Detection of the Schwarzschild precession in the orbit of the star S2 near the Galactic centre massive black hole

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ABSTRACT

The star S2 orbiting the compact radio source Sgr A* is a precision probe of the gravitational field around the closest massive black hole (candidate). Over the last 2.7 decades we have monitored the star’s radial velocity and motion on the sky, mainly with the SINFONI and NACO adaptive optics (AO) instruments on the ESO VLT, and since 2017, with the four-telescope interferometric beam combiner instrument GRAVITY. In this paper we report the first detection of the General Relativity (GR) Schwarzschild Precession (SP) in S2’s orbit. Owing to its highly elliptical orbit (e = 0.88), S2’s SP is mainly a kink between the pre- and post-pericentre directions of motion ≈ ±1 year around pericentre passage, relative to the corresponding Kepler orbit. The superb 2017-2019 astrometry of GRAVITY defines the pericentre passage and outgoing direction. The incoming direction is anchored by 118 NACO-AO measurements of S2’s position in the infrared reference frame, with an additional 75 direct measurements corresponding Kepler orbit. The superb 2017-2019 astrometry of GRAVITY defines the pericentre passage and outgoing direction. The incoming direction is anchored by 118 NACO-AO measurements of S2’s position in the infrared reference frame, with an additional 75 direct measurements of the S2-Sgr A* separation during bright states (‘flares’) of Sgr A*. Our 14-parameter model fits for the distance, central mass, the position and motion of the reference frame of the AO astrometry relative to the mass, the six parameters of the orbit, as well as a dimensionless parameter \( \delta f_p \) for the SP (\( f_p = 0 \) for Newton and 1 for GR). From data up to the end of 2019 we robustly detect the SP of S2, \( \delta f_p \approx 12' \) per orbital period. From posterior fitting and MCMC Bayesian analysis with different weighting schemes and bootstrapping we find \( f_p = 1.10 \pm 0.19 \). The S2 data are fully consistent with GR. Any extended mass inside S2’s orbit cannot exceed \( \approx 0.1\% \) of the central mass. Any compact third mass inside the central arc-second must be less than about 1000 \( M_\odot \).

Key words. black hole physics – Galaxy: nucleus – gravitation – relativistic processes.

1. Introduction

1.1. Testing GR and the Massive Black Hole Paradigm

The theory of General Relativity (GR) continues to pass all experimental tests with flying colours (Einstein1916; Will2013). High-precision laboratory and solar system experiments, and observations of solar mass pulsars in binary systems (Kramer et al.2006; Kramer2016) have confirmed GR in the low curvature regime. Gravitational waves from several stellar mass, black hole (sBH) candidate in-spirals with LIGO (Abbott et al.2016) have tested the strong curvature limit.

GR predicts black holes, that is, space-time solutions with a non-spinning or spinning central singularity cloaked by a communication barrier, an event horizon (c.f. Schwarzschild1916; Kerr1965). The LIGO measurements mentioned above provide presently the best evidence that the compact in-spiralling binaries are indeed merging sBHs, but see Cardoso & Pani (2019).

Following the discovery of quasars (Schmidt1963), evidence has been growing that most massive galaxies harbour a central compact mass, perhaps in the form of a massive black hole (MBH: \( 10^6 \approx 10^{10} M_\odot \)) (Lynden-Bell & Rees1971; Kormendy & Ho2013; McConnell & Ma2013). Are these compact mass concentrations truly MBHs as predicted by GR? Evidence in favour comes from relativistically broadened, redshifted iron Kα line emission in nearby Seyfert galaxies (Tanaka et al.1995; Fabian et al.2000), from stellar or gas motions very close to them (e.g. Moran et al.1999), and high resolution millimetre imaging (Event Horizon Telescope Collaboration et al.2019).

The nearest MBH candidate is at the centre of the Milky Way (\( R_0 \approx 8 \) kpc, \( M_\odot \approx 4 \times 10^6 M_\odot \)) (Genzel et al.2010; Ghez et al.2008). It is coincident with a very compact, variable X-ray, infrared and radio source, Sgr A*, which in turn is surrounded by a very dense cluster of orbiting young and old stars. Radio and infrared observations have provided detailed information on the distribution, kinematics and physical properties of this nuclear star cluster and hot, warm and cold interstellar gas interspersed in it (c.f. Genzel et al.2010; Morris et al.2012; Falcke & Markoff2013). Groups in Europe at the ESO NTT & VLT and in the USA
at the Keck telescopes have carried out high-resolution imaging and spectroscopy of the nuclear star cluster over the past two and a half decades. They determined increasingly precise motions for more than 100 stars, and orbits for \( \approx 50 \) (Schödel et al. 2002; Ghez et al. 2003, 2008; Eisenhauer et al. 2005; Gillessen et al. 2009b; Schödel et al. 2009; Meyer et al. 2012; Boehler et al. 2016; Fritz et al. 2016; Gillessen et al. 2017). These orbits, in particular the highly eccentric orbit of the \( m_k \approx 14 \) star S2 (or ‘S02’ in the UCLA nomenclature), have demonstrated that the gravitational potential is dominated by a compact source of \((4.25 \times 10^6 M_\odot)\), concentrated within \( S2 \)’s pericentre distance. S2 appears to be a slowly rotating, single, main sequence B-star of the gravitational potential. While the RS occurs solely in wavelength-space, the superior astrometric capabilities of GRAVITY serve to set much tighter constraints on the orbital geometry, mass and distance, thus decreasing the uncertainty of \( f_{RS} \) more than three times relative to data sets constructed from single telescope, AO imaging and spectroscopy.

