirrors made of AlAs and GaAlAs. For the 1.3- and 1.5-µm region, SESAMs have been developed from InGaAsP on InP substrates.

Nanoparticles of InAs in silica have also been demonstrated as easy-to-fabricate effective saturable absorbers [Bilinsky1999a, Bilinsky1999b], and 25-fs pulses from a Ti:sapphire laser have been obtained with this modelocking mechanism.

The discovery of saturable absorption of single-wall carbon nanotubes (CNTs) in the near-IR region with saturation times of approximately 1 ps led to a new type of solid saturable absorber material. Recently, 178-fs pulse generation in an CNT-modelocked erbium-doped fiber laser has been demonstrated [Rozhin2006].

3.3.3 Performance of bulk solid state frequency comb generators

Octave-spanning optical frequency comb from a KLM laser

There are basically two approaches to achieve octave-spanning optical frequency combs. One method exploits the speed of passive KLM to obtain particularly short and spectrally broad pulses directly from the laser [Morgner2001, Ell2001, Bartels2002, Fortier2003, Matos2004, Mücke2005]. The other method is to propagate somewhat longer femtosecond pulses from the laser through an external highly nonlinear medium, which then broadens the comb to the desired spectral width. This method is less demanding in cavity design and KLM performance, and is widely used for broadband comb generation (see section 3.6).

The first method, optimizing the KLM laser for particularly short pulses, generally leads to a cavity design with a critical self-starting behaviour of the laser. A stable and self-starting optical frequency comb generator based on KLM requires advance knowledge or educated assumptions of the Kerr-lensing parameters of the gain material and the resulting spatio-temporal intra-cavity field. The optimum performance design for the shortest pulses may result in a cavity that does not yield sufficient modulation depth for easily initiating the pulsed operation. A stronger spiking “kick” for the initiation of the KLM mechanism is then needed and the pulsed operation may be less robust.

Self-starting KLM for ultrashort pulses and octave-spanning frequency combs can also be achieved with the introduction of a slow saturable absorber such as a SESAM, which reacts to the pulse energy rather than the peak power. The SESAM bridges the gap from cw to (long) pulsed operation, helps the KLM mechanism to come into effect and to further shorten the pulses down to the sub-6-fs regime [Jung1997a].
The second method requires a sufficiently nonlinear medium and/or strong power confinement. Most commonly a photonic crystal fiber (PCF) or cobweb fiber is used. In these fibers the particularly strong power confinement from a large effective index contrast leads to rapid spectral broadening in a few cm propagation distance, even though glass is one of the weakest nonlinear media as far as the magnitude of the Kerr coefficient $n_2$ is concerned. The nonlinear spectral broadening of the comb is discussed in section 3.6.

**Ti:sapphire laser combs**

State-of-the-art commercially available Ti:sapphire lasers deliver several 100 mW of 10 – 50-fs pulses from a few Watts pump power. External nonlinear broadening typically broadens the combs to a spectral width of 500 – 1100 nm, a line spacing of 50 – 500 MHz and a typical comb line width of 100 kHz [www.menlosystems.com].

A detailed comparison between four different Ti:sapphire frequency combs from three laboratories showed that the generation and control of optical frequencies at least over neighboring modes is accurate with no systematic error down to a level of $1.4 \times 10^{-19}$ [Ma2004].

In an effort to achieve a particularly high repetition rate with Ti:sapphire lasers, optical combs with 0.5 to 3.5 GHz have been demonstrated and such Ti:sapphire lasers are also commercially available [www.gigaoptics.com].

Octave-spanning spectra produced directly from a Ti:sapphire laser is also state-of-the-art technology that matured from laboratory experiments and development to production, as is demonstrated with commercially available sub-10-fs systems [www.femtolasers.com, www.nanolayers.com]

Extremely broadband phase-coherent optical frequency synthesis from 0.57 to 1.45 µm has been demonstrated by linking the two combs of femtosecond lasers with different gain media. The laser repetition rates and the carrier-offset frequencies of a Ti:sapphire and a Cr:forsterite femtosecond laser have been phase locked to each other to achieve such a bandwidth in a laboratory experiment [Bartels2004].

**Alternatives to Ti:sapphire combs**

Colquirites such as Cr$^{3+}$:LiSAF, Cr$^{3+}$:LiSGaF, and Cr$^{3+}$:LiCAF, are a class of laser crystals that offer low-cost high-performance alternatives to Ti:sapphire because they can be directly pumped by laser diodes at 670 nm. These crystals have only a slightly smaller gain bandwidth and a 2-times-lower saturation intensity than Ti:sapphire. However, the mechanical, thermal and nonlinear properties of the colquirites are far inferior to Ti:sapphire.

White-light frequency-comb generation with a diode-pumped Cr:LiSAF laser operating with 115-mW 57-fs KLM power at 93 MHz with a spectral width of 24 nm (FWHM) centered at a wavelength of 894 nm was reported [Holzwarth2001]. The pulses were launched into a photonic crystal fiber and a broad spectrum with peaks near 530 and 1060 nm for self-referencing was generated. The system is particularly compact and efficient in that it has been pumped by battery-powered laser diodes and is perfectly suited for optical frequency metrology applications.
A Cr\(^{(4+)}\):forsterite laser, pumped by a Yb-fiber laser was demonstrated to generate 30-fs pulses at a 420-MHz repetition rate with nearly 500mW of average power. A highly nonlinear optical fiber led to octave-spanning spectra ranging from 1.06 to 2.17 µm [Thomann2003] and has been used for absolute frequency measurements [Corwin2004].

Other state-of-the-art frequency comb sources are based on ultrafast fiber lasers and semiconductor lasers, and will be discussed in section 3.4 and 3.5 below.

### 3.4 Fiber lasers for frequency combs

#### 3.4.1 Basic considerations

Compared to Ti:sapphire lasers, fiber based laser systems provide several advantages, as they can be efficiently pumped by laser diodes and can be realized in very compact set-ups. Additionally, the light guiding in the fiber results in a well-controlled beam shape which is insensitive to thermal effects or external disturbances. The guiding of the pump and laser light in the fiber results in excellent mode-matching between these two signals and the active dopants in the fiber. The interaction length is not limited by the beam focusing but only by the active fiber lengths. This enables the use of active dopants with low absorption and emission cross sections.

Compact femtosecond fiber laser systems are nowadays commercially available Imra Inc. [www.imra.com], Menlo Systems GmbH [www.menlosystems.com], Toptica Photonics AG [www.toptica.com], Precision Photonics Inc. [www.precisionphotonics.com] and from other suppliers. Typical output parameters are up to 170 mW output power and pulse duration of about 60 fs at 100 MHz repetition rate directly from the oscillator (Menlo Systems’ high power fiber laser version). These lasers are microprocessor-controlled and by this way self-optimising. Even higher output powers or multiple high power output ports can be realized by combining a femtosecond fiber laser source with fiber amplifiers. Such fiber based laser sources are well suited for the realisation of compact frequency comb systems, which are actually far more compact than Ti:sapphire based systems. Furthermore, the power consumption is much lower. For example, approximately 6 W of electric power to the pump diodes (thermoelectric cooler and control electronics are not included) yield 200 mW of optical output power.

Laser and amplifier systems based on erbium (Er\(^{3+}\)) doped fibers have the significant advantage that the required fiber optic components like pump sources, WDM-couplers, isolators and high-speed amplitude modulators are widely used for telecommunication applications. Therefore, these components are reliable, relatively cheap and readily available. Moreover, due to the broad bandwidth and the high gain, these fibers are well suited for generation and amplification of femtosecond pulses. An additional benefit of this gain material is the availability of fibers with normal as well as with anomalous dispersion at the emission wavelength 1.55 µm. The latter is due to the fact that the material dispersion of silica fibers is anomalous at this wavelength while the waveguide dispersion is normal and can be adjusted by the design of the refractive index profile in the core. Hence, the dispersion in the laser cavity can be adjusted by
the incorporation of fibers with opposite dispersion, and there is no need for additional (non-fiber) dispersive elements, which is highly advantageous for a reliable and compact laser set-up.

