THE LRIS POLARIMETER

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PRINCIPLES OF OPERATION

The LRIS polarimeter is a dual-beam instrument in which orthogonally polarized components are measured simultaneously. This largely eliminates errors due to seeing and tracking. It also contains a rotating waveplate with which the different throughputs in the two beams can be internally calibrated. A system of this type was first described by Miller, Robinson and Goodrich (1988), see also Goodrich (1991), Walsh (1992) and Goodrich, Cohen & Putney (1994). In this Manual a somewhat pictorial and heuristic discussion is used. A full analytic treatment may be made with Meuller matrices.

The polarimeter sits immediately under the slit mask, and the ray paths are shown in Figure 1. The beam passes successively through the calibration wheel, the rotating wave plate, and a beamsplitter which separates the ordinary and extraordinary rays (o and e) with a total internal reflection of the o-ray. The two beams then go to the dichroic reflector and the cameras, where they make parallel spectra in each camera. (At present the dichroic and the blue camera of LRIS are not available.) The o and e rays (with intensities X and Y, respectively) are polarized perpendicular and parallel to the plane of the figure. The LRIS coordinate system is used: x is from the optic axis to the slit (the field center) and z is "down", towards the cameras. The o-ray is polarized along the x-axis, and the long slit aperture is parallel to the y axis.

The ray has intensity I_o at the top of the atmosphere and intensity I at level a below the slit; $I = tI_o$ where t is the transmission coefficient and includes the effects of seeing, opacity, and tracking. The calibration wheel normally contains linear polarizers and a quarter-wave plate in addition to an open aperture. The Stokes parameters at level a are $I(1, q, u, v)$ in the (x, y) coordinate system. (A positive q-wave is polarized along the x-axis.)

The half-wave plate normally sits in the carousel between levels b and c and its orientation is defined in Figure 2. All angles increase in the counterclockwise direction when looking up, and are defined relative to the x-axis. The nominal (computer) setting of the fast axis of the waveplate is Φ, but the true (wavelength dependent) angle is $\phi = \Phi + \xi$. Figure 3 shows $(90° - 2\xi)$ which is used in reducing raw measurements to celestial coordinates (see later). The shape of this curve is accurate to about 0.2°. The absolute level is set by reference to the standard star HD155528, whose position angle we define as PA $\equiv$ 92.6°, in V band. (Schmidt et al, 1992) The zero point is set relative to the polarimeter frame with an optical switch, but may vary especially if the carousel is disassembled. The zero point should be checked against a standard star during every run.

The CCD counts B and T are related to the x- and y- intensities below the waveplate by $B = g_x X$ and $T = g_y Y$, where g_x and g_y are gains or throughputs, and include the effects of flat fielding and spectrum extraction. Thus the g's are functions of wavelength and may vary if, for example, the object moves along the slit. The symbols B and T stand for the Bottom and Top spectra if they are displayed using VISTA, or the reverse if FIGARO is used.

1. Linear Polarization

A half-wave plate has retardance $\tau = 180°$ and its effect is to rotate the polarization to its mirror position about the fast axis. Thus a 45° rotation of the waveplate interchanges x and y components:

$$\frac{X_1}{t_1} = \frac{Y_2}{t_2} \quad , \quad \frac{Y_1}{t_1} = \frac{X_2}{t_2} \tag{1}$$

$$\frac{B_1}{g_{x1}t_1} = \frac{T_2}{g_{y2}t_2} \quad , \quad \frac{B_2}{g_{x2}t_2} = \frac{T_1}{g_{y1}t_1} \tag{2}$$

where subscripts (1,2) refer to a pair of exposures taken 45° apart; eg $\Phi_1 = 0°$, $\Phi_2 = 45°$. From (2) we have two useful quantities

$$\Omega \equiv \sqrt{\frac{B_1 T_1}{B_2 T_2}} = (\frac{t_1}{t_2})\beta_x \beta_y \tag{3}$$

$$G \equiv \sqrt{\frac{B_1 B_2}{T_1 T_2}} = \frac{\sqrt{g_{x1} g_{x2}}}{\sqrt{g_{y1} g_{y2}}} \tag{4}$$

where β is the (square-root) ratio of successive gains of the same beam; ie $\beta_x = \sqrt{g_{x1}/g_{x2}}$, etc. In normal operation the β's are close to unity. Ω measures the ratio of sky transparencies in successive exposures, and G is the x/y gain ratio and can be monitored as a check on the system. Normally the exposure times are the same for each exposure of a pair; if they are not, then Ω also contains the ratio of exposure times. Several examples of G and Ω are shown in Figure 4.

The Stokes Parameters are measured in a primed coordinate system rotated by 2ξ from the (x,y) system:

$$x'_i = R_{2\xi} x_i \tag{5a}$$

where

$$R_{2\xi} = \begin{bmatrix} cos2\xi & sin2\xi \\ -sin2\xi & cos2\xi \end{bmatrix} \tag{5b}$$

To see this, consider an incident wave which has q'-polarization only. When $\Phi = 0$, $\phi = \xi$, and the q' wave, which at level b is polarized at 2ξ degrees, becomes polarized along 0° at level c. Thus, as the waveplate is rotated, the intensities X and Y vary as shown in Fig 5. The arrows at $\Phi = 0°$ and 45° show q':

$$q' = \frac{X_0 - Y_0}{X_0 + Y_0} = -\frac{X_{45} - Y_{45}}{X_{45} + Y_{45}} \tag{6}$$

Setting $X_0 = \sqrt{X_0 Y_{45}}\sqrt{t_0/t_{45}}$ and $Y_0 = \sqrt{Y_0 X_{45}}\sqrt{t_0/t_{45}}$ (From Eq 1) gives

$$q' = \frac{\sqrt{B_1 T_2} - \sqrt{B_2 T_1}}{\sqrt{B_1 T_2} + \sqrt{B_2 T_1}} \tag{7}$$

where $(1,2) = (0°, 45°)$, and we have assumed $\beta_x = \beta_y$. Eq (7) shows the power of the dual-beam method – only the four measured spectra appear on the right-hand side, provided the gains are constant during the two exposures. Eq. (7) can also be written in the suggestive forms

$$q' = \frac{B_1 - GT_1}{B_1 + GT_1} = \frac{B_1 - \Omega B_2}{B_1 + \Omega B_2} \tag{8}$$

A pure u'-wave produces the same response as in Fig (5), except that it is shifted by 22.5°. Thus Eq (7) also gives u', with $(1,2) = (22.5°, 67.5°)$. Note that q and u are each measured where the other is zero; thus their measurements are independent and Eq (7) can be used for an arbitrary polarized wave. The presence of circular polarization does not contaminate this measurement because the v component is indistinguishable from an unpolarized component. This is no longer true when $\tau \neq 180°$. The general case (rotation errors, or errors in τ), is discussed by Goodrich (1991).

