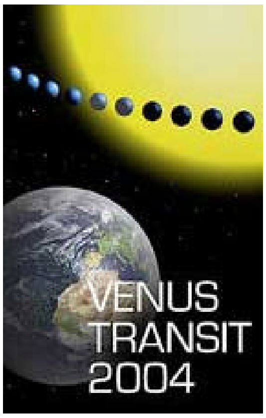


Analysis of the Observations of Contacts

Calculation of the AU by means of the Entire Data Set

Explanatory Note No. 2 - by Patrick Rocher and Jean-Eudes Arlot (IMCCE)

Introduction

After the deadline on July 10, 2004, for the delivery of contact timings by the participating groups in the VT-2004 Observing Campaign, we are now in a position to analyse the complete dataset of contact observations. Contrary to the procedure introduced for the on-line calculations (see Explanatory Note 1), we may now calculate the value of the Astronomical Unit (AU) using a non-constrained system or by means of Delisle's method while employing selected observations only.

For this purpose, we must first develop a criterion that will allow us to select the "good" (and hence, exclude the "bad") contact timings observations in the database, i.e. to identify those timings that shall be used for the definitive calculation of the AU. In a first step, we will assume that the "good" observations are those that are close in time to the corresponding (theoretical) predictions that we made. We will verify afterwards that this hypothesis is indeed correct.

Then, we will select several sets of "good" observations and calculate the value of the AU by means of the non-constrained method. We will also perform the calculation after weighting the observations according to the geographical location of the observation site, in the expectation that this will lead to a more correct value of the AU. Finally, we will apply Delisle's method on the "best" dataset in order to learn whether this method may be used in the same way as it was in the past, even though the observers in 2004 were not organized as were their predecessors during the past centuries.

The calculation of the AU on the basis of the entire database of contact timings

First, we have to ensure that the starting value of the parallax (or of the AU) that we initially enter into the calculation for the prediction of the contact times is sufficiently "good". This means that the average value of the AU that we then calculate on the basis of the recorded contact timings within a certain time interval around the predicted times must **converge** as we decrease this interval of time.

For instance: we select all timing observations that deviate no more than, say, 30 seconds of time from the corresponding predictions and we then compute the average value of the AU by obtaining the mean of all the values calculated from the individual observation. Then, we decrease this interval to 16s, 8s and finally 4s and again compute the average of the calculated AU for each of these intervals and their progressively smaller timing samples. If our initial value of the AU used for the prediction is good, then the successive averages of the AU will converge towards this initial value. At the same time, the number of observed contact timings that are earlier than the predicted time (i.e. fall in the first half of the time interval) will approach the number of contact timings that are later (i.e. fall in the second half of the time interval). If the distribution of errors is Gaussian, then the average of the (observed – predicted) time differences will move towards zero. As will be seen below, the analysis of the data that we received confirm this.

We hereafter list the results obtained from the dataset as this was available on June 18, 2004.

Characteristics of the database:

Number of registered (groups of) observers who delivered observations : 1440.

Number of registered (groups of) observers who observed the first contact : 722.

Number of registered (groups of) observers who observed the second contact : 1139.

Number of registered (groups of) observers who observed the third contact :1336.

Number of registered (groups of) observers who observed the fourth contact :1170.

Number of registered (groups of) observers who measured the external duration : 639.

Number of registered (groups of) observers who measure the internal duration: 1014.

Number of registered (groups of) observers who measured all four contacts: 616.

Total number of timing observations delivered: 4367.

In Table 1 below, we show for an adopted size of the time interval 2Δt around the predicted times of the contacts used for the calculation (column 1), first the corresponding number of observations that fall in this time interval (column 2), the average of the corresponding, calculated values of the AU found by solving the non-constrained system with these observations (column 3), the difference from the real value of the AU (column 4) and finally, the corresponding value of the solar parallax (column 5).

Size of the time interval	Number of observations	Average of the calculated AU	Shift to the true AU	Corresponding parallax
±30s = 60s	2459	148511434 km	-1086436 km	8,858482"
±15s = 30s	1719	148789697 km	-808172 km	8,841915"
±8s = 16s	1066	149421803 km	+176067 km	8,804510"
±4s = 8s	583	149608708 km	+10838 km	8,793511"

Table 1

This analysis of the observations confirms that the initial value of the parallax (or AU) used for the predictions is correct and in particular, the good quality of the predictions made with this value. This result is apparent from the fact that the data base contains many observations that are quite close to the prediction, e.g., there are still 1066 timings within the quite narrow interval of ±8s around the predicted times.