In principle femtosecond fiber lasers based on Ytterbium (Yb$^{3+}$) doped fibers are also promising. This is due to the high optical pump efficiency (up to 80%), the broad gain bandwidth and the high possible doping concentration of the Yb$^{3+}$-ions in silica fibers. The latter basically enables the realization of shorter cavities and therefore higher fundamental repetition rates. Another possible advantage of Ytterbium doped fiber based systems would be that Ytterbium-doped double clad fibers could be used for pulse amplification. Such fibers can be pumped by high-power laser diodes with low beam quality, which enables output powers of up to several Watts. The main drawback of this gain medium is that all conventional silica fibers have normal dispersion at the emission wavelength of Yb$^{3+}$-fibers (~ 1.05 µm). Dispersion compensation thus requires the incorporation of additional free space elements or of elaborate photonic crystal fibers into the cavity.

In cavity design, ring cavities are commonly used to realize unidirectional operation of a fiber laser. The basic elements are the gain fiber, a mirror-less fiber ring which incorporates polarization controllers and an optical isolator, as well as a wavelength-division multiplexing (WDM) coupler for launching the pump light into the ring and for coupling out a fraction of the circulating laser power. Alternatively, linear fiber cavities with Bragg mirrors, fiber loop mirrors and/or free-space coupled mirrors are possible.

A number of changes and adaptations to this scheme were demonstrated to achieve femtosecond passive modelocking and high output power. To this end, dispersion control for stretched-pulse amplification reduces the pulse peak power thus allowing the pulses to gain significantly more energy before nonlinear effects limit the output power.

### 3.4.2 Passive modelocking

As already discussed before, passive mode locking is also the preferred mode-locking scheme for femtosecond fiber lasers, as the switching speed of active elements is far below the desired pulse width due the limited frequency bandwidth of the control electronics. Furthermore, an active mode-locking element is a potential source for additional phase noise. Passive schemes require no additional control electronics which reduces the complexity of the set-up. They are controlled by the light intensity in the cavity and show an increasing mode-locking power with decreasing pulse duration in the cavity. This is due to the fact that at constant pulse energy the peak intensity increases with decreasing pulse duration.

The ultrafast far off-resonant Kerr nonlinearity of the fiber is most commonly exploited for effective saturable absorption mechanisms basically in two different ways: by nonlinear polarization rotation and polarization-dependent loss (PDL) or by a nonlinear optical fiber loop mirror (NOLM), both allowing an all-fiber closed-loop mirror-less cavity design. Alternatively, saturable semiconductor absorbers can be used (as discussed in the section on bulk laser modelocking above) which then require a free-space path in the laser cavity.
Passive modelocking with NOLMs uses the power-dependent transmission of a gain-unbalanced or splitting-ratio-unbalanced fiber loop Sagnac interferometer. These lasers are called figure-8 lasers because of their cavity layout. The additional fiber loop (NOLM) required for self-amplitude modulation makes this cavity configuration longer than a fiber laser without NOLM. Higher repetition rates are therefore more readily achieved with nonlinear polarization rotation modelocked lasers.

Passive modelocking with nonlinear polarization rotation and PDL is in its basic principle identical to NOLM modelocking except that orthogonally polarized components of the same pulse are used instead of the counter-propagating pulses of the NOLM. The ongoing development of nonlinear polarization rotation femtosecond fiber lasers has already resulted in considerable improvement of the modelocking performance, and dispersion management for intra-cavity stretched-pulse amplification led to higher pulse energies.

### 3.4.3 Performance of fiber laser frequency comb generators

Erbium-fiber based frequency combs in the near-IR region from 1000 to 2200 nm operate at repetition rates of 50 – 250 MHz and use nonlinear spectral broadening of the amplified output to achieve an octave-spanning spectrum. Higher repetition rates are harder to achieve because of the overall fiber cavity length which has to accommodate the active and passive functional fiber-optic elements. Commercially available fiber laser combs also deliver comb lines in the second-harmonic domain from 500 – 1100 nm, accessible through frequency conversion and spectral broadening.

Phase-locked frequency combs emitting from 1100 to 2200 nm have been demonstrated allowing for frequency metrology experiments in the near infrared in a compact, fiber-laser-based system [Tauser2003, Washburn2004, Kubina2005]. A phase-locked two-color erbium-doped fiber laser system demonstrated that the outputs of two independently configurable amplifier branches provide a wide spectral range, where one branch is used for carrier-envelope-offset-frequency stabilization whereas the other is used for frequency metrology [Adler2004].

Pump-induced spectral shifts remain significant for fiber-laser frequency combs [Rauschenberger2002, Haverkamp2004] and have been subject to detailed investigations [Washburn2005]. Compared to Ti:sapphire laser-based combs, the fiber laser comb has a smaller bandwidth of the response of the carrier-envelope-offset frequency $f_0$ to a change in laser pump power.

Fiber lasers still exhibit significantly more high-frequency phase noise than Ti:sapphire lasers [Hong2003]. The excess noise source is still under investigation. Nevertheless, modelocked erbium fiber lasers are excellent comb generators for optical frequency metrology. In a detailed comparison between different erbium fiber frequency combs a frequency measurement accuracy of $6 \times 10^{-16}$ has been found [Kubina2005].
3.5 Other optical comb generators

In “pre-femtosecond” times an even more compact setup to measure frequency gaps on the order of a few 10 THz in the 1.55-µm wavelength region was pioneered by M. Kourogi and co-workers in the group of M. Ohtsu (see also section 2.2, the brief historical overview).

The coupled-cavity optical frequency comb generator consists of an extended-cavity 1.55-µm laser diode and a monolithic lithium niobate cavity coupled to the laser and locked to the laser frequency [Kourogi1994, Kourogi1995, Kourogi1996]. The modulator crystal is placed inside a resonant microwave cavity the resonance frequency of which was adjusted to a few GHz, to resonantly enhance the generated sidebands. The primary optical frequency comb span that can be achieved is of the order of 10 THz.

Enhancement of the comb width to 30 THz was achieved by amplifying the 6-GHz 1-ps pulse train to 2 W average power and launching it into a 1-km dispersion-flattened fiber [Imai1998].

To further extend the span of the coupled-cavity optical frequency comb, second-harmonic generation at 1.55 µm was demonstrated [Widiyatmoko1999]. The fundamental comb was expanded to 45 THz in the dispersion-flattened fiber as demonstrated earlier. The second-harmonic comb’s span was 3.2 THz, tunable over a 30 THz range by changing the quasi-phasematching period in the periodically-poled lithium niobate (PPLN) used for second-harmonic generation. Phasematching, or equivalently the group-velocity walk-off, limits the bandwidth of the second-harmonic comb.

Recently, the coupled-cavity based optical frequency comb generator (C-C-OFCG) became commercially available for turn-key frequency gap measurement in the telecommunications C-band (1530 – 1565 nm) [www.optocomb.com]. The apparatus compares a cw laser frequency with the frequency of the internal light source within a bandwidth of a few THz and achieves a fractional accuracy of $10^{-8}$ in its standard version.

3.6 Nonlinear spectral broadening of the comb

The output spectrum of a femtosecond laser can be broadened significantly via self-phase modulation, thereby increasing its useful width far beyond the time-bandwidth limit of the laser. Techniques for nonlinear spectral broadening either employ optical fibers, such as photonic crystal fibers (PCFs), or nonlinear crystals, such as the Continuum Generation Chips (CGCs) from Mesophotonics [www.mesophotonics.com].

At the high peak intensities of femtosecond laser pulses and the strong power confinement of single-mode waveguides, nonlinear effects due to the $\chi^{(3)}$-nonlinear susceptibility are strong even in standard silica fibers with a weak Kerr coefficient. Power confinement is also used in CGCs, which are waveguide structures grown on the top surface of a silicon wafer.

For self-phase modulation to be dominant, the nonlinear length $L_{NL}$ defined in section 3.2 must be much shorter than the group-velocity dispersion length $L_D$. We are considering of course
the case of a physical waveguide longer than either $L_D$ or $L_{NL}$. In this situation almost
dispersion-free spectral broadening of the comb is achieved.

Various fibers have been used, depending on the wavelength and peak power of the laser
pulses. Long pieces of dispersion-flattened fibers work well in the 1.55-µm domain
[Imai1998], whereas short pieces of highly nonlinear photonic crystal fiber (PCF) or cobweb
fiber (Fig. 3.4) are used in the 0.8-µm domain for octave-spanning frequency combs.

![Image](image_url)

**Figure 3.4:** Top: An electron micrograph of a photonic crystal fiber (PCF) and close up of the
core area (top, right) (Courtesy of J. Knight). Bottom: Core area of a 1-micron cobweb fiber.

