## First Sagittarius A＊Event Horizon Telescope Results．VIII．：Physical interpretation of the polarized ring

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#### Abstract

In a companion paper, we present the first spatially resolved polarized image of Sagittarius $\mathrm{A}^{*}$ on event horizon scales, captured using the Event Horizon Telescope, a global very long baseline interferometric array operating at a wavelength of 1.3 mm . Here, we interpret this image using both simple analytic models and numerical general relativistic magnetohydrodynamic (GRMHD) simulations. The large spatially resolved linear polarization fraction ( $24-28 \%$, peaking at $\sim 40 \%$ ) is the most stringent constraint on parameter space, disfavoring models that are too Faraday depolarized. Similar to our studies of M87*, polarimetric constraints reinforce a preference for GRMHD models with dynamically important magnetic fields. Although the spiral morphology of the polarization pattern is known to constrain the spin and inclination angle, the time-variable rotation measure (RM) of Sgr A* (equivalent to $\approx 46^{\circ} \pm 12^{\circ}$ rotation at 228 GHz ) limits its present utility as a constraint. If we attribute the RM to internal Faraday rotation, then the motion of accreting material is inferred to be counter-clockwise, contrary to inferences based on historical polarized flares, and no model satisfies all polarimetric and total intensity constraints. On the other hand, if we attribute the mean RM to an external Faraday screen, then the motion of accreting material is inferred to be clockwise, and one model passes all applied total intensity and polarimetric constraints: a model with strong magnetic fields, a spin parameter of 0.94 , and an inclination of $150^{\circ}$. We discuss how future 345 GHz and dynamical imaging will mitigate our present uncertainties and provide additional constraints on the black hole and its accretion flow.


Keywords: Black Hole Physics - Galaxies: Individual: Sgr A*- Radio interferometry - Very long baseline interferometry - Polarimetry - Supermassive black holes - Magnetohydrodynamics (MHD)

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of the mass accretion rate $\dot{M} \sim 10^{-8} M_{\odot} / \mathrm{yr} \sim 10^{-3} \dot{M}_{\mathrm{B}}$
and a luminosity that is $L \lesssim 10^{36} \mathrm{erg} / \mathrm{s} \sim 10^{-9} L_{\text {Edd }}$ (see
e.g., Paper V, and references therein). Here, $\dot{M}_{\mathrm{B}}$ is the
Bondi mass accretion rate and $L_{\mathrm{Edd}} \equiv 4 \pi G M c m_{p} / \sigma_{\mathrm{T}}$
${ }_{28}$ is the Eddington luminosity, with $G, c, m_{p}$, and $\sigma_{\mathrm{T}}$ be-
ing the gravitational constant, speed of light, proton
$\circ$ mass, and Thomson cross-section, respectively. Previ-
ously, measurements of linearly polarized emission near
Sgr A* gave strong evidence for this low accretion state
(e.g., Agol 2000; Quataert \& Gruzinov 2000). In addi-
34 tion, the emission ring morphology including the lack
5 of a pronounced brightness asymmetry in EHT images
favors a viewing angle in $\mathrm{Sgr} \mathrm{A}^{*}$ that is at a low-to-
moderate inclination $\left(\lesssim 50^{\circ}\right)$ relative to the angular mo-
mentum of the inner accretion flow (see, e.g., Figure 9
in Paper V).
Event Horizon Telescope Collaboration et al. (2023a,
hereafter Paper VII) reports the first polarized images
of Sgr A*, using EHT observations at 230 GHz taken
in 2017. These images show a prominent spiral polar-
ization pattern in the emission ring that is temporally
stable, strongly linearly polarized $(\approx 25 \%)$, and dom-
inated by azimuthally symmetric structure. Both the image-averaged polarization fraction $\left(m_{\text {net }} \sim 5 \%\right)$ and the resolved polarization fraction $(\langle | m\rangle \approx 25 \%)$ are significantly higher in Sgr A* than in the EHT's observations of M87* (Event Horizon Telescope Collaboration et al. 2021a, hereafter M87* Paper VII). In M87*, this polarization pattern was explained by coherent and dynamically important magnetic fields, depolarized by Faraday effects (Event Horizon Telescope Collaboration et al. 2021b, hereafter M87* Paper VIII).
In this paper, we provide the theoretical modeling and interpretation to accompany Paper VII. In Section 2, we summarize the new polarimetric observational constraints on $\mathrm{Sgr} \mathrm{A}^{*}$. In Section 3, we provide general arguments about what these constraints imply for $\mathrm{Sgr} \mathrm{A}^{*}$ through comparison with three simple models: one-zone physical models to evaluate the plasma properties, geometrical ring models to evaluate the degree of coherence in the polarized image, and semi-analytic emission models to evaluate the interplay between space-time and emission parameters in determining polarized image structure. In Section 4, we describe a large library of GRMHD simulations for Sgr A*. In Section 5, we evaluate which of these GRMHD models are compatible with the observational constraints. In Section 6, we summarize our findings and describe the prospects for improved constraints from future observations of Sgr A*.

## 2. SUMMARY OF POLARIMETRIC OBSERVATIONS

In Paper VII, static polarimetric images are constructed from the Sgr A* EHT data taken on April 6 th and 7 th, 2017 between 226.1 and 230.1 GHz (see Section 2 of Paper VII for more details). For theoretical interpretation, we adopt 8 observational constraints derived from images generated by the THEMIS and the m-ring reconstruction methods (note that " $m$ " is the azimuthal/angular mode number here, not polarization fraction, see Johnson et al. 2020). Of the 4 methods included in Paper VII, these are the only methods which provide Bayesian posteriors, from which we compute $90 \%$ confidence intervals. These methods make drastically different assumptions, and in a sense, bracket the possible spatial and temporal variability. In brief, the m -ring method fits a ring model to each snapshot independently, but the allowed spatial variability is very limited by construction ( $m \leq 2$ for total intensity, $m \leq 3$ for linear polarization, and $m \leq 2$ for circular polarization). In contrast, THEMIS attempts to optimize a single static image most consistent with the full data over time, with a noise budget attributed to time variability. Despite the vast differences between these models, they recover key image quantities with similar accuracy in synthetic data tests and arrive at mostly consistent observables (Paper VII).
Throughout this work, the large and time-variable rotation measure ( RM ) of $\mathrm{Sgr} \mathrm{A}^{*}$ poses a significant
systematic uncertainty. Defined as $\mathrm{RM} \equiv \Delta \chi / \Delta \lambda^{2}$, where $\chi$ is the electric vector position angle (EVPA), the RM of $\mathrm{Sgr} \mathrm{A}^{*}$ may originate from Faraday rotation internal to the emitting region, an external screen, changes in the plasma probed as a function of optical depth, or a combination of these effects. Examining the polarized light curves for the same two days as our EHT observations, Wielgus et al. (2023) arrive at $\langle\mathrm{RM}\rangle=-4.65_{-1.18}^{+1.25} \times 10^{5} \mathrm{radm}^{-2}$. We reserve a


Figure 1. Polarized images of Sgr A* used for physical interpretation in this work. Two methods from Paper VII, snapshot m-ring and THEMIS, are included. Top and center: Total intensity is shown in grayscale, polarization ticks indicate the electric vector polarization angle (EVPA), the tick length is proportional to the linear polarization intensity magnitude, and color indicates fractional linear polarization. The dotted contour levels correspond to linearly polarized intensities of 25,50 , and $75 \%$ of the polarization peak. The solid contour levels indicate total intensity at 25,50 and $75 \%$ of the peak brightness. The top row shows images without derotation and the center row shows images with a derotation of 46.0 deg to account for Faraday rotation. Bottom: Total intensity is indicated in solid colored contours at 25, 50 and $75 \%$ of the peak brightness, and the Stokes $\mathcal{V}$ brightness is indicated in the diverging colormap, with red/blue indicating a positive/negative sign.
lengthy discussion of the RM of Sgr A* in both observations and theory for Appendix C. In summary, the fraction of the RM that can be attributed to an external Faraday screen is currently unresolved. Thus, throughout this work, we consider the recovered image statistics both with and without RM derotation. Derotating the image corresponds to an interpretation where the timeaveraged RM is attributed to a relatively stable external Faraday screen, separate from our models, which can be corrected for. Refraining from doing so corresponds to an interpretation in which all of the RM is generated internally, within our models. Our GRMHD simulations can reproduce the intra-day variability of the RM, but not its stability of sign (see Appendix C).
For each of these methods, 8 observational constraints explored in this paper are computed, listed in Table 1. To generate these ranges, a large quantity of images consistent with the data were generated from each method's posterior distribution. We computed the relevant observables for each of these images, and then inferred $90 \%$ confidence regions. The m-ring method does not provide independent values of $v_{\text {net }}$, which is fixed to the mean ALMA-inferred value for circular polarization analysis (see Paper VII). When combining the two methods for theoretical interpretation, we adopt the minimum and maximum of the union of both $90 \%$ confidence regions (see Figure 10 in Paper VII for a visualization).
The quantities $m_{\text {net }}$ and $v_{\text {net }}$ correspond to the net linear and circular polarization that would be inferred from a spatially unresolved measurement for the timeaveraged image. These are given by

$$
\begin{gather*}
m_{\text {net }}=\frac{\sqrt{\left(\sum_{i} \mathcal{Q}_{i}\right)^{2}+\left(\sum_{i} \mathcal{U}_{i}\right)^{2}}}{\sum_{i} \mathcal{I}_{i}}  \tag{1}\\
v_{\text {net }}=\frac{\sum_{i} \mathcal{V}_{i}}{\sum_{i} \mathcal{I}_{i}} \tag{2}
\end{gather*}
$$

where $\sum_{i}$ denotes a summation over each pixel $i$. For the time-resolved light curves, which are distinct from the values inferred from our static image reconstructions, Wielgus et al. (2022a, 2023) find $2.6 \%<m_{\text {net }}<$ $11 \%$ and $-2.1 \%<v_{\text {net }}<-0.7 \%$ respectively, where we quote the central $90 \%$ of the values observed during the same two days of observation. Interestingly, we find that the m-ring method arrives at much lower values of $m_{\text {net }}$ than THEMIS, which may be attributable to temporal cancellations of fluctuating electric vector position angle (EVPA) patterns.
The remainder of our constraints are structural quantities, beginning with $\langle | m\rangle$ and $\langle | v|\rangle$, the imageaveraged linear and circular polarization fraction. These are given by

$$
\begin{align*}
\langle | m\rangle & =\frac{\sum_{i} \sqrt{\mathcal{Q}_{i}^{2}+\mathcal{U}_{i}^{2}}}{\sum_{i} \mathcal{I}_{i}}  \tag{3}\\
\langle | v\rangle & =\frac{\sum_{i}\left|\mathcal{V}_{i} / \mathcal{I}_{i}\right| \mathcal{I}_{i}}{\sum_{i} \mathcal{I}_{i}} \tag{4}
\end{align*}
$$