From now on, the definition of a “good” observation (timing) is easy to make: it is an observation that is near in time to the corresponding prediction. We now possess an objective criterion for the selection of observations in the data base.

This result was obtained by analysis of observations from 1440 observers and 4367 contact timings. It would not have been possible to obtain such a result in real-time on June 8, since we did not know in advance how to identify "good" observations and whether the errors will follow a normal law (Gaussian distribution).

Results for an interval of 16s (±8s) for each contact:

Table 2 provides, for each contact successively and for all contacts together, the number of observations corresponding to the time interval of 16s, the number of timing observations that are up to 8 seconds earlier than the predicted time (T_c), and the number of observations which are up to 8 seconds later, then the average of the calculated values of the AU using these observations, the difference from the true value of the AU, the standard deviation, and the parallax corresponding to the calculated AU.

Contact	Number of observations	Number of observations earlier than T_c	Number of observations later than T_c	Average calculated value of 1 AU in km	Diff. from true value in km	Standard deviation in km	Parallax in arcsec
T_1	104	49	55	149443844	+154026	186773	8.803212"
T_2	262	128	134	149590268	-7602	108359	8.794595"
T_3	421	187	234	149226725	-371145	324822	8.816020"
T_4	279	130	149	149549752	-48118	70599	8.796978"
All	1066			149421803	-176067	252081	8.804510"

Table 2

So, based on all 1066 timing observations which are within ±8s of the prediction (i.e. within a time interval of 16 seconds), we obtain the following result:

1 AU = 149421803 km ± 252081 km

This value is **176067 km** smaller than the true value of the AU, as determined by radar observations, i.e., a deviation of only 0.12 %.

We calculate the average $\overline{a}$ of the a_i (the calculated AU for each observation) as

$$\overline{a} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

Then we calculate the experimental variance s^2 and the experimental standard deviation s of these measurements of the AU:

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}\left(a_i - \overline{a}\right)^2$$

$$s = \sqrt{s^2}$$

Finally, assuming that $\overline{a}$ is a random variable following a normal law (Gaussian distribution of the errors), i.e. that the observations are without biases, then $\overline{a}$ is a good estimate of the AU and the standard deviation s_a on this estimate is given by:

$$s_a = \frac{s}{\sqrt{n}}$$

Do not confuse the experimental standard deviation σ of the measurements - which is independent of the law of distribution of the observations - with the standard deviation s_a on the estimate which depends on the law of distribution of the observations.

We find, as expected, a rather good distribution of the observations before and after the predicted values. We also note that the calculated value of the AU using the third contact timings is the one that is most off (the "worst") and has the largest standard deviation. This is due to the fact that the corresponding observations were mostly made when the Sun was near zenith for the observers (and the diurnal parallax was correspondingly small).

Table 3 is identical to Table 2, but now we only select observations corresponding to a time interval of half the size, i.e. within ±4s of the prediction (the full width of the internal is 8s).

Contact	Number of observations	Number of observations earlier than T_c	Number of observations later than T_c	Average calculated value of 1 AU in km	Diff. from true value in km	Standard deviation in km	Parallax in arcsec
T_1	60	23	37	149725155	+127285	131387	8.786672"
T_2	148	67	81	149618152	+20282	69271	8.792956"
T_3	225	102	123	149267460	-330410	217813	8.813614"
T_4	150	76	74	150064685	+466815	55667	8.766792"
All	583			149608708	+10838	11835	8.793511"

Table 3

So, using all the observations within ±4s of the prediction (i.e. an interval of 8 sec), we get the following result:

1 AU = 149608708 km ± 11835 km

This value is **10838 km** larger than the true value of the AU, as determined by radar observations, i.e., a deviation of only 0.007 %.

We note that when each contact is taken separately, the results tend to be degraded and that the calculated values of the AU obtained using timings of the contacts T_3 and T_4 display standard deviations that are smaller than the difference from the true value of the AU: a sign that the distribution of the errors may not be Gaussian. Contrarily, the result using all contacts is definitely better.

Why are the results based on timings from the third contact less good? Quite simply because very many observation sites had a small diurnal parallax at the time of the third contact (i.e. the Sun was very high above the horizon) and because they are close to the intersection of the shadow cone at the time of the third geocentric contact with the terrestrial ellipsoid. One can visualize this on a map by plotting on the terrestrial sphere the four curves that represent the intersections of the shadow cones and penumbra with the terrestrial ellipsoid at the moments of contacts as observed from the Earth's center.