Provided that the coupling efficiency into the fiber is stable, the periodicity of the pulse train is
maintained. The discussion of section 3.1 is thus equally valid if the electric field $E(t)$ is
measured for example at the fiber output facet instead of the laser output coupler.

As an example Fig. 3.5. illustrates spectral broadening of 73-fs pulses from a Mira 900 system
(Coherent Inc.) in a standard single mode fiber (Newport FS-F). The low-power curve in the
left part of the figure resembles the input pulse with very little nonlinear interaction with the
fiber. The high power curve illustrates the broadening when 280 mW average power is
propagated through the fiber. The right part of Fig. 3.5. illustrates how the spectral width
defined by the “$-53\text{dB}$” point that has been shown to supply enough power per mode to
enable phase locking of cw sources, spreads out with increasing power.
Figure 3.5: Spectral broadening of 73 fs pulses in a standard single mode fiber. Left: no broadening with 12 mW average power coupled through the fiber, with 280 mW broadening to more than 50 THz. Right: the broadening is almost symmetrical to the center of the initial pulse.

This broadened frequency comb has been thoroughly tested for uniformity as it has been questioned whether our simple picture of this precisely equally spaced comb will fail due to dispersion or other effects. It has been found that the frequency comb is equally spaced even after further spectral broadening in a standard single mode fiber.

An example of an octave-spanning comb from a photonic crystal fiber, pumped by a high-repetition-rate Ti:sapphire laser, is displayed in Figure 3.6.

Figure 3.6: Spectral broadening of femtosecond pulses in a photonic crystal fiber. The narrow peaked curve in the middle (bold) denotes the initial pulse directly from the femtosecond laser (25 fs, 170 mW average power, 625 MHz repetition rate). The broadened spectrum stretches from 520 nm to 1100 nm (−10 dB width).
In a comparison between four different frequency comb synthesizers, two of which were externally broadened, demonstrated an accuracy in frequency measurement with no systematic error down to a level of $1.4 \times 10^{-19}$ [Ma2004]. This is one essential thing to a frequency comb used as a ruler in frequency space, while stabilization of its lines is another and will be discussed below.

### 3.7 Offset-frequency detection and comb stabilization

Now one question remains to be answered: how do we detect and control experimentally the frequencies $f_r$ and especially $f_0$? The repetition rate $f_r$ is readily picked up with a fast photo detector. The question is how to access $f_0$.

#### 3.7.1 Offset-frequency detection

With an octave spanning frequency comb as shown in Fig 3.6. one can directly access the interval between an optical frequency $f$ and its second harmonic $2f$. This significantly simplifies the detection procedure and has first been reported by the group of J.Hall at the JILA in Boulder/USA and at MPQ in Garching/Germany [Riehle1996, Bagayev1986, vonZanthier2000, Bernard1999, Schnatz1996] (see also section 2.2, Historical overview).

![Figure 3.7: The principle of the self-referenced optical synthesizer](image)

The experimental realization of this approach is known as an $f_2f$ interferometer, and can be set up basically in either a two-arm or one arm configuration. In the two arm interferometer, the octave spanning spectrum is divided up in a red arm which is frequency doubled and subsequently spatially overlapped with the green part of the spectrum out of the green arm. A
delay line in the green arm, e.g. glass prisms, is necessary to adjust the temporal overlap of the short pulses out of the two arms. As explained above, the offset frequency is given by the beat note between the interfering modes. The equation holds for all the modes which are frequency doubled and have a respective counterpart in the green part of the spectrum. A grating or filter is used to block all other parts of the spectrum from falling on the detector, thus increasing the S/N ratio. The setup shown in Fig. 3.8. is used to detect the offset frequency from a Ti:sapphire based optical frequency comb generator.

The one arm interferometer shown in Fig. 3.9 is a somewhat simplified version where the green and red arms are collinear in space and are not spatially separated. A PPKTP crystal is used for SHG of the red spectral part, the green part is just transmitted. By suitable dispersion management of the incident octave spanning spectrum, temporal overlap of both parts after the SGH can be achieved. Again, a filter is used to block the remaining parts of the spectrum which do not contribute to the offset frequency detection.

Both types of $f$:$2f$ interferometer can be used for Ti:sapphire based as well as for fiber laser based optical frequency combs. The one arm configuration has first been demonstrated by [Schibli2004, Jiang2005].

Recently, another approach for offset frequency detection using quantum interference control (QIC) of photocurrents in a semiconductor device have been demonstrated in Ref. [Roos2005]. Other possibilities with very short laser pulses (shorter than 8fs) include a combination of SHG and self phase modulation in one crystal. ZnO has been used for this task [Mücke2002] and also periodically poled lithium niobate (PPLN) [Rauschenberger2005]. Instead of using SHG to compare $f$ and $2f$ the difference frequency $(2f-f)$ can be generated and compared with the frequency $f$ [Zimmermann2005, Rauschenberger2005].

If the spectrum does not cover a full octave but only a fraction e.g. 2/3 (or a/b) of an octave, more complicated schemes involving several nonlinear steps to generate the second harmonic ($a^{th}$ harmonic) of the green and the third harmonic ($b^{th}$ harmonic) of the red spectral parts have been demonstrated [Morgner2001, Ramond2002].

With these set-ups the two radio frequencies $f_r$ and $f_0$ are determined. The pulse repetition rate $f_r$ is simply measured with a photo detector anywhere in the beam and $f_0$, which is smaller in value than $f_r$, is derived as explained in Fig. 3.7. To obtain a stable comb for absolute optical frequency measurements, it is advantageous to phase lock both $f_r$ and $f_0$ to a precise radio frequency reference such as a cesium atomic clock or GPS controlled quartz oscillator. The mode number $n$ may be determined by a coarse measurement of the mode in question, for example with a wavemeter. As soon as $n$, $f_r$ and $f_0$ are known, every frequency contained in the comb is known with basically the same precision: a full optical octave worth of reference frequencies at once.
3.7.2 Comb stabilization

We have seen from the previous discussion that a femtosecond frequency comb has two free parameters, i.e. the repetition frequency $f_r$ and the offset frequency $f_0$. For most applications it is desirable to fix one of the modes in frequency space and phase-lock the pulse repetition rate simultaneously. Furthermore, the laser spectrum is subject to acoustic and other technical noise that needs to be suppressed. For this purpose it is necessary to control the phase velocity
(more precisely the round trip phase delay) of that particular mode and the group velocity of the pulses (more precisely the round trip group delay) independently.

There are basically two cavity configurations, with prism-delay-line or with chirped-mirror dispersion compensation, for which different techniques of tuning the offset frequency $f_0$ are used. In all cases, the repetition frequency $f_r$ is controlled by changing the cavity length.

**Prism-delay-line compensated cavities**

A piezo-driven folding mirror as depicted in Fig. 3.10 changes the cavity length but leaves $\Delta \phi$ approximately constant as the additional path in air does have a negligible dispersion. Also the offset frequency $f_0 = f_r \Delta \phi / 2 \pi$ is almost untouched by changing the cavity length as $f_r$ is usually changed by a few 100 Hz in order to reach every desired position in frequency space with one of the modes. Figure 3.11 illustrates this in a exaggerated way with a simple “rubber band” model.

![Figure 3.10: Setup of a prism-compensated femtosecond laser (following a Mira 900 system (Coherent Inc.)). The possibilities to stabilize the frequency comb include changing the cavity length and tilting the end mirror.](image)

![Figure 3.11: A decreasing cavity length pulls the modes apart like mounted on a rubber band. Note that the pulling is not uniform.](image)
A mode-locked laser that uses two intracavity prisms to produce the negative group velocity dispersion \((\partial^2 \alpha / \partial k^2 < 0)\) necessary for Kerr-lens mode-locking provides us with a means for independently controlling the pulse repetition rate. We use a second piezo-transducer to slightly tilt the mirror at the dispersive end of the cavity about a vertical pivot that ideally corresponds to the mode \(f_n\) (see Fig. 3.10). We thus introduce an additional phase shift \(\Delta \phi\) proportional to the frequency distance from \(f_n\), which displaces the pulse in time and thus changes the round trip group delay. In the frequency domain one could argue that the length of the cavity stays constant for the mode \(f_n\) while higher (lower) frequency modes experience a longer (shorter) cavity (or vice versa, depending on the sign of \(\Delta \phi\)). This leads to changes in \(f_n\) and \(f_r\) but leaves the mode on the pivot axis constant as shown in Fig. 3.12. The first fs laser system that was used for such experiments was a Mira 900 system (Coherent Inc.). This delivers 73 fs pulses at a repetition rate of 76 MHz. It uses prisms for GVD compensation and can be tuned with a Lyot filter (and different mirror sets) between 750 and 1000 nm. It is pumped by a frequency doubled diode pumped Nd:YVO\(_4\) laser (model Verdi, Coherent Inc.).

\[ \text{Figure 3.12: Slightly tilting the end mirror at the dispersive end of the laser cavity changes the mode spacing.} \]

\[ \text{Figure 3.13: Changing the pump power efficiently changes the offset frequency } f_0 \]

**Prism-less chirped-mirror compensated cavities**

In the case where only dispersion compensation mirrors are used to produce the negative group velocity dispersion one can modulate the pump power or manipulate the Kerr lens by slightly tilting the pump beam. Primarily this changes the pulse energy of the fs laser and via the Kerr
lens nonlinearity mainly the phase delay is affected. This can be derived with the master equation for nonlinear pulse evolution [Haus1993].

