A Software based de-rotation algorithm concept for the New Adaptive Optics Module (NAOMI) for the Auxiliary Telescopes of the VLTI

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ABSTRACT

The New Adaptive Optics Module for Interferometry (NAOMI) is the future low order Adaptive optics system to be developed for and installed at the ESO 1.8 m Auxiliary Telescopes (ATs). The four ATs are designed for interferometry which they are essentially dedicated for. The project goal is to equip the telescopes with a low-order Shack-Hartmann system operating in the visible in place of the current tip-tilt correction. The deformable mirror (DM) for NAOMI is rotating with the AT azimuth axis whereas the wavefront sensor (WFS), which signals are used to control the DM, has a fixed position in the telescope basement. It is not co-rotating with the DM. The result is that the projection of the actuator pattern is rotating with respect to the WFS when the telescope is tracking an object on sky. In order to avoid the use of an optical de-rotator we developed an algorithm to de-rotate the commands to the DM in software. This paper outlines the concept of the software de-rotation as well as the performance obtained from end-to-end simulations.

Keywords: adaptive optics,Interferometry, VLTI, ESO, deformable mirror

1. NAOMI SYSTEM OVERVIEW

NAOMI\textsuperscript{1},\textsuperscript{2} is the future adaptive optics system for the 1.8 m Auxiliary Telescopes (ATs) of the Very Large Telescope Interferometer (VLTI). NAOMI will substitute the Tip-tilt (TT) correction stage consisting of a TT mirror (M6) and the STRAP tip-tilt sensor. NAOMI baseline consists of a low-order Shack-Hartman WFS with 4x4 subapertures on the visible, a real time computer based on the ESO Sparta-light and the corrective optics.

Figure 1 shows an overview of the AT telescope and optical path. The Corrective Optics (CO) are placed in the coude train (replacing the steering mirror M6 (2) ) after the Nasmyth focus (1) in a diverging beam and operating facing down. This location corresponds to an intermediate pupil plane of 28 mm.

In the other hand, the WFS is placed in the coude focus after the dichroic M9 in the transmitted path and it is fixed with the basement. When tracking an object on-sky, due to the rotation around azimuth axis the M6 image rotates with respect the WFS plane.

All the coude optics are placed in an independent mechanical box, the so-called, Relay Optical Structure (ROS) (3). When the ATs need to travel, the ROS is recovered by the AT from the current station and moves together up to the new station. On this point, the ROS is placed in the station and the AT position is readjusted to match the ROS axe.

NAOMI will be designed for the new dual feed ROSs. The upper part of the ROS (4) consists of the field derotator, the M9 and the Star Separator which offers two interferometrical channels to the VLTI. The lower ROS (5) volume is fed with the visible light (after M9 in transmission) and it is foreseen to allocate the AO WFS.

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2. PUPIL ROTATION PROBLEM

In NAOMI the DM is located at M6 and it is rotating with the AT azimuth axis. The WFS, which signals are used to drive the DM, is not fixed with respect to the telescope basement, therefore not co-rotating with the DM. The net result is that when the telescope is tracking an object on the sky the DM image on the WFS is rotating depending on the azimuth angle of the telescope (in the dual feed case also depending on the FoV coude derotator). The azimuth angle position and speed depend on the sky object equatorial coordinates and observing time.

In an AO system the correspondence between the WFS signals and the DM actuators is provided by the Interaction Matrix (IM) and its inverse, the Reconstructor, is used to control the AO loop. The IM is recorded for a given fixed geometry between the WFS and the DM. If this geometry is not longer valid the Reconstructor will not control properly the AO loop anymore.

In the case of NAOMI in principle the DM rotation mismatch is not known by the WFS. Therefore the atmospheric disturbance measured by the WFS cannot be applied to the DM as is, because it will result in a wrong correction with a destructive effect on the final performance of the AO system. In Figure 2 a schematic view of the problem is shown.