In the following we report the first measurement of the next relativistic effect in S2’s orbit, namely the in-plane, prograde precession of its pericentre angle, the Schwarzschild precession (Misner et al. 1973).

2. Observations

Following on from Gravity Collaboration et al. (2018a, 2019) we expand in this paper our analysis of the positions and K-band spectra of the star S2 by another year, to fall of 2019. This yielded five additional NACO points, six SINFONI points and, especially, eleven crucial GRAVITY points. We now have

- 118 measurements with the VLTI assisted infrared camera NACO (Lenzen et al. 1998; Roussel et al. 1998) between 2002 and 2019.7 of the position of S2 in the K or H-bands, relative to the ‘Galactic Centre infrared reference system’ (Plewa et al. 2015, rms uncertainty \( \approx 400 \) mas). This means that between the 2002.33 pericentre passage until 2019.7 we have seven to sixteen NACO positional measurements per year. Between 1992 and 2002 we also used the speckle camera SHARP @ NTT (Hofmann et al. 1993), but the astrometry of the speckle data on a 3.5 m telescope is an order of magnitude worse than the AO imagery on the 8 m VLTI (rms uncertainty \( \approx 3.8 \) mas);

- 75 NACO measurements between 2003.3 and 2019.7 of the direct S2-Sgr A* separation during bright states of Sgr A* (typical rms uncertainty 1.7 mas);

- 54 GRAVITY measurements between 2016.7 and 2019.7 of the S2-Sgr A* separation (rms uncertainty \( \approx 65 \) mas). During the pericentre passage year 2018 the sampling was especially dense with 25 measurements;

- 92 spectroscopic measurements of the 2.167 \( \mu \)m HI (Bry) and the 2.11 \( \mu \)m HeI lines between 2003.3 and 2019.45 with the AO assisted integral field spectrometer SINFONI at the VLTI (Eisenhauer et al. 2003; Bonnet et al. 2003), with an uncertainty of \( \approx 12 \) km/s (Gravity Collaboration et al. 2019). This means that we typically have three to six spectroscopic measurements per year, and more than 20 in 2018. We also added two more NACO AO slit spectroscopic measurements from 2003, and three more Keck-NIRC2 AO spectroscopic measurements between 2000 and 2002 (Do et al. 2019).

The SHARP/NACO data deliver relative positions between stars in the nuclear star cluster, which then are registered in the radio frame of the Galactic Centre (Reid et al. 2009; Reid & Brunthaler 2020) by multi-epoch observations of nine infrared stars common between the infrared and radio bands. Another important step is the correction of spatially variable image distortions in the NACO image, which are obtained from observations of an HST-calibrated globular cluster (Plewa et al. 2015). The radio calibrations still allow for a zero-point offset and a drift of the radio-reference frame centered on Sgr A* (strictly speaking on the mass-centroid) with respect to the infrared reference frame, which we solve for empirically in our orbit fitting. For this purpose we use the Plewa et al. (2015) radio-to-infrared reference frame results as a prior (\( \chi = -0.2 \pm 0.2 \) mas, \( \gamma = 0.1 \pm 0.2 \) mas, \( \nu_{\chi} = 0.05 \pm 0.1 \) mas/yr, \( \nu_{\gamma} = 0.06 \pm 0.1 \) mas/yr). These reference frame parameters (\( \chi_0, \gamma_0, \nu_{\chi0}, \nu_{\gamma0} \)) are now the limiting factor in the precision of the detection of S2’s SP.
The situation is different for GRAVITY. Here we detect and stabilize the interferometric fringes on the star IRS16C located ≈ 1″ NE of Sgr A*, and observe S2 or Sgr A* within the second phase-referenced fibre (see [Gravity Collaboration et al. 2017]), such that the positional difference between S2 and Sgr A* can be determined to < 100 mas levels (see Appendix A.1). To obtain this accuracy the measurements of S2 and Sgr A* are made within a short time interval and linked together interferometrically (Appendix A.2). Between the end of 2017 and throughout 2018, S2 and Sgr A* are simultaneously detected in a single fibre-beam positioning as two unresolved sources in > 95% of our individual integrations (5 mins each), such that the S2-Sgr A* distance is even more directly obtained in each of these measurements (Appendix A.3). The development over time of the astrometric and spectroscopic measurement uncertainties are summarized in Figure A.2. For more details on the data analysis of all three instruments we refer the reader to Gravity Collaboration et al. [2017, 2018ab, 2019] and Appendix A.

3. Results

3.1. Schwarzschild precession in the S2 orbit

Figure 1 shows the combined single-telescope and interferometric astrometry of the 1992-2019 sky-projected orbital motion of S2 and the line-of-sight velocity of the star.

The almost hundred-fold improvement of statistical astrometric measurement precision in the last 27 years is one key for detecting the SP in the S2 orbit. As discussed in Section 2, the accurate definition of the reference frame for the NACO data is the second key. The robustness of the detection of the SP strongly correlates with the precision of knowing \((\Delta_0,\Delta_0,\nu_0,\nu_0,\nu_0,\nu_0)\), as this sets the angle of the orbit at the last apocentre (2010.35). Using the priors from Plewa et al. [2015], we fit these four reference frame parameters in our posterior fitting, but the additional constraints obtained from Sgr A*-S2 flare offsets in NACO turn out to be very helpful. To this end, we include in the calculation of \(\chi^2\) the constraint that the flare positions are tracing the mass centre.