To correct (q', u') to the LRIS coordinate system, use the inverse transformation $R^{-1}_{4\xi}$; ie rotate (q', u') by -4ξ degrees.

$$\begin{aligned} q &= q' cos4\xi - u' sin4\xi \\ u &= q' sin4\xi + u' cos4\xi \end{aligned} \tag{9}$$

This is equivalent to adding 2ξ to $\theta' \equiv 0.5 \arctan(u'/q')$. The reduction of (q, u) to sky (celestial) coordinates requires a further rotation, to correct for the position angle of the LRIS, θ_L. (In the LRIS FITS header, θ_L = ROTPOSN − 90°.) We have used conventional definitions of q and u relative to x and y, but when $\theta_L = 0°$ the x-axis has a sky position angle of 90°. The final reduction to sky coordinates is $\theta_{sky} = \theta' - \theta_c$, where $\theta_c = (90° - 2\xi - \theta_L)$, or

$$\begin{aligned} q_{sky} &= q' cos2\theta_c + u' sin2\theta_c \\ u_{sky} &= -q' sin2\theta_c + u' cos2\theta_c \end{aligned} \tag{10}$$

The instrumental correction angle, $\theta_{corr} \equiv (90° - 2\xi)$, is shown in Fig 3. This curve should be checked against a standard star at every observing session, since it changes if the zero point of the encoder is changed.

The fractional linear polarization, p, is defined in the absence of noise as $p \equiv \sqrt{q^2 + u^2}$. However, q and u are intrinsically noisy, and when they have a Gaussian distribution p has a Rice distribution, whose mean is biased (Simmons & Stewart 1985). When the SNR is low (≤ 5) the errors associated with the usual debiasing routines are large, and it often is preferable to estimate p by rotating q_{sky} and u_{sky} by a smooth approximation to $2\theta_{sky} \equiv \arctan(u_{sky}/q_{sky})$. The new q is the estimate of p, and the new u should be zero to within noise. This estimate is sometimes called the "Rotated Stokes Parameter" and it is biased low by a factor of order $cos(2\Delta\theta)$, where $\Delta\theta$ is the error in the approximation to θ_{sky}. Although p is always biased, q, u, and θ are not, provided the S/N is high enough. However, when the signal is weak q and u are not normally distributed and may be biased. In this case the S/N should be improved by first averaging the fluxes over some wavelength interval, rather than by averaging the Stokes parameters themselves. This is discussed by Clarke et al (1983) but mainly for the case where the Stokes parameters are calculated with Eq (6). Our limited experience suggests that when the recommended Eq (7) is used, the bias is small if a bin in the extracted B or T spectrum contains a thousand photons or more.

2. Circular Polarization

a) Rotating Quarter-Wave Plate (This has not yet been tested in the LRIS)

The most efficient and stable way to measure circular polarization uses a rotating quarter-wave plate. In the LRIS polarimeter this is accomplished by replacing the half-wave in the carousel by the quarter-wave plate. The cells of the waveplates are pinned, so that interchanging waveplates should not affect the calibrations.

The quarter-wave plate has a wavelength-dependent fast axis, with orientation ψ offset by η from the nominal position Φ: $\psi = \Phi + \eta$. (See Fig 2, ψ and η are analogous to ϕ and ξ.) The numerical value of η is not well known yet.

The responses of a rotating quarter-wave plate to q, u, and v can be shown to be as follows.

$$q: \quad X = \frac{1}{2}I(1 + qcos^2 2\psi) = \frac{1}{2}I[(1 + \frac{1}{2}q) + \frac{1}{2}qcos4\psi] \tag{11}$$

$$u: \quad X = \tfrac{1}{2}I(1 + \tfrac{1}{2}usin4\psi)$$

$$v: \quad X = \tfrac{1}{2}I(1 + vsin2\psi)$$

and, in all cases, $Y = I - X$. These responses are sketched in Fig 6. Note in Fig (6c) that v produces maximum signal at $\psi = 45°$ and $135°$, where the q and u signals are zero. Thus measurements at $45°$ and $135°$ give v with maximum efficiency, and with no contamination from q and u. Eq (7) can be used to calculate v, with $(1,2) = (45°, 135°)$, and the angles now represent ψ, not Φ. Φ should be set to first $(45° - \eta)$ and then $(135° - \eta)$. The effect of η varying with wavelength can be seen in Fig. 6c. The arrows are displaced by an angle $\Delta\psi$ and are too small by a factor $cos(2\Delta\psi)$. The correction factor is small but may be applied to Eq. 7. When $\tau = 90°$ there is no contamination from q and u because their response is periodic in 4ϕ and their contributions cancel for a $90°$ rotation.

In principle it is possible to measure q and u with a quarter-wave plate, but there are problems. Fig (6a) shows that there is no information for q at $45°$, and $90°$ merely repeats $0°$. There is no way to make successive observations which flip the arrows and provide internal calibration of the gains. Fig (6b) shows that u can be found from measurements at $22.5°$ and $67.5°$; but q and v are not zero there and will contaminate the value of u if there are any angular offsets.

b) Dual-Waveplate Method

The polarimeter contains a novel method for measuring circular polarization, using the quarter-wave plate over the half-wave plate. The quarter-wave plate is inserted into the beam with the calibration wheel, and its fast axis is now fixed at ζ; an approximation to ζ is shown in Fig 7.

