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THE SOURCE FUNCTIONS OF SOME STRONG LINES IN LATE-TYPE STELLAR ATMOSPHERES

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SUMMARY

The classification of the source functions of some strong spectral lines in the atmospheres of late-type stars is discussed. Using Arcturus as an example it is shown that the H_ϵ emission seen in some late-type giants is the result of a photoelectrically controlled source function. In the cool dwarfs however the emission sometimes seen in the Balmer series and some strong neutral metal resonance lines results, like the Ca II H and K emission, from a chromospheric temperature rise coupled with a collisionally controlled source function.

INTRODUCTION

From the pioneering work of Jefferies, Thomas and collaborators in the late 1950's and early 1960's we can now understand in general terms the behaviour of the source function of a strong line in an atmosphere with a chromospheric temperature rise. The depth dependance of the line source function is known to depend in a sensitive way on the relative importance of direct collisional and indirect photoionization–recombination processes as sources and sinks of line photons. The prime example of this distinction is the case of the solar H_α and Ca II H and K line profiles. The H_α source function is marginally photoelectrically controlled and never shows an emission core on the disk (except in flares) while the high second ionization potential of calcium renders the H and K source functions strongly collisionally controlled, resulting in the characteristic self-reversed emission cores.

This non-LTE classification scheme has to take into account not only the energy level structure of the ion under consideration but also the thermodynamic structure of the atmosphere of which it is a part. The distance over which the ion can interact radiatively with its surroundings is called a thermalization length the value of which is set by the probability per scattering of a line photon being destroyed by some process which may be either a collisional de-excitation, absorption into some overlying continuum or ionization of an atom in the upper state. The thermalization length in a strong line is typically many photon mean free paths (Jefferies 1968). A line which is classified as photoelectrically controlled in the solar chromosphere may therefore behave quite differently in a star of a different spectral type.

In this paper we consider two interesting examples of the line formation problem in stellar atmospheres, namely the Balmer lines in a K-giant (Arcturus K2IIIp) and the Balmer and strong neutral metal resonance lines in M-dwarfs.

THE LINE SOURCE FUNCTION

Using the equation of statistical equilibrium the frequency independent line source function $S_l(\tau)$ can be written

$$S_{\mathrm{l}}(\tau) = \int_0^\infty \frac{J_\nu(\tau)\,\phi_\nu(\tau)\,d\nu + \epsilon B(T_{\mathrm{e}}) + \eta B^*}{1+\epsilon+\eta} \tag{1}$$

where J_ν is the mean intensity of the radiation field and ϕ_ν the normalized profile of the line absorption coefficient (Thomas 1957). $B(T_{\mathrm{e}})$ is the Planck function at the electron temperature T_{e}.

For this discussion it will be sufficient to consider the simplest meaningful model atom having just two bound levels 1 and 2 and a continuum k whereupon the other quantities in (1) can be written

$$\epsilon = \frac{C_{21}}{A_{21}} \tag{2}$$

$$\eta = \frac{R_{2k}R_{k1}}{(R_{k1}+R_{k2})\,A_{21}} \tag{3}$$

$$B^* = \frac{2h\nu^3}{c^2}\frac{g_1}{g_2}\frac{R_{1k}R_{k2}}{R_{2k}R_{k1}} \tag{4}$$

where the C's and R's are collisional and radiative rates respectively and the other symbols have their usual meaning. For optical transitions in cool stars we have ignored stimulated emissions and it is also implicit in (3) and (4) that densities are low enough to neglect three-body recombinations. The ratio of collisional ionizations to collisional excitations is assumed to be $\ll 1$.

The response of S_{l} to the kinetic temperature and density structure of an atmosphere will depend on the relative importance of those terms in (1) which represent photon sources, the second and third in the numerator; and photon sinks, the corresponding terms in the denominator. Thus if we wish to consider the expected behaviour of a strong line in stars of different spectral type, it is useful to examine the values of the ratios

$$\frac{\epsilon B(T_{\mathrm{e}})}{\eta B^*} = \frac{C_{12}}{R_{1k}}\left(1+\frac{R_{k1}}{R_{k2}}\right) \tag{5}$$

$$\frac{\epsilon}{\eta} = \frac{C_{21}}{R_{2k}}\left(1+\frac{R_{k2}}{R_{k1}}\right) \tag{6}$$

