Monostatic and bistatical schemes and optimal algorithm for tilt correction in ground-based adaptive telescopes

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One of the most promising trends in the modern astronomy is a creation of ground-based adaptive telescopes which employ a signal of a laser guide star (LGS) [1-3]. To my mind, the most complete literature revue of the modern state of the problem was made by R. Ragazzoni [2] (regretfully, works of Soviet and Russian authors were not included in the revue)

The present paper is a continuation of researches performed in the USSR in years 1978-86 [4-10] and resent researches made in Russia [11-15]. Theoretical investigations into the problem of laser employment for generation of guide stars and its practical applications which are very popular today induced my to speak again about results obtained more that 15 years ago. The most completely these results were presented by me in Refs. 5 and 9. After publication of my book on adaptive optics in English (Ref. 10) the results becomes known wide. The fourth and fifth chapters of this book (Ref. 10) are relevant to the problems of a beams and image adaptive correction with reference beacons.

In Ref. 5 the problem of stabilization direction of the diagram for a laser beam in real time is considered. As a method to solve this problem, the detection of an image of a reference source in a focal plane of a telescope was proposed. In particular, a mutual correlation function for a vector characterizing random shifts of the beam centroid and vector defining coordinates of image centroid of a reference signal was computed for a turbulent atmosphere. It was assumed that as a reference signal an image of some beacon can be used as well as a laser beam reflected from an object. A natural guide star can also be employed. In Ref. 9 the results characterizing the efficiency of correction are presented for a finite-size object seen through the turbulent atmosphere.

In the present paper different types of laser guide stars are analyzed. According to the modern authors working in the field there are two main schemes of the laser guide star generation: monostatic and bistatic [16, 17]. The laser used for this purposes is ground-based so the optical radiation travels two times through atmospheric inhomogeneties. First, upward, to form the laser guide star itself. Second, downward, in result of backscattering (secondary emission, or elastic aerosol scattering) by atmospheric inhomogeneities. In monostatic scheme it is assumed that correlation of fluctuations for the upward and downward propagation (for direct and secondary beams) are maximum. Quite different conditions are characteristic for the bistatic scheme for which is assumed that the correlation is practically absent.

«General» scheme of a laser guide star generation

It is possible to realize an optical scheme where mutual correlation for fluctuations for optical radiation under the propagation in direct and backward directions takes some intermediate value. In the present paper equations for "general" scheme of a laser guide star generation are drawn into which an arbitrary value of correlation between fluctuations of a random angular shifts of an image can be substituted. We presume that these equations are valid for monostatic scheme as well as for bistatic.

The geometry of the experiment is illustrated in Fig. 1. Parameters of the problem are as follows: R_0 is a telescope aperture radius, X is an altitude of (a distance from) a laser guide star (input pupil of the telescope is placed in the plane $x = 0$), a_0 is radius of the telescope transmitting aperture, $\vec{\rho}_0$ is a vector of displacement of the center of a radiation source relatively to the telescope optical axis.

We presume that adaptive correction is performed in the main telescope with the use of a laser guide star formed at an altitude X above the input aperture exactly at the optical axis of the telescope. According to the schematic drawn in Fig. 1 the main telescope is pointed at the zenith and a weak natural star and the artificial star are both at the telescope axis. The zenith angle of the laser beam is $\left|\vec{\rho}_0\right| / X$ (in the assumption that $\left|\vec{\rho}_0\right| << X$).

Let us also assume that the observed star is placed in infinity and has a plane wave front. The vector characterizing a random tilt of this wave front due to atmospheric turbulence is (see Ref. [18]):

$$\vec{\varphi}_F^{\,pl} = -\frac{1}{\Sigma}\iint\limits_{\Sigma} d^2\rho \nabla_\rho S^{\,pl}(0, \vec{\rho}), \tag{1}$$

where $S^{\,pl}(0, \vec{\rho}) = k\int\limits_0^{\infty} d\xi \iint d^2 n(\vec{\kappa}, x - \xi)\exp(\imath\vec{\kappa}\vec{\rho})$ are phase fluctuations for a plane wave, $\sum$ is area of the input aperture, k is the radiation wave number. In the last equation it is also taken into account that the wave from natural star travels from above to down and the following spectral expansion is used for fluctuations of the atmospheric index of refraction

$$n_1(\xi, \vec{\rho}) = \iint d^2 n(\vec{\kappa}, \xi)\exp(\imath\vec{\kappa}\vec{\rho}).$$

Random angular shifts of the centroid for a focused beam formed with a auxiliary telescope at the altitude X can be written as [18]:

$$\vec{\varphi}_{lb}(\vec{\rho}_0) = \frac{1}{P_0}\int\limits_0^{x} d\xi \iint d^2 R I\left(\xi, \vec{R} + \vec{\rho}_0(1 - \xi / x)\right)\nabla_R n_1\left(\xi, \vec{R}\right), \tag{2}$$

$$P_0 = \iint d^2 R\, I\left(0, \vec{R}\right),$$

where $n_1(\xi, \vec{\rho})$ denotes the fluctuations of the refractive index at the point $(\xi, \vec{\rho})$, $I(\xi, \vec{\rho})$ is the optical laser field intensity at the point $(\xi, \vec{\rho})$ from the a source placed at the coordinate origin in the initial plane (for $\xi = 0$). In Eq. (2) the fact is taken into account that the optical axis of the laser beam is shifted and this shift defined by the vector $\vec{\rho}_0$. The optical axis is also tilted at the angle $|\vec{\rho}_0| / X$.