Incident circular polarization at level a (Fig 1) is converted to linear at level b, where it is analyzed with the rotating half-wave plate. To discuss this we use two subscripts, a and b, to refer to the Stokes parameters at levels a and b. We also need three coordinate systems. The LRIS coordinates (x, y) are unprimed, and those based on the fast axis of the half-wave plate have a single prime (Eq 5). Those based on the fast axis of the quarter-wave plate have a double prime: $x_i'' = R_\zeta x_i$ ie they are rotated by ζ (not 2ζ) from (x, y).

The incident wave has Stokes parameters (q_a, u_a, v_a) which can be rotated to the coordinate system of the quarter-wave plate:

$$(q_a'', u_a'') = R_{2\zeta}(q_a, u_a) \tag{12}$$
$$v_a'' = v_a$$

The component q_a'' is polarized along an axis of the quarter-wave plate and is unchanged at level b: $q_b'' = q_a''$. The component u_a'' is polarized at 45°to the axes and is converted to circular polarization: $v_b = -u_a''$. (v is positive if the wave rotates clockwise when looking up.) The component v_a is converted to linear polarization: $u_b'' = -v_a$. The rotating half-wave plate analyzes the linear polarization at level b, and pairs of observations with $\Phi = 0°$ and 45°, and $\Phi = 22.5°$ and 67.5°, give (q_b', u_b'). When these are rotated back to the quarter-wave plate system we have (q_b'', u_b'') and then $v_a = -u_b''$. This final rotation can be made by first rotating to (q, u) using Eq (9) and then to (q_b'', u_b'') using Eq (12): $(q_b'', u_b'') = R_{2\zeta} R_{4\xi}^{-1}(q_b', u_b')$, or

$$q_b'' = q_b' cos2(\zeta - 2\xi) + u_b' sin2(\zeta - 2\xi) \tag{13a}$$
$$-v_a = u_b'' = -q_b' sin2(\zeta - 2\xi) + u_b' cos2(\zeta - 2\xi) \tag{13b}$$

Note that if $(\zeta - 2\xi) = 0°$ or 45° then v_a can be found from u_b' or q_b' alone; ie the circular polarization can be measured with two exposures rather than four, and the efficiency is the same as for the rotating quarter-wave plate. In the polarimeter the waveplates and the computer zero are normally set so that $(\zeta - 2\xi) \approx 38°$ (Fig 7). This means that the optimum procedure for determining v_a from a single pair of exposures is to use Eq(7) with $(1, 2) = (-7°, 38°)$:

$$v_a \approx \frac{\sqrt{B_1 T_2} - \sqrt{B_2 T_1}}{\sqrt{B_1 T_2} + \sqrt{B_2 T_1}} \tag{14}$$

The wavelength dependences of ζ and 2ξ are not the same, and so the curve of $(\zeta - 2\xi)$ is not flat. The settings therefore may be optimally tailored to a particular wavelength range by using Fig 7. The use of 38° as a typical value means that there will be errors of 3° to 5° at most wavelengths. The corresponding values of v will be too small by approximately $\cos 6°$ or $\cos 10°$, which for many purposes is negligible. The errors will be larger if the incident wave has linear polarization in addition to circular. These errors of course can be eliminated by making the full set of four exposures.

The dual-waveplate method has the great advantage that it obviates interchanging waveplates in the carousel, and allows easy mixing of linear and circular measurements. However, the user is cautioned that it has been tested to only a limited extent. The method is described in more detail in Goodrich, Cohen & Putney (1995).

3. Calibration

A null standard (unpolarized) star should be observed at least once during each night, as a check on the system. The values of q and u (and v if circular measurements are being made) should be less than 0.1%.

A standard polarized star should also be observed at least once during each observing run. The measured polarization is compared with the standard value, and with other measurements of the same object with LRIS, as a check on reliability and stability. This measurement also checks the zero point of the PA reference curve (Fig 3). The polarizers in the calibration wheel are useful intermediate standards. They should show nearly 100% polarization. They also give a low-noise PA correction curve. The detailed shape of the curve should be set by the polarizers and not by a star, since stars usually have a PA which varies with wavelength. However, the zero-point of the curve should be set by a standard star.

There are no standard circularly-polarized stars, although a number of magnetic white dwarfs are strongly polarized and have stable polarization. Figure 8 shows the Stokes parameter v for Grw+70° 8247, which may be useful as a check on circular polarization measurements.

The offset ζ can be measured with the dual–waveplate method, by looking at a linearly–polarized star. The star has $v_a = 0$, hence $u''_b = 0$, and from Eq 13b,

$$tan2(\zeta - 2\xi) = \frac{u'_b}{q'_b} \tag{15}$$

This gives $2(\zeta - 2\xi)$ which is needed for Eq (13), and ζ itself can be found if ξ is known. For this method to be efficient, u'_b and q'_b should be as large as possible; that is, the incident radiation should remain nearly linearly polarized to level b, and not be converted to circular. This can always be arranged by rotating the LRIS to an appropriate position angle. The position angle of the stellar radiation is immaterial, but it should be strongly polarized and must have $v = 0$. This procedure works because the radiation at level b is elliptically polarized and the axes are always along the principal axes of the quarter-wave plate.

4. Imaging Polarimetry

In imaging mode the observing procedures are the same. The wavelength dependence of ξ is averaged over the filter, and so the PA correction angle θ_c (see Eq 10) has an average small error. In April 1996 we measured $(\theta_c + \theta_L) = -8.0°$ in B band, and $\theta_c(\mathrm{B}) - \theta_c(\mathrm{V}) = 3.6°$, $\theta_c(\mathrm{B}) - \theta_c(\mathrm{R}) = 6.8°$. In addition, the calculated Stokes parameters will be slightly biased, because the various wavelength components are added together with a range of angles.