as functions of T_{e}, n_{e} and the relevant continuum radiation fields. The value of (5) represents the relative importance for the production of line photons of direct collisional to indirect photoionization processes while (6) expresses the corresponding quantity for photon destruction. Using the dipole approximation for the collisional cross-section (Seaton 1962) and the hydrogenic formula for the continuous absorption coefficient (Gaunt 1930), the ratios (5) and (6) can be written approximately

$$\frac{\epsilon B(T_{\mathrm{e}})}{\eta B^*} \simeq K n_{\mathrm{e}} \frac{T_{\mathrm{e}}^{0\cdot18}}{T_{\mathrm{r}}} \exp\left[\frac{a}{kT_{\mathrm{r}}} - \frac{c}{kT_{\mathrm{e}}}\right] \tag{7}$$

$$\frac{\epsilon}{\eta} \simeq K n_{\mathrm{e}} \frac{T_{\mathrm{e}}^{0\cdot18}}{T_{\mathrm{r}}} \exp\left[\frac{b}{kT_{\mathrm{r}}}\right] \tag{8}$$

where

$$K = 3{\cdot}4\times10^{-20}\left(\frac{c}{k}\right)^{-1\cdot68}\left(\frac{b}{k}\right)\left(\frac{13{\cdot}6}{c}\right)^2 A_{21}Z^{-4}n_2{}^5\left(1+\frac{g_2}{g_1}\left(\frac{n_1}{n_2}\right)^5\frac{a}{b}\right) \tag{9}$$

(Fosbury 1973b) and a, b and c are respectively the ionization energy from state 1, from state 2 and the $1 \rightarrow 2$ excitation energy in eV, k is measured in eV $\deg^{-1}$ and n_1, n_2 are the principal quantum numbers of the two bound states. The radiation temperatures in the two continua are assumed to be the same and equal to T_r. These ratios can be used to decide if a given resonance* line in a particular type of star is going to be collisionally or photoelectrically controlled, that is if (7) and (8) are $\gg 1$ or $\ll 1$ respectively.

THE dM AND THE dMe STARS

The classification of the H_α source function in the solar chromosphere has been a matter of some controversy since Jefferies & Thomas (1959) and Pottasch & Thomas (1959) first suggested photoelectric control. Athay (1972) has raised objections saying that the $2 \rightarrow 3$ collisional cross-section could be sufficiently large to favour collisional control, the reason for the absence of an emission core being simply insufficient line opacity. Recently however Gebbie & Steinitz (1974) have rediscussed the problem and concluded that Athay's assumed collisional cross-section is unrealistically large with the result that they still favour photoelectric control.

The argument becomes less critical however if we consider stars further down the main sequence. Here the radiation temperatures in the Balmer and Paschen continua will at some point become sufficiently low to change the source function classification to one of collisional control. In Fig. 1 are plotted the source and sink ratios (7) and (8) appropriate to H_α as functions of a radiation temperature characteristic of these two continua. The sink ratio is almost independent of T_e but (7) is plotted for three values of the kinetic temperature. This diagram can now be used to interpret observations of dM stars in terms of chromospheric electron density. We know that most if not all dM stars have chromospheric temperature rises because of the existence of strong Ca II H and K emission cores. If we see emission in the Balmer lines in an early M dwarf (say $T_r \simeq 3500$ K) then we know that the electron density in some part of the chromosphere must be $> 10^{11}$ cm^{-3}. If it is assumed that the relevant radiation temperature and T_{eff} are not too dissimilar then we can begin to understand the increasing incidence of emission dwarfs amongst the later types (see Woolley *et al.* 1970). This is an important point to consider when discussing the evolutionary distinctions between the dM and the dMe stars.

A good idea of the behaviour of H_α down the temperature sequence can be obtained by studying the M dwarf scanner observations of Spinrad (1973). For the majority of field stars the correlation between Spinrad's H_α equivalent width parameter and his temperature index is quite tight. Hyades members and flare stars however branch from this sequence at a temperature corresponding to a spectral type of about M2V or $T_{\mathrm{eff}} \simeq 3000$ K, indicating stronger emission relative to the continuum among the cooler stars. From Fig. 1 it can be seen that this branch point is just at the temperature where we would expect the H_α source function to show strong signs of collisional control if the chromospheric electron densities are similar to that in the Sun (say $n_e \simeq 10^{11}$ cm^{-3}).