Presuming that the initial laser beam is wide enough (so that $\Omega^{-1} = ka_0^2 / X >> 1$) and additional broadening due to turbulence is small. Under these conditions the laser guide star can be taken as a point source (for the large main telescope). For the downward beam additional random shifts appear in a backscattered wave similar to shifts in a spherical wave [18]

$$\vec{\varphi}_F^{\,sph} = -\frac{1}{\Sigma}\iint\limits_{\Sigma} d^2\rho \nabla_\rho S^{\,sph}(0, \vec{\rho}). \tag{3}$$

Whether a scheme in question is bistatic or monostatic is defined by the level of mutual correlation the data of measurements for angles in a reference channel

$$\vec{\varphi}_m = \vec{\varphi}_{lb}(\vec{\rho}_0) + \vec{\varphi}_F^{\,sph} \tag{4}$$

and by an angular shift of a natural star image $\vec{\varphi}_F^{\,pl}$.

To define conditions of the monostatic scheme transformation to the bistatic, let us obtain a mutual correlation function between random angular shifts of the image of a natural star (function $\vec{\varphi}_F^{\,pl}$, Eq. (1)) formed by the main telescope and shifts of the centroid of a focused beam formed by an auxiliary laser system (function $\vec{\varphi}_{lb}(\vec{\rho}_0)$, Eq. (2)). We are going to based on results of such computations, which were wide published in papers (Refs. 5, 9, 10, 17, 14). Deviation of the turbulence spectrum from a power function in the domain of large scales was also accounted for [18, 20-24]:

$$\Phi_n(\kappa, \xi) = 0{,}033 C_n^2(\xi)\kappa^{-11/3}\{1 - \exp(-\kappa^2 / \kappa_0^2)\}, \tag{5}$$

where $C_n^2(\xi)$ is turbulence intensity on the path of propagation and $\kappa_0^{-1}(\xi)$ is the outer scale of turbulence. If these properties are correctly allowed for, it is possible to obtain the following equation for a correlation function:

$$\left\langle \vec{\varphi}_{lb}(\vec{\rho}_0)\vec{\varphi}_F^{pl} \right\rangle = (-2\pi^2 0{,}033\Gamma(\frac{1}{6}))2^{1/3} R_0^{-1/3} \int_0^X d\xi C_n^2(\xi)(1-\xi/X)$$

$$\{[1+b^2(1-\xi/X)^2]^{-1/6}\,{}_1F_1(\frac{1}{6},1;-\frac{d^2(1-\xi/X)^2}{(1+b^2(1-\xi/X)^2)}) - \quad . \quad (6)$$

$$-[1+b^2(1-\xi/X)^2+4c^2]^{-1/6}\,{}_1F_1(\frac{1}{6},1;-\frac{d^2(1-\xi/X)^2}{(1+b^2(1-\xi/X)^2+4c^2)})\}$$

Here the designations were used: $b = a_0 / R_0$, $d = |\vec{\rho}_0| / R_0$, $c = \kappa_0^{-1} R_0^{-1}$ and a_0 is the initial radius of a laser beam, ${}_1F_1(\ldots)$ is a confluent Gaussian hypergeometric function.

It can readily be shown that the second term in the braces is related with the outer scale of turbulence. And if the outer scale approaches infinity, this term can be omitted so the correlation function takes the form:

$$\left\langle \vec{\varphi}_{lb}(\vec{\rho}_0)\vec{\varphi}_F^{pl} \right\rangle = (-2\pi^2 0{,}033\Gamma(\frac{1}{6}))2^{1/3} R_0^{-1/3} \int_0^x d\xi C_n^2(\xi)(1-\xi/X)\times$$

$$.\left[1+b^2(1-\xi/X)^2\right]^{-1/6}\,{}_1F_1(\frac{1}{6},1;-\frac{d^2(1-\xi/X)^2}{(1+b^2(1-\xi/X)^2)}) \quad . \quad (7)$$