Figure 1. Auxiliary telescope layout. The optical path is shown as well as the main interface components.
3. SOFTWARE DEROTATION IN THE DM SPACE

The viable solution we propose to compensate for the DM-WFS rotation is to implement it softwarewise in the DM space. We propose to control the AO loop using a modal basis (modal control) having the property to be easily mathematically rotated.

The algorithm proposed is based in the projection of the WFS slopes measurements into a modal base to recover the projected coefficients. A rotation matrix is applied to this coefficients to recover the correct coefficients in the DM orientation. The new derotated coefficients are as well projected into the DM space to recover the DM commands. The derotation matrix can be easily computed if the Zernike modal base is used due to the symmetric properties over rotation.

Let’s define the hereafter matrices:

- \( M_{s2m} \): The Reconstructor obtained by inverting the IM recorded for a known fixed geometry between the WFS and the DM which converts the WFS signals \( \tilde{s} \) into the modes \( \tilde{m} \) of the selected modal basis.
- \( M_{m2a} \): The matrix transforming the modes \( \tilde{m} \) of the selected modal basis in the commands of the single DMs actuators \( \tilde{a} \).

During the AO closed loop operations the WFS measures the signals \( \tilde{s} \) which are converted in coefficients for the modal basis as seen by the WFS (which is ignoring the relative rotation w.r.t. DM):

\[
\tilde{m}_{wfs} = M_{s2m} \cdot \tilde{s}
\]

In order to counter-rotate to modal basis to compensate for the DM rotation angle \( \theta \) (i.e. matching the modal basis of the DM) a derotation matrix \( R(\theta) \) has to be applied to the modal basis coefficients measured by the WFS:

\[
\tilde{m}_{dm} = R(\theta) \cdot \tilde{m}_{wfs}
\]

Finally the set of commands \( \tilde{a} \) to be sent to the DM actuators is given by:

\[
\tilde{a} = M_{m2a} \cdot \tilde{m}_{dm}
\]

Merging the above equations we can obtain:

\[
\tilde{a} = M_{m2a} \cdot (R(\theta) \cdot (M_{s2m} \cdot \tilde{s}))
\]

or:

\[
\tilde{a} = (M_{m2a} \cdot R(\theta) \cdot M_{s2m}) \cdot \tilde{s}
\]
finally the command vector to be applied to the DM can be expressed as:
\[ \tilde{a} = D_{\text{swd}}(\theta) \cdot \tilde{s} \]  
(6)

where:
\[ D_{\text{swd}}(\theta) = M_{m2a} \cdot R(\theta) \cdot M_{s2m} \]  
(7)

is the rotated reconstructor which can be updated for any rotation angle upon need.

It is worth noting that the Reconstructor \( M_{s2m} \) is unique and it is recorded for a nominal fixed angle between the WFS and the DM (i.e. non need to calibrate it for different angles). Also the matrix \( M_{m2a} \) can be easily pre-computed using the DM influence function and doesn’t depend on a particular geometry of the WFS.

3.1 Choice of the modal base
As mentioned before the key point is to identify a modal basis for which the modal derotation matrix \( R(\theta) \) has a simple and predictable representation.

We identified the Zernike polynomials as suitable basis for our purpose. The Zernike polynomials have an interesting property which makes them friendly w.r.t. rotation: at each given radial order the Zernike polynomial identifying a specific aberration are always coupled and one of the two polynomials is always equal to other plus a given rotation angle.

Therefore an aberration with any rotation angle can be described simply by a linear combination of the related polynomials couple. The linear combination turns to be a simple law of sinus-cosines. The only exception is represented by the radially symmetric polynomials which do not change with rotation.