This argument of course not only applies to H_α but also to the neutral metal

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* H_α can be considered a resonance line if L_α and L_β are in detailed balance where it is being formed; see Thomas (1965).

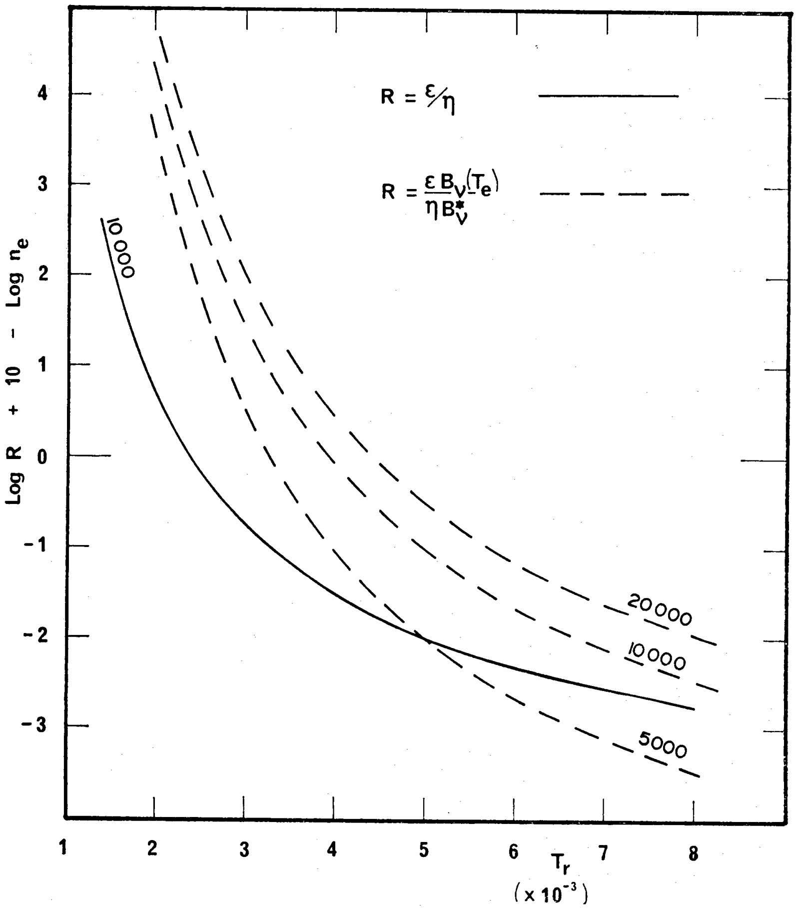

FIG. 1. *The dependence of the H_α source function photon source-sink terms on atmospheric parameters. The numbers on the curves refer to T_e.*

resonance lines which have relatively low ionization potentials and are probably photoelectrically controlled in the Sun. As examples we might consider the Ca I resonance line λ 4227 and the Na I D-lines although the latter are in fact more or less collisionally controlled even in the Sun because of the rather small photoionization cross-section characteristic of certain alkali metals including sodium (Seaton 1951; Johnson 1965; Mugglestone 1965). For the case of λ 4227 the ratios (7) and (8) have been evaluated for $n_e = 10^{11}\ \text{cm}^{-3}$ and $T_e = 6000$ K to find that

$$\frac{\epsilon B(T_e)}{\eta B^*} \simeq 1 \quad \text{for} \quad T_r = 3400\ \text{K}$$

$$\frac{\epsilon}{\eta} \simeq 1 \quad \text{for} \quad T_r = 2500\ \text{K}$$

suggesting collisional control in the later M dwarfs. In Fig. 2 is shown a microdensitometer tracing of part of an electronographic spectrogram of AD Leo (M4 Ve) obtained with a Spectracon image-tube on the 2·5 m (98-in.) Isaac Newton Telescope. The resolution is 0·5 Å. The exposure time was 85 min during which the star did not appear to exhibit any major flares. This clearly shows the Ca I emission core together with quite strong Ca II and Balmer line emission. The resolution is sufficient to show that the Balmer lines H_γ–H_8 are broader than the H and K line cores. Since the H and K lines are strongly in emission with an intensity ratio close to unity we deduce that the chromosphere is very optically thick in

1974MNRAS.169..147F

these lines (τ_0 chrom > thermalization length). If the hydrogen and calcium lines are formed in approximately the same regions then the Balmer line widths must be dominated by thermal Doppler broadening, a result which is consistent with the H_α and K-line width–luminosity relations extrapolated to these faint stars (Fosbury 1973a).