Note that these quantities depend on the effective resolution of our images. Throughout this work we quote values from our simulations corresponding to $20 \mu$ as resolution to mimic EHT resolution. We treat the resolved circular polarization fraction $\langle | v\rangle$ as an upper limit, and thus the combined range extends to 0 in Table 1. This is due to the fact that the circularly polarized images presented in Paper VII show structural differences that we attribute to noise (see also Event Horizon Telescope Collaboration et al. 2023b, hereafter M87* Paper IX). Because of the absolute magnitude inherent to the definition of this quantity, it is biased high when the signal-to-noise is too low.
Complex $\beta_{m}$ modes correspond to Fourier decompositions of the linear polarization structure, where $m$ refers to the number of times that an EVPA tick rotates with azimuth (Palumbo et al. 2020). These coefficients are defined

$$
\begin{align*}
\beta_{m} & =\frac{1}{I_{\mathrm{tot}}} \int_{0}^{\infty} \int_{0}^{2 \pi} P(\rho, \varphi) e^{-i m \varphi} \rho \mathrm{~d} \varphi \mathrm{~d} \rho  \tag{5}\\
I_{\mathrm{tot}} & =\int_{0}^{\infty} \int_{0}^{2 \pi} I(\rho, \varphi) \rho \mathrm{d} \varphi \mathrm{~d} \rho \tag{6}
\end{align*}
$$

where $\rho$ and $\varphi$ correspond to polar coordinates in the image, and $P=Q+i U$. The rotationally invariant mode, $\beta_{2}$, has natural connections to what we believe are azimuthally symmetric disk/jet structures, in particular the magnetic field geometry. Its amplitude encodes the strength of this mode, while its phase encodes the pitch angle and handedness of EVPA ticks. We observe $\angle \beta_{2}$ closer to $\pm 180^{\circ}$ than $0^{\circ}$, which corresponds to tick patterns that are more toroidal than radial.
When considering observational constraints without RM derotation, we simply adopt the range of $\angle \beta_{2}$ as observed on the sky. When considering observational constraints with RM derotation, we derotate $\angle \beta_{2}$ assuming that there is an external Faraday screen between us and the emitting region that we can characterize by the mean RM over time. Since $\angle \beta_{2}$ depends on twice the EVPA, we therefore add $-2\langle\mathrm{RM}\rangle \lambda^{2}$ to $\angle \beta_{2}$, where $\langle\mathrm{RM}\rangle$ is the mean RM observed on April 6th and 7th. Therefore, the range on $\angle \beta_{2}$ had been significantly shifted by the Faraday screen by $2 \mathrm{RM} \lambda^{2}=-92.0_{-23.4}^{+24.7}$ degrees. Applying this derotation both shifts and broadens the constraint.

Mean images from the posterior distributions generated by each method are plotted in Figure 1. Two sets of linearly polarized images are shown, corresponding to images without and with derotation respectively. Note that derotation reverses the handedness of the polarization spiral, which has important implications for the flow structure. In the first two rows, total intensity is shown in gray scale, with contours drawn at 25,50 , and $75 \%$ of

| Observable | m-ring | THEMIS | Combined |
| :--- | :---: | :---: | :---: |
| $m_{\text {net }}[\%]$ | $(2.0,3.1)$ | $(6.5,7.3)$ | $(2.0,7.3)$ |
| $v_{\text {net }}[\%]$ | - | $(-0.7,0.12)$ | $(-0.7,0.12)$ |
| $\langle \| m\rangle[\%]$ | $(24,28)$ | $(26,28)$ | $(24,28)$ |
| $\langle \| v\rangle[\%]$ | $(1.4,1.8)$ | $(2.7,5.5)$ | $(0.0,5.5)$ |
| $\left\|\beta_{1}\right\|$ | $(0.11,0.14)$ | $(0.10,0.13)$ | $(0.10,0.14)$ |
| $\left\|\beta_{2}\right\|$ | $(0.20,0.24)$ | $(0.14,0.17)$ | $(0.14,0.24)$ |
| $\angle \beta_{2}[\mathrm{deg}]$ (as observed) | $(125,137)$ | $(142,159)$ | $(125,159)$ |
| $\angle \beta_{2}[\operatorname{deg}]$ (RM derotated) | $(-168,-108)$ | $(-151,-85)$ | $(-168,-85)$ |
| $\left\|\beta_{2}\right\| / / \beta_{1} \mid$ | $(1.5,2.1)$ | $(1.1,1.6)$ | $(1.1,2.1)$ |

Table 1. Polarimetric constraints derived from the static reconstruction of $\mathrm{Sgr} \mathrm{A}^{*}$. These two methods each provide posteriors, from which $90 \%$ confidence regions are quoted. As constraints on our models, we conservatively adopt the minimum and maximum of these $90 \%$ confidence regions from both of these methods combined (rightmost column), with the exception of $\langle | v\rangle$ which is treated as an upper limit. Derotation assumes that the mean RM can be attributed to an external Faraday screen, for which a frequency of 228.1 GHz is adopted.

## 3. ANALYTIC MODELS

As discussed in the previous section, the linearly po${ }_{531}$ larized image of Sgr A* exhibits three salient features:

1. It has a large resolved polarization fraction of 24$28 \%$, with a peak of $\sim 40 \%$, much higher than M87*
2. The linear polarization structure is highly ordered.
3. The ordered structure exhibits a high degree of rotational symmetry, which appears to spiral inwards with counter-clockwise handedness after derotating by the apparent RM, or clockwise without derotating.

Before exploring more physically complete GRMHD models, we demonstrate that each of these features can be understood in the context of simple analytic models.

### 3.1. One-zone Modeling

We use the basic assumptions described in Paper V that $\operatorname{Sgr} \mathrm{A}^{*}$ is an accreting black hole with extremely
small Eddington ratio, and follow M87* Paper VIII to include polarimetry. This polarized one-zone model validates the more complicated numerical models shown later in this paper, and offers a natural explanation for the high polarization fraction of $\operatorname{Sgr} \mathrm{A}^{*}$ relative to M87*.

We model the accretion flow around Sgr $\mathrm{A}^{*}$ as a uniform sphere of plasma with radius $r=5 r_{g}$, where $r_{g}=G M / c^{2}$, comparable to the observed size of Sgr A* at 230 GHz (Paper III, Paper IV), with uniform magnetic field oriented at a fiducial $60^{\circ}$ inclination relative to the line-of-sight. The outcomes of our one-zone model depend only weakly on the field orientation. Note that the plasma velocity and the gravitational redshift are neglected.
In Paper V, we assumed that the plasma is optically thin, the ion-electron temperature ratio is 3 , the ions are subvirial by a factor of 3 , and plasma $\beta \equiv$ $P_{\text {gas }} / P_{\text {mag }}=1$. Adopting the observational flux constraint $F_{\nu}=2.4 \mathrm{Jy}$ (Wielgus et al. 2022b), we obtained the self-consistent solution $n_{e} \simeq 10^{6} \mathrm{~cm}^{-3}$ and $B \simeq 29 \mathrm{G}$. Using this solution, we can estimate the strength of the Faraday rotation at 230 GHz with the optical depth to Faraday rotation $\tau_{\rho_{V}}$ :

$$
\begin{equation*}
\tau_{\rho_{V}} \approx r \times \rho_{V} \simeq 0.98\left(\frac{r}{5 r_{g}}\right) \tag{7}
\end{equation*}
$$

where $\rho_{V}$ is the Faraday rotation coefficient (e.g., Jones \& Hardee 1979). In contrast, similar modeling arrived at $\tau_{\rho_{V}} \sim 5.2\left(r / 5 r_{g}\right)$ for M87* (M87* Paper VIII). The value inferred for $\mathrm{Sgr} \mathrm{A}^{*}$ suggests that the internal Faraday rotation may not be negligible (see also Wielgus et al. 2023), but also may not necessarily lead to substantial depolarization.

By including optical depth effects and using Dexter (2016)'s polarized synchrotron emission and transfer coefficients, we relax some assumptions such as ionelectron temperature ratios and virial factor and plot


Figure 2. Allowed parameter space in electron number density ( $n_{e}$ ) and dimensionless electron temperature ( $\Theta_{e}$ ) for the onezone model described in Section 3.1. The panels correspond to different assumed values of plasma $\beta=P_{\text {gas }} / P_{\text {mag. }}$. We require that the total flux density $2 \mathrm{Jy}<F_{\nu}<3 \mathrm{Jy}$ (gray region) and optical depth $\tau<1$ (green region). Corresponding magnetic field strengths are shown as red dotted lines. In blue, we plot the Faraday thick region, $\tau_{\rho_{V}}>2 \pi$. Unlike for M87* we find that the model is Faraday thin wherever there is intersection between our two constraints.
the allowed parameter space as in (M87* Paper VIII). Specifically,

- we relax the flux constraint to $2 \mathrm{Jy}<F_{\nu}<3 \mathrm{Jy}$ to include the effect of variability; and
- we require the same assumption that $\operatorname{Sgr} \mathrm{A}^{*}$ is optically thin, i.e., $\tau<1$.

The above requirements are marked by the gray and green regions in Figure 2, respectively. The magnetic field strengths are shown as red dotted contour lines, and the different panels assume different plasma $\beta$. In blue, we plot the contour corresponding to $\tau_{\rho_{V}}>2 \pi$, beyond which internal Faraday depolarization becomes increasingly important. Unlike for M87* (see Figure 2 of M87* Paper VIII), we find that the regions where the total flux and optically-thin constraints are satisfied only occur in Faraday thin regions of parameter space. We note that this is compatible with multi-frequency RM measurements that suggest $\tau_{\rho_{V}} \sim 1$ (Wielgus et al. 2023). Again, this is enough to noticeably rotate the EVPA pattern, but not enough to cause substantial depolarization.
In summary, the total flux and optical depth constraints of $\mathrm{Sgr} \mathrm{A}^{*}$ naturally require small Faraday depths, which explains the large inferred values of $\langle | m\rangle$.