Equation 8 shows the modal derotation matrix \( R(\theta) \) for the first 15 zernikes:

\[
\begin{bmatrix}
Z_2 & Z_3 & Z_4 & Z_5 & Z_6 & Z_7 & Z_8 & Z_9 & Z_{10} & Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} \\
Z_2 & \cos \theta & \sin \theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_3 & -\sin \theta & \cos \theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_4 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_5 & 0 & 0 & 0 & \cos 2\theta & -\sin 2\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_6 & 0 & 0 & 0 & \sin 2\theta & \cos 2\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_7 & 0 & 0 & 0 & \cos \theta - \sin \theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_8 & 0 & 0 & 0 & \sin \theta & \cos \theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_{10} & 0 & 0 & 0 & 0 & 0 & \cos 3\theta & -\sin 3\theta & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_{11} & 0 & 0 & 0 & 0 & 0 & \sin 3\theta & \cos 3\theta & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos 2\theta & \sin 2\theta & 0 & 0 \\
Z_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin 2\theta & \cos 2\theta & 0 & 0 \\
Z_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos 4\theta & \sin 4\theta & 0 & 0 \\
Z_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin 4\theta & -\cos 4\theta & 0 & 0 \\
\end{bmatrix}
\]  
(8)

This matrix provides the linear combination law to express a rotated aberration with the linear combination of the two related polynomials. As an example if we consider the first Coma of Zernike \( Z_6 \) and \( Z_7 \) rotated by an angle \( \theta \) it can be expressed as:
\[
\begin{align*}
Z_7(\theta) &= Z_7(0) \cos \theta - Z_8(0) \sin \theta \\
Z_8(\theta) &= Z_7(0) \sin \theta + Z_8(0) \cos \theta
\end{align*}
\]  
(9)

The modal derotation matrix \( R(\theta) \) is very easy to compute once known the rotation angle (for example from the telemetry of the telescope) and can be simply obtained on demand and calculated in anticipation before that specific angle is reached.

It is worth noting any modal basis with modes having similar rotation properties can serve to our purpose.
3.2 Zernike modal calibration

As explained before the SWD algorithm requires to work in the space of the zernike base to project the zernike coefficients in each rotation angle space. This requires to do a Zernike Modal Calibration (ZMC). In a classical zonal calibration each of the actuators of the DM is pocked and the corresponding signal view by the WFS is recorded. In the modal calibration each of the zernike modes is projected into the DM and measured by the WFS. The resulting modal IM measured in this way corresponds to the matrix $M_{m2s}$ (mode to slopes).

For the calibration we need to know the matrix $M_{m2a}$. Each column of this matrix gives the DM actuators pattern to recreate each of the zernike modes. This matrix is easily computed knowing the influence functions of the DM.

4. STATIC VERIFICATION

The first test we performed was to check whether the SW derotation was introducing additional errors besides the normal aliasing in the reconstruction of the wavefront (4x4 subapertures Shack-Hartmann with 12 useful ones).

For this analysis we use the AO simulation code PSIM$^3$ under Matlab. We used as reconstruction modal basis the Zernike polynomials and we compared two sets: 15 and 21 zernike modes (corresponding to full radial orders). The corresponding IMs have been recorded and inverted to generate the reconstructors.

A set of 20 independent phase screens with known Zernike modal decomposition up to mode 528 has been created (the Kolmogorov statistical dependence between Zernike polynomials has been taken into account). Then each phase screen has been rotated from 0 to 350 degrees, with a step of 10 degrees, and injected in the WFS. The coefficients of the first 15 or 21 Zernike polynomials have been retrieved, then rotated using the modal derotation matrix and subtracted from the ones injected in the WFS.

Finally the mode by mode variance of the residuals has been computed and compared to the pure WFS aliasing. The WFS aliasing has been obtained applying the above process only for the non rotated phase screens.

Figure 3. SHS aliasing. The plots show the comparison between the real zernike coefficients used to construct the input WF and the coefficients decomposition seen by the WFS. We can see that if a 15 zernike modal base (left figure) is used the aliasing has not impact while if the 21 zernike modal base (right figure) is used the aliasing introduces an important error in the WF estimation.