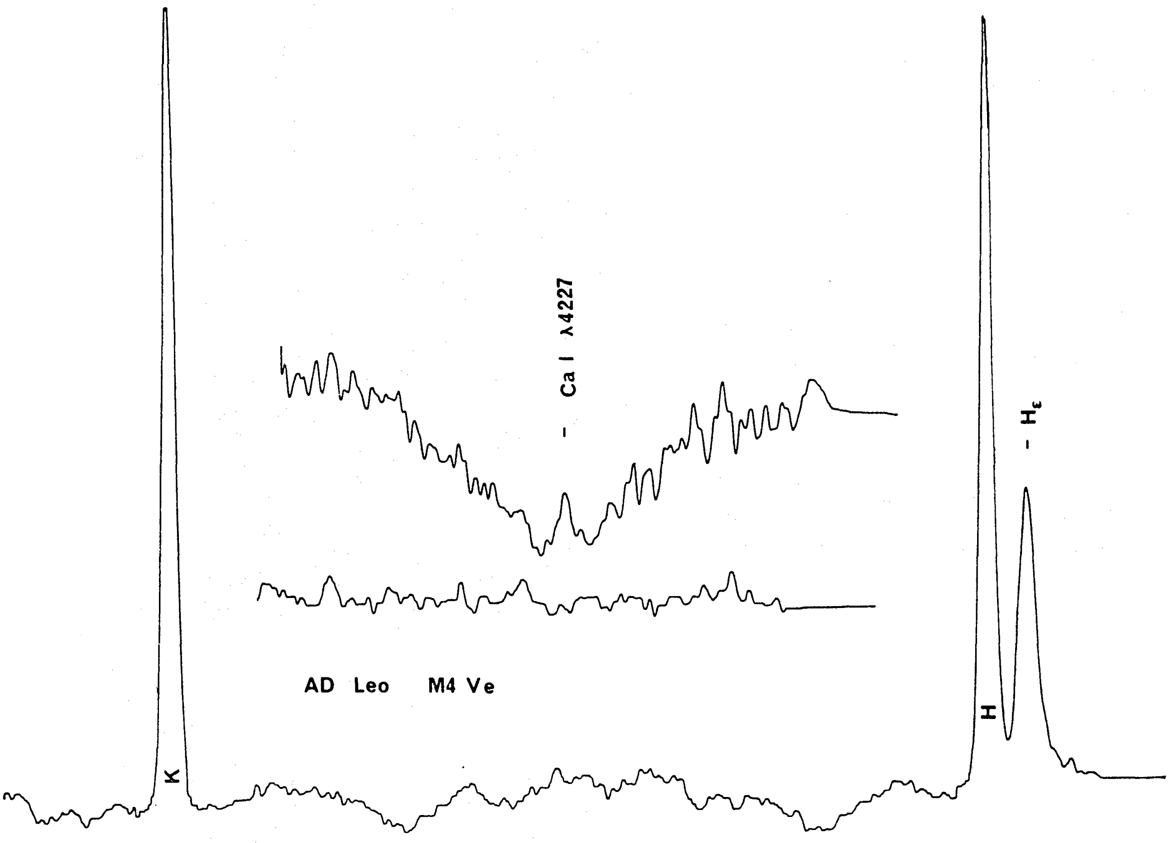

FIG. 2. *Part of the spectrum of a dMe star taken with a Spectracon image-tube on the Isaac Newton telescope.*

THE BALMER LINES IN ARCTURUS

In the atmospheres of the K giants the electron density is expected to be sufficiently low to place the H_α source function clearly in the photoionization controlled class ($n_e \ll 10^{11}$ cm^{-3}). In this section we look at the Balmer lines in Arcturus to see if this expectation is justified.

Ideally we should like to use observations to infer the run of the H_α source function with height in the atmosphere. There are a number of difficulties in the path of such an investigation but we can however make two simple observations which give some indication of the prevailing situation. We know that the core of H_α is formed in the chromosphere of Arcturus (Simon 1971) and so the absence of emission is consistent with this source function classification. It is not however a sufficient condition since the chromospheric opacity may not be large enough compared to a thermalization length in the line. Another classical characteristic of a photoelectric source function is an excitation temperature which can lie above the kinetic temperature over some range of height in the atmosphere (Jefferies & Thomas 1959). It is suggested here that the appearance of H_ϵ 'emission' within the wing of the Ca II H-line is a clear manifestation of such a characteristic.