### 3.2. Ordered Polarization: Ordered Fields

Because beam depolarization can only decrease the observed polarization fraction, measurements of the linear polarization at varying angular scales provides information about the degree of order in the underlying polarization. A priori, it could be possible that the the underlying magnetic field is significantly tangled on
scales much smaller than the beam. However, the combination of unresolved ( $m_{\text {net }} \approx 0.07$ ) and EHT-resolved $(\langle | m\rangle \approx 0.25)$ linear polarization measurements constrains the degree of order in the true, underlying polarization pattern on scales smaller than our beam size, disallowing significant spatially unresolved disorder.
As a simple toy model, we analyzed a thin, circular ring with polarization confined to two azimuthal Fourier modes, labeled with index $\ell .{ }^{1}$ First, we include a constant $(\ell=0)$ mode that defines $m_{\text {net }}$. We fix the amplitude of this mode to be 0.07 to match unresolved observations of Sgr A*. Next, we add a second mode with varying index $\ell>0$ and an amplitude of 0.7 , similar to the peak fractional polarization expected for synchrotron emission. By varying $\ell$, we can crudely assess the allowed degree of coherence in the polarization of Sgr A*.
Figure 3 shows the resolved fractional polarization $\langle | m\rangle$ at an angular resolution of $20 \mu$ as as a function of the secondary mode index $\ell$. Both a perfectly ordered polarization field $(\ell=0)$ and a highly disordered polarization field $(\ell \gg 1)$ will have $m_{\text {net }} \approx\langle | m| \rangle$. For the former, there is no beam depolarization; for the latter, the beam depolarization eliminates all small-scale polarized power, even at the resolution of the EHT. Hence, the high value of $\langle | m\left\rangle\right.$ relative to $m_{\text {net }}$ that we observe is a powerful diagnostic of coherent polarized structure.

As expected, small values of $\ell$ produce resolved polarization fractions that are too high, while large values of $\ell$ produce resolved polarization fractions that are

[^1]

Figure 3. The combination of unresolved ( $m_{\text {net }}$ ) and EHTresolved $(\langle | m\rangle)$ linear polarization measurements (at $20 \mu$ as resolution) constrains the degree of order in the underlying polarization image. In this schematic example, a polarized m -ring has a fixed net polarization, $m_{\text {net }} \equiv 0.07$ (denoted with the black dashed line), together with a single strongly polarized mode at higher order, $\ell$, that controls the degree of disorder. For small values of $\ell$, the resulting image is too ordered, with $\langle | m\rangle$ exceeding our observed value for Sgr A* (denoted with the upper yellow band). For large values of $m$, the resulting image is too disordered, with beam depolarization eliminating the highly polarized image structure. In this example, the fields must be substantially ordered to be consistent with our observations of $\mathrm{Sgr} \mathrm{A}^{*}$, with polarized structure that is coherent on scales of the $\ell \approx 4$ mode, corresponding to angular scales of $\theta \approx 4 \theta_{\mathrm{g}} \approx 20 \mu$ as.

Semi-analytic models enable computationally inexpensive investigation of the effects of model parameters on images. For example, semi-analytic models of radiatively inefficient accretion flows have been used for decades to gain tractable yet physically motivated insights into accretion flows (Bromley et al. 2001; Broderick et al. 2009, 2011, 2014, 2016; Pu et al. 2016; Pu \& Broderick 2018; Vincent et al. 2022). Here, we explore a very simple model, KerrBAM (or Kerr Bayesian Accretion Modeling), a semi-analytic model for equatorial, axisymmetric synchrotron emission around a Kerr black hole (Palumbo et al. 2022). This modeling framework carries out ray-tracing in a Kerr space-time to produce a model image assuming an equatorial ring of emission with a specified fluid velocity, magnetic field geometry, and radial emission profile. Here, we use this simple model to illustrate the effects of inclination and spin on polarized image structure.
As our starting point, we average ${ }^{2}$ magnetic fields and velocity fields in three KHARMA GRMHD simulations (to be discussed in Section 4) in both time and azimuth. We specify a ring of emission centered at a radius of $6 r_{g}$ and use the values of the fluid velocity and magnetic field extracted from the GRMHD midplane at this radius. ${ }^{3}$ To give the emission ring a realistically finite width, the emission is spread in a Gaussian spanning approximately 4 to $8 r_{g}$, keeping the velocity and magnetic field vectors constant. With these values, KerrBAM is able to capture the effects of beaming, frame-dragging, and lensing on the resultant image. Note that this model excludes the likely contribution of emission off the mid-plane (e.g., Falcke et al. 1993; Markoff et al. 2007).

For three different MADs with spins of $0,+0.5$, and +0.94 , we plot several polarimetric quantities of interest (leftmost column) and their model images (subsequent columns) in Figure 4. Along with the polarimetric observables, we overlay our constraints in gray, where for $\angle \beta_{2}$ the range without RM derotation is shown as a hatched region. Since this model places emission exactly at the mid-plane by construction, images produced at inclinations too close to $90^{\circ}$ are misleading and therefore not included. The KerrBAM prescription does not include Faraday effects, only crudely models optical depth (in this case applying a midplane-normal crossing optical depth $\tau_{\perp}=0.5$ applied uniformly to $\mathcal{I}, \mathcal{Q}$, and $\left.\mathcal{U}\right)$, and assumes a pre-specified emission model confined to the mid-plane, so detailed agreement with the GRMHD models is neither expected nor achieved. Nevertheless, this model is useful for understanding several qualita-

[^2]

Figure 4. Left column: Image quantities determined from simplified analytic KerrBAM models evaluated using MAD GRMHD fluid velocities and magnetic fields of three spins. In this and subsequent plots, we plot our observational constraints as gray bands for reference, with the $\angle \beta_{2}$ constraint prior to RM derotation shown as a hatched region. We use this model to understand key trends, but caution that more physically complete GRMHD models are necessary for quantitative comparison. Right three columns: corresponding KerrBAM images evaluated at four example inclinations.
tive trends in our GRMHD library that are successfully reproduced.
First, the net polarization is minimized at low inclination, since the symmetry of the accretion flow causes cancellation of polarization in the integrated image. The amplitude of the rotationally invariant mode $\beta_{2}$ is always high, due to the underlying azimuthal symmetry of the system. Meanwhile, the amplitude of $\left|\beta_{1}\right|$ is stronger at higher inclinations, as it is sensitive to asymmetries
${ }_{723}$ in the polarized image. Finally, we highlight the spin724 dependence of $\angle \beta_{2}$, which this modeling demonstrates is 725 driven by the evolution of the magnetic field and velocity 726 structure in the GRMHD simulations due to frame drag727 ging (see also Ricarte et al. 2022; Emami et al. 2023a; 728 Chael et al. 2023). The $a_{*}=0$ model has $\angle \beta_{2} \sim-180^{\circ}$, 729 corresponding to a very toroidal EVPA pattern and thus ${ }_{730}$ radial magnetic fields. Meanwhile, the higher spin mod${ }_{731}$ els acquire $-180 \lesssim \angle \beta_{2} \lesssim 0$ due to their more spiral

732 EVPA structures. Interestingly, $\angle \beta_{2}$ remains strikingly

## 4. GRMHD MODELS

While semi-analytic models provide qualitative insights and intuition about black hole accretion flows, they do not enforce conservation laws or capture timedependent phenomena such as turbulence and shocks that play a crucial role in determining the detailed system structure. Thus, we generate dynamical source models using numerical ideal general relativistic magnetohydrodynamic (GRMHD) simulations. A fluid approximation would appear to conflict with the fact that the rate of Coulomb collisions is small, leading to mean-free paths well exceeding the system size, implying that a collisionless kinetic treatment of the plasma may be necessary (Mahadevan \& Quataert 1997). However, kinetic instabilities can produce small-scale inhomogeneities in the magnetic field that produce an effective collisionality through particle-wave interactions (Kunz et al. 2014; Sironi \& Narayan 2015; Riquelme et al. 2015; Meyrand et al. 2019). We implicitly assume that radiative effects like cooling are not dynamically important for the fluid evolution. This assumption is well-motivated given the low accretion rate of Sgr $\mathrm{A}^{*}, \dot{M} \lesssim 10^{-6} \dot{M}_{\text {Edd }}$, for which the radiative cooling timescale is long compared to the accretion timescale (Porth et al. 2019; Dibi et al. 2012; Ryan et al. 2017; Chael et al. 2018; but see also Yoon et al. 2020).
In Paper V, to compare with total intensity EHT and multi-wavelength constraints, we generated a suite of GRMHD-derived images sampling a range of initial conditions and parameterizations of the electron temperature and distribution function. We simplify our exploration in this work, limiting ourselves to simulations with untilted torus-like initial conditions, relativistic thermal electron distribution functions (eDFs) lacking non-thermal contributions, and electron temperatures prescribed via the Mościbrodzka et al. (2016) $R-\beta$ prescription (see Equation 8 below). Radiative transfer is integrated within a radius of $100 r_{g}$, explicitly ignoring material in highly magnetized regions with $\sigma \equiv b^{2} / \rho>1$, within which mass density is artificially
injected to keep the simulation stable. We briefly test the impact of our choices of outer integration radius, the $\sigma$ cut, and eDF in Appendix D, Appendix E, and Appendix F respectively. While departures from these assumptions are both interesting and physically justified, we defer a thorough investigation of these topics to future work.

Our GRMHD library samples a 5 -dimensional parameter space. The first parameter is the magnetic field state, either a magnetically arrested disk (MAD) model (Bisnovatyi-Kogan \& Ruzmaikin 1976; Igumenshchev et al. 2003; Narayan et al. 2003; Tchekhovskoy et al. 2011) or standard and normal evolution (SANE) model (De Villiers et al. 2003; Gammie et al. 2003; Narayan et al. 2012; Sądowski et al. 2013). These describe models in which the magnetic flux threading the horizon for a given accretion rate has saturated and become dynamically important (MAD) or not (SANE). The second is the BH spin, which we denote as $a_{*} \in[-1,1]$, where a negative sign indicates a retrograde disk with respect to the spin vector. Third is the inclination, which uniformly samples $i \in\left[0^{\circ}, 180^{\circ}\right]$, instead of only $i \in\left[0^{\circ}, 90^{\circ}\right]$ as probed in Paper V, because Faraday rotation and emission of circular polarization break the symmetry when polarization is considered. Our fourth parameter is $R_{\text {high }}$, which sets the asymptotic value of the ion-to-electron temperature ratio as plasma $\beta \rightarrow \infty$ (Mościbrodzka et al. 2016). Specifically,

$$
\begin{equation*}
\frac{T_{i}}{T_{e}}=R_{\mathrm{low}} \frac{1}{1+\beta^{2}}+R_{\mathrm{high}} \frac{\beta^{2}}{1+\beta^{2}}, \tag{8}
\end{equation*}
$$

where $T_{i}$ and $T_{e}$ are the ion and electron temperatures respectively. While the potential importance of electron cooling for M87* motivated models with cooler electrons, $R_{\text {low }}=10$, here we only consider $R_{\text {low }}=1$ due to the much smaller Eddington ratio of Sgr A*. Finally, our fifth parameter is the magnetic field polarity with respect to the angular momentum vector of the disk, either aligned or reversed, which affects the direction of Faraday rotation and the handedness of circularly polarized emission. This last degree of freedom only matters for polarized radiative transfer and was ignored in Paper V. We produce a library of images for each combination of these parameters, tabulated in Table 3.