Into this equations (6), (7) the value of $d = 0$ corresponds to the monostatic scheme of a laser guide star generation, and then

$$\left\langle \vec{\varphi}_{lb}(\vec{\rho}_0)\vec{\varphi}_F^{pl} \right\rangle = (-2\pi^2 0{,}033\Gamma(\frac{1}{6}))2^{1/3} R_0^{-1/3} \int_0^x d\xi C_n^2(\xi)(1-\xi/X) \quad . \quad (8)$$

$$\left[1+b^2(1-\xi/X)^2\right]^{-1/6}$$

For the bistatic scheme (condition $d >> 1$) correspond to asymptotic of a hypergeometric function ${}_1F_1(\ldots)$. Thus,

$$\left\langle \vec{\varphi}_{lb}(\vec{\rho}_0)\vec{\varphi}_F^{pl} \right\rangle = (-2\pi^2 0{,}033\Gamma(\frac{1}{6}))2^{1/3} R_0^{-1/3}\Gamma^{-1}(\frac{5}{6})d^{-1/3} \int_0^x d\xi C_n^2(\xi)(1-\xi/X)^{2/3}$$

(9)

from the analysis of the last equation it is possible to conclude that correlation between the plane wave and the beam decrease down to 0.1 for $d \geq 10^3$. This value is characteristic for the bistatic scheme for infinite outer scale of turbulence.

Numerous data obtained experimentally [Refs. 10, 20, 22, 25-27] justify the assumption that the outer scale $\kappa_0^{-1}(\xi)$ is a finite number. Moreover, in result numerical estimations performed with the use of different altitude profiles of $C_n^2(\xi)$ and $\kappa_0^{-1}(\xi)$ we come to the conclusion [28] that it is possible to

introduce some effective outer scale of turbulence for the whole atmospheric column. As it was turned out, under «moderate» conditions of seeing [29] the value of this effective outer scale is 5 - 60 meters. So for a telescope with R_0=4 м parameter $c = \kappa_0^{-1} R_0^{-1}$ <10.

Let us perform asymptotic analysis of the outer scale influence on the correlation function expressed by Eq. (6). The most important is the term in the braces which we denote as

$$\Psi(d,b,c,X) = \left[1 + b^2(1-\xi/X)^2\right]^{-1/6} {}_1F_1\left(\frac{1}{6},1;-\frac{d^2(1-\xi/X)^2}{(1+b^2(1-\xi/X)^2)}\right) -$$

$$-\left[1 + b^2(1-\xi/X)^2 + 4c^2\right]^{-1/6} {}_1F_1\left(\frac{1}{6},1;-\frac{d^2(1-\xi/X)^2}{(1+b^2(1-\xi/X)^2+4c^2)}\right) \quad (10)$$

As an argument of the function we take a variable *d* and as parameters variables *b, c,* and *X*. Simple estimations show that when parameter *d* = 0 and *c* < 5, the correlation with the finite outer scale is two-three times lower than with the infinite outer scale. With increase of *d* (for *d* > 1) correlation computed by Eq. (10) is not greater that 0.2. For *d* > 2*c* the correlation is 17 times lower than that for the infinite outer scale. And at last for *d* >> *c* the sign of correlation function $< \vec{\varphi}_{lb}(d)\vec{\varphi}_F^{\,pl} >$ changes for reverse and dependence on *d* can approximately be written as $\approx d^{-7/3}$.

So from the analysis performed we can conclude that for *c* < 5 and for the difference of the telescopes axis $\rho_0 \geq 2\kappa_0^{-1}$, the considered scheme is bistatic.

To confirm this conclusion let us estimate numerically a correlation coefficient

$$K(d,b,c,X) = \frac{< \vec{\varphi}_{lb}(\vec{\rho}_0)\vec{\varphi}_F^{\,pl} >}{\sqrt{< (\vec{\varphi}_{lb}(\vec{\rho}_0))^2 >< (\vec{\varphi}_F^{\,pl})^2 >}}. \quad (11)$$

The coefficient can be expressed through correlation function (6) and corresponding variances:

$$\left\langle (\vec{\varphi}_F^{\,pl})^2 \right\rangle = (2\pi^2 0{,}033\Gamma(\frac{1}{6}))2^{1/6} R_0^{-1/3} \int_0^x d\xi C_n^2(\xi)[1 - \left[1 + 4c^2\right]^{-1/6}], \quad (12)$$

$$\left\langle (\vec{\varphi}_{lb}(\vec{\rho}_0)^2 \right\rangle = (2\pi^2 0{,}033\Gamma(\frac{1}{6}))2^{1/6} R_0^{-1/3} \int_0^x d\xi C_n^2(\xi) \times$$

$$\{(b^2(1-\xi/X)^2)^{-1/6} - (b^2(1-\xi/X)^2 + 4c^2)^{-1/6}\} \quad (13)$$

Computations were performed with a model of $C_n^2(\xi)$ [29] corresponding to «moderate» conditions of seeing. As an argument the variable *d* was taken which characterize separation between axis of the main and auxiliary telescopes. The altitudes corresponding to Rayleigh and sodium artificial stars (10 and 100 km) were included into initial conditions. Parameter *b* was chosen as follows: *b* = 0.1, 0.3, 0.7, 1.0, 3.0, and 5.0.

The values of *b* greater than unity are typical in situations when a big telescope forms an artificial star for a small one. Such ratios of parameters can be realized in observatories where telescopes of different sizes are placed. For example, when in Mauna Kea observatory the ten-meter Keck telescope used as auxiliary one for a small telescope.