The nearer the geographical site of a group of observers is to one of these curves, the smaller will be the effect of the diurnal parallax for the corresponding contact and the more likely it is that the dispersion of the measurement will be large, unless, of course, the recorded contact timing is exceptionally accurate. Moreover, the observation of the exact moment of the interior contact T3 is a bit more difficult than that of the external contact T4 because of the Black Drop phenomenon. One would expect that contacts T3 and T4 were easier to observe than the contacts T1 and T2, because the end of the transit was seen high in the sky by the majority of the observers - and this is indeed confirmed by the number of "good" observations: 225 such timings for T3 against only 148 for T2. The tables show, however, that the timings of contacts T3 and T4 produce less accurate values of the AU than those of contacts T1 and T2, and this is because the Sun was close to the zenith and the parallax angle correspondingly small for most of the observers.

Weighted average calculations

In the preceding calculation, we took the average of all the calculated values of the Astronomical Unit and we allotted the same weight to the values determined from each timing observation. However, it is known that the errors of observation, for a given contact and from an observing site that is "badly" located may generate significant variations in the results. Thus, depending on the observing site and the time of the observation (i.e. which contact), a random error of a few seconds in the timing measurement of a contact can have more or less strong effects on the resulting value of the calculated AU.

We will therefore try to weight these results by assigning a weight to each observation. The weight will be smaller if the observing site is badly located for the contact in question.

If it is assumed that the observations are made without bias (i.e. the distribution is not skew) with a random error τ. then the standard deviations of the parallax or the astronomical unit can be estimated for each observation by :

$$s_a = a_0 \frac{t}{|t_c - t_G|}$$

$$s_p = p_0 \frac{t}{|t_c - t_G|}$$

where t_c is the instant of the topocentric contact calculated and t_G is the instant of the geocentric contact.

One may then take as the weight of each observation : $w(k) = 1 / s^2(k)$

Then the weighted averages of the Astronomical Unit and the parallax are calculated starting from the individual values *a(k)* or *p(k)* calculated for each observation *k*, by using the following two equations :

$$a = \sum_{k=1}^{n} w_a(k).a(k) / \sum_{k=1}^{n} w_a(k)$$

$$p = \sum_{k=1}^{n} w_p(k).p(k) / \sum_{k=1}^{n} w_p(k)$$

and the standard deviations of the weighted averages are given by :

$$s_a^2 = \frac{\sum_{k=1}^{n} w_a(k)^2 s_a(k)^2}{\left(\sum_{k=1}^{n} w_a(k)\right)^2}$$

$$s_p^2 = \frac{\sum_{k=1}^{n} w_p(k)^2 s_p(k)^2}{\left(\sum_{k=1}^{n} w_p(k)\right)^2}$$

The corresponding results are shown in Table 4a, as they were obtained from the sample of 1066 contact timings inside a 16-second interval around the predicted values and assuming that the random error on each observation is ±5s. This table may be compared with Table 2.

Contact	Number of observations	Average calculated value of 1 AU in km	Diff. from true value in km	Standard deviation in km	Parallax in arcsec	Standard deviation on the parallax
T_1	104	149491052	-106818	194889	8,800432"	0,011457"
T_2	262	149564790	-33080	114908	8,796093"	0,006755"
T_3	421	149424892	-172978	231528	8,804328"	0,013610"
T_4	279	149312924	-284946	285616	8,810931"	0,016790"
All	1066	149507347	-90523	86718	8,799473"	0,005098"

Table 4a

If we instead assume that the random error on each observation is of +/- 10s, then the results shown in Table 4b are obtained:

Contact	Number of observations	Average calculated value of 1 AU in km	Diff. from true value in km	Standard deviation in km	Parallax in arcsec	Standard deviation on the parallax
T_1	104	149491052	-106818	389778	8,800432"	0.022913"
T_2	262	149564790	-33080	229816	8,796093"	0.013510"
T_3	421	149424892	-172978	463056	8,804328"	0.027221"
T_4	279	149312924	-284946	571233	8,810931"	0.033580"
All	1066	149507347	-90523	173437	8,799473"	0.010196"

Table 4b

It is evident that the weighted averages of the calculated values of the AU do not change if the assumed random error is doubled, but that the standard deviations on these averages are doubled. This illustrates the importance of the estimate of the error of measurement in the contact timing observations.

Broadly speaking, the weighted average calculation (Tables 4a and 4b) gives better results than the unweighted one (Table 2) since the badly situated observing sites are given less weight. But this does not hold when the contacts are considered individually, for the above explained reasons.

Use of Delisle's method.