We retain the use of multiple codes to assess numerical systematic differences. For scoring, we generate libraries spanning $15,000 t_{g}\left(t_{g} \equiv r_{g} / c\right)$ equivalent to about 10 8 -hour nights of observation for the parameter combinations listed in Table 3 using two code combinations: KHARMA $^{4}$ (Prather et al. 2021) + IPOLE $^{5}$ (Mościbrodzka \& Gammie 2018) and BHAC ${ }^{6}$ (Porth et al. 2017;

[^3]| Setup | GRMHD | GRRT | $a_{*}$ | Mode | $\Gamma_{\text {ad }}$ | $t_{\text {final }}$ | $r_{\text {out }}$ | Resolution |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| torus | KHARMA | IPOLE | $0, \pm 0.5, \pm 0.94$ | $\mathrm{MAD} / \mathrm{SANE}$ | $\frac{4}{3} / \frac{4}{3}$ | 50,000 | 1,000 | $288 \times 128 \times 128$ |
| torus | BHAC | RAPTOR | $0, \pm 0.5, \pm 0.94$ | $\mathrm{MAD} / \mathrm{SANE}$ | $\frac{4}{3} / \frac{4}{3}$ | 30,000 | 3,333 | $512 \times 192 \times 192$ |
| torus | H-AMR | IPOLE | $0, \pm 0.5, \pm 0.94$ | $\mathrm{MAD} / \mathrm{SANE}$ | $\frac{13}{9} / \frac{5}{3}$ | 35,000 | $1,000 / 200$ | $348 / 240 \times 192 \times 192$ |

Table 2. Summary of the Sgr A* GRMHD simulation library used in this work. The last column is $N_{1} \times N_{2} \times N_{3}$, with coordinate $x_{1}$ monotonic in radius, $x_{2}$ monotonic in colatitude $\theta$, and $x_{3}$ proportional to longitude $\phi$. Times are given in units of $t_{g}$ and radii in units of $r_{g}$. Different settings may be adopted for MAD models compared to SANE ones, as denoted by a $/$.

| Parameter | Values |
| :---: | :---: |
| Magnetic Field State | MAD, SANE |
| $a_{*}$ | -0.94, -0.5, 0.0, 0.5, 0.94 |
| $i\left[{ }^{\circ}\right]$ | 10, 30, 50, 70, 90, 110, 130, 150, |
| $R_{\text {high }}$ | 1, 10, 40, 160 |
| Magnetic Field Polarity | Aligned, Reversed |

Table 3. Summary of parameters sampled by our GRMHD libraries. We coarsely sample a 5 -dimensional parameter space. For each combination of parameters and for each of the KHARMA and BHAC codes, we ray-trace the equivalent of 10 nights of observations.

In Figure 5, we plot a selection of time-averaged GRMHD snapshots from our library, blurred to EHT 857

Olivares et al. 2019a) + RAPTOR ${ }^{7}$ (Bronzwaer et al. 2018, 2020), where the first and second code in each pair corresponds to GRMHD and GRRT respectively. As a further consistency check, a third set is generated with H-AMR ${ }^{8}$ (Liska et al. 2022) + IPOLE for a subset of parameter space (only $i \leq 90^{\circ}$, aligned fields, and $5,000 t_{g}$ ) that we do not use for scoring.
Each simulation is initialized with a torus of gas in constant-angular-momentum hydrodynamic equilibrium (Fishbone \& Moncrief 1976). These tori are perturbed with a weak, poloidal magnetic field. The simulations vary in their initial radius of maximum pressure (from $\sim 15 r_{g}$ to $40 r_{g}$ ) and adiabatic index, $\Gamma_{\mathrm{ad}}$. Codes differ in their choice of $\Gamma_{\text {ad }}$ because $\Gamma_{\text {ad }}=4 / 3$ applies to a fluid of relativistic electrons and $\Gamma_{\text {ad }}=5 / 3$ applies to a fluid of non-relativistic ions, but only one fluid is evolved in these models. Depending on the torus size and initial magnetic field configuration, the simulations develop into a MAD or SANE state (see e.g., Wong et al. 2022).

[^4]the line-of-sight magnetic field direction, and this signal disappears as the viewing angle decreases (Ricarte et al. 2021; Tsunetoe et al. 2021).

## 5. GRMHD MODEL SCORING

We introduce a novel methodology to score each of our GRMHD models using the 8 polarimetric constraints in Table 1. Our new scoring scheme acts on time-averaged GRMHD images and attempts to accommodate variations between codes. Note that we only include quantities inferred from our polarimetric images in these constraints, but we will discuss comparisons with total intensity and multi-frequency constraints derived in (Paper V).

- First, each model time series of images is split into 10 windows, each with 1500 M duration. Within each window, we produce a time-averaged image by averaging each of the Stokes parameters. Then, we blur the average image with a Gaussian kernel with a FWHM of $20 \mu$ as and compute each of the 8 observables for scoring.


Figure 5. Gallery of example time averaged simulations in our library. Each panel displays a time-averaged and blurred (with a $20 \mu$ as FWHM Gaussian kernel) MAD $a_{*}=0.94 R_{\text {high }}=40$ aligned models at three different inclinations. The first panel of each set displays total intensity and linear polarization, while the second panel of each set displays total intensity and circular polarization. Tick lengths scale the total polarized flux density in a given pixel, while their colors scale with the polarization fraction. H-AMR models are ray-traced only for a subset of models for comparison and are not used for scoring.

- For each combination of parameters, we combine the values of the observables predicted by the KHARMA and BHAC codes. Since there are 10 windows and 2 sets of codes, this results in 20 different samples. From these values, we compute the $90 \%$ quantiles ${ }^{9}$ of each observable to capture the time variability.
- A model passes an individual observational constraint if there is overlap between its $90 \%$ quantile region and that of the observations. A model passes a set of observational constraints if it passes all of the constraints in the set simultaneously.

The most important differences compared to the scoring system utilized in Paper V are that this new system operates on time-averaged images and combines the results from multiple codes into a single theoretical range. We tested performing scoring using only one simulation set at a time. Since KHARMA model electron temperatures are assigned systematically hotter than those of

[^5]the BHAC models (see Appendix H), KHARMA passes models with larger $R_{\text {high }}$. There is more disagreement between the codes for SANE models than for MAD models. The constraints with the most disagreement between the two codes are $\angle \beta_{2},\left|\beta_{2}\right| /\left|\beta_{1}\right|$, and $m_{\text {net }}$, with the KHARMA simulations ruling out more SANE models than the BHAC simulations in each case.
Each of the observational constraints has known connections with the underlying physics. For brevity, we defer a pedagogical exploration of how each of our free parameters is imprinted onto the observables to Appendix A. We study how each individual constraint affects model selection in Appendix B. Here, we summarize the highest level scoring results, first excluding $\angle \beta_{2}$, then including $\angle \beta_{2}$ either as observed or after performing RM derotation.

### 5.1. Constraints Independent of $R M$

In Figure 6, we plot a pass/fail table combining all polarimetric constraints, with the exception of $\angle \beta_{2}$. These plots combine both polarities of the magnetic field, showing a pass as long as either polarity passes. These tables are slightly but not systematically different as a function of magnetic field polarity.

We find that the tight constraint on $\langle | m\rangle(24-28 \%)$ is the most powerful, driving most of the trends shown in this figure. It is much more constraining on parameter space than $m_{\text {net }}$, for which a much larger range (2.0-

# Constraints Without $\angle \beta_{2}$ 



Figure 6. Combined polarimetric constraints on the GRMHD model library excluding $\angle \beta_{2}$. Orange models fail, green models pass at both the given and its supplementary angle, and blue regions only pass with the given or supplementary angle as indicated. SANE models are plotted on the top half, and MAD models are plotted on the bottom. Different columns correspond to different spins from -0.94 to 0.94 . Within each wedge, the radial direction corresponds to $R_{\text {high }}$ and the azimuthal direction corresponds to observer inclination.
$7.3 \%$ ) is allowed. The $\left|\beta_{2}\right|$ constraint rules out a few additional typically edge-on models, but does not provide too much more additional constraining power because $\langle | m\rangle$ and $| \beta_{2} \mid$ are correlated. Without $\angle \beta_{2}$, Figure 6 reveals no significant preference between $i>90^{\circ}$ and $i<90^{\circ}$ models.
While our total intensity constraints generally favored larger values of $R_{\text {high }}$ (due largely to multi-wavelength constraints; Paper V), our polarimetric constraints usually prefer more moderate values. This is because larger values of $R_{\text {high }}$ usually lead to larger internal Faraday rotation depths (see Section A.4), which is the most important physical driver of depolarization in our models. However, an interesting trend with respect to spin allows one of the best bet models of Paper V to continue to pass with $R_{\text {high }}=160$. This is the MAD $a_{*}=0.94 R_{\text {high }}=160 i=30^{\circ} / 150^{\circ}$ model. MAD models with larger spin have smaller Faraday rotation depths (see Appendix H), allowing them to pass the $\langle | m\rangle$ constraint for larger values of $R_{\text {high }}$. We refer readers to Appendix B for a more detailed breakdown of each constraint considered individually.

### 5.2. Constraints Including $\angle \beta_{2}$ Without $R M$ Derotation

First, we discuss the $\angle \beta_{2}$ constraint if RM derotation is not performed. It is possible that the RM may be attributed entirely to Faraday rotation captured within
our simulation domain. GRMHD models are capable of producing the correct magnitude of RM from Faraday rotation on event horizon scales, but tend to produce RM sign flips that are not consistent with decades of Sgr A* observations that produce negative values of the RM (Ricarte et al. 2020; M87* Paper VII; Wielgus et al. 2023). However, it is possible that this problem is related to the excess variability in our models identified in Paper V. We further discuss the uncertainties surrounding our interpretation of the RM in Appendix C.
If one attributes the RM entirely to internal Faraday rotation, then our constraint on $\angle \beta_{2}$ spans the interval $\left(125^{\circ}, 160^{\circ}\right)$. Adding this constraint to Figure 6 results in Figure 7. A selection for $i<90^{\circ}$ arises because the handedness of the polarization spiral is opposite that of the magnetic field, which inherits the handedness of the inflowing and emitting gas (see Section 3.3 and Section A.3). This corresponds to counter-clockwise motion, which disagrees with hot-spot interpretations of polarized flares both in the NIR (GRAVITY Collaboration et al. 2018, 2020a,b), and in the sub-mm (Wielgus et al. 2022a; Vos et al. 2022). That is, consistency with clockwise motion would require $-180^{\circ}<\angle \beta_{2}<0^{\circ}$ if we assume that $\angle \beta_{2}$ traces magnetic field lines with outgoing Poynting flux (Chael et al. 2023), which does not agree with the linearly polarized morphology as observed on the sky.