Since centre-limb observations are not yet available for Arcturus the best way to proceed is to infer $S_l(\tau_0)$ from the central intensities of the first few members of the Balmer series making the assumption of a common excitation temperature for these lines. The problem of Balmer line excitation temperature equality in the Sun has been extensively discussed in the past (see for example Athay & Thomas 1958; Jefferies 1968) and although it is not clear that the assumption is strictly true it does appear to be good to within about 100 K for the first four lines.

Making the assumption of complete redistribution throughout the Doppler core (Thomas 1957), the line source function can be expressed

$$S_l = B(T_{ex}). \tag{10}$$

Now if the depth variation of the source function can be written in the linear form

$$S_l = a + b\tau_0 \tag{11}$$

where τ_0 is measured in the centre of the line, we have the well-known result for measurements of the radiation flux that

$$T_B = T_{ex} \quad \text{at} \quad \tau_0 = \tfrac{2}{3}. \tag{12}$$

Care has to be exercised when using the Eddington–Barbier relation where the line absorption coefficient is changing rapidly with frequency near the edge of the Doppler core but it is probably a good approximation near the line centre (Athay 1972).

If $\bar{N}_2$ is the radial column density of hydrogen atoms in the second level,

$$\tau_0 = \bar{N}_2 \alpha_0 \tag{13}$$

where

$$\alpha_0 \propto \frac{f\lambda_0}{v_D}$$

is the line centre absorption coefficient and v_D is the Doppler velocity. Observations of the brightness temperatures at the centres of the lines coupled with some assumption about $v_D(\tau_0)$ then will result in T_{ex} as a function of $\bar{N}_2$.

Arcturus is the only late type star other than the Sun for which suitably calibrated observations exist. The central residual intensities of the lines are taken from Griffin's (1968) photometric atlas and the continuum is calibrated by measuring the blocked fraction in 50 Å band passes containing the lines and comparing with the photoelectric scans of Willstrop (1964). The fluxes are made absolute using the angular diameter of Arcturus of 0″·022 ± 0″·003 measured using speckle interferometry by Gezari, Labeyrie & Stachnik (1972). Willstrop's fluxes are reduced by 6 per cent following the recalibration of the energy distribution of Vega by

Table I

The Balmer line brightness temperatures in Arcturus

Line	T_B (continuum) K	T_B (line centre) K
H_α	4180	3180
H_β	4290	3280
H_γ	4300	3450
H_δ	4300	3490
H_ϵ	4160	3440

Oke & Schild (1970). The results for the first five lines are summarized in Table I where it has been assumed that v_D is the same for them all. This assumption should not be too bad for hydrogen, at least over the range of depths encompassing the centres of the higher members of the series since thermal broadening should dominate. The absolute uncertainty in these temperatures is about ±100 K primarily due to the uncertainty in the angular diameter. The relative errors should be rather less however.