Since we have now in hand all the observations made on June 8, we may select those which are suitable for the calculation of the AU by means of Delisle's method.

Consider the database consisting of the timing observations within the 16-sec time interval, as outlined above. We have then 104 observations of the first contact, 262 observations of the second contact, 421 observations of the third contact and 276 observations of the fourth contact. All of these observations are independent.

Delisle's method implies that, for each contact, the observations are combined two by two, pairing observations that have a large difference between the predicted times of the same contact. So, we will build, for each contact, a series of observations which are no longer independent since each observation may be combined with numerous other ones.

We combined only those observations that had a difference in the timing of the contact larger than 6 minutes of time. There were 103 such combinations of observations for the first contact, 1531 combinations of observations for the second contact, 1979 combinations of observations for the third contact and 773 combinations of observations for the fourth contact, that is to say, a total of. 4386 combinations of observations.

This combination of observations necessitates a weighting of the pairs to take into account the fact that the same observation may be used several (many) times. Each pair of observations will receive a certain weight. If we suppose that the timing observations are made without biases and with a random error τ, then the error on the difference in the time of contact is $\sqrt{2}t$ and the standard deviation on each parallax or AU calculated is then given by :

$$s_a = a_0 \frac{\sqrt{2}t}{dt_c}$$

$$s_p = p_0 \frac{\sqrt{2}t}{dt_c}$$

where a_0 and p_0 are the reference AU and parallax and dt_c is the difference between the predicted contacts. The choice of an optimal statistical combination is not a simple matter, but a good compromise consists in taking an average weight between the combinations by giving a weight $w(k) = 1 / s^2(k)$ to the k th pair.

Then the values of the AU and the parallax are calculated by starting from the individual values *a(k)* or *p(k)* calculated for each combination k using the following two equations:

$$a = \sum_{k=1}^{n} w(k).a(k) / \sum_{k=1}^{n} w(k)$$

$$p = \sum_{k=1}^{n} w(k).p(k) / \sum_{k=1}^{n} w(k)$$

where n is the number of combinations for a given contact.

These equations are no longer independent and we must then build the correlation matrix linking all the various combinations of observations. For each contact, this matrix is of the nth order, n being the number of pairs for the given contact. In this matrix, the coefficient of correlation $r(k,k')$ between the result k obtained from the combination (i,j) of two observations and the result k' obtained from the combination of two observations (i',j') is equal to 0, if (i,j) are different from $(i'j')$ (no common observation); is equal to 0,5, if (i,j) is combined with (i,j') or (i',j); and is equal to –0,5 if (i,j) is combined with (j',i) or (j,i'). The matrix is then symmetrical and the standard deviations on the weighted averages are given by :

$$s_a^2 = \frac{\left(\sum_{k=1}^{n} w_a^2(k).s_a^2(k) + 2\sum_{k=1}^{n} \sum_{k'>k} w_a(k)\, w_a(k')\, r(k,k')\, s_a(k)\, s_a(k') \right)}{\left(\sum_{k=1}^{n} w_a(k) \right)^2}$$

and

$$s_p^2 = \frac{\left(\sum_{k=1}^{n} w_p^2(k).s_p^2(k) + 2\sum_{k=1}^{n} \sum_{k'>k} w_p(k)\, w_p(k')\, r(k,k')\, s_p(k)\, s_p(k') \right)}{\left(\sum_{k=1}^{n} w_p(k) \right)^2}$$

The following table provides the results obtained with the sample described above, assuming that the random error on the observation of each contact is +/-5s.

Contact	Number of observation pairs	Weighted average AU in km	Diff. from true value in km	Standard deviation in km	Parallax in arcsec	Standard deviation in arcsec
T_1	103	149593369	-4501	1308668	8.794413"	0.076930"
T_2	1531	149604208	+6338	535661	8.793775"	0.031489"
T_3	1979	150623168	+1025298	423861	8.734286"	0.024917"
T_4	773	148904105	-693765	534664	8.835121"	0.031430"
All	4386	149840958	+243088	310577	8.779881"	0.018257"

Table 5

Using all the contacts, we thus find that: **1 AU = 149840958 km ± 310577 km**; this result is to be compared with the value obtained using the non-combined observations: **1 AU = 149421803 km ± 252081 km** or rather with the result obtained by making the weighted average : **1 AU = 149507347 km ± 86718 km**.