## All Polarimetric Constraints



Figure 7. As in Figure 6 but including the constraint on the phase of $\beta_{2}$ without RM derotation. Only models with counterclockwise motion $\left(i<90^{\circ}\right)$ pass. There is no model that passes all polarimetric and total intensity constraints utilized in Paper V.

Without RM derotation, no model can simultaneously 1002 This is because the $a_{*}=0.94$ best bet model of Pa${ }_{1003}$ per V produces an EVPA pattern that is too radial (see 1004 Section A.2). All models that pass our polarization con1005 straints in Figure 7 fail multiple constraints on the total 1006 intensity. In particular, all eight models shown in Fig1007 ure 7 produce too much flux in the infrared to match 1008 observations, and all but the SANE model at $a_{*}=0.94$ 1009 overproduce the X-ray flux (Paper V). Both of these are 1010 serious failures, as both the IR and X-ray fluxes esti1011 mated by our models are lower limits due to our lack 1012 of non-thermal electrons and small simulation domain 1013 relative to the X-ray emitting area. Five of the models 1014 additionally fail to match the observed size and flux of 1015 the source at 86 GHz (Issaoun et al. 2019). All of these 1016 models also fail at least one total intensity structural 1017 constraint (m-ring and visibility amplitude morphology 1018 tests in Paper V). In conclusion, we cannot find a con1019 cordance model of $\mathrm{Sgr} \mathrm{A}^{*}$ without RM derotation.

1020 5.3. Constraints Including $\angle \beta_{2}$ With RM Derotation
1021 Alternatively, in this section we interpret the mean 1022 RM as an external Faraday screen, motivating derota1023 tion. As discussed in Section 2, $\angle \beta_{2}$ depends on twice 1024 the RM , for which a mean value of $\langle\mathrm{RM}\rangle=-4.65_{-1.18}^{+1.25} \times$ $102510^{5} \mathrm{rad} \mathrm{m}^{-2}$ has been obtained. This potentially re1026 sults in a shift in $\angle \beta_{2}$ of $2\langle\mathrm{RM}\rangle \lambda^{2}=-92.0_{-23.4}^{+24.7} \mathrm{deg}$ 1027 if this RM is interpreted an external Faraday screen. ${ }_{1028}$ In this picture, a relatively stable external screen ex-10331034 V

1035 If one attributes the mean RM of a given day entirely 1036 to an external screen, then our constraint on $\angle \beta_{2}$ spans $1037\left(-168^{\circ},-85^{\circ}\right)$. Adding this constraint to Figure 6 results 1038 in Figure 8. Performing this cut requires inclination 1039 angles $>90^{\circ}$, corresponding to clockwise motion on the 1040 sky, which now agrees with the aforementioned models 1041 of polarized NIR and sub-mm flares.
1042 With RM derotation, one of the best bet models from 1043 our total intensity analysis passes all applied total in1044 tensity and polarimetric constraints. This is the MAD ${ }_{1045} a_{*}=0.94 R_{\text {high }}=160 i=150^{\circ}$ aligned model. The sec1046 ond best bet model from Paper V had $a_{*}=0.5$ and oth1047 erwise identical parameters. This second model passes 1048 all constraints except $\langle | m\rangle$ which it underproduces by $1049 \sim 3 \%$. In order for the $a_{*}=0.94$ best bet model to 1050 pass, at least $97 \%$ of the measured RM must arise from 1051 an external screen. Notably, the best bet model fails if 1052 the smaller RM measured at 86 GHz a few days prior, $1053-2.14 \pm 0.51 \times 10^{5} \mathrm{rad} \mathrm{m}^{-2}$ (Wielgus et al. 2023), is 1054 instead interpreted as the external screen.
1055 In Figure 9, we visualize the best bet model (BHAC 1056 shown) that survives with RM derotation. In the left 1057 two columns, we plot its full polarimetric image in the 1058 style of Figure 5. No blurring is applied in the left-

## All Polarimetric Constraints



Figure 8. As in Figure 6 but including the constraint on the phase of $\beta_{2}$ with RM derotation. Only models with clockwise motion $\left(i>90^{\circ}\right)$ pass. A best bet model from Paper V passes all total intensity and polarimetric constraints: MAD $a_{*}=0.94$ $R_{\text {high }}=160 i=150^{\circ}$ aligned.

1059 most column, and a $20 \mu$ as FWHM Gaussian kernel is 1060 convolved with the image in the second column to ap1061 proximate EHT resolution. This model features a bright 1062 photon ring, and in our image without blurring, we omit 1063 total intensity contours from the circular polarization 1064 map to reveal a photon ring sign inversion, (discussed 1065 in Mościbrodzka et al. 2021; Ricarte et al. 2021).
1066 On the right, we produce a map of the density of the 1067 observed emission in the equivalent KHARMA simula1068 tion (using Kerr-Schild coordinates). The emission den1069 sity map is normalized such that its peak value is unity, 1070 and it is visualized in logarithmic scale with 3 orders 1071 of magnitude in dynamic range. Our line of sight is 1072 indicated by the green arrow, and a white contour en1073 closes the $90 \%$ of the total emission. This reveals that 1074 while the emission is peaked at small radius near the 1075 disk mid-plane, a substantial fraction of the emission 1076 originates from a more diffuse jet funnel region. Com1077 puting an emission-weighted characteristic emission ra${ }_{1078}$ dius $\bar{x} \equiv \int x \epsilon d V / \int \epsilon d V$, where $\epsilon$ is the emission density 1079 and $x$ is the radius in cylindrical coordinates, we find ${ }_{1080} \bar{x}=7.3$. We note that our choices to include only ther1081 mal electron distribution functions and cut out regions 1082 with $\sigma>1$ in this work minimize the potential contri1083 bution of a jet to the total emission (e.g., Figure 12 of 1084 Fromm et al. 2022). A significant jet component may be 1085 necessary to reproduce the flat spectral index at these 1086 frequencies (Falcke et al. 1993; Falcke \& Markoff 2000; 1087 Mościbrodzka \& Falcke 2013).

1088 At a radius of $7.3 r_{g}$, we compute a mass-weighted av1089 erage magnetic field strength of $26_{-4}^{+3} \mathrm{G}$, where the range 1090 quoted here corresponds to the 16th to 84th percentile 1091 values obtained in the time series. This value agrees rea1092 sonably well with the one-zone model discussed in 3.1, 1093 although we note that this value evolves substantially 1094 with radius, reaching $67_{-9}^{+8} \mathrm{G}$ at a radius of $4 r_{g}$, and $1095560_{-80}^{+80} \mathrm{G}$ at the horizon.
1096 This model produces an outflow power of $4 \times$ $109710^{38} \mathrm{erg} \mathrm{s}^{-1}$ and has an accretion rate of $5 \times$ ${ }_{1098} 10^{-9} M_{\odot} \mathrm{yr}^{-1}$. This model has a very large jet effi1099 ciency of approximately $150 \%$ powered by the Blandford ${ }_{1100} \&$ Znajek (1977) mechanism. Yet despite its efficiently, 1101 the jet's power is not high enough to expect global effects 1102 on the evolution of our Galaxy (e.g., Su et al. 2021).

## 1103 6. DISCUSSION AND CONCLUSION

1104 The first polarized image of $\operatorname{Sgr} \mathrm{A}^{*}$ on event horizon 1105 scales exhibits a high resolved polarization fraction of $110624-28 \%$ and an ordered, rotationally symmetric EVPA 1107 pattern. Through semi-analytic arguments and compar1108 isons to GRMHD simulations, we come to the following 1109 conclusions:

- The large resolved polarization fraction implies 1111 that the magnetic field on event horizon scales can1112 not be very tangled on scales smaller than beam, nor can Faraday rotation add too much additional disorder to the EVPA structure. The disparity between the spatially resolved (24-28 \%) and un-


Figure 9. The best bet model of Sgr A*: MAD $a_{*}=0.94 R_{\mathrm{high}}=160 i=150^{\circ}$ aligned. In the left two columns, we plot its simulated image in the style of Figure 5. Images in the first column are unblurred, and images in the second column are blurred with a Gaussian with a FWHM of $20 \mu$ as, approximating EHT resolution. In the right panel, we provide a map of the emission in this model. The white contour encloses $90 \%$ of the total emission, the dashed white circle demarcates the horizon, and the green arrow indicates our viewing angle. While the emission peaks close to the BH in the mid-plane, a significant fraction of emission originates from a more diffuse region, including the jet sheath.
${ }_{133}$ While our ideal GRMHD simulations containing only 1134 thermal electron distributions have done remarkably ${ }_{1134}{ }_{1135}$ well at reproducing many of the observed quantities of ${ }_{136} \mathrm{Sgr} \mathrm{A}^{*}$, they nevertheless have many known imperfec1137 tions. Most of these models over-estimate time vari1138 ability, including the best bet model (Paper V), and we 139 caution that the values inferred from our best-bet model
resolved (2.0-7.3 \%) linear polarization fractions can be attributed to cancellations due to the symmetric nature of the image.

- Driven mostly by the spatially resolved polarization fraction, our constraints strongly favor MAD models over their SANE counterparts, as in M87* Paper VIII.
- If we rely on internal Faraday rotation to produce the observed RM and do not perform derotation, then there is no model that passes all total intensity and polarimetric constraints.
- On the other hand, if we assume that the RM can be attributed to an external screen and derotate the EVPA pattern, then we find one model that passes all applied total intensity and polarimetric constraints: MAD $a_{*}=0.94 R_{\text {high }}=160 i=150^{\circ}$ aligned.

140 should not be interpreted as measurements. Known ar1141 eas where these simulations can be improved include the 1142 following:

1143 - Initial Conditions: All of our simulations are 1144 initialized with tori that are either perfectly 1145 aligned or anti-aligned with the BH angular mo1146 mentum axis. Simulations feeding the BH via stel1147 lar winds have different variability characteristics 1148 (Murchikova et al. 2022) and can self-consistently 1149 predict an external Faraday screen (Ressler et al. 1150 2019, 2023). Tilted disk models (e.g., Fragile et al. 11512007 ; Liska et al. 2018; Chatterjee et al. 2020) may 1152 lead to different Faraday rotation characteristics 1153 due to their geometry at large radii. 1155 brodzka et al. (2016) prescription that we adopt 1156 to set the electron temperature broadly captures 1157 the trends seen in kinetic simulations that explic1158 itly model heating and cooling (e.g., Chael et al. 1159 2018; Dexter et al. 2020; Mizuno et al. 2021; Di1160 hingia et al. 2023), but does not reproduce them in 1161 much detail. More generally, a non-thermal con1162 tribution to the electron distribution function is 1163 believed to be necessary to reproduce the spectral 1164 energy distribution (Özel et al. 2000; Markoff et al.