Confusion of S2 with nearby other sources is the final key issue (see also Gillessen et al. [2009b, 2017], Plewa & Sari [2018, 2019], Do et al. [2019], Ghez et al. [2003] and Schödel et al. [2002] already had noted that the NACO/NIRC2 AO astrometry at times was unreliable and biased over longer periods of time (0.5 – 1.5 years). These systematic position excursions turn out to be mainly caused by confusion, i.e. the positional pulling of the apparent sky position of S2 by a passing nearby background object. This issue is especially detrimental when the variable Sgr A* emission source is within the diffraction limit of the telescope (Ghez et al. [2003, 2005], Plewa & Sari [2018, 2019]), making the 2002 and 2018 AO astrometry more uncertain or even unusable. Fortunately GRAVITY removed any need for AO imagery during the 2018 pericentre passage, so we excised most of the 2002 and 2018 NACO astrometry from our data set. We identified further confusion events with fainter stars passing close to S2 on a number of occasions (e.g. 1998, 2006, 2013/2014) and removed these questionable data points.

At pericentre \(R_{\text{peri}}\), S2 moves with a total space velocity of \(\approx 7700\) km/s, or \(\beta = v/c = 2.56 \times 10^{-2}\). The SP of the orbit is a first order \((\ell^{2N}, N = 1)\) effect in the parameterized post-Newtonian (PPN, c.f. Will & Nordvedt [1972]) expansion, PPN(1) ≈ \(\beta^2 \approx R_S / R_{\text{rem}} \approx 6.6 \times 10^{-2}\). We use the post-Newtonian expansion of Will [2008] and add a factor \(f_{\text{SP}}\) in the equation of motion in front of the Schwarzschild related terms (see Appendix B). This corresponds to (e.g. Misner et al. [1973])

\[
\Delta \phi_{\text{per orbit}} = f_{\text{PPN1SP}} = \frac{3 \pi R_S}{a (1 - e^2)} \text{ for } S2 \approx f_{\text{SP}} \times 12.1'.
\]

Here \(a\) is the semi-major axis and \(e\) the eccentricity of the orbit. The quantity \(f_{\text{SP}}\) can then be used as a fitting parameter, similar to our approach for the RS (Gravity Collaboration et al. [2018b, 2019]). Appendix B explains what effects the SP should have on the measured parameters of S2’s orbit.
3.2. Posterior Analysis

The six parameters describing the Kepler orbit \((a,e,i,\omega,\Omega,b_0)\), the distance and central mass, and the five coordinates describing the position on the sky and the three-dimensional velocity of the reference frame (relative to the AO spectroscopic/imaging frame) all have uncertainties. In particular, distance and mass are uncertain and correlated. Following Gravity Collaboration et al. (2018a, 2019) we find the best-fit value of the parameter \(f_{SP}\) a posteriori, including all data and fitting for the optimum values of all parameters with the Levenberg-Marquardt \(\chi^2\)-minimization algorithm (Levenberg 1944; Marquardt 1963), including prior constraints. It is essential to realize that the inferred measurement uncertainties are affected and partially dominated by systematic effects, especially when combining the evidence from three or more, very different measurement techniques.

Figures 2 and 3 show the fit results when fitting simultaneously the data and the flare positions. As priors we used the Plewa et al. (2015) reference frame results (see Section 2). All data prior to the 2018.3 pericentre passage are fit by \(f_{SP} \approx 0\) (Kepler/Newton, plus Rømer effect, plus SRT, plus RS). The residuals in this period are consistent with 0 (bottom panels of Figure 2). The GRAVITY data between 2017 and 2019.7 clearly show that the post-pericentre orbit exhibits a sudden kink, mainly in RA. The data are compatible with a pure in-plane precession. This is shown in the upper right panels of Figures 2 and 3, where we have computed the residuals in the projected angle of the SP-fitted orbit on the sky \(\delta \phi (r)\), relative to the \(f_{SP} = 0\) orbit. This is exactly as expected from a \(f_{SP} \approx 1\) GR orbit (Figure B.2). The more subtle swings in \(\delta RA, \delta \phi, \delta v_z\) and \(\delta \phi\) predicted by GR (Figure B.2) are detected as well (see Appendix B for a more detailed discussion).

Table E.1 lists the best fit parameters and their 1\(\sigma\) uncertainties. Depending on the weighting of different data sets and the choice of priors we find that the best fitting \(f_{SP}\) parameter varies between 0.9 and 1.2, with a fiducial value of \(f_{SP} = 1.1\). The formal statistical fit uncertainty of this parameter does not depend much on the selection of astrometric and spectroscopic data of S2. The value of its rms uncertainty \(\chi^2\) does depend on the methodology of error treatment. The distribution of the NACO flare position residuals shows significant non-Gaussian outliers. There are \(\approx 6\) (or 75 data points) \(> 4\sigma\) outliers above the rms of \(\approx 1.7\) mas. If the \(\chi^2\)-distribution and the weighting of these points are treated as if they had a normal distribution, the reduced \(\chi^2\) of our overall fits is driven up to \(\approx 1.65\), for a total \(\chi^2\) of 995. In that case \(\Delta f_{SP} = 0.204\). These outliers can be down-weighted by replacing the penalty function \(p(r) = r^2\) in the calculation of \(\chi^2 = \Sigma p((data-model)/error)\) with \(p(r,s) = r^2 \cdot s^2 / (r^2 + s^2)\), \(s = 10\). This introduces a soft cut-off around 10\(\sigma\) in how much a data point can maximally contribute to the \(\chi^2\). With this scheme, \(\chi^2\) of the overall fit drops to 1.50, and \(\Delta f_{SP} \approx 0.194\).