That is just what is needed to control the pulse repetition rate $f_r$ and the offset frequency $f_0$ separately: a separate control of the round trip group delay $T$ and the round trip phase delay [Reichert1999]. Another method that we have used to stabilize the offset frequency $f_0$ was to translate the laser crystal along the axis of the cavity mode. This changes the integrated power inside the crystal with the same effect but it is reversed at the point where the focus is centered inside the crystal.

Yet another possibility to adjust the offset frequency $f_0$ is to insert additional glass into the laser cavity e.g. by moving prisms already present for GVD compensation or by inserting a wedge into the laser cavity. The phase difference between carrier and envelope $\Delta \phi$ experiences a shift during propagation through a dispersive transparent medium $\delta \phi = \left(k(\omega_b) - \omega_b v_g^{-1}\right)l$ where $l$ is the propagation length, $v_g$ the group velocity and $k$ the propagation constant $k(\omega_b) = \omega_b n(\omega_b)/c$. With $v_g^{-1} = \frac{\omega}{\omega_b} = (n + \omega \frac{dn}{d\omega})/c$ we arrive at $\delta \phi = -\left(\frac{\omega}{\omega_b}\right) \frac{dn}{d\omega}$ and finally $\delta \phi = 2\pi (\frac{dn}{d\omega}) \delta \lambda$ where $\frac{dn}{d\omega} = -0.018 \mu m^{-1}$ for fused silica at $\lambda = 790$ nm [Spielmann1996]. According to $f_0 = f_r \Delta \phi / 2\pi$ we expect $\delta \phi_0 (\delta \lambda) = -0.018 f_r \delta \lambda$. This has actually been verified in our Vienna experiments [Apolonski2000]. Note that putting more glass into the cavity moves the comb in the same direction as increasing the power.

The second type of femtosecond laser system that we used in several experiments is based on a Ti:sapphire 25 fs ring laser with a high repetition rate (GigaOptics, model GigaJet). We have modified the original setup by mounting one of the mirrors on a translation stage for coarse control of the repetition rate and another mirror on a piezo transducer for fine tuning and phase locking of the repetition rate. Furthermore we have inserted an electro optic modulator into the pump beam for fine adjustment and phase locking of the offset frequency. For this purpose we used an electro optic amplitude modulator (EOM) from Gs’anger (model LM 0202). The pump power can only be changed be about 10 % without terminating mode–locked operation. This changes the slipping frequency beat signal $f_0$ by about 60 MHz. This range is enough for phase locking but it is not enough to place $f_0$ at any desired frequency within $f_r/2$ of a few hundred MHz. Therefore we included in our setup a fused silica wedge at Brewster’s angle. This also gives access to the offset frequency $f_0$ and can be used to preset $f_0$ to the desired position (e.g. 64 MHz as in most our experiments) and phase lock it via pump modulation. By double folding the cavity more bounces are obtained on the chirped mirrors so that we can actually add the fused silica wedge in the first place. The full system is depicted in Fig. 3.14.
Figure 3.14: The high repetition rate laser. This has been operated with repetition frequencies of 625, 750 and 950 MHz, piezo transducer (PZT) and translation stage (TS) are used for coarse adjustment and locking of the repetition rate fr, fused silica wedge (W) and an electro optic intensity modulator (EOM) are used for coarse adjustment and locking of the offset frequency f0. All mirrors except the output coupler (OC) are chirped.

Note that a distinct advantage of the pump beam modulation technique is a much higher servo bandwidth than is attainable with piezo transducers. It should in fact only be limited by the life time of the upper level in Ti:sapphire of 2.2 μs. Furthermore, stabilizing $f_0$ with the help of pump power modulation even reduces amplitude noise from the femtosecond laser.
4 Enabling technology for high-repetition-rate combs

4.1 Basic considerations

Basically, three aspects have to be considered in obtaining a design for a 15-GHz optical frequency comb system in the desired wavelength range: 1) the choice of a femtosecond source, 2) the nonlinear conversion to the desired wavelength range, and 3) a repetition multiplier such as a mode filter cavity, due to the lack of 15-GHz femtosecond pulse sources.

In principle there are several different ways to design a comb system with 15 GHz mode spacing:

First, using a 15-GHz comb generator (fs laser) with subsequent high-power amplification to provide sufficient power for nonlinear broadening in an external highly nonlinear fiber is a potential straightforward, yet challenging approach (see Fig. 4.1). The advantage of this approach is the possibly compact and simple overall system layout. However, the challenges and disadvantages of this approach are the lack of suitable and technologically mature off-the-shelf components. In fact, promising passively modelocked sources at > 10 GHz have been demonstrated, e.g. with microchip lasers and semiconductor lasers, with fundamental and harmonic mode locking, as will be discussed below. A practical low-noise source for broadband comb generation and stabilization at > 10 GHz has yet to be identified and assessed. Part of the problem is the difficulty of achieving passive modelocking with femtosecond linear and nonlinear pulse shaping at these high repetition rates. Another challenge is the required output power to achieve nonlinear frequency conversion and broadening. Fortunately, there are high-power amplifiers available in the telecommunication wavelength bands around 1550 nm, making a frequency-tripled infrared HRR source a possible candidate for the desired comb system.

Figure 4.1: A reference system layout using a high-repetition rate femtosecond laser and strong power amplifier. The laser and amplifier emission would most probably be centered near 1550 nm, where compact 10 – 40 GHz repetition rate sources and high-power amplifiers are developed for communication applications. The main challenges of this layout are the availability of femtosecond HRR sources and the strong amplification required for third-harmonic generation and spectral broadening.

Second, using a medium repetition rate fs laser (200 MHz – 1 GHz) with an external repetition rate multiplier such as a mode-filtering cavity is another potential approach that avoids the extreme power amplification for nonlinear conversion (see Fig. 4.2). Another advantage of this approach is the availability of mature femtosecond comb generators in this repetition rate range. The challenge with this approach is the broadband mode filtering and side mode suppression requirement. Multiplying the repetition rate by a factor of 15 to 75 (starting with a
1-GHz or 200-MHz source, respectively) external to the laser requires e.g. a perfectly dispersion-compensated cavity over the desired bandwidth. The dispersion matching and stabilization requirements become more stringent as the linewidth of the mode filter cavity decreases (higher finesse) due to requirements of sufficient side mode suppression (see discussion in mode filter section). Also, the repetition rate (or equivalently, the mode spacing enhancement factor) directly translates into a reciprocal loss, i.e. a power penalty of 13 to 19 dB. Another problem to be considered in detail is the nonlinear mixing of the (suppressed) side modes and of noise with the strong main modes in a post-filtering nonlinear \( \chi^{(2)} \) or \( \chi^{(3)} \) conversion process.

![Diagram](image.png)

**Figure 4.2:** A reference system layout using a medium-repetition rate femtosecond laser and an external mode filter. Post- or pre-filtering nonlinear conversion can be considered (upper or lower design, respectively). The main challenge of this layout is the matching and stabilization of a high-finesse mode-filter cavity. (If the mode filter is coupled to or integrated in the active laser cavity to achieve harmonic mode locking, the layout in Fig. 4.1 applies.)