In the calculations values of *c* were the following: 1, 3, 5, 10, 100, 1000. The case when *c* = 1000 practically corresponds to the infinite outer scale. The results obtained are shown in Figs. 2 and 3.

It should be noted that the results obtained numerically confirm the conclusions of the above analytic analysis. Also possible to made the next conclusions:

1. With the large outer scales (c =100 and 1000) the scheme for the laser guide star formation can be considered as bistatic one if the difference between axis of the main and auxiliary telescopes is (200 - 1000) R_0, i.e., if d > 200.

2. With a finite outer scale (c < 5) the value for differences d in two or three outer scale times means substitution of the monostatic scheme to bistatic one.

3. Smaller values for differences d correspond to intermediate (between monostatic and bistatic) schemes.

4. It should also be noted that the results of our calculations are not coincide with results reported in (Refs. 30 and 31). These authors considered mutual correlation functions for two plane waves traveling from infinity with different tilts. In particular, in our computations the correlation coefficient K for $d = 0$ is not equal to unit. Only with decrease of X, the coefficient K approaches unity asymptotically. For the first time these computations were performed in 1980 (Ref. 5), now we have also the results [17].

5. It is interesting to consider the behavior of K in the region of small values of c (c = 1, 3, 5) and relatively large values of b (b =3, 5). This situation is characteristic for the case when a large telescope generates an artificial star for a small one. The results obtained (Fig. 2 and 3) show that in this scheme the correlation in the region d < c is practically constant (correlation is equal, correspondingly, to 0.4, 0.5, 0.7).

6. With increasing of c the characteristic scale of correlation function K increases, i.e., the correlation radius increases. This effect was noted (for two plane waves) in [17].

7. Increasing of the correlation radius is not infinite, gradual saturation of the function was observed in our numeric experiments. For small values of c (the outer scale is small) correlation function $K(d)$ decreases down to 0.1 for $d = c$. But even for c = 100 the correlation falls down to 0.1 if $d = c/2$. When c = 1000 such fall occurs for $d = c/10$. This is possible for altitude X=10 km as well as for X=100 km.

8. The predicted change of the sign for correlation function made on the basis of the asymptotic analysis (the conclusion was made comparing Eqs. (6) and (7)) is due to a finite size of the outer scale of turbulence. When the outer scale is small (c = 1, 2, 3) and d > (2 - 3)c, the correlation function change its sign to opposite. For large c this effect is impossible to register. When values of the outer scale are large, coefficient K (Eq. (7)) keeps its sign.

Really, it is interesting to find a relationship between the value of radius of mutual correlation for two plane waves [30, 31] and radius of correlation for a plane wave and a slanted laser beam (the last characteristic is presented in Figs. 2 and 3). If found, these data would allow one to use the results of direct astronomical observations of images of two stars seen at different angles in prognoses about the value of correlation (for tip-tilt correction) for a system «telescope-laser guide star» and also to make more correct conclusions concerning the mode of operation of this system (whether the scheme is monostatic or bistatic). To my regret, it is impossible to make a simple comparison between Figs. 2 and 3 and the data presented in Ref. 31 because in algorithms of computations different models of the atmospheric turbulence were used.

«Optimal» algorithm of tip-tilt correction

A laser guide star allows one to expand the domain of stable operation for an adaptive optics system. The star is formed at some finite distance, so the problem arouses to correct the data of optical measurements performed with a guide star to ensure efficient compensation for aberrations of a image of a real astronomical object. This correction of data is possible with the use of atmospheric models [13, 15]. The models of atmospheric turbulence allow one: to estimate the level of turbulent aberrations above the reference star, i.e., to compute optimal altitude of the star generation, to compensate partially focal anisoplanarity in a system with a reference star placed at a finite altitude, to make estimations of efficiency for a «tip-tilt» correction more precise.

Doubtless, when we use the laser guide star formed in the atmosphere by backscattered signal in the algorithm of correction for random wandering [2, 5, 14, 16, 17] of a real star image, the problem arouses of data processing optimization. Let us try to construct the algorithm of angular natural star image correction, using data of laser guide star angular position measurements, as following [14]:

$$\vec{\varphi}_F^{\,pl} - A\vec{\varphi}_m \quad . \tag{14}$$

This algorithm ensures the minimum of the variance

$$\left\langle \beta^2 \right\rangle = \left\langle \left(\vec{\varphi}_F^{\,pl} - A\vec{\varphi}_m \right)^2 \right\rangle = \left\langle \left(\vec{\varphi}_F^{\,pl} \right)^2 \right\rangle + A^2 \left\langle \left(\vec{\varphi}_m \right)^2 \right\rangle - 2A \left\langle \vec{\varphi}_F^{\,pl} \vec{\varphi}_m \right\rangle. \quad (15)$$