Figure 3 shows the comparison between the real zernike coefficients used to construct the input WF for a single phase screen and the coefficients decomposition seen by the WFS (for angle zero, no rotation applied to the phase screen). We can see that if a 15 zernike modal base is used the aliasing has not impact while if the 21 zernike modal base is used the aliasing introduces an important error in the WF estimation.
Figure 4. Left plot shows the reconstructed coefficients for the first 15 Zernike polynomials for a single phase screen and 36 rotation angles. The right plot shows the new coefficients obtained by applying the modal derotation matrix to the coefficients of the previous plot. We can see that the coefficients for rotation zero are recovered.

Figure 4 (left) shows the measured zernike coefficients seen by the WFS for the same phase screen when this rotates between 0 and 360 deg (for the case of 15 reconstructed Zernike polynomials). In figure 4 (right) we can see that if we apply the matrix derotation matrix (computed for the corresponding angle) to the previous coefficients the original coefficients for rotation zero are recovered.

Figure 5. Variance of the residual of first 15 Zernike modes (left) and of the first 21 Zernike modes (right). As comparison, the averaged variance of the injected phase screens is shown. The application of the modal derotation matrix doesn’t add any additional contribution w.r.t. the normal aliasing. On the contrary for the case of the 21 zernike modes the wavefront reconstruction is poor due to the limited number of useful sub-apertures in the WFS.

Figure 5 shows the variance of the residuals per Zernike mode compared to the normal aliasing. Here all 20 phase screen and 36 rotations have been considered. The application of the modal derotation matrix doesn’t add any significant error besides the normal aliasing demonstrating that the principle works.
5. END-TO-END SIMULATIONS

To validate the concept of Software Derotation (SWD) we proceeded with an AO E2E simulation under realistic atmospheric conditions as shown in table 1. For this purpose we use the YAO code (YAO is Monte-Carlo simulation tool for Adaptive optics under Yaorick language, http://frigaut.github.io/yao/).

The code was modified to include our specific needs. The rotation between DM pupil and WFS pupil is done in the DM space. This means that for each rotation the influence functions of the actuators DM are generated in a new position. Then we can initiate the simulation taking a new interaction matrix (IM) for each actuators rotation configuration or let rotate the IF while keeping the zero rotation IM.

The zernike modal calibration (ZMC) module was included in the code to calibrate the deformable mirror (DM) applying the zernike modes. Then the derotated reconstructor is computed for each angle to compute the right commands to the DM.

Table 1. Parameter configuration of the AO simulation

<table>
<thead>
<tr>
<th>WFS</th>
<th>N subapertures</th>
<th>4x4 (12 subapertures)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N pixels</td>
<td>6x6</td>
</tr>
<tr>
<td></td>
<td>Pixel scale</td>
<td>0.6 arcsec/pix</td>
</tr>
<tr>
<td></td>
<td>RON</td>
<td>1 e- (Excess noise considered)</td>
</tr>
<tr>
<td>DM</td>
<td>N actuators</td>
<td>5x5 (21 act), 9x9 (69 act), Pure zernike DM (15 -21 modes)</td>
</tr>
<tr>
<td></td>
<td>Coupling</td>
<td>20 %</td>
</tr>
<tr>
<td>Control loop</td>
<td>Gain</td>
<td>Optimized for each simulation</td>
</tr>
<tr>
<td></td>
<td>Integration time</td>
<td>500 Hz</td>
</tr>
<tr>
<td></td>
<td>Loop delay</td>
<td>2 simulation frames</td>
</tr>
<tr>
<td>Observational conditions</td>
<td>Star magnitude</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Band</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>Seeing</td>
<td>1 arcsec</td>
</tr>
<tr>
<td></td>
<td>( T_0 )</td>
<td>3 ms</td>
</tr>
</tbody>
</table>

Along our simulations we study 3 DM configurations:

- Pure Zernike DM. In this case perfect Zernikes are projected into the WFS in the calibration. It is equivalent of a DM with infinite number of actuators and therefore without fitting error. This case gives us the maximum theoretical performance of the SWD.

- 5x5 actuators DM. Real DM with 5x5 actuators corresponding with a Friend geometry for the 4x4 sub WFS. 21 actuators are used always the same for all rotation angles. The zernike modes used for the calibration are computed from the IF of the actuators.