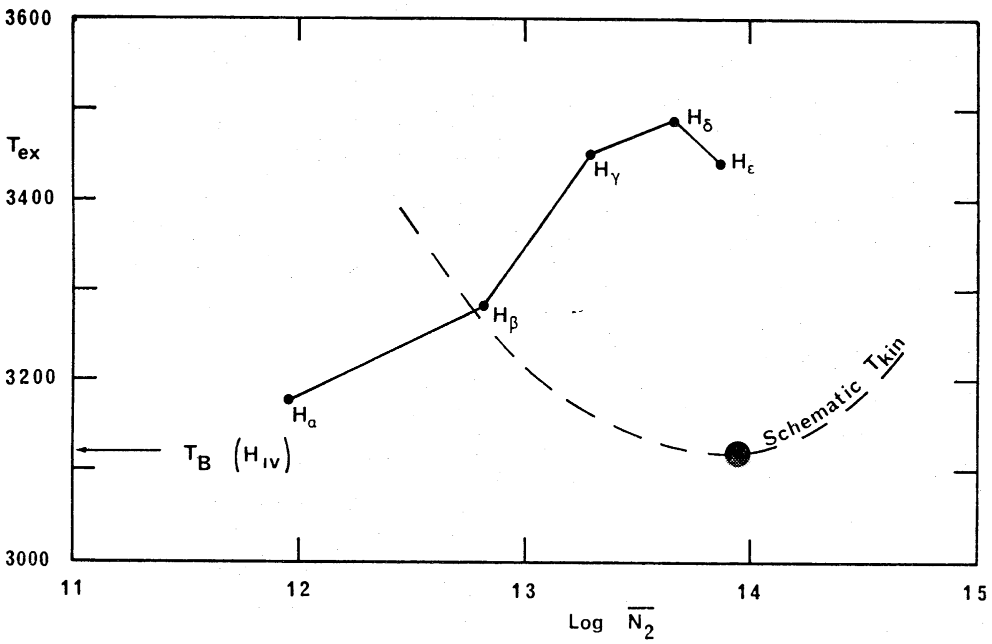

FIG. 3. *The excitation temperature as a function of the column density of second level hydrogen atoms measured from the central intensities of the first five Balmer lines in Arcturus (K2IIIp). The approximate position of the temperature minimum is shown by the large dot.*

In Fig. 3 T_{ex} has been plotted against log $\bar{N}_2$ for the five lines. To make a full comparison with the kinetic temperature–height profile would require sophisticated model calculations but we can in fact get all the information we need by placing the temperature minimum on this diagram. Following the idea of Linsky & Ayres (1973) we identify the minimum temperature with the H_1 or K_1 minima in the Ca II line profiles. In fact we use the absolute intensity of the H_{1V} minimum to deduce a kinetic temperature of 3120 K*. In order to fix the position of the temperature minimum on the log $\bar{N}_2$ axis we use the opacity of the chromosphere measured in the centre of H_α by Fosbury (1973a). A value of $\tau_0(H_\alpha) \simeq 10^2$ is suggested by a comparison of the H_α and K-line width–luminosity relations but even an error of an order of magnitude in this estimate would not invalidate our conclusions. The turnover of the T_{ex} curve at H_ϵ is presumably due to the failure for this line of out implicit assumption that the ratio of line to continuous opacity is $\gg 1$. This is partly due to the extra pseudo-continuous opacity of the Ca II H-line wing.

Having now fixed the position of the temperature minimum we can draw in part of a schematic kinetic temperature column density profile. Fig. 3 now shows the classic form of a photoelectrically controlled source function, lying above the

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[*] This differs from the result of Linsky & Ayres (1973) by 290 K primarily because of a difference of a factor very close to π between our deduced fluxes.

Planck function in the upper photosphere and dropping below it in the low chromosphere. The H_ϵ emission in Arcturus is then not a direct manifestation of a chromospheric temperature rise, it is in fact formed very near the temperature minimum and has a source function which is fixed by the photospheric radiation field in the Balmer and Paschen continua, not by the local kinetic temperature.

CONCLUSIONS

Wilson (1938) first called attention to the fact that H_ϵ appears in emission in Arcturus and later (Wilson 1957) made visual estimates of the emission intensity from spectrograms of the large sample of stars used for the investigation of the K-line width–luminosity relation. Although the sample is large and does contain a wide range of spectral types, it is difficult to draw any very solid conclusions from these measurements. Wilson suggests that there may be a slight correlation of H_ϵ emission intensity with K-line intensity, but this is certainly not well established.

We now believe that it is possible for two quite distinct line formation processes to be at work in this sample of stars. In the very cool dwarfs it has been shown that it is reasonable to expect the Balmer line source functions to show signs of collisional control, resulting in emission lines in the presence of a chromospheric temperature rise and a sufficiently high electron density. In the more luminous stars however, where we expect lower electron densities in both photosphere and chromosphere, the Balmer line source functions are still controlled by photo-ionization–recombination processes and thus are dependent on the radiation fields in the Balmer and higher continua. In these stars the H_ϵ emission is thus an effect rather analogous to that responsible for the rare-earth emission lines found within the wings of H and K in the Sun (Canfield 1971a, b) and in Arcturus (Fosbury 1971).

It is tempting to explain the rather erratic appearance of H_ϵ emission in Wilson's sample as being due to the fact that the chromosphere and temperature minimum have an optical depth of about one measured in the line centre. The chromospheric optical depth in H_α lies in the range $50 \lesssim \tau_0(H_\alpha) \lesssim 500$ (Fosbury 1973a) and so the appearance of H_ϵ is critically dependent on this quantity.

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