From Eq. (15) we obtain

$$\left\langle \beta^2 \right\rangle_{\min} = \left\langle \left(\vec{\varphi}_F^{\,pl} \right)^2 \right\rangle - \left\langle \vec{\varphi}_F^{\,pl} \vec{\varphi}_m \right\rangle^2 / \left\langle \left(\vec{\varphi}_m \right)^2 \right\rangle. \quad (16)$$

The coefficient of correlation A is expressed through a (nonrandom) determined functions as

$$A = < \vec{\varphi}_F^{\,pl} \vec{\varphi}_m > / < \varphi_m^2 >. \quad (17)$$

The form (17) of this coefficient allows one to conclude that the coefficient A can be calculated from data in direct optical experiments. It should also be noted that the minimum of variance (17) is impossible to obtain with traditionally used correction algorithms (Eq. (14) with $A = 1$). To confirm this let us perform intercomparison of residual variance for optimal and nonoptimal algorithms of correction.

In a real experiment we can obtain only data on $\vec{\varphi}_m$ because vector $\vec{\varphi}_F^{\,pl}$ characterizing angular wandering of a star image is impossible to detect due to insufficient intensity of light from a real star. But we can estimate this coefficient A by formula (17) using [15] the models of atmospheric turbulence.

In our designations the minimum variance of residual fluctuations of a star image angular shifts in a scheme presented in Fig. 1 can be estimated as:

$$< \beta^2 >_{\min} = < (\vec{\varphi}_F^{\,pl})^2 > \left\{ 1 - \frac{2^{1/3} f(X, b, d, C_n^2)}{\left[1 + \varepsilon^{-1/3} - 2^{7/6} \left(1 + \varepsilon^2 \right)^{-1/6} {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2}{(1+b^2)} \right) \right]} \right\}, \quad (18)$$

where the function

$$f(X, b, d, C_n^2) = \left(\int_0^X d\xi C_n^2(\xi) (1 - \xi / X) \{ [1 + (1 - \xi / X)^2]^{-1/6} - \right.$$

$$\left. - [1 + b^2 (1 - \xi / X)^2]^{-1/6} {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2 (1 - \xi / X)^2}{1 + b^2 (1 - \xi / X)^2} \right) \right\})^2 \times \quad . \quad (19)$$

$$\left[\left(\int_0^X d\xi C_n^2(\xi) (1 - \xi / X)^{5/3} \int_0^\infty d\xi C_n^2(\xi) \right]^{-1} \right.$$

depends upon parameters of the optical experiment as well as upon the used atmospheric model (the above equations are written assuming that the outer scale of turbulence is infinite). Numerical analysis of these equations showed [14] that in result of optimal correction the residual angular distortions can be less than residual distortions of traditional methods of control.

To illustrate advantages of the optimal algorithm in comparison with nonoptimal (common) one we include in the paper Table I where the values of residual angular distortions are presented for a telescope with a bistatic reference star (it was assumed that the star is a point source). (In the Table we put the values of normalized residual variance). In the fifth column we also placed the values of correction coefficient A computed for the model of turbulence [29]. It is seen from the data presented in the Table that optimal correction with properly chosen coefficient A allows one to decrease considerably the residual distortions.

So the conclusion can be drawn about high efficiency of optimal correction with the use of information concerning altitude profiles of turbulence. At the same time intensity of distortions can even increase in the result of nonoptimal correction.

As it seems, that the most promising method of correction for general tilts of a wavefront is employment of LGS adaptive optics systems with hybrid algorithms of control [3, 14, 16, 17, 32].

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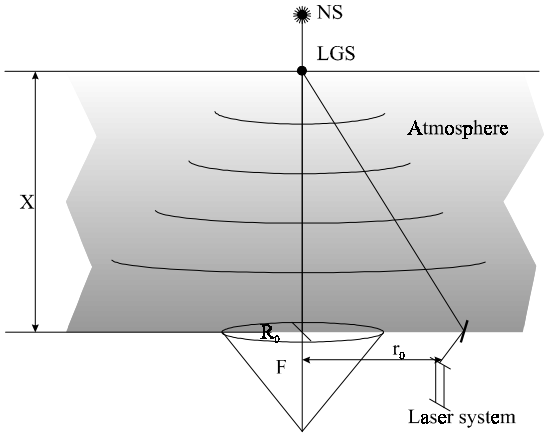

Fig.1. Laser guide star formation scheme .