- 9x9 actuators DM. Same as before but with a real DM with 9x9 actuators. This case is computed to study the impact of a DM with higher number of actuators than subapertures.

5.1 Performance analysis

First we consider the standard case where a derotation is not required. We compare the performance of the 2 methods used to obtain the reconstructor: the zonal calibration (with SVD reconstructor, Single Value Decomposition), and the zernike modal calibration (15 zernikes). For the second case we consider 2 configurations: a perfect zernike DM and a 5x5 actuators DM. This comparison gives us the possible lost of performance when using a zernike modal calibration (limiting the number of degrees of correction) respect to the zonal calibration case that would be used in case the system will not rotate. This simulations is done in two ways:
• Static system that is not rotating. The Actuators are placed in different configurations, so not following the Fried geometry. For each configuration the interaction matrix is measured and the real reconstructor retrieved. This allows us to understand the loss of performance due to the fact that the actuators are not well placed in the corners of the subapertures (some actuators could not be well sensed).

• As well we want to understand what is the lost of performance for the 2 reconstruction methods when the system is calibrated at a defined angle and a delta rotation is introduced. Like this we can know which is the limit of stability and at which frequency we would need to update the reconstructor due to the pupil rotation.

Then we compare the software derotation performance for different angles where the reconstructor is computed with the information of a single IM at zero rotation with the performance that we will obtained if it would be possible to measure the zonal interaction matrix for each angle.

5.2 Pupil rotation impact: no reconstructor updating

This case corresponds with a misregistration in rotation. The DM (IFs position in the pupil) rotates while the loop is closed and the reconstructor is not updated meantime. Since the configuration for each angle does not corresponds with the calibration at zero position the loop is unstable after a certain angle. Figure 6 shows the performance as a function of the rotation angle. The drop of performance is slower for the cases of Zernike calibrations with respect the standard SVD mode. The performance is quite stable up to 10 degrees (going from 51% to 48.5% SR). After 10 degrees we can show an acute drop of performance.

The stability of the system can be interfered also from the optimal gain at each angle. Higher gain means higher stability. The gain is constant up to 10 degrees.

Following the above analysis we can set the maximum tolerated departure angle $\phi_{tol}$ beyond which the reconstructor has to be updated. The system will be stable up to 10 degrees rotation error, so we can consider for it a conservative value of $\phi_{tol} = 5^\circ$. 

![Figure 6. Plot showing the loss of performance when the DM is rotating and no update of the reconstructor is performed.](image)
5.3 Impact of breaking the Fried geometry

Prior to study the performance of the SWD algorithm it is necessary to understand the possible loss of performance due to the fact that the Fried geometry is not kept during the rotation.

For this we rotate the DM and we compute a new IM for each angle. In the case of the modal calibration the reconstructor is computed by SVD but for each angle the number of eigenmodes used in the reconstructor has to be optimized. This is due to the fact that for each angle the eigenmodes of the SVD decomposition are different (visibility of the actuators respect the WFS are completely different during the rotation).

In the case of the zernike modal calibration there are no impact since we work in the modal basis space.

Figure 7 shows the behavior of the performance as a function of the angle. For the ZMC the performance is approximately constant independently of the orientation while for the zonal calibration the performance drop depending on the angle.

The ticks of the Y-axis represents one percent of SR, where the error bar of the points is estimated to be this value. It is worth noting that differences lower than 2-3 percent would be difficult to be measured in real sky due to the fact that the turbulence is not constant.

![Figure 7. Performance vs rotation angle. The DM is rotating and for each angle the reconstructor is updated. The reconstructor is computed from the true interaction matrix measured at each angle. The two calibration methods are compared (zonal and modal).](image)

5.4 Software derotation performance

We compute the performance of our AO system as a function of the angle rotation. In this case we measure a single modal IM at the calibration step for rotation zero. At this step we compute the matrices $M_{m2a}$ and $M_{s2m}$.