It is obvious that in this case, i.e. by pairing observations for which the difference of the contact moments is larger than 6 min, the **method of Delisle does not improve the results**. The averages calculated using contacts T1 and T2 are close to the true value, but they have large standard deviations. That is because we only combine some observations that have a large difference of time of contact. In fact, only one (very good) timing for T_1 and six for T_2 from the East are paired with all the European observations. The difference between the times of contacts are very large, (more than 12 minutes) and all the pairs have approximately the same weight, but on the other hand there is a very strong correlation in these pairings, so that the results have very large standard deviations (especially in the case of T_1). We observe an effect of the same order for contacts T_3 and T_4; also here there are very few observations that have a large difference from the European observations (six for T_3 and three for T_4), but this time the differences in the times of contacts are smaller (from 6 to 9 minutes).

The results would have been different if, as this was done during the past centuries, we had "sent" observers to carefully selected observing sites (or having found such observers already located near those places). Indeed, our base of observers has two important deficits: firstly, a very strong asymmetry with many European observers and rather few observers with the associated large difference in the times of contacts, and secondly, there is very large proportion of observations of the third contact and fourth contacts with the correspondingly small diurnal parallax (i.e. the Sun was high above the horizon) and for which the observing sites are near the above mentioned intersections of the shadow cone at the times of the geocentric contacts with the terrestrial ellipsoid shadow.

In spite of this, the above stated results are quite satisfactory because we determined the value of the AU and the solar parallax with a precision that corresponds to that expected by using the present methods of observations. It is also proves that the vast majority of the observations were well made. We are also pleased to report that it seems that very few, if any at all, seems to have been "cheating", in fact, no group reported all its contact timings with a "too good" a precision.

The four maps included at the end of this note provide, for each contact, the curves corresponding to a contact at a given instant t. We plotted in bold the curves C_1, C_2, C_3 and C_4 corresponding to the sites on Earth that experienced the contacts at the same time as did (a fictive observer at) the centre of the Earth. We also plotted on these maps the observing sites that we selected for our calculations as having a difference of less than 8 seconds from the predictions (the 16-second time interval).

Although the method of Delisle is thus not very powerful in this connection from a statistical point of view, it is certainly of great didactic interest. We have therefore extracted from the entire set of data, a smaller database made up of the "good observations". From this data base, students and teachers will be able to use the method of Delisle on any two observations of their choice and thus to understand that the geographical locations of the observing sites play an important role. In other words, two observing sites are not equivalent, even if the corresponding timings were made with the same measuring accuracy.

Conclusions

In conclusion, our best result on the AU is obtained by the non-constrained solution using 583 timings within 4 seconds of time (interval of 8s) from the prediction:

1 AU = 149 608 708 km ± 11 835 km with a difference from the true value of **+10 838 km**

while Delisle's method with all contacts (4386 pairs of timings from 1066 observations in an interval of 16s) gives:

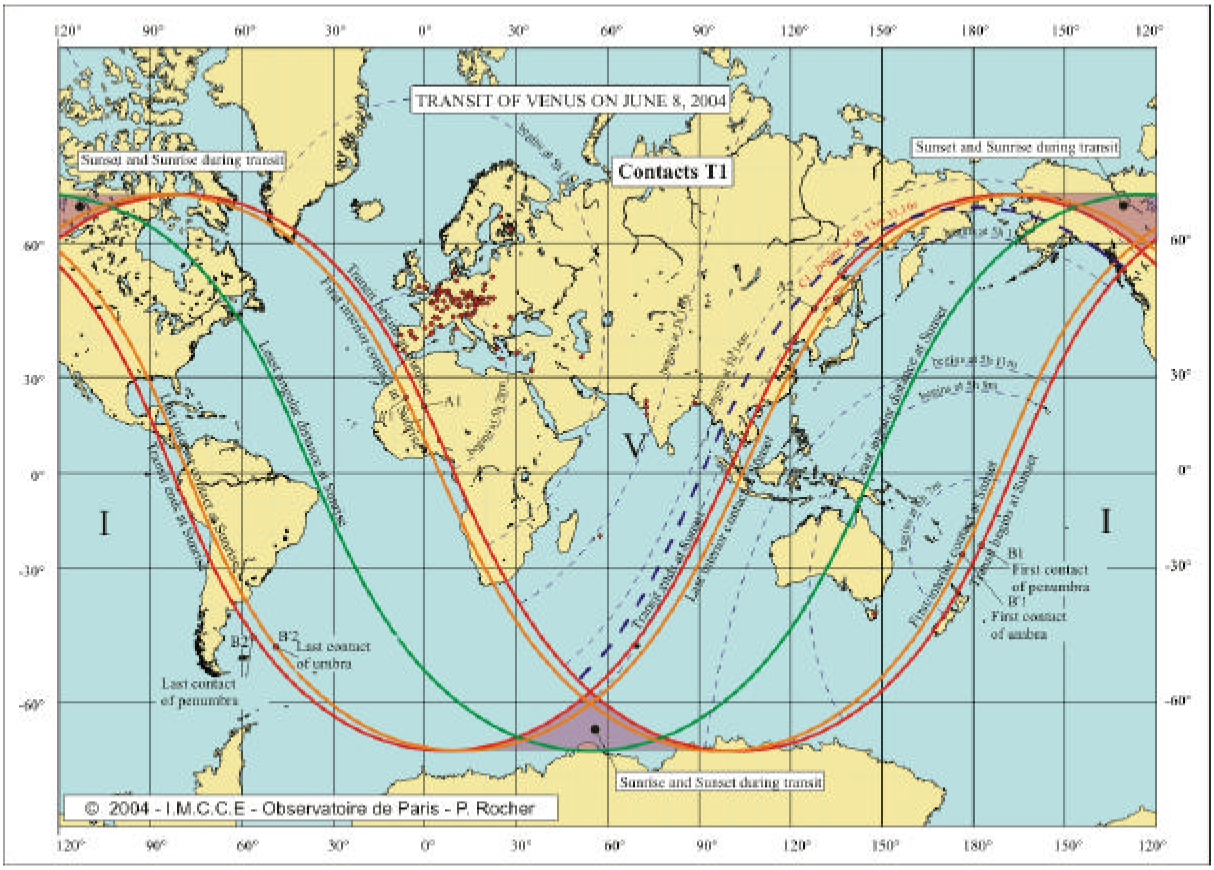
1 AU= 149 840 958 km ± 310 577 km with a difference from the true value of **+243 088 km**

