PARTIAL CORRECTION FOR TURBULENT DISTORTIONS IN TELESCOPE

V.P.Lukin and B.V.Fortes

Institute of Atmospheric Optics

Siberian Branch of the Russian Academy of Sciences, Tomsk

INTRODUCTION

The spatial frequency spectrum of turbulent distortions is sufficiently wide, and although the lowest frequency wavefront aberrations (slopes and quadratic aberrations) give the largest contribution to the phase fluctuation variance for a telescope of large aperture a compensation for these aberrations does not improve the image quality essentially. Therefore to achieve a sufficiently high level of compensation the wavefront correctors with a large number of degrees of freedom ought to be used. The complexity and cost of such devices grows rapidly with requirements to their spatial resolution. Therefore it is necessary to choose such a configuration of the corrector which, on the one hand, should not lead to excessive complication and cost rise of an adaptive optical system (AOS) and, on the other hand, should be able to provide a sufficient increase in the image quality. Moreover, as the compensation efficiency depends also on the operation of other units of AOS, the characteristics of a compensating device (configuration, spatial resolution, and frequency range) must be coordinated and balanced with the parameters of other units of the adaptive optical system, for example, a reference source and wavefront sensor.

1. WAVEFRONT CORRECTORS AND PARTIAL CORRECTION OF THE PSF

At present three basic types of wavefront correctors have been developed for adaptive telescopes: a modal corrector, a zonal corrector, and a segmented one. Let us consider the requirements to a spatial resolution of these devices based on simple analytical estimations and a more detailed simulation.

1.1. Modal corrector

Modal corrector is such a hypothetical compensation device whose response functions make up an analytical basis, and Zernike polynomials are normally chosen as this basis. Since, the first terms of the Zernike series coincide with the analytical representation of the classic wavefront aberrations (distortion, defocusing, astigmatism etc.) the theory of modal corrector allows one to simply obtain estimations of the efficiency of the low-order adaptive compensation. Moreover, such correctors as bimorphic mirrors (see, for example, Ref. 1) allows us to reproduce up to 30 of Zernike aberrations with an admissible accuracy.

A number of theoretical studies[2,3,4,5] dealt with the theory of modal corrector in application to compensation for turbulent distortions. Approximate estimations of the modal corrector efficiency can be done based on the results from Ref. 3 where the variance of residual distortions in the phase of a wave corrected was calculated as a function of the normalized diameter D/r_0 of a telescope aperture and of the number of Zernike polynomials which are compensated for with a modal corrector. Corresponding formula is as follows $= C_N\,(D/r_0)^{5/3}$, where N is the number of aberrations compensated for. Values of the coefficients C_N are presented in Table I where $N = 1$ corresponds to the compensation for a constant component, $N = 3$ corresponds to the compensation for a constant component and slopes (linear aberrations), $N = 6$ corresponds to the compensation for up to quadratic aberrations inclusive etc. For large values of N an approximate formula can be written $C_N \approx 0.2944\,N^{-\sqrt{3}/2}$.

TABLE I.

N	1	3	6	10	15	21
C_N	1.03	0.134	0.0648	0.0401	0.0279	0.0208

This theoretical results allow one to estimate the presence of residual distortions for a given parameter N of a modal corrector and vice versa, for a given level of the residual distortions, to determine the number of polynomials which ought to be compensated for. For example, to reach the level of residual distortions corresponding to the criterion $\lambda/6$ ($\sigma^2_{,N} \approx 1$) we obtain the estimation

$$N = 0.244\,(D/r_0)^{1.92}.$$

Table II illustrates the dependence of N on the normalized aperture diameter. For such a number of degrees of freedom of the modal corrector the Strehl ratio approximately equals to $S = \exp(-\sigma^2_{,N}) = 1/e \approx 0.37$.

TABLE II.

D/r_0	10	20	30	40	50
N	20	78	170	295	454

Obviously the bimorphic mirrors available now are not able to provide such compensation level in the visible range (when the most probable value of $D/r_0 = 40–50$) but their use for the compensation in the infrared range (for $D/r_0 = 10–15$) seems to be quite reasonable. Moreover, as it will be shown below, and for higher values of the residual error variance the angular resolution close to the diffraction one (defined as the width of PSF at half maximum) can be obtained. It turns out that partially corrected image (PCI) of a point object seems as if composed of two component. One of the components has the width approximately equal to the width of the turbulent PSF while the other component has a diffraction–limited dimension. The above theory which only considers the variance of wavefront residual distortions and does not describe this effect.