1165

1221 internal Faraday rotation may exhibit more spatial vari1222 ation and potentially sign flips due to turbulence in the 1223 inner accretion flow (Ricarte et al. 2020).
1224 Given the vastness of parameter and modeling space 1225 available to theoretical interpretation, we expect the po1226 larized image of Sgr A* to continue to constrain models 1227 for many studies to come. This growing EHT dataset 1228 will continue to challenge theoretical models and pro1229 vide insights into the nature of black holes, accretion, 1230 and plasma physics.

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1548

## APPENDIX

## A. KEY TRENDS: BRIDGING THEORY AND

 OBSERVATIONSUsing our GRMHD models, we explore a 5dimensional parameter space, constrained by 8 observable aspects of the polarized image that we believe are tied to the models in physically understood ways. Below, we highlight the most salient trends in our simulated image library to explain their physical origins. We

1557 focus on illustrative examples in this section, but pro1558 vide exhaustive distributions of observables calculated 1559 from our GRMHD models in Appendix H.

## 1560

## A.1. Magnetic Field State

By construction, SANE models have weaker magnetic 1562 fields near the horizon than their MAD counterparts 1563 at a given accretion rate. As a result, once the fluid

1564

1574 MAD and SANE models with fixed $a_{*}=0.5 R_{\text {high }}=40$ ${ }_{1575} i=50^{\circ}$ and aligned fields. In the upper panels, we plot 1576 the time-averaged KHARMA images in total intensity 1577

1582 are calculated based on differences between codes, time-
1583 variability, and nearest-neighbors in parameter space.
1584 The details of how these theoretical error bars are cal1585 culated are provided in Section 5.
1586 Here, we see that the SANE model has much lower 1587 linear polarization fraction $\left(\langle | m\rangle\right.$ and $\left.| \beta_{2} \mid\right)$, which can ${ }_{1588}$ be attributed to a much larger Faraday depth $\left(\left\langle\tau_{\rho_{V}}\right\rangle\right) .{ }^{10}$ ${ }_{1589}$ Much larger Faraday depths in SANE models than their 1590 equivalent MADs drive most of the differences between 1591 these two classes of models. SANE models can also 1592 produce larger circular polarization $(\langle | v\rangle)$ due to Fara1593 day conversion (M87* Paper IX). Palumbo et al. (2020) 1594 showed that $\left|\beta_{2}\right|$ is a strong discriminant between MAD 1595 and SANE models of M87*. As expected, $\left|\beta_{2}\right|$ is signif1596 icantly larger for the MAD model than for the SANE. 1597 Interestingly, while SANE models of M87* usually ex1598 hibit $\angle \beta_{2} \sim 0$, corresponding to radial EVPA patterns, 1599 the EVPA pattern in this SANE model acquires some 1600 twist due to a tilted forward-jet that we view in projec1601 tion (top left side).

1602

## A.2. Spin

1603 The BH spin is a particularly interesting quantity to 1604 constrain due to implications for its cosmic assembly and 1605 feedback processes. A number of EHT-related studies 1606 have recently explored signatures of spin, and resolved 1607 linear polarization structure has been shown to be one 1608 of the most promising and accessible probes (Palumbo 1609 et al. 2020; Emami et al. 2023a; Qiu et al. 2023; Ricarte 1610 et al. 2023; Chael et al. 2023).
1611 In Figure 11, we plot the phase and amplitude of $\beta_{2}$ as 1612 a function of spin for the subset of the MAD $R_{\text {high }}=10$ ${ }_{1613} i=30 / 150^{\circ}$ reversed models. The outer accretion disk
${ }^{10}$ Faraday rotation depth is obtained by integrating the radiative ${ }^{1665}$ transfer coefficient of Faraday rotation, $\rho_{V}$, along each geodesicic666 then performing an intensity-weighted average across the imag ${ }_{6667}$ (see e.g., M87* Paper VIII).

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## A.4. $R_{\text {high }}$ (Electron Temperature)

increasing $R_{\text {high }}$ indirectly increases the Faraday rotation depth (Mościbrodzka et al. 2017; Jiménez-Rosales \& Dexter 2018; Ricarte et al. 2020; M87* Paper VIII). ${ }_{1668}$ Increasing $R_{\text {high }}$ also shifts emission away from the mid-


Figure 10. Comparison of the MAD and SANE $a_{*}=0.5 R_{\mathrm{high}}=40 i=50^{\circ}$ aligned models (KHARMA images plotted). As in Figure 5, the length of the ticks scale with the polarized flux in each pixel, normalized for each model individually. A selection of polarimetric observables are shown with theoretical error bars, along with our observational constraints in gray. The constraint on $\angle \beta_{2}$ prior to RM derotation is shown with a hatched band instead of a filled band. With other parameters held fixed, SANE models typically have lower resolved linear polarization due to higher Faraday depths and can sometimes reach large values of circular polarization. Large Faraday depths in SANEs result in lower values of $\langle | m\rangle$ and $| \beta_{2} \mid$.

1669 plane and concentrates it towards the jet funnel region ${ }_{1670}$ (Paper V; Wong et al. 2022). This effect is much weaker 1671 for MADs than for SANEs, since MAD models intrinsi1672 cally have smaller plasma $\beta$ on horizon scales.
1673 In Figure 13, we plot time-averaged BHAC MAD $a_{*}=$ $16740.5 i=130^{\circ}$ aligned field models as a function of $R_{\mathrm{high}}$, 1675 as well as several of their linear polarization observables. 1676 Increasing Faraday depolarization explains the declines 1677 in $\langle | m\rangle$ and $| \beta_{2} \mid$ with $R_{\text {high }}$. The polarization grows 1678 more asymmetric as $R_{\text {high }}$ increases, because at this in1679 clination, the Faraday thick mid-plane is at the top half 1680 of the image. This, combined with increased Faraday 1681 rotation that slightly turns ticks clockwise ${ }^{11}$, leads to 1682 a shift in $\angle \beta_{2}$. In addition, $\left|\beta_{2}\right| /\left|\beta_{1}\right|$ decreases as the 1683 polarization grows more asymmetric.

## 1684 A.5. Magnetic Field Polarity

1685 In ideal GRMHD, the equations governing the evolu1686 tion of a magnetized fluid are invariant to a sign flip of

[^6]1687 the magnetic field direction. However, the equations of ${ }_{1688}$ GRRT are not, leading to potential polarimetric signa1689 tures of the poloidal field direction. When performing 1690 radiative transfer, $j_{V}$ (intrinsic circular polarization of 1691 emitted radiation) and $\rho_{V}$ (Faraday rotation) are each 1692 sensitive to the direction of the field with respect to the 1693 photon wave-vector. The historically negative Stokes V 1694 of $\mathrm{Sgr} \mathrm{A}^{*}$ is suggestive of a magnetic field oriented away 1695 from us. However, M87* Paper IX discusses how flipping 1696 the magnetic field direction can have non-trivial effects 1697 on the circularly polarized image (beyond a simple sign 1698 flip) as well as noticeable effects on $\angle \beta_{2}$ due to Faraday 1699 effects (see also Ricarte et al. 2021; Emami et al. 2023a). 1700 In Figure 14, we highlight the differences between 1701 aligned and reversed field models for the time-averaged 1702 KHARMA MAD $a_{*}=0.5 R_{\text {high }}=160 i=130^{\circ}$ models. 1703 Each model is blurred with a $20 \mu$ as FWHM Gaussian 1704 beam shown in total intensity and linear polarization 1705 ticks on the left, and circular polarization and total in1706 tensity contours on the right. We write $\angle \beta_{2}$ and $v_{\text {net }}$ 1707 for each model on the bottom left corner, revealing sigis $_{708}$ nificant and unpredictable differences, motivating inde-


Figure 11. Rotationally symmetric linear polarization structure as a function of spin, encapsulated in the phase and amplitude of $\beta_{2}$. For this plot, MAD $R_{\text {high }}=10 i=30 / 150^{\circ}$ reversed models are included, with either $i=30^{\circ}$ in blue or $i=150^{\circ}$ in red. Our observational constraints are shown as gray bands, and the constraint prior to RM derotation is shown as a hatched region. In this slice of parameter space, prograde models with spin values that are too large tend to produce polarization patterns that are too azimuthally symmetric and radially oriented compared to our observations.


Figure 12. A selection of polarimetric observables plotted as a function of inclination in a slice of our parameter space corresponding to MAD $a_{*}=0.94 R_{\mathrm{high}}=10$ reversed models. In very ordered models such as this one, symmetry and cancellation leads to the smallest net linear polarization fractions for face-on viewing angles at the same time that the resolved linear polarization fraction is highest. In this model, $\angle \beta_{2}$ encodes the direction of motion, and $v_{\text {net }}$ encodes the direction of the magnetic field with respect to the line of sight.


Figure 13. Time-averaged images and a selection of polarimetric observables as a function of $R_{\mathrm{high}}$, for the slice of our parameter space corresponding to MAD $a_{*}=0.5 i=130^{\circ}$ aligned models (BHAC images plotted). In this slice of parameter space, Faraday rotation has a clear effect, since increasing $R_{\text {high }}$ leads to smaller linear polarization fractions and correspondingly $\left|\beta_{2}\right|$. At this inclination, sight lines at the top of the image pass through the Faraday thick disk mid-plane, increasing the polarization asymmetry as $R_{\mathrm{high}}$ increases, which is reflected in $\left|\beta_{2}\right| /\left|\beta_{1}\right|$. Both line-of-sight Faraday rotation and changing emission regions lead to a trend in $\angle \beta_{2}$.
${ }_{1709}$ pendent ray-tracing for each magnetic field polarity. In 1710 linear polarization, the difference comes from reversing 1711 the direction that Faraday rotation shifts the EVPA pat1712 tern. The magnitude of this effect is larger than that 1713 reported in M87* Paper IX because M87* models are 1714 oriented almost completely face-on, viewed through an 1715 evacuated funnel (Ricarte et al. 2020). Models of Sgr A* 1716 can accumulate larger Faraday rotation depths as radi1717 ation passes through more of the disk at larger inclina1718 tions. In circular polarization, this particular model is 1719 mostly characterized by an overall sign-flip, but this is 1720 not uniform across the image, leading to a small differ1721 ence in $v_{\text {net }}$. This is because the coefficient of Faraday 1722 conversion, which exchanges linear and circular polar1723 ization, is invariant to a sign flip in the magnetic field 1724 direction.