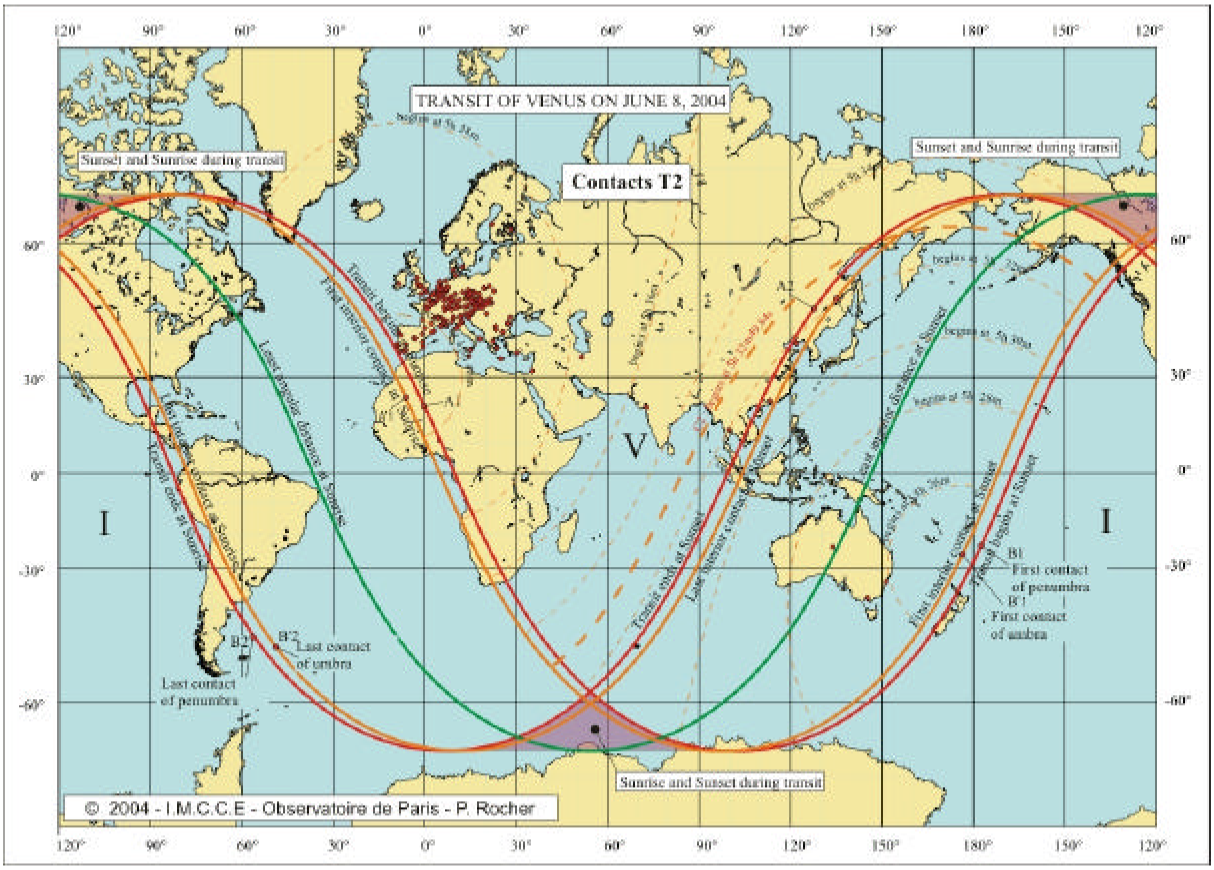
The result obtained by the non-constrained method is better than the one obtained in real time since we were now able to discard select the “good” observations (and discard the “bad” ones). It was also better than what was obtained with Delisle’s method since the observing sites were not very well distributed on the surface of the Earth.