We also fitted the data by solving simultaneously for \(f_{SP}\) and \(f_{RS}\), without fixing the RS term to 1. In that case we find \(f_{SP} = 0.99 \pm 0.24\) and \(f_{RS} = 0.965 \pm 0.042\) (with the outlier damper on), again fully consistent with GR.

An alternative approach is to put the reference frame constraints obtained from the flare positions into a combined prior with the one from Plewa et al. (2015). In that case the prior for the location of Sgr A* in the NACO infrared frame is \(x_0 = -0.42 \pm 0.15\) mas, \(y_0 = 0.30 \pm 0.15\) mas, \(v_{x0} = -0.02 \pm 0.05\) mas/yr, \(v_{y0} = 0.015 \pm 0.05\) mas/yr. If we use this prior to fit only the S2 data, we obtain \(f_{SP} = 0.92 \pm 0.22\) and \(\chi^2 = 0.88\) (\(\chi^2 = 398\)).
Fitting the orbit with \( f_{SP} = 0 \) fixed yields \( \chi^2 = 932.3 \), compared to 906.4 with \( f_{SP} = 1 \) fixed. The corresponding difference in Bayesian information criterion (Claeskens & Hjort 2008) \( \Delta \text{BIC} = 25.9 \) yields very strong evidence that the GR model describes the data better than the best-fitting Kepler (with SRT, RS, and Rømer delay included) orbit.

Gravity Collaboration et al. (2018) showed that the near-IR emission of Sgr A* during bright flares exhibits clock-wise loop motions of excursions 50–100 \( \mu \)as. The typical flare duration and the orbital time scale are \( \approx 1 \) hour. A stationary offset between the infrared emission and the mass centroid of that size would induce a change of up to \( \pm 0.2 \) in \( f_{SP} \), which is comparable to the overall uncertainty in the SP parameter. During a typical time of several hours making up a GRAVITY data point in this work, these fluctuations should average out to less than 10 \( \mu \)as such that the additional error on \( f_{SP} \) is well below the statistical error.

Next we carried out a Markov-Chain Monte Carlo analysis. Using 200,000 realizations we find that the distribution of \( f_{SP} \) is well described by a Gaussian centred on \( f_{SP} = 1.11 \pm 0.21 \) (Figure 2 and see Appendix B for more details). The largest relative uncertainty in the determination of the Schwarzschild term originates in the degeneracy of \( f_{SP} \) with the pericentre time (see Appendix B) and with the zero point \( x_0 \) of the long-term reference frame (mass vs. NACO imaging coordinates). This is not surprising, given that the precession is largest in the EW direction.

Further we compared our first order post-Newtonian code with fully relativistic GR orbits using the GYOTO ray-tracing code \( ^1 \) (Vincent et al. 2011; Grould et al. 2017). As expected, the deviations are small. The largest differences over the full data range are \( \Delta RA = 62 \mu \text{as}, \Delta \text{Dec} = 41 \mu \text{as} \) and \( \Delta v_z = 11.4 \text{km/s} \), occurring for a short time around pericentre. Also, the Bayesian comparison between the best-fitting full-GR and Kepler (with SRT, RS, and Rømer delay included) orbits strongly prefers the GR model.

Finally, we also included the data from Do et al. (2019) (excepting the 2018 astrometry) using the scheme in Gillessen et al. (2009a) allowing for an additional offset in position and velocity for the Keck reference system. The 18-parameter fit yields a consistent result, but no further improvement.

4. Conclusions

We have presented the first direct detection of the in-plane Schwarzschild precession around Sgr A* in the GC. Our results are fully consistent with GR. We detect the precession of S2 robustly at the 5 to 6\( \sigma \) level in posterior fitting and MCMC analysis. Our result is independent of the fit methodology, data selection, weighting and error assignments of individual data points. The significance of our result depends mostly on how accurately we can constrain \( (x_0, y_0, v_{x0}, v_{y0}) \). The success rests crucially on the superior GRAVITY astrometry during and past pericentre passage on the one hand, and on 75 measurements of the Sgr A*-S2 separation for the NACO AO data between 2003 and 2019 on the other. The flare data allow us to independently constrain the zero point of the NACO reference frame.

Additional masses in the GC would lead to Newtonian perturbations of the S2 orbit. An extended mass component (for example composed of stars or remnants, but also of other particles) would result in a retrograde precession. The presence of a second, massive object would lead to short excursions in the smooth orbit figure. Our data place tight constraints on both, which we detail in Appendix D.

We expect only modest further improvement of the significance of our result as our monitoring continues and S2 moves away from pericentre, since our result already now is limited by the precision with which we have measured the pre-pericentre orbit with AO data.

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Appendix A: Experimental Techniques

Appendix A.1: GRAVITY Data Analysis

Our result crucially depends on the use of GRAVITY, the VLTI beam combiner, which due to its extremely high angular resolution of \( \approx 3 \text{ mas} \) yields very accurate astrometry with errors well below 100 \( \mu \text{as} \) (Gravity Collaboration et al. 2017 and Figure A.3 next page). Depending on the separation between S2 and Sgr A* there are two fundamentally different ways to retrieve the separation vector between S2 and Sgr A*.