Calculations of the optical transfer function (OTF) for the modal compensation were performed in a number of papers (for example, in Ref. 4) but the OTF bears no complete information on the angular resolution. Let us consider the results of the numerical experiment we have carried out using our own software. Detailed description of the procedure of image forming simulation in the system "atmosphere – telescope" can be found in our publications.[6,10] Figure 1 presents PSFs of a modal adaptive optical system obtained using the simulations. As earlier the wavefront distortions were considered known (the model of an "ideal" sensor was used). A control by the corrector was determined the based on minimization of integrated square error of the compensation. Calculations was performed for several values of the normalized aperture diameter $D/r_0 = 10,20,30$. The simulation of the modal compensation was performed for the values $N = 3,10,15,21,28$ that corresponds to compensation for the wavefront aberrations of the 1st to 5th radial degree inclusive. Figure 2 presents the radial cross–sections of the normalized PSF as functions of the telescope normalized diameter and the compensation parameter N. Figure 3 shows the Strehl ratio as a function of the variance of the residual phase distortions.

It is seen that the quite contrast diffraction limited core of the PCI is observed up to the values of the Strehl ratio of the order of 0.01. This fact is not obvious and could be established only by direct calculation of the PSF. Really, it could have been expected that a decrease in the axial image intensity by the factor of 10 leads to its, approximately threefold broadening, since the intensity is inversely proportional to the square of effective dimension with the invariable "form" of the intensity distribution in the image plane. However, when compensating for the lowest aberrations the spatial spectrum of wavefront distortions changes essentially. The small–scale aberrations not compensated for cause a redistribution of the corrected PSF power in the far "wings" (as compared with the turbulent PSF for the same phase distortion variance). In this case the PCI effective dimension is more than it could have been expected but the width of PSF at half maximum only slightly differs from the diffraction one. This gives a possibility to carry out astronomic observations connected with the measurement of object angular positions with the accuracy close to the diffraction one, even for the comparatively “poor” (by the Strehl ratio) compensation.

The following practical conclusions can be drawn. The use of bimorphic mirrors (the most close to a modal corrector by their characteristics) can be recommended at the first stages of the development of adaptive optics . Bimorphic mirrors will be able to provide a high level of the compensation in the far and middle infrared ranges and will provide partial correction in the near infrared range. It is hardly worth expecting the creation of bimorphic correctors in the visible region unless a good mirror is manufactured with about 100 to 200 control channels.

When the final decision on the location of the observatory will be made and after obtaining data on astro–climate it will be necessary to perform additional calculations in order to determine the parameters of bimorphic correctors required for facilitating partial correction providing a sufficient contrast of the diffraction core in spectral windows of the near infrared range.

1.2. Zonal and segmented correctors

Resolution close to the diffraction limited one, at small values of Strehl ratio, is also achievable with other types of wavefront correctors. Let the deformable mirrors of a zonal type and segmented correctors be considered from this point of view.

In contrast to the modal corrector it is characteristic of zonal correctors that application of a signal at one of the control points effects the shape of deformable mirror surface only in the part, under the influence of a given control channel. The deformable mirrors actuated with the piezoelectric elements stack to the rear side of the mirror are most widely used correctors of the zonal type. The “effect zone” dimension of a control element is determined by the distance from the nearest adjacent drive. Outside this zone the “response” of a mirror plate rapidly decreases.

Segmented adaptive mirrors also can be referred to the zonal class in the sense that the control of one mirror element does not affect the state of the rest its segments provided that no special connection is arranged. A distinguishing peculiarity of segmented mirrors is the presence of the surface breaks which are caused by the non–joint of adjacent segment edges.

For zonal type correctors as well as for the segmented mirrors the residual error variance of the phase correction is described by the formula $\sigma^2 = C\,(d/r_0)^{5/3}$, where d is the corrector characteristic scale, i.e., the distance between the control points of a zonal corrector or dimension of a segmented mirror element, and the coefficient C depends on the peculiarities of a corrector performance. This expression is obtained within the Kolmogorov turbulence theory and can be used in the case when the outer turbulence scale exceeds the corrector scale d. Otherwise this formula overestimates residual distortions. The value of the coefficient C for a segmented corrector can be estimated approximately based on the modal compensation theory considered in the previous section neglecting the fact that the shape of a controlled element is different from a circle. Since, for the isotropic turbulent distortions the variance of the residual correction error coincides with the variance of errors on a separate segment then $C = 1.03$ for the control of a segment location and $C = 0.134$ for the control of location and slopes of segments. For a deformable mirror with the Gaussian response function the estimation $C = 0.2$ can be used.[7]

The number of control elements, which are necessary for obtaining a given level of residual distortions can be estimated by the following formula $N = (D/r_0)^2 (C/\sigma^2)^{6/5}$.