Raferences

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Goodrich, R.W. 1991, PASP, 103, 1314.

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Miller, J.S., Robinson, L.B. and Goodrich, R.W. 1988, in Instrumentation for Ground-Based Astronomy, ed. L.B. Robinson (New York, Springer), p 157.

Schmidt, G.D., Elston, R. and Lupie, O.L. 1992, AJ, 104, 1563.
Simmons, J.F.L. and Stewart, B.G. 1985, A&A, 172, L11.
Walsh, J.R. 1992, in 4^{th} ESO/ST-ECF Data Analysis Workshop ed P.J. Grosbol and R.C.E. de Ruijsscher, ESO Conf. and Workshop Proc. No. 41, p. 53.

OPTICS

Calibration Polarizers

The calibration wheel contains 6 positions, normally occupied as follows.

1. IR polarizer, Meadowlark Optics. NIR-850 sheet polarizer,
   BK-7 cover plates, AR coated.
2. Open
3. Quarter-wave plate (see below)
4. Spare (Open)
5. UV polarizer. Polaroid HN-22, 3800-7600 A.
   Will have to be replaced when Blue camera is added.
6. Shut (Closed)

Half-wave Plate

Normally sits in the carousel but can be interchanged with the quarter-wave plate.
Made by B. Halle, Berlin Catalog RSU 2.30
Pancharatnam superachromatic design, 310-1100 nm. see Fig 3.
Single-layer MgF_2 AR coating
Free aperture 29.5 mm, field of view 35 arc sec.

Quarter-wave Plate

Normally sits in Position 3 of the calibration wheel, but can be interchanged with the half-wave plate.
Made by B. Halle, Berlin Catalog RSU 4.25
Pancharatnam superachromatic design, 310-1100 nm.
Single-layer MgF_2 AR coating
Free aperture 24.5 mm, field of view 29 arc sec.

Beamsplitter

The beamsplitter was manufactured by Karl Lambrecht Corp. in Chicago. This item required special design, which was supervised by Dr. Vinod Vats with consultation by Dr. Robert Goodrich. A schematic sketch is shown in Fig 1. Orthogonally polarized components are separated by total internal reflection of the ordinary wave. The dimensions of the ultran prisms are chosen to equalize the paths of the two beams, so that they focus together on the CCD chip. The focal shift introduced by the calcite and ultran is compensated by the horizontal displacement, so that the net focal shift is near zero.

In practice the two paths are not exactly the same, and the user should take care to focus the camera carefully, so as to optimize the two foci.

Addresses

Bernhard Halle Nachfl, Hubertusstrasse 10-11, D-1000 Berlin 41, Germany.

Karl Lambrecht Corp, P.O. Box 97268, Chicago IL 60690.

Meadowlark Optics, 7460 Weld County Rd 1, Longmont, CO 80504.

User Commands

The following polarimetry commands are available. They typically are preceded by “s” (show) or “m” (modify).

plangle (PA)	shows (modifies) the position angle of the waveplate.
calnum (N)	shows (modifies) the calwheel position, N = 1-6.
calname (A)	shows (modifies) the calname, A = ir, open, etc.
object (B)	shows (modifies) the object name.
pol-acq	runs a procedure making 4 exposures at position angles 0, 45, 22.5, and 67.5 degrees. The exposure time is set in the main window, as always. Do not use “s” or “m” before pol-acq.

focusloop_pol_100	Telescope focus, move secondary mirror in 100-micron steps, with 5″ steps to the north.
focusloop_pol_50	Same with 50-micron steps.