A third approach is to synthesize a design from the previous two approaches. The basic idea is to start out with a higher repetition rate than 1 GHz, which relaxes the requirements of matching and stabilizing the external mode filter cavity and reduces the power loss. For example, a multiplication factor of 60 is required to generate a 15-GHz comb from a 250 MHz erbium-doped fiber laser. Also, nonlinear frequency tripling from 1550 nm to 516 nm is required to achieve the desired comb spectral range. In this example it seems advantageous and feasible to multiply the repetition rate by a factor of 6 with a 1.5-GHz external filter cavity before power amplification and nonlinear frequency conversion is performed. With this repetition rate, nonlinear conversion processes are still efficient with average powers of approx. 1 W (instead of > 10 W for 15 GHz repetition rate). In a second step, the 1.5-GHz comb, now available in the desired wavelength range, is filtered by a factor of 10 (instead of 60) to obtain the desired mode spacing. This will be discussed in more detail below.

![Diagram](image.png)

**Figure 4.3:** A generic system layout using a high-repetition rate femtosecond laser and an external mode filter. Pre-filtering nonlinear conversion, the matching and the stabilization of the mode-filter cavity can be more readily achieved than in the layout of Fig. 4.2.
There are several technology risks involved, particularly with the designs shown in Figs. 4.1 and 4.2. In the following, we will examine the sub-units individually, the femtosecond sources, mode-filter techniques and nonlinear conversion issues, before we identify promising solutions for the 15-GHz comb generation.

4.2 High-repetition-rate (HRR) femtosecond lasers

Passive continuous-wave (cw) mode-locking at high repetition rates beyond 1 GHz presents several challenges, two of which are as follows:

First, passive mode locking schemes using a fast saturable loss mechanism are based on a nonlinear effect. These schemes are, e.g., Kerr-lens mode locking (KLM), nonlinear polarization rotation mode locking (in fiber lasers) or semiconductor saturable absorber mirror (SESAM) mode-locking with soliton pulse shortening as discussed in sections 3.3.2 and 3.4.2. In any case, nonlinear effects are greatly reduced when several pulses circulate in the laser cavity (harmonic modelocking) instead if a single pulse, or when the cavity is shortened to yield higher repetition rates at same average power. Compared to a traditional single-pulse standard-cavity laser, the pulse energy of a high-repetition rate laser is reduced approximately by the number of pulses circulating in the cavity (multi-pulse standard-cavity laser) or by the enhancement of repetition rate in the fundamentally-modelocked single-pulse short-cavity laser. Even worse, the pulse peak power is further reduced by the fact that shorter cavities may provide less nonlinear phase shift, which makes Kerr-lens (and comparable) mode-locking ever harder to achieve. For example, in the case of SESAM-soliton mode locking it has been observed, that multiplying the repetition rate of a 124-MHz Cr:forsterite laser by a factor of 5 (to 620 MHz) still supports 168-fs solitons, while multiplying by a factor of 10 (to 1.24 GHz) yields pulses of 2 ps width. At such a high repetition rate, the relatively low pulse peak power cannot generate enough self-phase modulation to support soliton pulse shortening, while the pulse energy is still sufficient to saturate the SESAM to initiate and maintain cw mode-locking. Indeed, this is a common problem that we observed in the review of high-repetition rate femtosecond lasers. In the GHz regime, modelocked lasers tend to produce pulses of several picoseconds duration. However, when pushing the limits, femtosecond laser operation has been achieved at repetition rates of up to 4 GHz (see below).

Second, the implementation of linear and possibly nonlinear pulse shaping optical elements becomes a challenge in particularly short cavities. For a fundamentally mode-locked standing-wave cavity, a 15-GHz free spectral range corresponds to only 1 cm separation of the end mirrors in air and less with a gain medium in between. The traditional solution of an intracavity prism sequence for sufficient negative group-delay dispersion (GDD) makes it difficult to reach repetition rates of more than 200 MHz. However, a particularly compact three-element cavity has been designed with a prismatic crystal and output coupler [Ramaswamy-Paye1994], and scanning the mode-lock frequency by more than 1 % of a 800-MHz femtosecond Ti:sapphire laser has been demonstrated with this design [Demers1999]. With this compact design, a 4-GHz repetition frequency of 81-fs pulses at a center wavelength of 1525 nm (Cr:YAG) has been achieved by Kerr-lens modelocking [Brown2004; Leburn2004]. Another promising cavity design uses double-chirped mirrors (see section 3.2.2) for compact group-delay compensation: Up to 3.5 GHz repetition rate of 20-fs pulses have been obtained from a Ti:sapphire laser with a four-mirror cavity. Geometrical constraints pose a limit to higher
repetition rates, with 10 GHz being feasible with short-gain crystals and lower output power [Bartels2005].

These two constraints show in the ultrafast laser performance diagram, compiled from 112 laser performance data. Due to geometrical constraints, the major fraction of lasers has a repetition rate of 200 MHz or less. Due to pulse shaping constraints in very small cavites, very high repetition rates are generally obtained in the picosecond regime. Surprisingly, new types of optically pumped high-repetition-rate semiconductor lasers are closest to the femtosecond regime, with one demonstration of sub-500-fs pulses from a 10-GHz system [Hoogland2005].

![Ultrafast solid state lasers](image)

**Figure 4.4**: State-of-the-art of ultrafast laser performance at various repetition speeds: the low (<50 MHz) range is shown in blue, the standard (50 – 200 MHz) range is green, the medium (200 MHz – 1 GHz) range yellow and the high-repetition (>1 GHz) range is shown in red. Clearly, very high repetition rates (>10 GHz) are achieved with picosecond pulses, whereas (sub-) 100-fs pulses are generated only to up to 4 GHz. The desired frequency comb parameters lie in the quadrant of >10-GHz femtosecond operation, for which no published demonstration has been found.

### 4.2.1 Fundamentally mode-locked HRR laser sources

**Compact Gigahertz Kerr-lens mode-locked Ti:sapphire lasers**

State-of-the-art GHz Ti:Sapphire lasers with compact cavities built from dispersion-compensating mirrors achieve repetition rates of up to 3.5 GHz [Bartels2005]. The lasers are Kerr-lens modelocked which produces pulse widths of 20 – 30 fs. The compact prism-less cavity design has been demonstrated in a 6-mirror and 4-mirror configuration [Bartels1999, Bartels2002, Bartels2005]. These lasers are commercially available, yielding approx. 700 mW of output power near a central wavelength of 810 nm.
Compactness has also been achieved with a three-element cavity consisting of a focusing misror, a prismatic crystal and a prismatic output coupler [Ramaswamy-Paye1994]. Repetition-rate tunability of more than 1 % of a 800-MHz femtosecond Ti:sapphire laser has been demonstrated with this design [Demers1999].

**Other crystal-based Kerr-lens modelocked femtosecond lasers**

It seems logical to extend the concepts of high repetition rate cavity designs to other gain media.

The above-mentioned prism-less 6-mirror configuration has been used in conjunction with a **Cr:forsterite** crystal [Thomann2003, Bartels2005]. The crystal was pumped with a 10-W Yb:glass fiber laser, and a mode-locked output power of 620 mW has been achieved, emitting 30-fs pulses at a 433-MHz rate with a center wavelength near 1.3 µm. When broadenend via SPM, an octave-spanning comb ranging from 1050 to 2100 nm is obtained. In a different experiment, the output of the Cr:forsterite laser was actively linked to a broadband Ti:sapphire laser to achieve a phase-coherent comb ranging from 570 to 1450 nm [Bartels2004].

The very compact three-element cavity design [Ramaswamy-Paye1994] with prismatic gain crystal and prismatic output coupler has been applied to a **Cr:YAG** laser [Mellish1998, Leburn2004]. One primary driver for this development has been the center wavelength around the communication window of 1550 nm. The emission band of Cr:YAG ranges from 1200 to 1600 nm and supports ultrashort pulses down to less than 20 fs. In a first demonstration, femtosecond pulses of 1 GHz repetition frequency have been produced [Mellish1998] by soft-aperture Kerr-lens mode-locking. This cavity design was refined to achieve 4.04-GHz repetition frequency of pulses as short as 80-fs with a tunability of the center wavelength of 1505 to 1550 nm [Leburn2004, Brown2004]. The few-component compact design, and the wavelength match for using erbium-doped fiber amplifiers [Chai2005] make this laser a possible candidate for a 15-GHz comb.