Table III presents the estimations of the number of elements N for both corrector types calculated for the level of residual distortions corresponding to $\sigma^2 = 1$. The upper line of the Table presents the values of the aperture normalized diameter (10 ... 50). Next two lines correspond to a segmented corrector, and the last line corresponds to a flexible mirror.

TABLE III.

D/r_0	10	20	30	40	50
$C = 1.03$	104	414	932	1657	2590
$C = 0.134$	9	36	81	143	224
$C = 0.2$	14	58	130	232	362

Figure 4 presents the PSFs in this configuration for different turbulent distortions. It is clear that the PSF normalized by its axial value practically does not differ from the diffraction one. It is clear from the plot presenting the PSF normalized by the diffraction maximum that the Strehl ratio varies within the range 0.1–0.85 as a function of D/r_0. Similar calculations were performed for a deformable mirror with the Gaussian response function. Results of this calculations are presented in Fig. 5.

Summing up the considered aspect of the partial correction we note that the appearance of a sharp diffraction core against the background of a turbulence distorted image occurs earlier (for large residual distortions) at higher level of the initial turbulent aberrations. It is explained by the fact that at lower intensity of the turbulent component in the partially corrected PSF its diffraction–limited part is characterized by a greater contrast for the same value of the Strehl ratio S. For example, if for an uncorrected image the Strehl ratio S is 0.001 then the partial correction increasing the image axial intensity up to the level $S = 0.01$ leads to appearance of diffraction component of a PSF with the contrast parameter (ratio of the axial intensity to the intensity at the level of the first diffraction fringe) of the order of 10. To obtain the same contrast at the initial distortion level corresponding to $S = 0.01$ it is necessary to compensate for turbulent distortions up to the level corresponding to $S = 0.1$ that is reached at an essentially lower value of the variance of phase residual distortion.

It may be expected that already at the first stages of the development of adaptive optics for this telescope it will be possible to achieve the angular resolution close to the diffraction one even at low values of the Strehl ratio. Although in this case a power portion concentrated in the diffraction circle will be much less than its diffraction–limited value the advantage as compared with an uncorrected image can be essential. First of all, this will provide a possibility to carry out accurate angular measurements in the visible region.

2. HARTMANN SENSOR AND PARTIAL CORRECTION

Even if we have got a wavefront corrector with a sufficiently high spatial resolution the performance of an adaptive correction is impossible without a device measuring aberrations of the optical wave to be corrected. Since, the correction quality will be determined by the weakest member of the chain “measurement–compensation” during the development of an adaptive system one should make a choice balancing the characteristics of both a corrector and wavefront sensor.

Measurement of atmospheric aberrations of optical waves coming from astronomic objects in a real time has a set of specific peculiarities. First of all, these are fast temporal variability of aberrations and low level of the light flux. Other peculiarity is a wide spatial spectrum of distortions. This spectrum is characterized by the presence of both small–scale and large–scale distortions that impose higher requirements to the dynamic range of wavefront distortion sensor (WDS).

The development of equipment for recording turbulent phase distortions led to appearance of certain new types of WDS which essentially differ from devices used in the problems of optics control while inheriting the basic principles of phase measurements.

At present the Shack–Hartmann[8] sensors are most widely used. Prototype of this sensor is the known classic Hartmann sensor widely used in the problems on testing astronomic optics. The Shack–Hartmann sensor consists of the diaphragm located in the plane which is conjugate to the telescope entrance aperture plane, set of photodetectors, and a computing device.

Diaphragm of the Shack–Hartmann sensor in contrast to the Hartmann diaphragm is not an amplitude transparency but the phase one and it is a microlens matrix as a rule. Also the use of a microprism matrix is known. Modern optical production process allows this matrices to be produced at a comparatively low costs. The most promising type of a photodetector for astronomic WDS is a CCD–matrix which is able to operate in the photon–counting mode.

Principle of reconstruction of the wavefront aberration pattern used in the Shack–Hartmann sensor is based on the solution of a problem on constructing an estimation of unknown function from its local slopes (gradients). In this case the wavefront local slopes are estimated from a displacement of the focal spots in the photodetector plane. These displacements can be determined from the position of the center of gravity of corresponding instantaneous intensity distribution or (for low spatial resolution of a photodetector) from the power redistribution among quadrants. There is also a version of such a sensor in which the micromovement devices provide equality of the light fluxes on elements of a quandrant photodetector.[9] In this case the local slope is determined by the value a photodetector displacement. Such a sensor is used in adaptive system for a solar telescope and is less suitable for use with the general–purpose telescopes operating under condition of low light fluxes.