Dual Beam Method. For separations larger than the single-telescope beam size (FWHM \( \approx 60 \text{ mas} \)), one needs to point the GRAVITY science channel fibre once to Sgr A* and once to S2, such that the respective target is the dominant source in the field. The phases of the complex visibilities of each pointing then yield an accurate distance to the fringe-tracking star, IRS16C in our case. By interferometrically calibrating the Sgr A* data with S2, the position of IRS16C drops out, and one gets a data set in which the six phases \( \Phi_i \) directly measure the desired separation vector \( \mathbf{s} = (\Delta \text{RA}, \Delta \text{Dec}) \) between S2 and Sgr A* via the basic interferometer formula for a unary model

\[
\Phi_{i,j} = 2\pi \mathbf{s} \cdot \mathbf{B}_i / \lambda_j ,
\]

where \( \mathbf{B}_i \) denotes the \( i \)-th of the six baselines. Since our data are spectrally resolved into 14 channels \( \lambda_j \) across the K-band (2.0 \( \mu \text{m} \) to 2.4 \( \mu \text{m} \)), the unknown \( s \) with two parameters is well constrained by a fit to the phases. This method applies mostly to the 2019 data, and partly to the 2017 data. In Figure A.1 we show an example for such a unary fit for one Sgr A* exposure.

Single Beam Method. For separations below the single-telescope beam size both sources are observed simultaneously and appear as an interferometric binary. In such a case, the amplitudes of the complex visibilities as well as the closure phases carry the signature, which is a beating pattern in each baseline along the spectral axis. We fit a binary model to these data, for which the complex visibilities are:

\[ C_{ij} = \frac{I_E + \sqrt{f_k f_l I_C}}{\sqrt{I_A + f_k f_B + f_BG I_D}} \sqrt{I_A + f_l f_B + f_BG I_D} . \]

In this expression we use the abbreviations

\[
I_A = I(\alpha_{\text{Sgr}}, 0) ,
I_B = I(\alpha_{S2}, 0) ,
I_C = I(\alpha_{S2}, \text{OPD}_{S2}) ,
I_D = I(\alpha_{BG}, 0) ,
I_E = I(\alpha_{Sgr}, \text{OPD}_{Sgr}) ,
\]

where

\[
I(\alpha, \text{OPD}) = \int_{\Delta \lambda} P(\lambda) I_{22}^{\text{e}} \cdot e^{-2\pi \text{OPD}/\lambda} d\lambda .
\]

The function \( P(\lambda) \) is the spectral band pass, for which we use a top hat function with a width corresponding to the measured spectral resolution. The \( f_k, f_i \) are the flux ratios of S2 to Sgr A* for telescope \( k, l \); \( f_{\text{BG}} \) is the flux ratio of unresolved background to Sgr A* flux. The model yields for all baselines and spectral channels a complex visibility, of which we fit the amplitudes and closure phases to the data. We used this analysis also in our previous work (Gravity Collaboration et al. 2018a,b) and also here for the 2018 data and for the 2017 data. In Figure A.2 we show an example how the binary model describes the visibility amplitudes for one exposure.

Appendix A.2: Details of the Unary Model Fits

The aim is to measure the separation vector between S2 and Sgr A*. GRAVITY measures the separation between science object and fringe-tracking star (IRIS16C in our case). The desired separation is obtained by measuring both S2 and Sgr A* with respect to IRS16C, and subtracting the two measurements.
This corresponds to interferometrically calibrating the phases of Sgr A* with those of S2.

By construction, the phases of the calibrator S2 frame are identical to 0, and ideally the phases for all other S2 frames are 0, too. In reality, this is not the case. At the time of observing, the separation vector \( r \) between fringe-tracking star IRS16C and S2 needs to be provided to GRAVITY for tracking the fringes with the differential delay lines. At that point, \( r \) is not known to the interferometric precision, but only from the AO-data based orbital motion of IRS16C (Gillessen et al. 2017). Hence, for a subsequent S2 file, when the projected baselines have changed by some value \( \Delta B \) due to Earth rotation, the error in pointing \( \Delta r \) leads to an additional phase \( \Delta \Phi = \Delta B \cdot \Delta r \). By observing S2 a few times per night, one obtains a set of constraints for \( \Delta r \), which allows fitting for \( \Delta r \) over the course of the night.

Therefore we can correct our data post-facto for this offset \( \Delta r \), and obtain phases for Sgr A* which directly relate to S2.

The GRAVITY pipeline determines the normalized interferometric visibility from the correlated flux of two telescopes divided by their respective individual fluxes. The field-dependent aberrations enter the Van Cittert-Zernicke theorem as

\[
\Psi_k = E_0 A_k^{\text{off}} \exp\left\{ i \omega t + i s \cdot x_k + i \Phi_k^{\text{off}} \right\},
\]

where \( k \) labels the telescope and \( x_k \) denotes its position, \( E_0 \) is the amplitude of the unperturbed electric field and \( s \) the source’s position on the sky. The scaling in amplitude \( A_k^{\text{off}} \) and the phase shift \( \Phi_k^{\text{off}} \) are functions of the source position with respect to the field centre and differ for each telescope.