Finally, it should be noted that the elimination of even more observations from the solution (e.g.selecting only observations within 3 or 2 seconds of time from those we predicted) does not provide better results – now there are simply too few data left.

Appendix: Circumstances of the four contacts

The four maps below provide, for each contact, the curves corresponding to a contact at a given instant t. We plotted in bold the curves C_1, C_2, C_3 and C_4 corresponding to the sites on Earth that experienced the contacts at the same time as did (a fictive observer at) the centre of the Earth. We also plotted on these maps the observing sites that were selected for our calculations as having a difference of less than 8 seconds from the predictions (the 16-second time interval).




TRANSIT OF VENUS ON JUNE 8, 2004
Contacts T2
Sunset and Sunrise during transit
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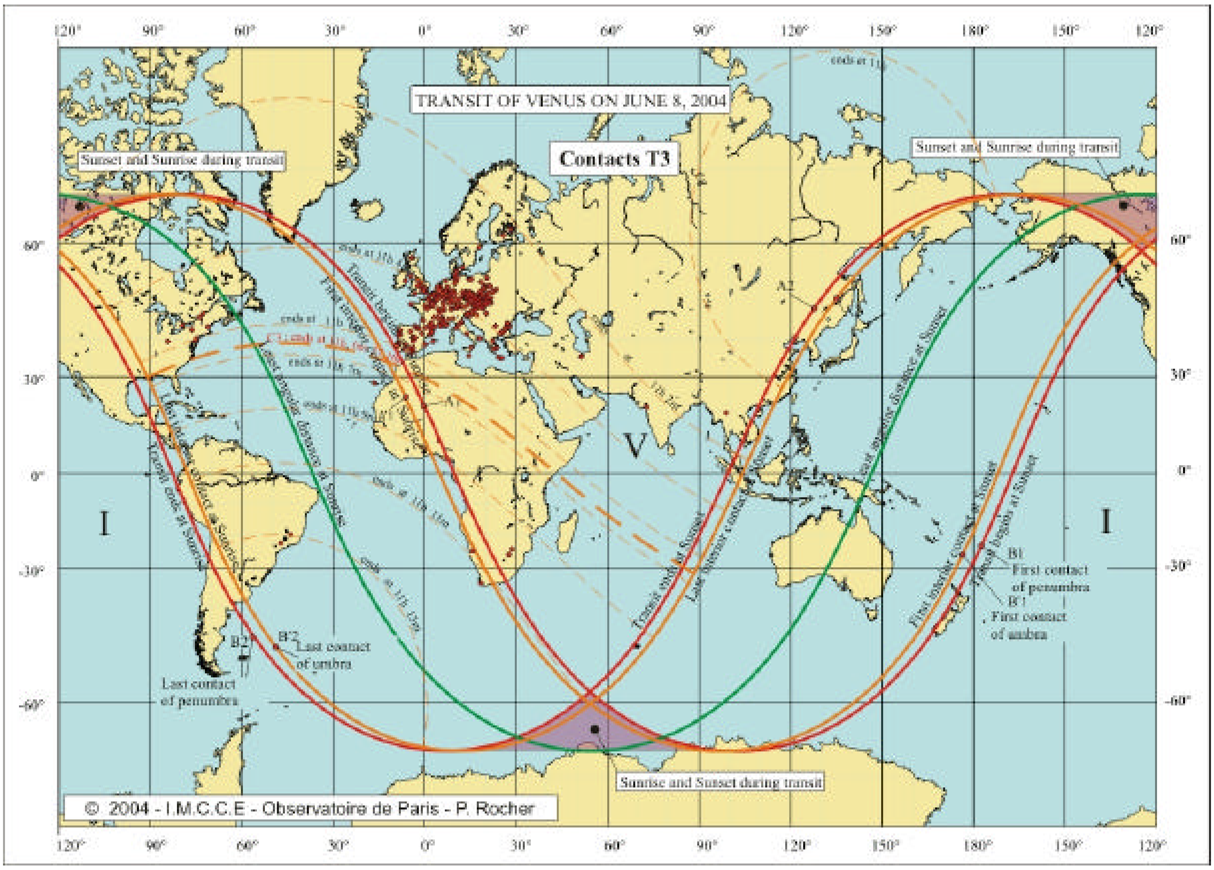
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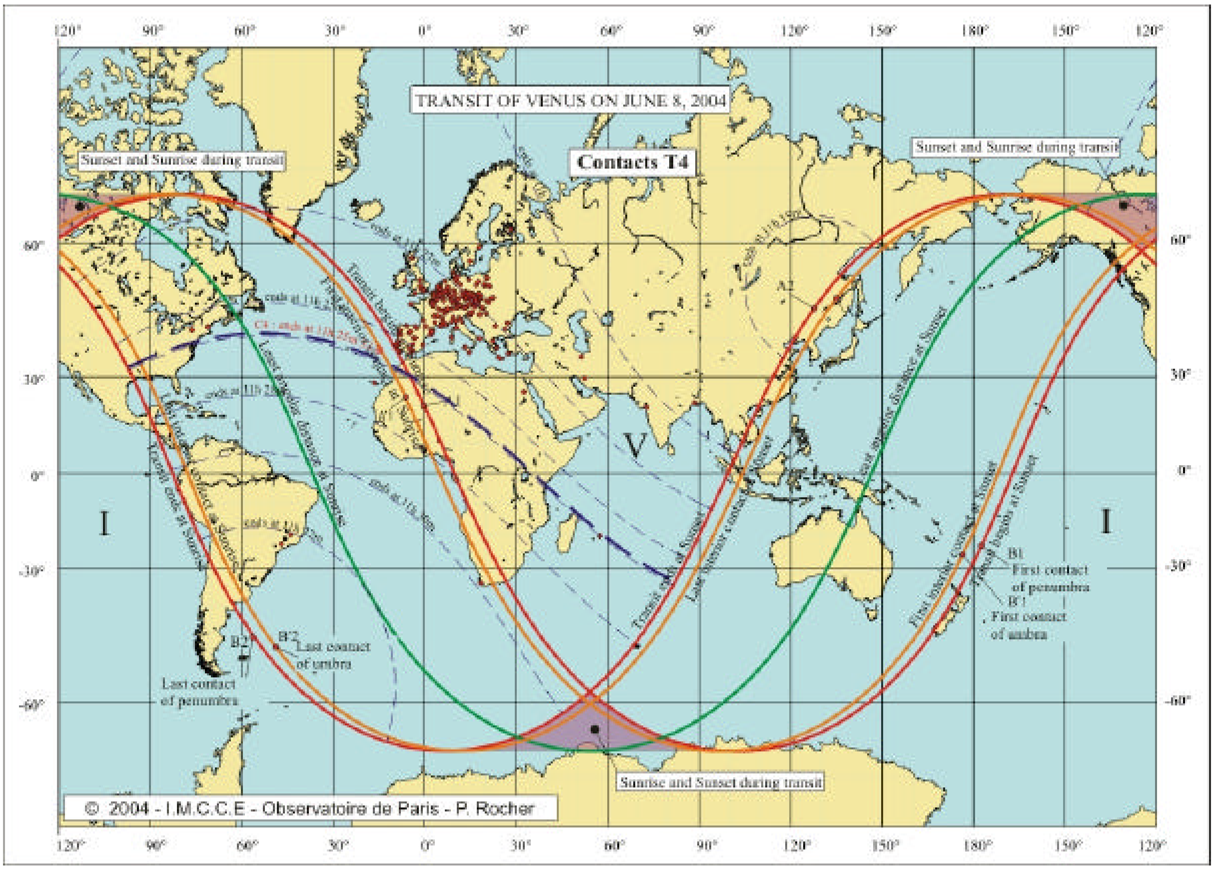
TRANSIT OF VENUS ON JUNE 8, 2004
Contacts T4
Sunset and Sunrise during transit
Sunset and Sunrise during transit
Sunrise and Sunset during transit
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B'1
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B2
B'2
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