## 1725 B. IMPACTS OF INDIVIDUAL OBSERVATIONAL

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## CONSTRAINTS

1727 In Section 5, we included a limited selection of plots 1728 reflecting which of our models passed each of the polari1729 metric observational constraints on Sgr A*. Here, we ${ }_{1730}$ break down the impact of each constraint individually.
${ }_{1731}$ In Figure 15, we plot the impact of our $\langle | m\rangle$ con1732 straint, which we find is the most important for model 1733 selection. Compared to the other constraints, $\langle | m\rangle$ is 1734 measured relatively precisely and the two methods agree 1735 very well. The Faraday rotation depth explains the 1736 trends in this figure (see Appendix H). More Faraday 1737 depolarization tends to occur if $R_{\text {high }}$ is larger, if the 1738 inclination is larger, or if the model is SANE. Of the 1739 models that fail the $\langle | m\rangle$ constraint, most are too de1740 polarized, but some low $R_{\text {high }}$, high-spin, face-on mod${ }_{1741}$ els are ruled out for predicting values of $\langle | m\rangle$ that are 1742 too large. We find that $\langle | m\rangle$ is much more constrain1743 ing than $m_{\text {net }}$ (Figure 16), which is measured much less 1744 precisely. Recall that $m_{\text {net }}$ is substantially lower (and 1745 less consistent with the light curve) in the $m$-ring model 1746 than Themis. We find that if the higher and tighter ${ }_{1747} m_{\text {net }}$ constraint from ThEMIS had been adopted on its 1748 own, then this would have ruled out many face-on mod1749 els (explained in Section 3.3 and Section A.3), including 1750 the $a_{*}=0.94$ best-bet model.
1751 Our circular polarization constraints are not very im1752 pactful. Our upper limit on $\langle | v\rangle$ rules out no mod1753 els (Figure 17), as all GRMHD models produce $\langle | v\rangle$ 1754 lower than the upper limit (similar to M87* Paper IX). ${ }_{1755}$ Our constraint on $v_{\text {net }}$ is also not very impactful (Fig1756 ure 18), but while not visible with our plotting scheme, it 1757 does rule out many retrograde models that have aligned ${ }_{1758}$ fields. These models produce preferentially positive $v_{\text {net }}$, 1759 while decades of $\operatorname{Sgr} \mathrm{A}^{*}$ observations produce $v_{\text {net }}<0$. ${ }_{1760}$ Our constraints on $\left|\beta_{2}\right|$ (Figure 20), $\left|\beta_{1}\right|$ (Figure 19), 1761 and $\left|\beta_{2}\right| /\left|\beta_{1}\right|$ (Figure 21) are impactful, but they are cor1762 related with each other and $\langle | m\rangle$. Compared to $\langle | m|\rangle$, ${ }_{1763}\left|\beta_{2}\right|$ additionally rules out some $i=90^{\circ}$ models. The 1764 ratio $\left|\beta_{2}\right| /\left|\beta_{1}\right|$ is not very constraining, as most models

1765 naturally produce $\left|\beta_{2}\right|>\left|\beta_{1}\right|$, in agreement with the ob-

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$$ 6 servations. While some methods in Paper VII produced 1767 1760 tained in this paper produced more modest values. In 1770 terestingly, a few face-on models are ruled out for being 1771 too dominated by the rotationally symmetric mode.

Finally, we consider the effect of $\angle \beta_{2}$ both with and 1773 without RM derotation in Figure 22 and Figure 23 re1774 spectively. In either case, models with preferentially ra1775 dial EVPA patterns are most likely to fail, such as face1776 on prograde MAD models (see Section A.2). With dero1777 tation, this constraint produces a preference for clock1778 wise motion on the sky $\left(i>90^{\circ}\right)$. Without derotation, 1779 the opposite is true, and more models fail outright since 1780 the constraint is tighter.

## C. ROTATION MEASURE

1782 The rotation measure (RM) of Sgr A* is a significant 1783 systematic uncertainty in our work, affecting our inter1784 pretation of $\angle \beta_{2}$. The RM is defined

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$$
\begin{equation*}
\mathrm{RM} \equiv \frac{\Delta \chi}{\Delta \lambda^{2}} \tag{C1}
\end{equation*}
$$

1786 where $\chi$ is the EVPA and $\lambda$ is the wavelength. If the ${ }_{1787}$ EVPA of the polarized emission does not intrinsically

$$
\mathrm{RM}=8.1 \times 10^{5} \mathrm{rad} \mathrm{~m}^{-2} \int_{\text {source }}^{\text {observer }} f_{\mathrm{rel}}\left(\Theta_{e}\right) \frac{n_{e}}{1 \mathrm{~cm}^{-3}} \frac{B_{\|}}{G} \frac{d s}{\mathrm{pc}},
$$

1794 where $n_{e}$ is the electron number density, $B_{\| \mid}$is the lo1795 cal magnetic field parallel to the photon wave-vector, 1796 and $f_{\text {rel }}\left(\Theta_{e}\right) \approx \log \left(\Theta_{e}\right) /\left(2 \Theta_{e}^{2}\right)$, a factor causing lower 1797 efficiency as electrons become too relativistic (Jones \& 1798 O'Dell 1977). If the two assumptions above are correct, 1799 then the "intrinsic" EVPA pattern can be easily recov1800


Figure 14. Impact of reversing the polarity of the magnetic field on the time-averaged KHARMA MAD $R_{\text {high }}=160 a_{*}=0.5$ $i=130^{\circ}$ model. In radiative transfer, the handedness of Faraday rotation and intrinsic circularly polarized emission flip sign when the magnetic field flipped. This can lead to changes in the morphologies of both linearly and circularly polarized images.


Figure 15. Individual impact of our $\langle | m\rangle$ constraint on model selection. This tight constraint is our most informative, ruling out models that are either overly or insufficiently Faraday depolarized.

1816 Carefully predicting the RM directly for all of our 1817 GRMHD simulations would increase the computational 1818 cost by factors of a few (more than 2) with the software 1819 utilized in this work. This is because ray-tracing must be 1820 performed at different frequencies at non-uniform spac1821 ings to resolve potential phase wrapping and non- $\lambda^{2}$ be1822 havior of the EVPA. Nevertheless, we check the RM for 1823 a few snapshots of our models in Figure 24, where the ${ }_{1824} \mathrm{RM}$ is estimated by ray-tracing at 213, 215, 227, and ${ }_{1825} 229 \mathrm{GHz}$ (emulating observations) and then fitting for 1826 the slope $\mathrm{RM}=d \chi / d \lambda^{2}$. MAD models are plotted in 1827 the top row, and SANE models are plotted in the bot1828 tom row. Three inclinations are shown, $30^{\circ}$ in blue, $50^{\circ}$

1829 in orange, and $90^{\circ}$ in grey. All models are at $R_{\text {high }}=40$ 1830 and in an aligned field configuration. Note that these 1831 simulations only include material within $100 r_{g}$, but ab1832 initio simulations of the accretion of $\mathrm{Sgr} \mathrm{A}^{*}$ from stellar 1833 winds suggest that a steady Faraday screen could po1834 tentially be situated at even larger radii (Ressler et al. 1835 2019, 2023).
${ }^{1836}$ We find that most of our models naturally produce ${ }_{1837}|\mathrm{RM}| \sim 10^{5} \mathrm{rad} \mathrm{m}^{-2}$ at at least one point in time, in 1838 rough agreement with the observed value. The SANE 1839 models, as well as the MADs at $90^{\circ}$, tend towards larger 1840 values, similar to models of M87* (Ricarte et al. 2020). ${ }_{1841}$ However, as in previous works, the RM flips sign in ev-


Figure 16. Individual impact of our $m_{\text {net }}$ constraint on model selection. This is less impactful than $\langle | m\rangle$, mostly because the allowed range is much larger.


Figure 17. Individual impact of our $\langle | v\rangle$ constraint on model selection, which is treated as an upper limit. All models naturally produce smaller resolved circular polarization fractions than this constraint.


Figure 18. Individual impact of our $v_{\text {net }}$ constraint on model selection. This is not very constraining, but does rule out models that whose distributions of $v_{\text {net }}$ are skewed towards positive values.


Figure 19. Individual impact of our $\left|\beta_{1}\right|$ constraint on model selection.


Figure 20. Individual impact of our $\left|\beta_{2}\right|$ constraint on model selection. This observable is correlated with $\langle | m\rangle$ and behaves similarly.


Figure 21. Individual impact of our $\left|\beta_{2}\right| / / \beta_{1} \mid$ constraint on model selection. This only rules out a few face-on models that are too rotationally symmetric.


Figure 22. Individual impact of our $\angle \beta_{2}$ constraint with RM derotation. This constraint produces a preference for $i>90^{\circ}$.


Figure 23. Individual impact of our $\angle \beta_{2}$ constraint without RM derotation. Compared to Figure 22, fewer models pass and there is now a preference for $i<90^{\circ}$.

## D. IMPACT OF OUTER INTEGRATION RADIUS

1874 We test the impact of the outer radiative transfer in1875 tegration radius in Figure 25, where we ray-trace a few 1876 KHARMA snapshots at a variety of radii ranging from 187730 to $300 r_{g}$. We focus on $\angle \beta_{2}$, which should be directly 1878 affected by Faraday rotation on large scales. Both incli1879 nations of $50^{\circ}$ and $90^{\circ}$ are considered, with $R_{\text {high }}$ values 1880 of both 10 and 160. Fortunately, we find that $\angle \beta_{2}$ ap188

1894 While $\angle \beta_{2}$ appears to show evolution for some mod1895 els, the other polarimetry metrics are well converged, 1896 and show minimal change for all models across integra1897 tion radius. However, although we have checked the

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## E. IMPACT OF CUTTING JET CENTER (" $\left.\sigma_{\mathrm{CUT}} "\right)$

The polar funnel in the GRMHD simulations is filled ,

## 1947 F. IMPACT OF NON-THERMAL ELECTRONS

1948 Throughout this work, we have considered only ther1949 mal electron distribution functions (eDFs) when per1950 forming GRRT. Here, we briefly explore the impact of 1951 non-thermal electrons in the polarimetric properties of


Figure 24. Rotation Measure (RM) as a function of time for a selection of KHARMA model snapshots, each with $R_{\text {high }}=40$ and aligned magnetic fields. Our models can roughly reproduce the observed magnitude of the RM, but predict rapid sign flips (colored vs. white markers) that are not observed.


Figure 25. $\angle \beta_{2}$ as a function of outer integration radius for a selection of KHARMA models. The GRRT in our work includes material at $r \leq 100 r_{g}$, encoded by the gray band. Lines transition from thick to thin at the first radius at which $\left|\beta_{2}\right|<0.05$. For models with $\left|\beta_{2}\right|>0.05, \angle \beta_{2}$ typically converges by $r=100 r_{g}$.