The GRAVITY pipeline determines the normalized interferometric visibility from the correlated flux of two telescopes divided by their respective individual fluxes. The field-dependent aberrations enter the Van Cittert-Zernicke theorem as

\[
V_{ij} = \int \frac{I(\sigma) A_{ij}^{\text{off}}(\sigma) A_{kl}^{\text{off}}(\sigma)}{\sqrt{\int I(\sigma) \left( A_{ij}^{\text{off}}(\sigma) \right)^2 d\sigma \times \int I(\sigma) \left( A_{kl}^{\text{off}}(\sigma) \right)^2 d\sigma}} d\sigma,
\]

where \( I(\sigma) \) is the source intensity distribution and \( b_{ij} \) is the projection of the baseline vector onto the plane perpendicular to the line of sight. The expression for a binary system follows from this equation and generalizes equation (A.2). The integrals in equation (A.3) read then as

\[
I(\alpha, \beta) = \int P(\lambda) I_{2\lambda}^{-1/2} e^{-2\pi i \Phi(\alpha, \beta)} d\lambda,
\]

and the flux ratios \( f_i, f_j \) in equation (A.2) are multiplied with the ratio of \( A_{ij}^{\text{off}} \) for the two sources.

Hence, the refined binary fitting requires maps of the amplitude and phase distortion as additional input. We obtained those from dedicated calibration runs, using the GRAVITY calibration unit, which simulates the light of an unresolved source. The offset between this source and the fibre can be controlled, and we scanned the FOV in order to measure the relative changes in phase and amplitude across (see Figure A.4 for an example).

In contrast to an astronomical observation, our calibration data are not affected by the smoothing effects of the AO residuals. We account for the atmospheric smoothing by applying a Gaussian kernel to the phase maps. The typical tip/tilt jitter for observations of the GC has an RMS per-axis of \( \approx 15 \) mas (Perrin et al. 2018).
The effect of the static aberration does not average out, since the orientation of the field inside GRAVITY is always the same for our observations. Also the projected baselines are not drawn from full tracks in the uv-plane, but we rather have a typical observing geometry. We expect thus a bias. Indeed, in comparison to binary fits neglecting static aberrations, we find that the position of S2 is offset systematically throughout 2017 by approximately $0.44 \times (t - 2018) - 0.10$ mas in RA and $-0.86 \times (t - 2018) + 0.28$ mas in Dec. As expected, the offset decreases as S2 moves closer to Sgr A*.

Finally, we note that our result for $f_{SP}$ does not depend in a significant way on this correction.

**Appendix B: Theoretical expectations for the precession of S2**

The 12.1' precession angle predicted by GR corresponds to a spatial shift between the GR and the Kepler orbit of 0.78 mas at apocentre, mostly in RA because of the current orientation of the orbit. To detect this shift with 5σ significance requires a positional measurement precision of 100 μas or less. We have more than 100 NACO measurements of the orbit, each with a statistical precision of 400 μas. If we did not have systematics (offset and drift of the infrared to mass/radio references frames) it should thus (have) be(en) possible to detect the SP with NACO or the Keck NIRC imager alone. While S2’s motion on the sky could be detected with NACO over periods of months, the GRAVITY observations detect the star’s motion over 0.5 – 2 days.

The precession angle projected on the sky depends on the geometric angles of the orbit and thus, on time. In the plane of the orbit the precession advances the angle $\delta \phi$ by 12.1' per orbital period of 16.046 yrs. The precession projected on the sky varies from $-17'$ to $-8.4'$ through each half SP period of $P_{SP} = 28,710$ yr (Figure B.1).

Figure B.1 illustrates the effects the SP should have on the measured parameters of S2’s orbit. Because of the strong dependence of $\delta \phi$ on radius, much of the 12.1' precession occurs within ±1 year of pericentre. In RA/Dec space the precession is seen as a ‘kink’ in the time change of the post-pericentre vs. pre-pericentre residuals. Very near pericentre passage, the precession acts to first order as a time shift between the precessing and the equivalent $f_{SP} = 0$ orbit (see also figure [E.2] top left panel). As a result in the residuals $\delta \text{RA}$, $\delta \text{Dec}$, $\delta v_z$, and $\delta \phi$ between the data and the $f_{SP} = 0$ short term excursions of order a few times $\beta^2$ appear in all these observables.
Appendix C: Parameterization of the Schwarzschild precession

We use the post-Newtonian limit for the equation of motion presented in [Will 2005], equation (1) therein. We parameterize the effect of the Schwarzschild metric (i.e. the prograde precession) by introducing an ad-hoc factor $f_{SP}$ in front of the terms arising from the Schwarzschild metric. The terms due to spin $J$ and quadrupole moment $Q_2$ are set to 0. This results in

$$a = - \frac{GM}{r^3} r + f_{SP} \frac{GM}{c^2 r^2} \left[ \left( \frac{GM}{r} - v^2 \right) \frac{r^2}{r} + 4r v \right] + O[J] + O[Q_2] .$$

(C.1)

In the (first order) parameterized post-Newtonian (PPN) expansion of GR, the second term becomes

$$\frac{GM}{c^2 r^2} \left[ \left( 2(\gamma + \beta) \frac{GM}{r} - \gamma v^2 \right) \frac{r^2}{r} + 2(1 + \gamma) r v \right] .$$

(C.2)

In GR, $\beta_{GR} = \gamma_{GR} = 1$. Two PPN parameters ($\beta, \gamma$) are needed to describe the equation of motion, and for no choice of $\beta$ and $\gamma$ can one recover the Newtonian solution. Hence using the PPN formalism is less well suited for our experiment than using $f_{SP}$.