**SESAM mode-locked high-repetition-rate lasers**

Very high repetition rates are achieved with passive SESAM modelocking schemes (see section 3.3.2) because one cavity mirror provides the nonlinear loss mechanism so that only two to three mirrors are needed. Also, in contrary to Kerr-lens modelocking it is not required that a tight focus is located in the gain medium, so that a larger mode diameter can be designed for a shorter, i.e. microchip laser crystal, which helps to achieve higher output power. However, if SESAM mode-locking is not supported by soliton dynamics, or if the pulses are so long that only weak soliton dynamics is achieved, then femtosecond operation is not feasible. In fact, the ultra-high repetition-frequency lasers in Fig. 4.4 are manily realized with SESAM mode-locking and high output power was achieved but no femtosecond operation.

An efficient and compact femtosecond **Cr:LiSAF** laser with a reduced component count has been realized with SESAM mode-locking. Transform-limited 136-fs pulses centered on 859 nm at a 470-MHz repetition rate and 146-fs pulses at 1 GHz have been reported. Due to direct diode pumping, the laser achieved an overall optical-to-optical conversion efficiency of over 20% [Agate2002].

The highest repetition rates of SESAM passively mode-locked microchip lasers have been produced with **Nd:YVO4**, **Er:Yb:glass** and **surface-emitting semiconductors** (see below) as
gain media [Paschotta2004]. Repetition rates of nearly 160 GHz have been demonstrated with a miniature Nd:YVO4 laser emitting 2.7-ps pulses centered at 1064 nm [Krainer2002]. The shortest pulses in the tens-of-GHz regime were about 2 ps from Er:Yb:glass lasers [Yamada2001, Zeller2003]. Compact passively modelocked Er:Yb:glass lasers are commercially available in a small package, with 10 GHz repetition rate and 2-ps pulses centered at 1535 nm. However, for broadband comb generation, (sub-) 100-fs pulses are needed. Probably the most promising laser would be an Nd:glass laser, if it can be developed to operate at high repetition rates. Commercially available \textit{Nd:glass} lasers deliver a pulse width of 150 fs, however at low repetition rates of 70 – 150 MHz [www.tbwp.com]. To achieve both, 100-fs pulses and 5 - 15 GHz repetition frequency, further research and development is needed. To this end, optically pumped semiconductor lasers also seem to have great potential.

**Passively SESAM mode-locked VECSELS: InGaAs/GaAsP/GaAs semiconductor lasers**

A 10-GHz repetition rate with 6.1-ps pulses (time-bandwidth product 0.42) at an average 1.4-W output was achieved with an optically pumped vertical-external-cavity surface-emitting laser design [Aschwanden2005]. The cavity contained a SESAM mode-locking mirror and a focusing output coupler. The active medium consisted of seven InGaAs quantum wells placed in the antinodes of the standing-wave pattern of the laser field. The output spectrum was centered around 960 nm with a bandwidth of 0.21 nm. Stability at 1.4 W output power, at the performance limit of the laser, is still an issue but improved stability was observed at greatly reduced power levels.

While the repetition rate may be increased to up to 100 GHz in an improved design [Aschwanden2005; Lorensen2004], the long pulses and the small associated bandwidth make it improbable to achieve the desired comb bandwidth.

Surprisingly, sub-500-fs operation of a passively mode-locked diode-pumped external-cavity surface-emitting semiconductor laser has been reported to deliver pulses at a rate of 10 GHz [Hoogland2005]. The optically pumped VECSEL has been mode-locked by a SESAM. It has been found that the ac Stark effect plays an important role in the generation of sub-ps pulses.

**Passively mode-locked integrated InGaAsP diode lasers**

Another recent development is semiconductor lasers that are passively mode-locked running at 10-40 GHz repetition rates, a regime that is of particular interest to the telecommunication industry [Barbarin2005; Ji2006; Yousefi2006]. With recent progress in optical pulse output quality, their application in integrated photonic Microsystems, where short unchirped pulses and low timing jitter are important, becomes a viable perspective.

The cavity can be realized in a linear configuration with cleaved facet mirrors or as a ring using photolithography. The latter has the advantage that the repetition rate of the laser is controlled more accurately in the production process.

A 27-GHz 1-ps semiconductor ring laser has been demonstrated that incorporates active and passive waveguide types in one ring, which is the first integrated extended cavity passively mode-locked ring laser [Barbarin2005]. However, stable mode-locking was difficult to reach which was attributed to residual intracavity reflections.
Transform-limited output of 1.84-ps pulses at a 10-GHz rate was achieved in a linear integrated waveguide semiconductor laser with an extended cavity that incorporated active and passive waveguide elements [Ji2006]. The clean output pulses resulted from suppressing the residual back-reflections, by implementing a lateral 45° tilt at the active-passive interfaces. In addition, the colliding-pulse mode-locking (CPM) mechanism is more effective in pulse narrowing because it produces a deeper saturation of the saturable absorber section of the laser. For synchronized low timing jitter laser output, such as used for stabilization of the comb repetition frequency to an external radio-frequency source, electrical hybrid mode-locking (HML) of the CPM laser was demonstrated. A 0.283-ps rms jitter was observed with an RF source noise (jitter) floor of 0.199 ps [Ji2006].

Further research with integrated 10, 20 and 40-GHz indicates that output pulses with a bandwidth of 4 – 5 nm and a repetition rate tuning range of 60 MHz can be achieved in the near future. However, the technology is still in a basic research stage, where individual samples may be available that are not custom-tailored for certain optical output specifications [Yousefi2006].

In conclusion, the RF spectra indicate a line width that seems still too large for highly stable frequency comb generation. Also, the pulse duration seems too large and spectral bandwidth too small for broadband comb generation. A broadening factor of 10 has been observed to be a practical limit so far, indicating that up to 40-50 nm can be expected from the integrated passively mode-locked lasers after amplification of the 1 – 2-ps pulses.

Also, it is to be expected that such lasers exhibit significant excess noise compared with e.g. fiber comb lasers due to carrier-density and relaxation oscillations. However, this could possibly be remedied by filtering in a high finesse resonator that is locked to sub-harmonic of the repetition frequency.

### 4.2.2 Harmonically mode-locked HRR laser sources

A mode-filter that is coupled to the active laser cavity may act as a multiplier for the pulse repetition rate to achieve harmonic mode locking. However, there are noise and stability issues involved that make a harmonically mode-locked laser seem a less attractive concept for stable frequency comb generation. These issues will be studied in more detail. A demonstration of a phase-insensitive pulse seeding technique suggests that an internal mode-matched sub-cavity is not necessary to multiply the repetition rate of a laser.

#### Pulse seeding technique using a low-reflectivity intracavity flat surface

A flexible and phase-insensitive method for multiplying the repetition rate of a femtosecond passively mode-locked laser has been demonstrated with a Cr:forsterite laser. By inserting a low-reflectivity flat surface inside the active laser oscillator, a multiplication factor of 10 has been observed. In contrast to coupled-cavity set-ups, the behaviour of the demonstrated method operates without mode matching and feedback control. The method is phase-insensitive and more robust to changes in environmental conditions. It is therefore suggested, that the reflection off the flat surface provides pulse seeding rather than mode filtering as a coupler of the two subcavities [Liu2005].

It remains to be seen whether this concept can be applied to other lasers and cavity configurations as well.
In the following, we discuss mode filtering with passive external cavities.

### 4.3 Mode filter techniques

A high-finesse empty cavity acts as an effective mode filter. The specified 15 GHz free spectral range (FSR) translates into a standing-wave geometry with 1 cm separation between the mirrors. A high finesse is required to achieve the specified side mode suppression of 50 dB and to minimize the ring-down intensity fluctuations. For keeping the external cavity in resonance, a side-mode stabilization scheme may be used. There are novel techniques for achieving large FSRs with fiber-gap microcavities [Hunger2006]. A finesse of $>37,000$ has been demonstrated with this very compact set-up.

The required finesse of a Fabry-Perot mode filter cavity is evaluated as follows.

The finesse is the ratio of the FSR to the full width at half maximum of the resonance. Therefore, the finesse is uniquely determined by the specifications parameters $q$ and $\rho_{db}$ (see section 5.1.4 for definition) through the relation

$$ F = \frac{\pi}{2} \left[ \frac{\rho_{\text{db}}}{10^{\frac{10}{10}}} - 1 \right] \sin \left( \frac{(1-q)\pi}{2} \right) \left( 1 - \frac{\rho_{\text{db}}}{10^{\frac{20}{10}}} \right) $$

with a good approximation for suppression ratios $>10$ dB and large mode suppression range (MSR). The approximation of the finesse is accurate to better than 1.6% for a $q \geq 0.8$.