X, km	b	Residual level of aberrations		A
		Optimal Algorithm	Traditional Algorithm	
8	0.3	0.640	1.291	-0.426
	0.5	0.603	1.105	-0.471
	0.7	0.578	0.999	-0.500
	1	0.552	0.899	-0.532
	2	0.500	0.736	-0.593
	3	0.471	0.656	-0.628
	5	0.434	0.570	-0.671
20	0.3	0.612	1.354	-0.420
	0.5	0.572	1.148	-0.463
	0.7	0.545	1.030	-0.492
	1	0.516	0.918	-0.523
	2	0.461	0.736	-0.583
	3	0.429	0.647	-0.618
	5	0.390	0.551	-0.660
40	0.3	0.602	1.406	-0.413
	0.5	0.561	1.187	-0.456
	0.7	0.533	1.062	-0.484
	1	0.504	0.944	-0.515
	2	0.447	0.751	-0.574
	3	0.414	0.657	-0.608
	5	0.374	0.556	-0.650
80	0.3	0.600	1.446	-0.408
	0.5	0.558	1.220	-0.450
	0.7	0.531	1.091	-0.478
	1	0.501	0.969	-0.508
	2	0.443	0.769	-0.567
	3	0.410	0.672	-0.600
	5	0.370	0.567	-0.641
100	0.3	0.599	1.455	-0.406
	0.5	0.588	1.227	-0.448
	0.7	0.530	1.097	-0.477
	1	0.500	0.974	-0.507
	2	0.443	0.774	-0.565
	3	0.410	0.676	-0.598
	5	0.370	0.570	-0.639

Table 1. Comparison of efficiency for "optimal" and "traditional" algorithms of tilt correction in the bistatic LGS scheme.

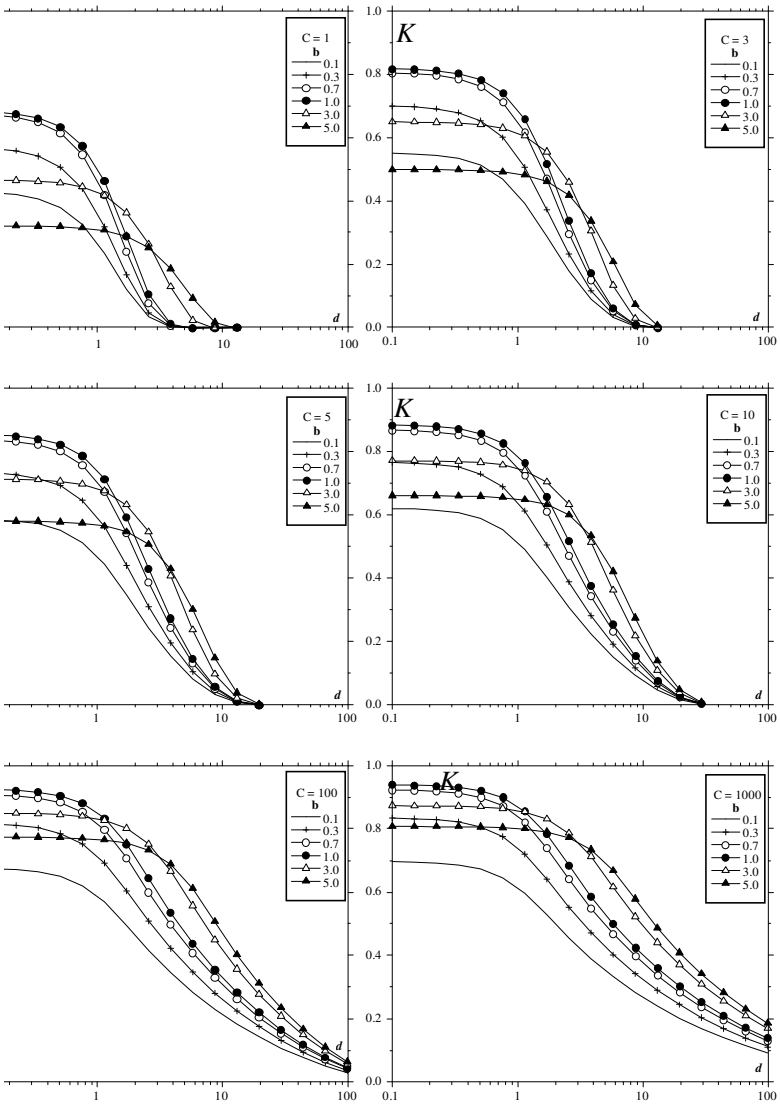

Fig.2. Correlation functions K from Eq.(14) for different sizes of outer scale of turbulence (parameter c), laser system aperture sizes (parameter b), and beacon altitude X=10 km.