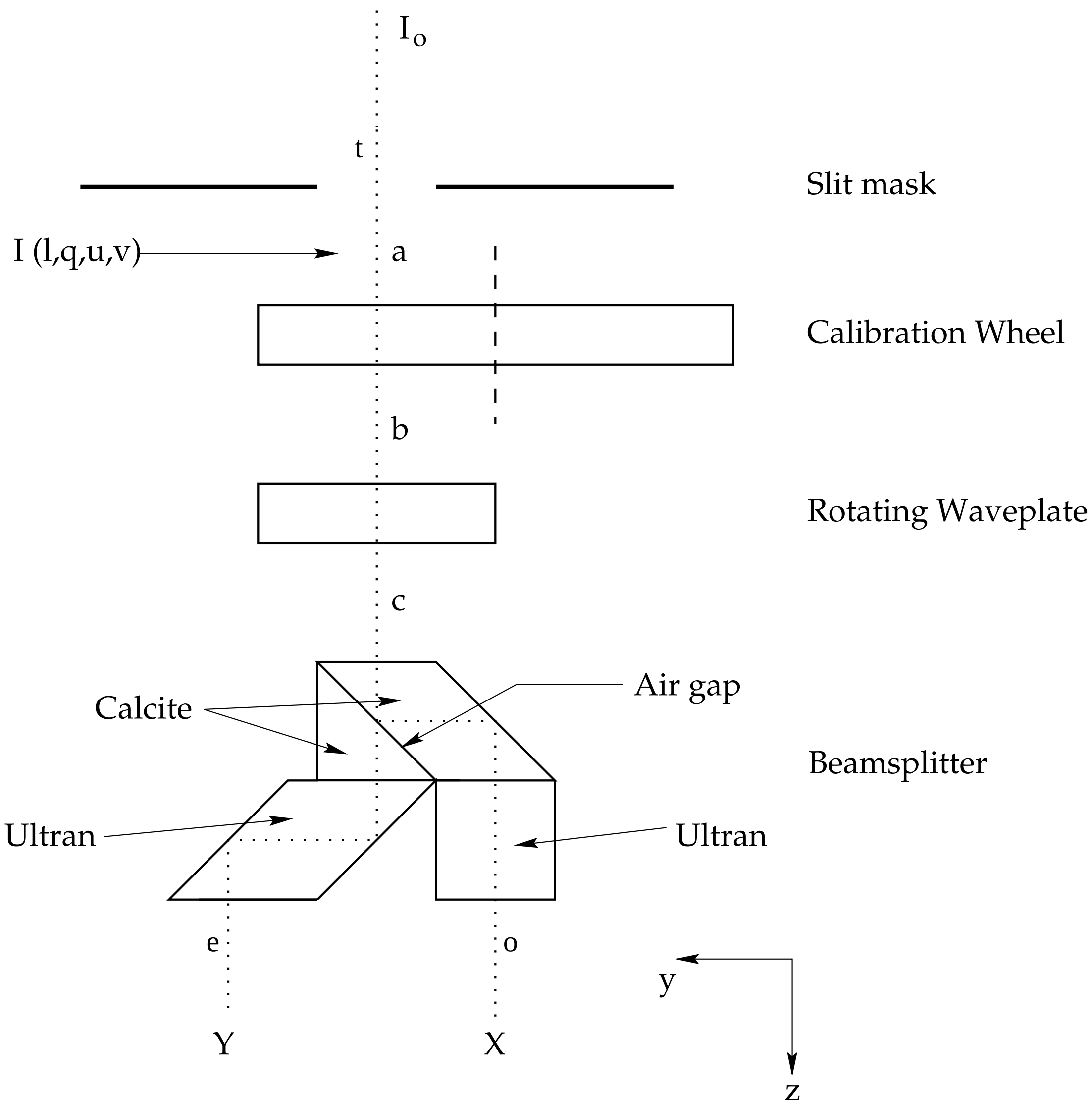

Figure 1: Schematic outline of the LRIS polarimeter.

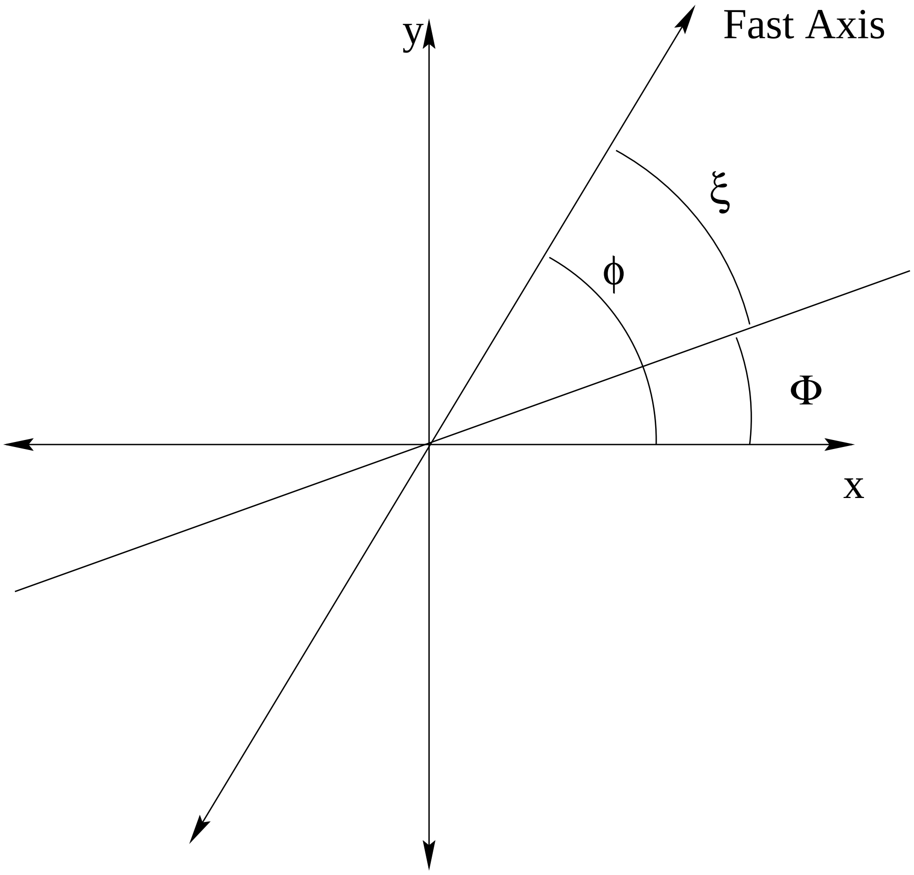

Figure 2: Coordinate system looking up. Φ is the nominal setting of the waveplate. The slit is along the y axis, x points from the optic axis to the slit and z is down.