The net effect of the precession amounts to

$$\Delta \phi_{precl} = 3 f_{SP} \frac{\pi R_s}{a(1 - e^2)} ,$$

(C.3)

for our parameterization, and to

$$\Delta \phi_{precl} = (2 + 2\gamma - \beta) \frac{\pi R_s}{a(1 - e^2)} ,$$

(C.4)

in the PPN formulation of GR [Will 2014]. Yet it is imprecise to identify the factor $f_{SP}$ with $(2 + 2\gamma - \beta)$. Our parameter $f_{SP}$ characterizes how relativistic the model is, as is probably easiest seen by the fact that its effect corresponds to changing the value of the speed of light in the equations, with the limit $c \to \infty$ for $f_{SP} \to 0$.

In the PPN formulation of GR, all orbits with $\beta = 2(1 + \gamma)$ have zero net precession per revolution, and all orbits with $\beta = 2\gamma - 1$ have the same amount of pericentre advance as GR.

Due to the large eccentricity of the S2 orbit, the precession shows up as an almost instantaneous change of the orbit orientation $\omega$ in its plane when the star passes pericentre. Hence, our result essentially compares the orbit orientations post- and pre-pericentre 2018. In this limit, we can indeed state that we have
measured \((2 + 2\gamma - \beta)/3 = 1.10 \pm 0.19\). Figure [C.1] illustrates our constraint in the plane spanned by \(\beta\) and \(\gamma\). Since there is no exact representation of the Keplerian orbit in the PPN formalism, we instead seek that PPN orbit, that most closely resembles the Keplerian one. This depends on the eccentricity, and for S2 we find \(\gamma_{\text{Kep}} = -0.78762\) and \(\beta_{\text{Kep}} = 0.42476\). Hence, changing \(f_{\text{SP}}\) corresponds to moving along a line from \((\gamma_{\text{Kep}}, \beta_{\text{Kep}})\) to \((\gamma_{\text{GR}}, \beta_{\text{GR}})\). With this we find \(\beta = 1.05 \pm 0.11\) and \(\gamma = 1.18 \pm 0.34\), and the two are fully correlated.

### Appendix D: Astrophysical implications

There are several important astrophysical issues our measurements can contribute to.

**Distributed mass component inside the orbit of S2:** An extended mass component would create retrograde Newtonian precession. Our data strongly constrain such a component. For simplicity we use spherically symmetric distributions of the extended mass. Using a Plummer (1911) profile with a scale parameter of 0.3 arc-seconds (Mouawad et al., 2003) and fitting for the normalization of that mass component assuming \(f_{\text{SP}} = 1\) shows that \((0.00 \pm 0.10)\%\) of the central mass could be in such an extended configuration. Changing the radius parameter to 0.2 or 0.4 arc-seconds yields \((-0.02 \pm 0.09)\%\) or \((0.01 \pm 0.11)\%\). Using instead a power-law profile with logarithmic slope between \(-1.4\) and \(-2\) results in a mass estimate of \((-0.03 \pm 0.07)\%\). Overall, we estimate that for typical density profiles the extended mass component cannot exceed \(0.1\%\), or \(\approx 4000 M_\odot\) (1σ limits). For comparison, modelling of the star cluster suggests that the total stellar content within the apocentre of S2 is \(<1000 M_\odot\), and the mass of stellar black holes within that radius is \(80 - 340 M_\odot\) (Figure D.1). We conclude that the expected stellar content within the S2 orbit is too small to significantly affect the SP.

Merritt et al. (2010) investigated for which configurations the Newtonian precession due to an extended mass component in the form of individual stellar mass objects exceeds the effects of spin and quadrupole moment of the MBH. They addressed a range of masses between 1 and \(10^5 M_\odot\) in the central milli-parsec. The above limits translate into a limit of \(\approx 200 M_\odot\) in that radial range. Figure 1 of Merritt et al. (2010) shows that for S2 itself, our limit on the extended mass would lead to perturbations almost on par with the expected spin effects for a maximally spinning MBH, giving some hope that Sgr A*’s spin eventually can be detected from S2 despite its large orbital radius. Zhang & Iorio (2017) caution, however, that already the Newtonian perturbation from S55/S0-102 (Meyer et al., 2012) Gillessen et al. (2017) might hide the spin’s signature. For stars on shorter period orbits or with larger eccentricities, detecting the higher-order effects of the metric is easier; and stellar perturbations have a different observational signature than the effect of the metric.
The distance to the GC: Our data set continues to constrain $R_0$ ever better. Setting $f_{\text{sp}} = f_{\text{KS}} = 1$ fixed during the fit we obtain our best estimate for $R_0 = 8248.6 \pm 8.8 \, \text{pc}$, where the error is the statistical error only. This is 25% more precise than our result in Gravity Collaboration et al. (2019), yet the values differ by $\approx 2\sigma$ when taking into account the systematic error from Gravity Collaboration et al. (2019). We adopt now conservatively, with the improved data set, a systematic error of 45 pc, twice as large as before. This reflects better the variations between Gravity Collaboration et al. (2018a), Gravity Collaboration et al. (2019) and this work. Our current best estimate is thus $R_0 = 8249 \pm 9_{\text{stat}} \pm 45_{\text{sys}} \, \text{pc}$. Because of the strong correlation between the best fit MBH mass $M_\bullet$ and $R_0$ (Figure D.2), the increase in $R_0$ is reflected in $M_\bullet$.