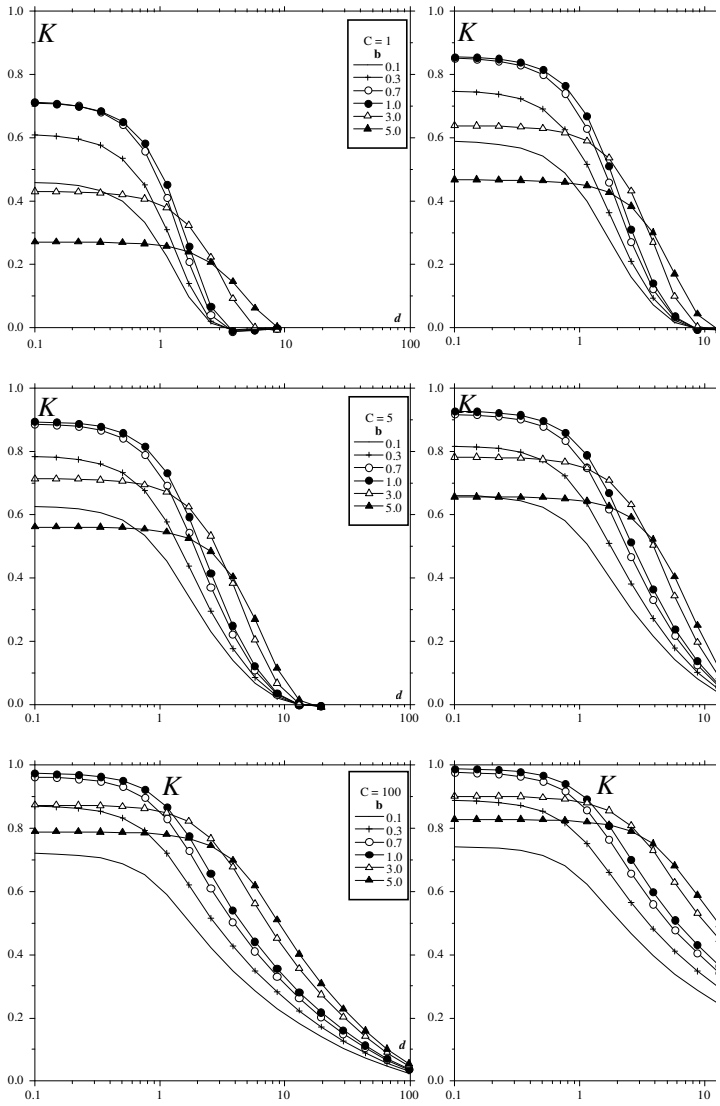

Fig.3. Correlation functions K from Eq.(14) for different sizes scale of turbulence (parameter c), laser system apertu (parameter b), and beacon altitude X=100 km.