1952 one GRMHD model: MAD $a_{*}=0 R_{\text {high }}=40 i=150^{\circ}$ 1953 aligned. Two non-thermal prescriptions are explored:

1954 - Variable $\kappa$ : In each cell, a $\kappa$ distribution (Vasyli-

1955
1956
$1971 \kappa$, and $\kappa=5$ models respectively.) Interestingly, $v_{\text {net }}$
1972 switches sign in the $\kappa=5$ model, while $\angle \beta_{2}$ varies only
1973 slightly, due to its link with the underlying field geom-
1974 etry. Overall, images with non-thermal eDFs will be 1975 useful to study in future work.

## 1976 G. AN INTERPOLATIVE SCORING SCHEME

1977 With our GRMHD models, we coarsely sample a five1978 dimensional parameter space. Here, we investigate the 1979 possibility that this sparse sampling misses potentially
1980 passing models by performing scoring using expanded
1981 theoretical error bars. We conceptualize each combina1982 tion of $a_{*}, R_{\mathrm{high}}$, and $i$ as a volume in three-dimensional $90 \%$ quantiles of the neighbor does not overlap,

1986 each observable to the midpoints of their nearest neigh-
1987 bors. This scheme helps mitigate sparse sampling, but
1988
el is lo ider
1990 this methodology fails to consider correlated evolution
1991 between observables.
1992 In Figure 27 and Figure 28, we show the results of our 1993 interpolative scoring scheme considering all polarimet-

1996 cases. The preference for clockwise motion with dero-
1997 tation or counter-clockwise motion without derotation
1998 is less dramatic with this scheme. Without derotation, 1999 both best-bet models still fail. With derotation, the 2000 second best-bet model from Paper V MAD $a_{*}=0.5$ ${ }_{2001} R_{\text {high }}=160 i=30 / 150^{\circ}$ also passes in this scheme. 2002 Without interpolation, this model had only failed by 2003 producing too little $\langle | m\rangle$.
2004 This interpolative scoring scheme does not produce 2005 as clear of a preference for MAD over SANE models.

2006 We find that this difference is driven by a shortcoming 2007 of this method: SANE models evolve very rapidly with ${ }_{2008} R_{\text {high }}$, especially between $R_{\text {high }}=1$ and $R_{\text {high }}=10$, 2009 leading to very large theoretical error bars. We explore 2010 one example in Figure 29, where a set of KHARMA 2011 SANE $a_{*}=-0.5 i=150^{\circ}$ aligned field models are ray2012 traced at intermediate values of $R_{\text {high }} \in\{3,5,8\}$. Each 2013 of our 8 polarimetric observables is plotted, and we bet2014 ter resolve the rapid evolution in these parameters with $2015 R_{\text {high. }}$. A noteworthy interaction occurs in our inter2016 polation scheme with $\langle | m\left\rangle\right.$ and $\angle \beta_{2}$, two of our most 2017 constraining observables. We see that at $R_{\text {high }}=1$, the 2018 model overproduces $\langle | m\left\rangle\right.$ but fails to reproduce $\angle \beta_{2}$, 2019 which is too radial. Meanwhile, SANE models with ${ }_{2020} R_{\text {high }}=10$ have too low $\langle | m\rangle$ and a uniformly dis2021 tributed $\angle \beta_{2}$. Interpolation allows models in this region 2022 to pass because our scoring system suggests there might 2023 be a model with intermediate $R_{\text {high }}$ that has both a cor2024 rect $\angle \beta_{2}$ and $\langle | m\rangle$. However, with better resolution in ${ }_{2025} R_{\text {high }}$, we do not find an individual model that would 2026 pass. Overall, this exercise shows that our main con2027 clusions are not likely driven by our sparse sampling of 2028 parameter space.

## 2029 H. GRMHD OBSERVABLE DISTRIBUTIONS

${ }_{2030}$ To visualize trends of our 8 observables in the 52031 dimensional parameter space that we explore, we provide "violin" plots of our observables from our models as a figure set, the complete version of which is available in the online journal. In each figure, we consider one observable and one magnetic field state (either MAD or SANE models). One figure, the distributions of $m_{\text {net }}$ for MAD models, is shown in Figure 30. Different spins 8 are shown in different columns, and different values of $R_{\text {high }}$ are shown in different rows. Within each panel, we plot distributions as a function of inclination, where only 5 of the 9 inclinations ray-traced in this work are included to improve readability. Aligned field models are shown on the left, and reversed field models are shown on the right. The distributions with opposite magnetic field polarity are usually very similar, with the notable exceptions of $v_{\text {net }}$ and, more subtly, $\angle \beta_{2}$. To display the relative agreement or disagreement between codes, we plot BHAC models in red and KHARMA models in blue. H-AMR models, which are ray-traced for a subset of models only for comparison here and not for scoring, are displayed as dashed distributions when available. Finally, the observational constraints are shown in gray, where as usual the allowed range for $\angle \beta_{2}$ without RM derotation is shown as a hatched region.
Our last set of plots, distributions of the Faraday rotation depth $\left\langle\tau_{\rho_{V}}\right\rangle$, are not directly observable, but drive many of our physical trends as well as differences between codes. For a detailed discussion of the physical trends present in these figures, we refer readers to Appendix A.


Figure 26. Comparison of thermal and non-thermal eDFs for MAD $a_{*}=0 R_{\text {high }}=40 i=150^{\circ}$ aligned models. Changes in the distributions of polarimetric quantities motivate future exploration in this area.

## All Polarimetric Constraints



Figure 27. As Figure 7, but using the interpolative scoring scheme described in Appendix G.

2061 Differences between our KHARMA and BHAC models 2062 inflate our theoretical error bars in Section 5. We find 2063 that at least part of these differences arise from physi-
2064 cal approximations regarding the assignment of electron 2065 temperature during the GRRT. One fluid with a single 2066 adiabatic index is evolved in our GRMHD codes, but 2067 it represents both relativistic electrons (with an adia2068 batic index of $4 / 3$ ) and non-relativistic ions (with an 2069 adiabatic index of $5 / 3$ ). During the GRRT step of our 2070 calculations, only the electron temperature is relevant 2071 for the synchrotron emission that we observe. When as-

2072 signing electron temperatures, RAPTOR adopts (see e.g., 2073 Davelaar et al. 2018).

2074

$$
\begin{equation*}
\Theta_{e}=\frac{u}{\rho} \frac{m_{p}}{m_{e}} \frac{1}{3(R+1)} \tag{H3}
\end{equation*}
$$

2075 where $\Theta_{e}$ is the electron temperature, $u$ is the internal 2076 energy, and $R=T_{i} / T_{e}$ given by Equation 8. Meanwhile, 2077 IPOLE accounts for the difference in adiabatic indices 2078 by adopting

## All Polarimetric Constraints



Figure 28. As Figure 8, but using the interpolative scoring scheme described in Appendix G.


Figure 29. Distributions of observables for a selection of SANE models ray-traced with greater resolution in $R_{\text {high }}$ between 1 and 10. These correspond to KHARMA SANE $a_{*}=-0.5 i=150^{\circ}$ aligned models. We find rapid evolution in this part of parameter space.

2079

$$
\begin{align*}
\Theta_{e} & =\frac{u}{\rho} \frac{m_{p}}{m_{e}} \frac{\left(\gamma_{p}-1\right)\left(\gamma_{e}-1\right)}{\left(\gamma_{e}-1\right) R+\left(\gamma_{p}-1\right)} \\
& =\frac{u}{\rho} \frac{m_{p}}{m_{e}} \frac{2}{3(2+R)} \tag{H4}
\end{align*}
$$

2081
2082 where $\gamma_{e}=4 / 3$ and $\gamma_{p}=5 / 3$. Equation H 4 is physi-
2083 cally justified, but it sacrifices internal consistency with
2084 the GRMHD simulations, where a single fluid with
${ }_{2085} \gamma=4 / 3$ is evolved (Wong et al. 2022). When we set ${ }_{2086} \gamma_{e}=\gamma_{p}=\gamma=4 / 3$ in Equation H4, we recover Equa-

2087 tion H3 used by RAPTOR. Electron temperatures assigned
2088 by RAPTOR are systematically colder, $3 / 4$ as hot as the
2089 IPOLE prescription at $R=1$, and $1 / 2$ as hot as $R \rightarrow \infty$.
2090 This explains the systematically larger Faraday depths
2091 in our BHAC models relative to both KHARMA and
2092 H-AMR, which are both ray-traced with IPOLE.
2093 Larger differences are seen between SANE models 2094 than MADs. A unique SANE model is not believed to 2095 exist, and differences are known to occur at the GRMHD 2096 fluid level (Porth et al. 2019).
2097 Fig. Set 30. Violin Plots

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Figure 30. Distributions of observables for either MAD or SANE models. Black hole spin $a_{*}$ varies in each column, and $R_{\text {high }}$ varies in each row. Inclination varies along the $x$-axis. BHAC and KHARMA GRMHD simulations are shown in red and blue in each case, with H-AMR shown as an unfilled dashed curve. Distributions plotted on the left represent aligned magnetic fields, while those plotted on the right represent reversed magnetic fields. Our observational constraint is shown in gray. The complete figure set (18 images) is available in the online journal.

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[^1]:    ${ }^{1}$ This toy model is equivalent to the "m-ring" model used in Pa per VII, but we label with the index " $\ell$ " here to avoid ambiguities.

[^2]:    ${ }^{2}$ Rather than four-vector components, we average the Hodge dual of the Faraday tensor, then reconstruct the averaged magnetic field vector from the condition $b^{\mu} u_{\mu}=0$.
    3 The velocity is computed in the frame of the zero angular momentum observer in Boyer-Lindquist coordinates, while the magnetic field is computed in the fluid frame.

[^3]:    ${ }^{4}$ https://github.com/AFD-Illinois/kharma
    ${ }^{5}$ https://github.com/moscibrodzka/ipole
    ${ }^{6}$ https://bhac.science

[^4]:    ${ }^{7}$ https://github.com/jordydavelaar/raptor
    ${ }^{8}$ https://www.matthewliska.com/home-1/project-four-zng9grd5bb

[^5]:    ${ }^{9}$ For $\angle \beta_{2}$, to evade problems with phase wrapping, we translate angles into unit vectors in the complex plane centered at 0 before computing $90 \%$ quantiles, then translate back. If the magnitude of the mean of these unit vectors is less than 0.05 , we set the lower and upper ranges of $\angle \beta_{2}$ to -180 and 180 degrees, respectively. This occurs predominantly when a model is so depolarized that its $\angle \beta_{2}$ is approximately uniformly distributed.

[^6]:    ${ }^{11}$ For an aligned field model with $i>90^{\circ}$, the poloidal field pointed away from us, leading to a systematic clockwise shift.