For the given specifications of a 15-GHz FSR, a side mode suppression of $\rho_{\text{db}} = 50$ dB in a range that eliminates 9 out of 10 modes of the frequency comb (MSR of at least 8 times the repetition rate and FSR of 10 times the repetition rate), and a $q = 14.5/15$ as calculated in section 5.1.4, we find that

$$ F = 9500. $$

Evaluating the output of the mode-filter behaviour in the time-domain, we may approximate (for such a high-finesse cavity) the ring-down power loss per round trip as

$$ \frac{\Delta P_{\text{rt}}}{P_{\text{circ}}} \approx \frac{2\pi}{F}. $$

Because only one out of 10 pulses leaving the cavity is driven by the 1.5-GHz pump pulse, the total power fluctuation (peak-to-peak) from periodic power decay is approximately

$$ \frac{\Delta P_{\text{p-p}}}{P} \approx 9 \frac{2\pi}{F} = 6 \cdot 10^{-3} $$

and is therefore negligible.

For stabilization on resonance, a side-mode detection stabilization scheme may be used (Pound-Drever-Hall scheme). There are several parameters that can be controlled to match the comb and the cavity spectrum:
The comb repetition rate (mode spacing) and the comb offset frequency. The independent control of the two comb parameters is only a part of the solution to the mode filtering technique. The cavity dispersion is another parameter that needs to be controlled for broadband match of comb and cavity.

Dispersion of the cavity originates from all optical elements. Therefore, we need to select elements with suitable dispersion properties, such as low-dispersion or negative-dispersion broadband mirrors to offset the positive dispersion of the intracavity optical and dispersion tuning elements and potentially of the intracavity gas.

Unfortunately, some requirements are in competition with each other. A high cavity finesse for strong side-mode suppression (or, expressing this requirement in the time domain, for low intensity ringdown loss and good intensity stability at low frequencies) must be balanced with the usable spectral bandwidth for a given mirror dispersion and reflectivity. Achieving a long optical storage time with high-reflectivity mirrors puts a more stringent requirement on dispersion compensation, as more chirp or broadening is accumulated during a longer storage time. In the frequency domain, the equivalent view is that the reduction of line width due to the high-reflectivity mirrors enhances the sensitivity to uncompensated dispersion, putting a more stringent requirement on achieving exact resonance over the desired spectral width. In any view of the problem, a higher finesse practically reduces the usable overall dispersion-compensated bandwidth or reduces the allowable residual dispersion of the desired spectral width. These issues will be assessed more quantitatively in conjunction with the comb-generating laser characteristics in the course of the study.

The technique of matching a comb with an external cavity not new. Such a cavity has been demonstrated as a synchronously pumped build-up resonator [Gohle2005]. A recent paper on cavity-ring-down spectroscopy (CRDS) using broadband optical frequency combs describes how an OFC and a Fabry-Perot cavity can be made to match [Thorpe2006] with a bandwidth of 100 nm, that was limited only by the bandwidth of the modelocked femtosecond laser.

4.4 Nonlinear processes

The average power requirement for nonlinear processes scales with the enhancement of the repetition rate, assuming constant pulse width. Therefore, it is advantageous to perform frequency conversion at lower repetition rates.

A typical pulse energy requirement for broadening a 100-fs pulse from an erbium fiber laser to an octave-spanning spectrum is 1.5 – 2 nJ. At a repetition rate of 15 GHz, this translates into 23 – 30 W of average power.

For second-harmonic generation of 100-fs (output) pulses, e.g. conversion from 1560 nm to 780 nm, a 25 – 50% conversion efficiency can be achieved with 300 mW of typically 45-fs (pump) pulses.

Depending on the concept used, the requirements for nonlinear conversion differ greatly. The overall power budget requirements will be calculated for each concept individually that this study presents.
4.5 Promising technical solutions for HRR combs

Possible system designs, to be evaluated and refined during the study, are shown below.

Figure 4.5: System layout based on a 250-MHz erbium-doped fiber laser emitting at a wavelength of 1560 nm.

Figure 4.6: System layout based on a 250-MHz erbium-doped fiber laser emitting at a wavelength of 1560 nm. If the spectrometer calibration procedure allows for covering spectral range of 400 to 680 nm by spectral tuning, a Raman-shift approach (or any other spectral tuning mechanism) can be used to cover the blue end of the spectrum. The red end is provided by frequency-doubling the 1000-nm end of a broad 1000 to 2000 nm spectrum from 1560-nm pulses.

Figure 4.7: System layout based on a 4-GHz femtosecond chromium-YAG laser emitting at a wavelength of 1560 nm. This layout is similar to the one for the fiber laser. Third-harmonic generation in two steps (second-harmonic generation and sum frequency generation) and spectral broadening in the visible is an alternative to second-harmonic generation followed by strong spectral broadening to visible. The advantage of spectral broadening at 780 nm is that it is a proven and mature technique.
Figure 4.8: System layout based on a harmonically modelocked 250-MHz erbium-doped fiber laser emitting at a wavelength of 1560 nm. Harmonic modelocking as an alternative to external filtering at 2 GHz is considered with special attention to stability and (250-MHz) fundamental-mode suppression.

The concepts will be evaluated and refined in the course of the study.
5 Proposed solution

5.1 Assessment of specifications

5.1.1 Preliminary specifications

<table>
<thead>
<tr>
<th>subsection in Ref.1</th>
<th>parameter</th>
<th>min</th>
<th>max</th>
<th>typical</th>
<th>units</th>
<th>Rem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>short wavelength limit (blue limit)</td>
<td>350</td>
<td>400</td>
<td>nm</td>
<td>single- or multi-comb system</td>
<td></td>
</tr>
<tr>
<td>6.1</td>
<td>long wavelength limit (red limit)</td>
<td>680</td>
<td>1000</td>
<td>nm</td>
<td>single- or multi-comb system</td>
<td></td>
</tr>
<tr>
<td>6.2</td>
<td>wavelength uncertainty</td>
<td>1.00E-11</td>
<td></td>
<td>absolute accuracy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.3</td>
<td>wavelength stability period</td>
<td>10</td>
<td>a</td>
<td>duration of CODEX experiment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.4</td>
<td>photon flux per line</td>
<td>1.00E-11</td>
<td>W</td>
<td>per line in the comb (at 550 nm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.4</td>
<td>photon flux per line</td>
<td>2.50E+08</td>
<td>s^{-1}</td>
<td>photon rate per line in the comb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td>intensity fluctuation</td>
<td>0.1</td>
<td></td>
<td>over a period (tbd)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.6</td>
<td>relative intensity between lines</td>
<td>5</td>
<td></td>
<td>of all lines</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.7</td>
<td>line distribution in the wavelength range</td>
<td>one gap</td>
<td></td>
<td>over the length of one order (tbd)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.8</td>
<td>comb line separation (free spectral range)</td>
<td>14</td>
<td>16</td>
<td>15 GHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.9</td>
<td>supermode suppression ratio</td>
<td>60 [50]</td>
<td>dB</td>
<td>in the report</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.9</td>
<td>supermode suppression frequency</td>
<td>100 [250]</td>
<td>MHz</td>
<td>side modes around each line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.9</td>
<td>supermode suppression range</td>
<td>14.8 [14.5]</td>
<td>GHz</td>
<td>q = 14.8/15=0.9867 in the report</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


5.1.2 Wavelength range

If the wavelength ranges specified are assumed to be obtained from a single-comb system by symmetric broadening of the frequency spectrum (which is the case in the absence of Stimulated Raman Scattering and higher-order dispersion [checked by experiment, see Fig. 3.5]), then the center wavelength derived from the center frequency of the range is given by

$$\lambda_{\text{center}(f)} = \frac{2 \lambda_1 \lambda_2}{\lambda_1 + \lambda_2}.$$  

Here, $\lambda_1$ and $\lambda_2$ are the lower and upper wavelength limit of the spectrum, given at the -3-dB point relative to the average spectral intensity.