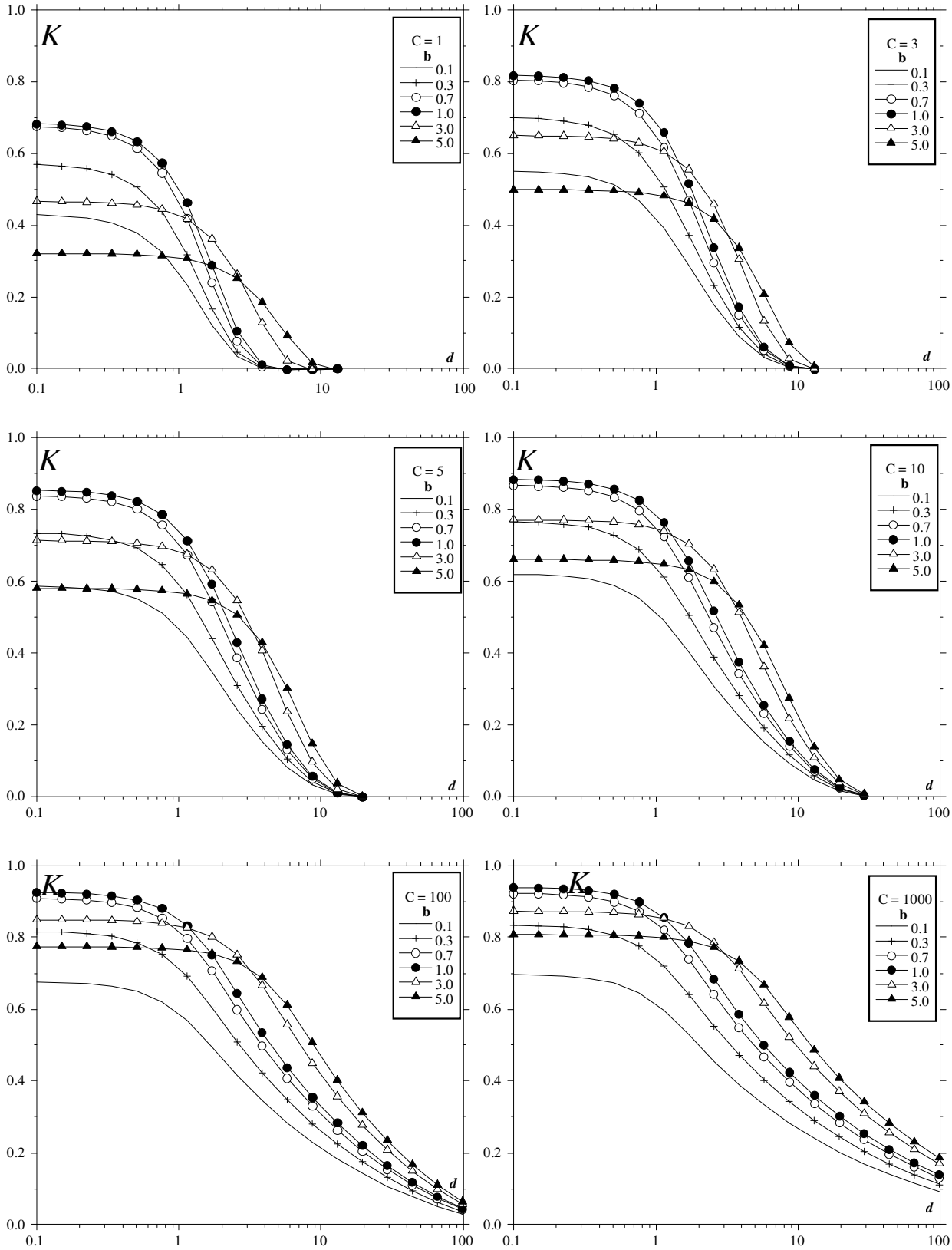

Fig.2. Correlation functions K from Eq.(14) for different sizes of outer scale of turbulence (parameter c), laser system aperture sizes (parameter b), and beacon altitude X=10 km.

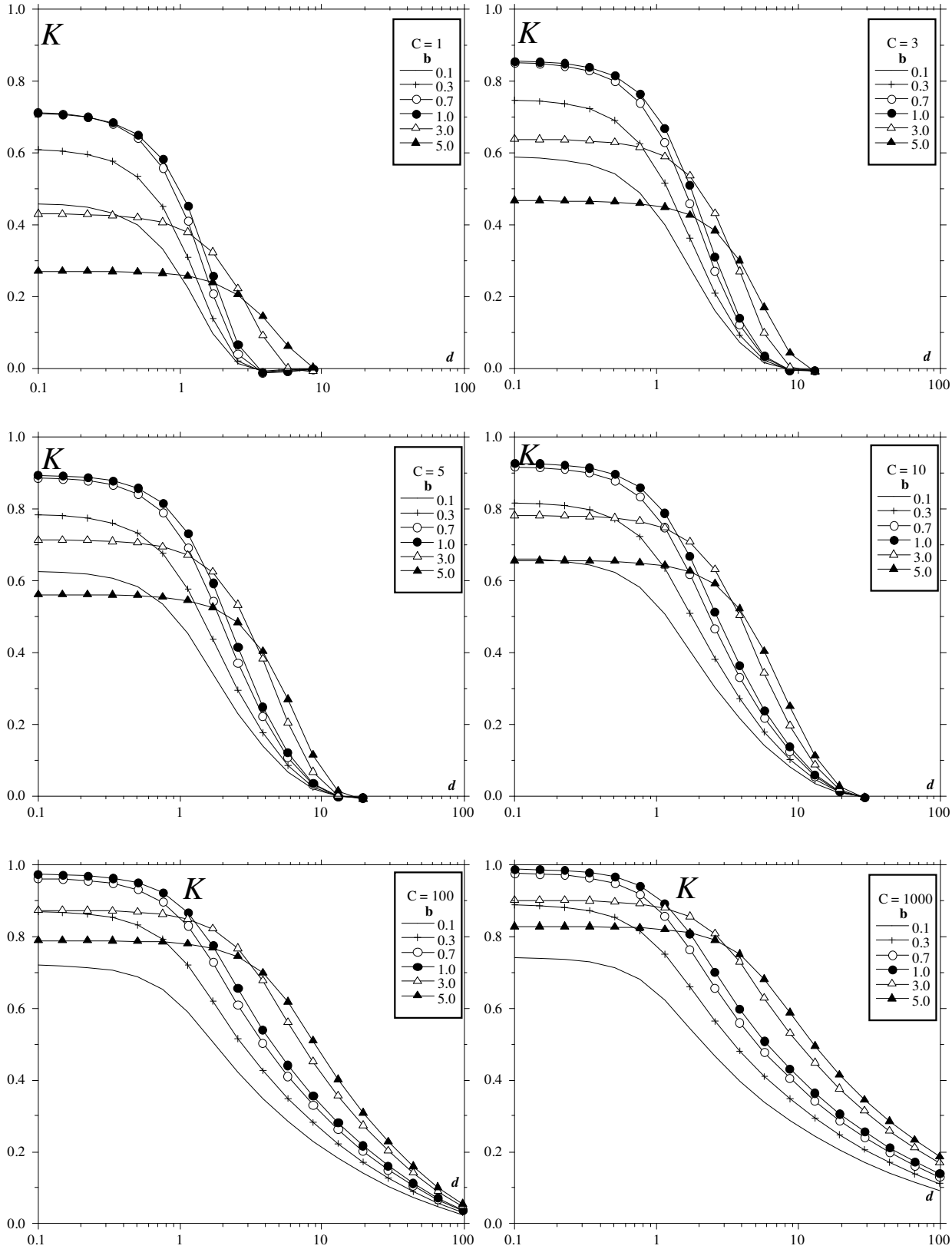

Fig.3. Correlation functions K from Eq.(14) for different sizes of outer scale of turbulence (parameter c), laser system aperture sizes (parameter b), and beacon altitude X=100 km.