The simulation is done rotating the DM by regenerating the position of the IFs. At each angle we update the reconstructor by computing the corresponding rotation matrix as explained in section 3.

We study the regular case where 15 zernike modes are controlled. Figure 8 shows the performance of the software derotation angle for the 3 types of DMs: pure zernike DM, 5x5 actuators and 9x9 actuators.
The first conclusion is that the SWD algorithm works for any rotation. The SR varies between 51.4 % and 50.2 %, means less than 2% (relative) and therefore we can assume there are not loss of performance by the fact that the pupils are rotating.

The plot shows that the case of 9x9 actuators (with more actuators than subapertures) behave in the same way that the Fried case of 5x5 actuators. Anycase we can see that for both cases there is only a small dependence to the rotation angle with respect the ideal case of the pure zernike DM.

Figure 9 compare for the case of the 5x5 actuators DM the performance between the SWD method acquiring a single IM at zero rotation with the standard zonal calibration where we acquire a new IM at each angle (number of SVD modes optimized).

The plot shown that the SVD method is more performant only for some rotation angles, but in the main range of angles the performance is equivalent.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>mean (SR [%])</th>
<th>std (SR [%])</th>
</tr>
</thead>
<tbody>
<tr>
<td>New IM acquired at each angle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5x5 act - zonal IM - SVD</td>
<td>51.7</td>
<td>1.0</td>
</tr>
<tr>
<td>5x5 act - modal IM (ZMC)</td>
<td>51.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Software derotation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>single IM at $\theta = 0^\circ$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pure zernike DM</td>
<td>51.2</td>
<td>0.1</td>
</tr>
<tr>
<td>5x5 act - ZMC</td>
<td>50.9</td>
<td>0.3</td>
</tr>
<tr>
<td>9x9 act - ZMC</td>
<td>50.8</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2. Mean and standard deviation of the SR performance over the rotation comparing the software derotation (single IM at zero rotation) and the case where a new IM is acquired at each angle.
6. CONCLUSION

The objective of our study is to find an algorithm that allow us to solve the problem of differential rotation between pupils at DM and WFS by taking into account the rotation in the computation of the reconstructor avoiding the increase of complexity (derotators, large calibrations ...).

We found an algorithm based on the modal base properties that has demonstrated its validity analytical and by E2E simulations. The algorithm is based in the projection of the slopes measurements into a modal base to recover the projected coefficients. A rotation matrix is applied to this coefficients to recover the correct coefficients in the DM orientation. The new derotated coefficients are as well projected into the DM space to recover the DM commands.

The Zernike modal base was chosen due to its symmetric properties allowing to compute easily the rotation matrix.

The Zernike base is used to apply a modal calibration where a single IM at a specific angle is measured. The modes to be used in the control are measured in the WFS by applying the shape of each mode in the DM. It is the so-called Zernike modal calibration (ZMC). We demonstrated that there are not performance difference between zonal calibration and zernike modal calibration (less than 1%, mean value over whatever rotated configuration).

In the analytical study (and then confirmed by the E2E simulations) we demonstrated that a modal base of 15 zernikes is the most suitable (complete radial order). Higher number of modes can not be used due to the limited number of subapertures. The aliasing error due to this subsampling introduces an error in the WF estimation that avoid the correct estimation of the derotation.

The E2E simulations show that the software derotation method proposed works with the followed properties:

- The performance is constant for any rotation. The standart deviation of the performance over a full rotation is 0.3 %.
• The mean loss of performance is 2% (averaged over a full rotation) respect to a classical zonal calibration and SVD reconstructor (where we can access the IM for all rotation configurations). This loss can be assumed without not problem in the error budget of the system.

To understand the update speed requirement of the reconstructor we studied the loss of performance when we leave the system rotating but not update of the reconstructor is performed. We showed that the system will be stable up to 10 degrees rotation error, for this rotation the loss of performance due to the calibration error is \( \approx 7\% \). This value is reduced to \( \approx 1.5\% \) if 5 degrees error is considered. For 1 degree rotation no appreciable loss of performance is observed.

REFERENCES