For the specifications above we obtain

<table>
<thead>
<tr>
<th>wavelength range</th>
<th>center wavelength</th>
<th>second harmonic</th>
<th>third harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$ to $\lambda_2$</td>
<td>$\lambda_{\text{center}(f)}$</td>
<td>$2 \times \lambda_{\text{center}(f)}$</td>
<td>$3 \times \lambda_{\text{center}(f)}$</td>
</tr>
<tr>
<td>min.</td>
<td>400 - 680 nm</td>
<td>504 nm</td>
<td>1007 nm</td>
</tr>
<tr>
<td>max.</td>
<td>350 - 1000 nm</td>
<td>519 nm</td>
<td>1037 nm</td>
</tr>
<tr>
<td>blue half-range</td>
<td>(i) 350 - 519 nm</td>
<td>418 nm</td>
<td>836 nm</td>
</tr>
<tr>
<td>red half-range</td>
<td>(ii) 519 - 1000 nm</td>
<td>683 nm</td>
<td>1367 nm</td>
</tr>
</tbody>
</table>

The second and third harmonic wavelengths in the table indicate possible laser system wavelength specifications that generate the comb at a subharmonic wavelength.

For the second, larger range, however, a single-comb system is unlikely to be found, so that at least two combs have to be combined to cover the entire range.
Using a second- or third-harmonic process, the gain media as discussed in sections 3 and 4 may be considered:

Alternatively, an OPO, four-wave mixing processes or stimulated Raman scattering may be used for converting ultrashort laser pulses to the desired wavelength range.

The additional requirement of extended-hands-off operation of the laser may give a preference to those materials that can make a waveguide or fiber integrated laser. Such materials are Nd:glass, Yb:glass and Er:glass as well as semiconductor lasers. Another practical consideration is the availability of high-power amplifiers, such as those developed for the telecommunication industry.

It remains to be clarified whether all lines of the comb have to be provided simultaneously over the entire spectrum or whether a swept-frequency comb and at which sweep rate and bandwidth is acceptable. The latter opens up alternative promising technical solutions derived from widely tunable fs laser sources (see Fig. 4.6).

5.1.3 Relative intensity between lines

The relative intensity between lines shall not vary by a factor of 5 or equivalently, 7 dB, due to the limited dynamic range of the CCD.

This specification requires special attention as most optical frequency combs are optimized for beat signal generation only at selected wavelengths in the comb spectrum. The requirement for the spectrograph calibration however, is challenging because it required a relatively uniform distribution of spectral intensity across the comb. Variations of 20 dB which are observed in typical situations of broadening in a highly nonlinear fiber or PCF are not acceptable.

To obtain a relatively smooth spectral distribution, the broadening process and its options have to be evaluated in more detail.

One technique to be evaluated is fast polarization scrambling within the detector integration time. Polarization of the light coupled into the highly nonlinear fiber is one parameter that determines the location and depth of spectral “holes” in the comb. The fast random or deterministic scanning of the Poincaré sphere, i.e. two polarization parameters, yields a more uniform integrated power distribution across the spectrum.

Another technique is to provide strong broadband amplification in combination with spectral equalization. In broadband telecommunication networks, such gain equalizers with a resolution of 50 GHz are commercially available around the center band (C-band) with insertion loss of approx. 8 dB). It remains to be evaluated whether such equalizers work equally well with a harmonic frequency (e.g., third harmonic of C-band, to cover the wavelength range shown in the table).
5.1.4 Attenuation of “supermodes”

The specifications determine a spectral width and a depth of mode suppression within the free spectral range (FSR).

The mode suppression range (MSR) is specified as a fraction \( q \) of the FSR

\[
q = \frac{\Delta f_{\text{MSR}}}{\Delta f_{\text{FSR}}}
\]

or as an absolute width \( \Delta f_{\text{MSR}} \) (in GHz).

Suppressing the “supermodes” at the 250-MHz repetition frequency of the comb-generating laser, this translates into a MSR of

\[
\Delta f_{\text{MSR}} = \Delta f_{\text{FSR}} - 2 \times 250 \text{ MHz}
\]

and therefore

\[
q = 1 - \frac{500 \text{ MHz}}{15 \text{ GHz}} = 14.5 / 15 = 0.9667.
\]

The mode suppression depth is specified as the ratio \( \rho_{\text{dB}} \) of side mode suppression relative to maximum transmission,

\[
\rho_{\text{dB}} = -10 \log \left( \frac{T_{\text{MSR}}}{T_{\text{max}}} \right) = -10 \log [\rho] = 50 \text{ dB}
\]

As shown in Fig.5.1, these specifications can be understood as a hard limit (red) under which the Airy-function of the Fabry-Perot transmitted power (relative to maximum transmitted power in resonance) has to be accommodated by choosing a proper finesse. (see section 4.3)

![Diagram of Side mode suppression specification](image-url)

**Figure 5.1:** Specification of the side mode suppression ratio; FSR and MSR are the free spectral range and the mode suppression range, respectively. The suppression depth is the ratio \( \rho_{\text{dB}} \) of side mode suppression relative to maximum transmission.
If a 15-GHz mode-filter cavity is used, which eliminates 9 out of 10 modes of the 1.5-GHz frequency comb, then the power transfer efficiency is at best 10%. In section 4.3 (mode filter techniques), the finesse and ring-down power fluctuations of such a cavity are discussed.

As discussed earlier, by averaging over 400 lines in one echelle order, one might gain a 23-dB reduction in uncertainty.

The following topics will be covered in the course of the study and will be presented in the Feasibility Study Report.

5.2 Assessment of competing technical solutions
5.3 System design proposal
5.4 Detailed technical description of the proposed solution
6 Schedule, cost and risk
6.1 Development and laboratory testing plan
6.2 Realization and initial verification
6.3 On-site system line-up and testing (SLAT), verification
6.4 Cost and time estimation
6.5 Risk analysis
7. References

7.1 Books and book chapters on femtosecond lasers and optical frequency combs


7.2 Journal publications on high-repetition-rate laser technology


[Arahira1998] Arahira, S., Kutsuzawa, S., Matsui, Y., Kunimatsu, D. and Ogawa, Y.,
Repetition-frequency multiplication of mode-locked pulses using fiber dispersion. 


[Harvey1993] Harvey, G. T. and Mollenauer, L. F., Harmonically mode-locked fiber ring laser


[Liu2005] Liu, T.-M., Kärtner, F. X., Fujimoto, J. G. and Sun, C.-K., Multiplying the repetition rate of passive mode-locked femtosecond lasers by an intracavity flat surface with low


(2002).


### 7.3 Journal publications on femtosecond optical frequency comb science and technology


[Hall2001] Hall, J. L., Ye, J., Diddams, S. A., Ma, L. S., Cundiff, S. T. and Jones, D. J.,
Ultrasensitive spectroscopy, the ultrastable lasers, the ultrafast lasers, and the seriously

[Hall2003] Hall, J. L. and Ye, J., Optical frequency standards and measurement. *IEEE

pedestal-free 10 GHz pulses from a comb-like dispersion profiled fiber compressor and
its application in supercontinuum generation. *Chinese Physics Letters*, 17, 806-808
(2000).

Spectroscopy of a Reflecting Reference Cavity. *Optics Communications*, 35, 441-444
(1980a).


[Hansch1990] Hansch, T. W., A Proposed Sub-Femtosecond Pulse Synthesizer Using Separate

[Hansch2005] Hansch, T. W., Alnis, J., Fendel, P., Fischer, M., Gohle, C., Herrmann, M.,
Holzwarth, R., Kolachevsky, N., Udem, T. and Zimmermann, M., Precision
spectroscopy of hydrogen and femtosecond laser frequency combs. *Philosophical
Transactions of the Royal Society A-Mathematical Physical and Engineering Sciences*,

Chen, Z., Long-wavelength continuum generation about the second dispersion zero of a

Modes Induced by Synchronous Intracavity Modulation ( Diffraction by Phonons in

J. K. and Windeler, R. S., Ultrahigh-resolution optical coherence tomography using
continuum generation in an air-silica microstructure optical fiber. *Optics Letters*, 26,

self-referenced frequency-comb laser based on a combination of fiber and waveguide

[Hashimoto2000] Hashimoto, T., Sotobayashi, H., Kitayama, K. and Chujo, W., Photonic
conversion of OC-192TDM-to-4 x OC-48WDM by supercontinuum generation. *Electronics

[Hauri2004a] Hauri, C. P., Kornelis, W., Helbing, F. W., Heinrich, A., Couairon, A.,
Mysyrowicz, A., Biegert, J. and Keller, U., Generation of intense, carrier-envelope