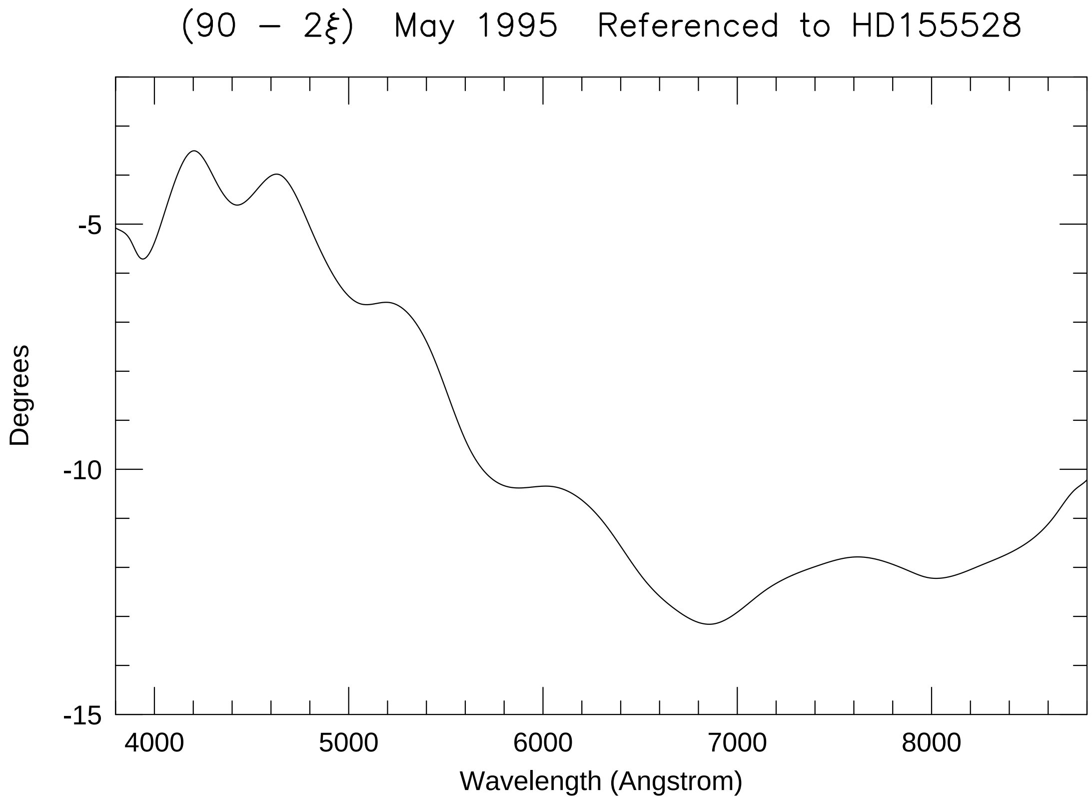

Figure 3: Instrumental correction to the measured raw position angle θ'. The wavelength dependence is inherent in the Pancharatnam design of the waveplate.

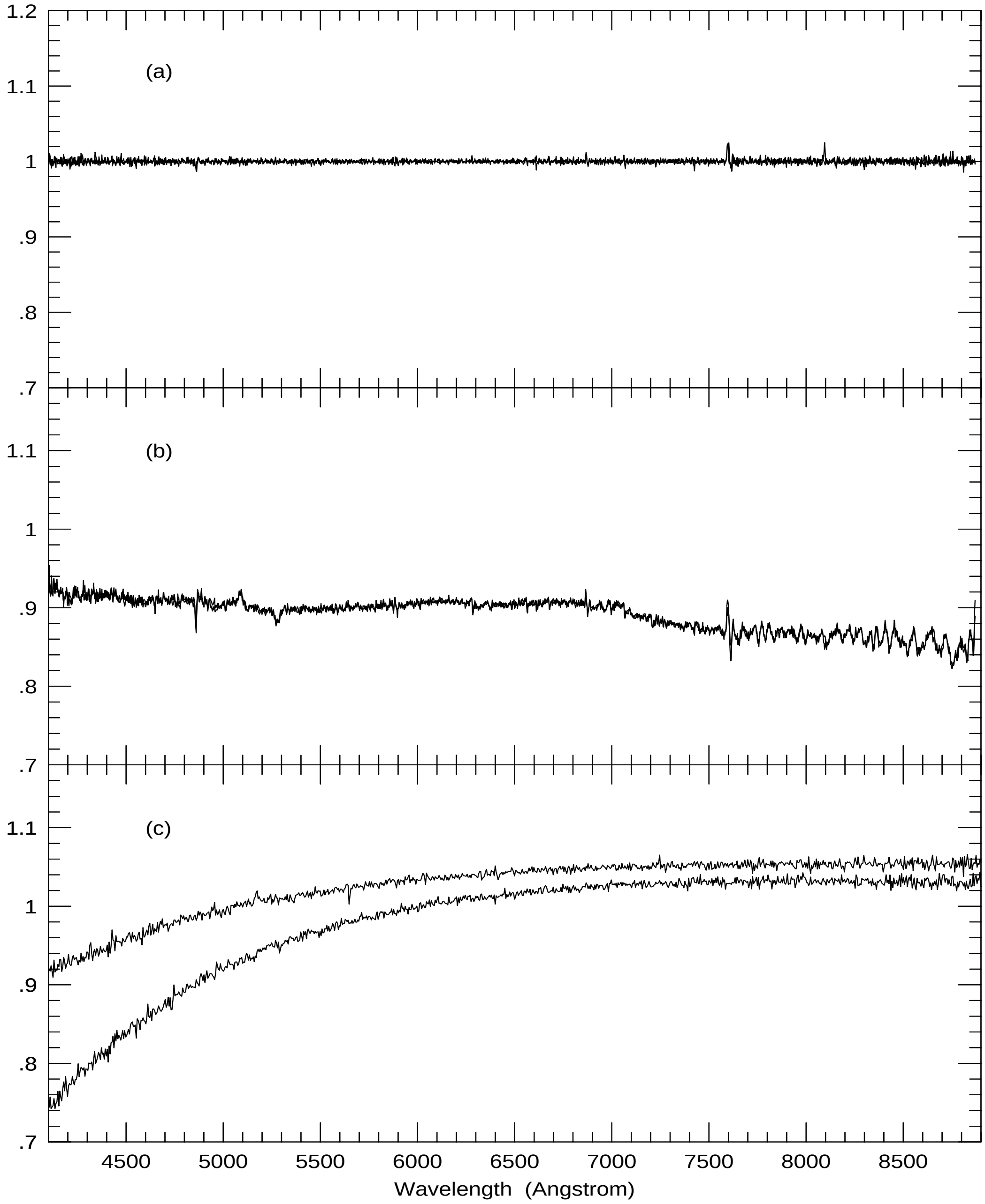

Figure 4: Examples of Ω and G (Eqs. 3 and 4). In each panel there are two curves, one for the q pair of exposures and the second for u. (a) Ω for Hiltner 102, August 1994; (b) G for Hiltner 102, August 1994; (c) Ω for Hiltner 102, October 1994. The exposures for (c) were taken through cirrus and evidently the atmospheric effects were wavelength-dependent and variable. The total elapsed time in (c) was 147 sec.