Constraints on PPN parameters: In Appendix C we derived our constraints on the PPN parameters $\beta = 1.05 \pm 0.11$ and $\gamma = 1.18 \pm 0.34$. These are consistent with GR, but not competitive with the results obtained in the solar system from spacecraft measurements. The deviation from the GR value of $\gamma = 1$ is best constrained via the Shapiro delay from the Cassini spacecraft to better than $2 \times 10^{-6}$, while VLBI measurements of the light deflection using the quasars 3C273 and 3C279 yield a factor 10 weaker constraints on $\gamma$ (Will 2014). Assuming the value of $\gamma$ from Cassini, the SP of Mercury’s orbit from the Messenger spacecraft yields a constraint on $\beta$ of $8 \times 10^{-5}$. While our constraints are weaker, they probe a completely different regime in mass (by a factor $4 \times 10^4$) and potential strength (by a factor $10^2$ to $10^3$) than the solar system tests (Figure D.3).

A second massive object in the GC: The presence of an intermediate mass black hole (IMBH) orbiting Sgr A* inside the orbit of S2 is constrained by our measurements (Gualandris et al. 2010) who calculated the effect of an IMBH on the distribution of stellar orbits. Their original box also extends to higher masses as indicated by the shaded area to right. A first constraint from the actual orbit data of S2 was given in (Gillessen et al. 2009a). Extending the orbital coverage in a simulation to the 2018 pericentre passage (Gualandris et al. 2010) concluded that from a lack of extra residuals one should be able to exclude the area right of the dotted line labeled GGM10. GRAVITY improved the accuracy compared to these simulations by a factor 4.6, which moves the limit further to lower masses (red-shaded region). All but a $10^{-4} \times 10^{-3} M_\odot$ IMBH inside the orbit of S2 or just outside of S2’s orbit is now excluded by the various measurements. This also excludes the configurations (Merritt et al. 2009) found to be efficient in randomizing the S-star orbits.

Beyond the standard model: We also derive limits on a Yukawa-like potential in the Galactic Centre (Hees et al. 2017). Our limits show the same sensitivity to the length scale parameter $\lambda$ as in Hees et al. (2017), but are a factor 20 more constraining in terms of the interaction strength $\alpha$. At our most sensitive $\lambda = 180 \, \text{AU}$ we constrain $|\alpha| < 8.8 \times 10^{-4}$ (95% C.L.).

Our data also constrain the possible parameters for an assumed dark-matter spike in the GC. Lucrèix (2018) showed that for an NFW profile (Navarro et al. 1996) with slope $\gamma = -1$ plus a spike with a power-law profile with slope $\gamma_{\text{sp}} = -7.3$ the data of Gillessen et al. (2017) constrain the spike radius to $R_{\text{sp}} \lesssim 100 \, \text{pc}$, corresponding to an enclosed mass of $\approx 5 \times 10^5 M_\odot$. Our new data set constrains $R_{\text{sp}} \lesssim 10 \, \text{pc}$ (corresponding to $\approx 3 \times 10^6 M_\odot$), which is just below the theoretical prediction from (Gondolo & Silk 1999), who took into account limits from the absence of a neutrino signal from the GC.
Table E.1. Best fit orbit parameters. The orbital parameters should be interpreted as the osculating orbital parameters. The argument of periapsis $\omega$ and the time of pericentre passage $t_{\text{peri}}$ are given for the epoch of last apocentre in 2010.

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Appendix E: Details of the fit

In Table E.1, we report the best-fitting parameters of our 14-parameter fit, together with the formal fit errors and the 1$\sigma$ confidence intervals from the MCMC. The two approaches agree, since our fit is well-behaved. There is a single minimum for $\chi^2$, and the posterior distribution is close to a 14-dimensional Gaussian (Figure E.3), with significant correlations though. Figure E.1 shows the posterior for $f_{\text{SP}}$.

In Figure E.2 we show selected correlation plots from the posterior distribution, which are worth discussing in the context of $f_{\text{SP}}$. The strongest correlation for $f_{\text{SP}}$ is with the pericentre time. This is not surprising, given the discussion in Appendix B where we showed that near pericentre the SP acts like a shift in time. The second strongest correlation for $f_{\text{SP}}$ is with the RA offset of the coordinate system. This explains why the inclusion of the NACO flare data helps determining $f_{\text{SP}}$: The flares essentially measure the offset of the coordinate system.

The parameter $f_{\text{SP}}$ is also weakly correlated with the semi-major axis $a$ and anti-correlated with the eccentricity $e$ of the orbit. The former can be understood in the following way: If the orbit were a bit larger on sky, one would need a stronger precession term in order to achieve the same amount of kink (in mas on sky) at pericentre. The latter is understood similarly: A larger eccentricity leads to a narrower orbit figure, and hence less of the precession term would be needed.

Interestingly, $f_{\text{SP}}$ is almost uncorrelated with the argument of periapsis $\omega$ (i.e. the angle describing the orientation of the orbital ellipse in its plane), despite the fact that the SP changes exactly that parameter.

The strongest correlation between any two parameters for our fit is the well known degeneracy between mass $M_\star$ and distance $R_0$ (Ghez et al. 2008 [2624]; Gillessen et al. 2009b [2625]; Boehle et al. 2016 [2626]; Gillessen et al. 2017 [2627]; Gravity Collaboration et al. 2018a [2628]; 2019 [2629]). The parameter $f_{\text{SP}}$ is only very weakly correlated with $R_0$. 

![Fig. E.1. Result of the MCMC modelling of our data, showing the posterior distribution of the Schwarzschild parameter $f_{\text{SP}}$](image1)

![Fig. E.2. Selected parameter correlations from the 14-dimensional posterior distribution as determined from MCMC modelling.](image2)
Fig. E.3. Full, 14-dimensional posterior distribution of our orbit fit.