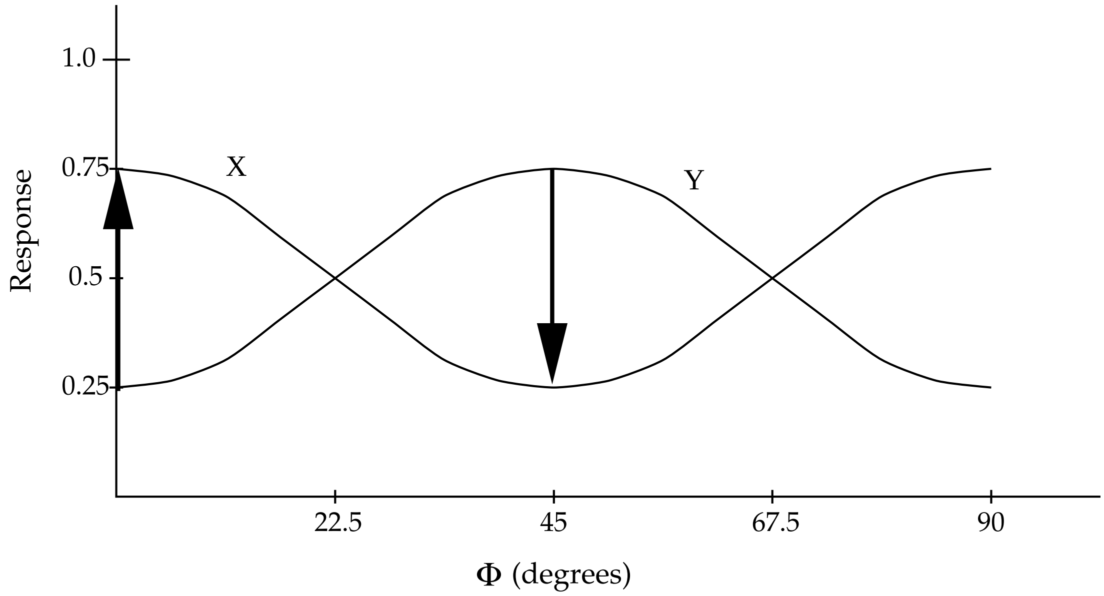

Figure 5: Variation of X and Y with Φ for an incident wave with q′=0.5, u′=0.0.

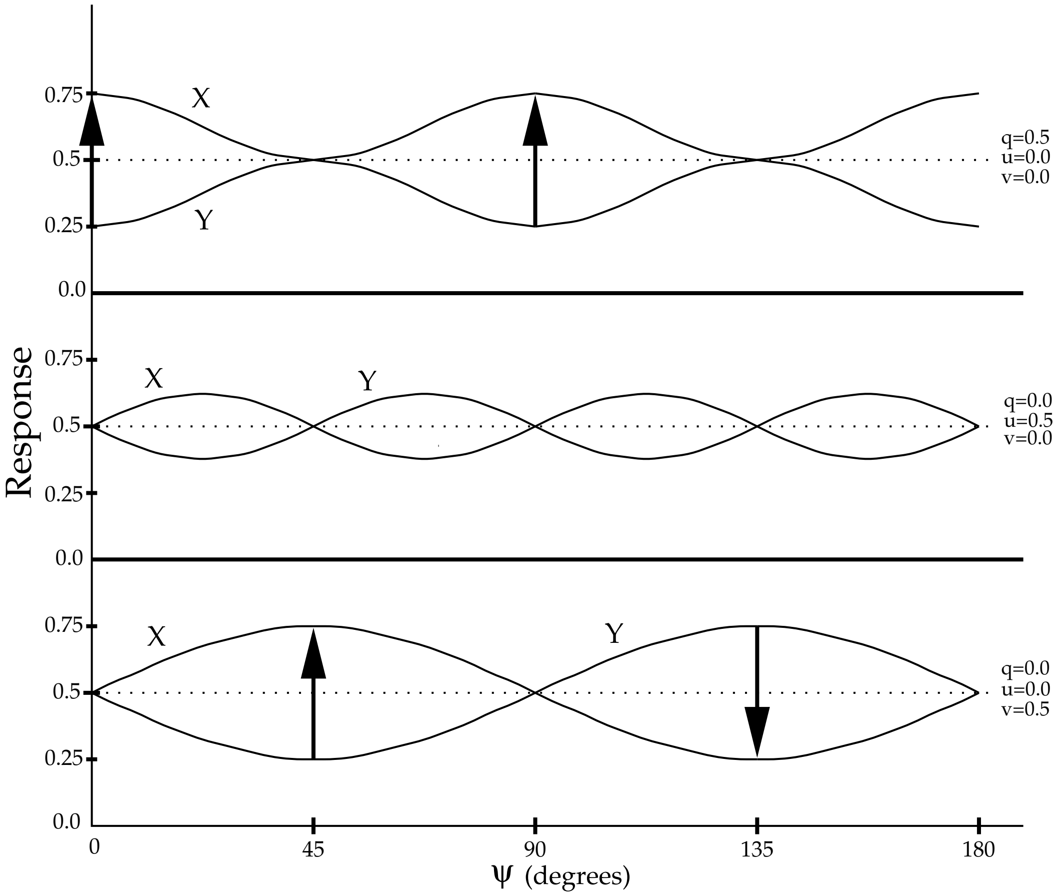

Figure 6: Response of the polarimeter to a rotating quarter-wave plate. The three panels are for incident waves with pure q, u and v Stokes parameters.

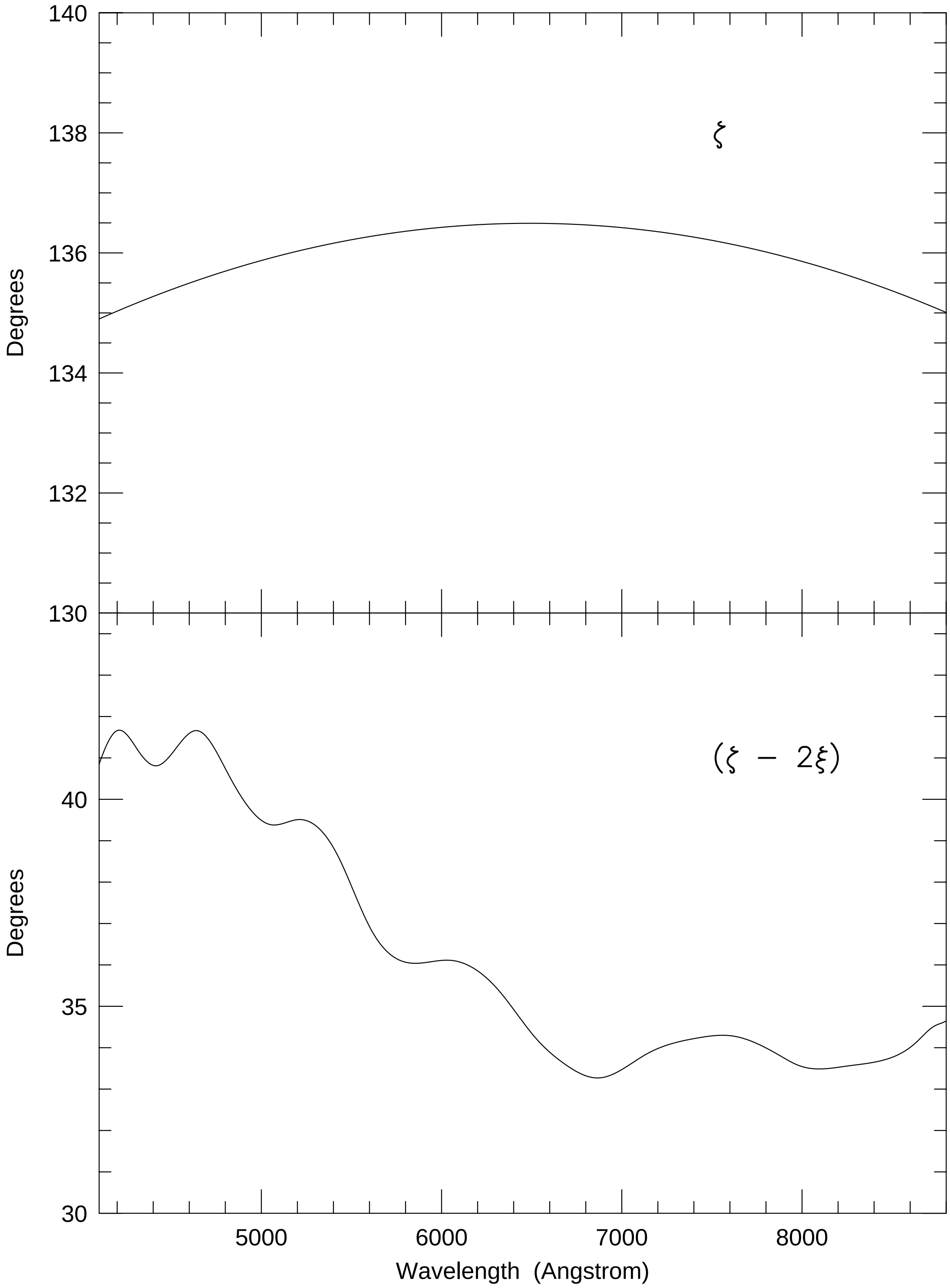

Figure 7: (Top) ζ, the position angle of the fast axis of the quarter-wave plate when it is mounted in the calibration wheel. (Bottom) (ζ-2ξ), the rotation angle needed in Eq. (13).

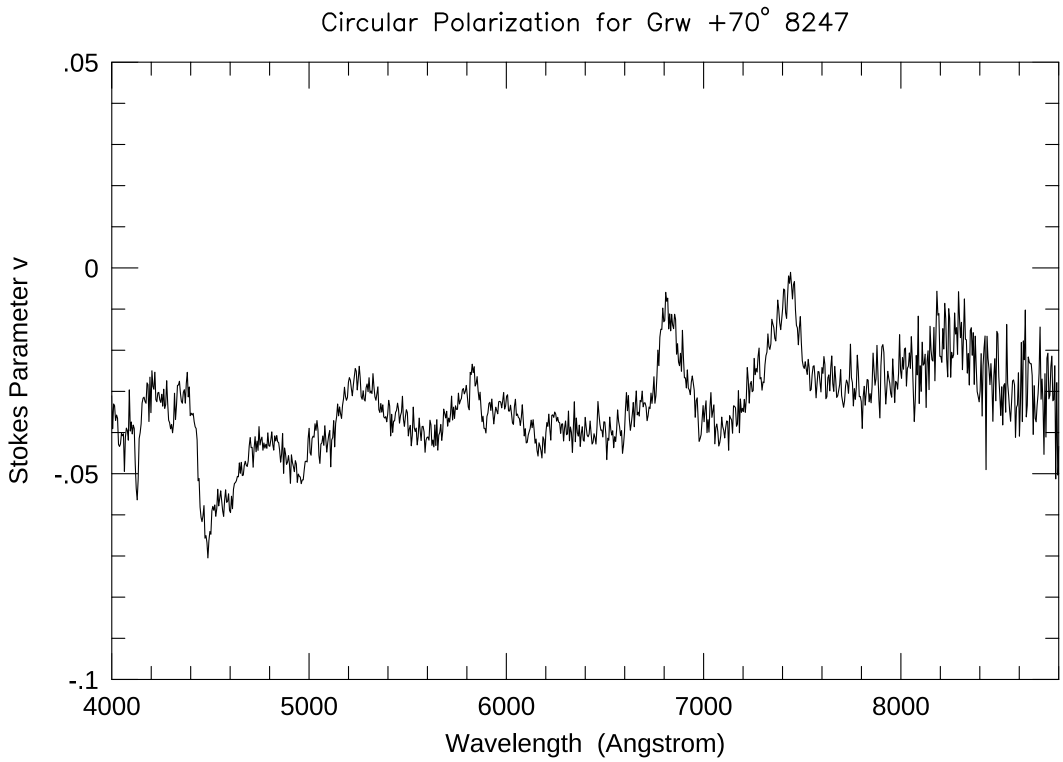

Figure 8: Stokes parameter v for the magnetic white dwarf Grw +70^{o} 8247, measured at the Keck telescope October 29, 1994.