Numerical model of the Shack–Hartmann sensor with a modal algorithm of retrieving from the Zernike polynomials[10] is included as a component in the applied software we have developed to study the problems on adaptive formation of beams and images in the atmosphere. This model allows us to investigate the accuracy of measurements of the wavefront aberrations depending on the number of subapertures, quantum noise level, relationship between subaperture dimension and coherence radius, and estimating algorithm.

Let us consider the results of calculation of the adaptive telescope PSF which illustrates the dependence of the compensation quality on the number of photons coming to a subaperture during the sensor exposure time. Figure 6 presents PSF obtained from simulation of the sensor which has 10×10 dimension of the Hartmann diaphragm. Calculations have been made for a circular aperture with the normalized diameter D/r_0=10. So, the subaperture dimension was taken to be equal to the atmospheric coherence radius. Construction of the wavefront aberration pattern by the local slope estimations was performed in accordance with the modal algorithm using 28 Zernike polynomials.

These results show that the effect of a quantum noise manifests itself at the reference wave intensity level corresponding to less than 100 photons per subaperture during the sensor exposure time ($N = 100$). Signal-to-noise ratio for this intensity level equals 10 in accordance with the Poisson photoelectron statistics. The Strehl ratio decreases twice relative to its value in the absence of quantum noise for the reference wave intensity corresponding to $N = 5$.

These requirements to the intensity level determine the minimal brightness of a reference source, and in combination with the isoplanatism angle value and distribution of stars with such brightness over the celestial sphere determine the portion of this sphere area where the effective compensation for turbulent distortions is possible. Different estimations show that this portion does not exceed several percent and depends on operation wavelength and turbulence altitude profile. To provide the qualitative compensation on all area of the celestial sphere it is necessary to create an artificial reference source.

2.1. Cone anisoplanatism and partial correction

Two basic types of the anisoplanatism are distinguished in the image correction schemes used at present: the angular anisoplanatism and cone anisoplanatism. The first one occurs for the object image correction using reference wave from a star located at a certain angular distance from this object. When correcting image of an extended object the angular anisoplanatism manifests itself in the fact that only a part of the object laying within isoplanatism zone is sharp.

Cone anisoplanatism takes place when a laser guide star (LGS) is used as a reference source. In this case the source angular position, as a rule, coincides with the angular position of the observed object, but the latter is at infinity and the LGS altitude is limited within the effectively scattering atmosphere. In contrast to the "classic" angular anisoplanatism the angle between the beam paths of the reference and corrected waves changes as a function of distance between the considered point and the center of receiving aperture. This angle equals zero at the aperture center and increases with the distance from it. Wavefront residual distortions increase with the increasing distance from the aperture center and also with a decrease in the distance between an aperture and LGS, i.e. at a decrease of its height.

It is characteristic of both types of anisoplanatism that the more turbulent distortions are concentrated in the upper part of the atmosphere the larger is the residual error, since the distance between the beam paths of a reference and corrected waves increases with the distance from the telescope aperture. For equal integral values $\tilde{N}^2{}_{,n}$ the compensation efficiency will be higher in the case when the value $\tilde{N}^2{}_{,n}$ decreases more rapidly with the increasing altitude.

Other problem arising in connection with the use of LGS is the compensation for general slope (i.e. image jitter). One way to solve this problem is the compensation for slope using a natural star. For a complete compensation for the input angle fluctuations the adaptation efficiency in a system with LGS will be determined by the cone anisoplanatism. In this case the residual distortions will decrease with the increase of a wavelength and increase of the LGS altitude.

Realization of the LGS numerical model which is included as a component in our applied software allows us to study this effect. Let us consider the results of calculation of the 10-meter telescope PSF using two LGS types: the Rayleigh LGS and a sodium one.

In the first case the reference radiation is formed by the Rayleigh scattering in the atmosphere. Scattered radiation intensity in the telescope aperture plane is inversely proportional to the square range to the scattering volume. Moreover, the Rayleigh scattering coefficient decreases exponentially with the altitude above sea-level that is caused by the decrease of air density. This limits the Rayleigh LGS altitude by about 10 km for a reasonable power of a laser source. Signal from large altitudes can be received using the resonance scattering of laser radiation in the layer with increased content of the sodium atoms which is at the altitude of the order of 100 km.

Figure 7 presents the PSF for this versions of the LGS. It is clear that the use of Rayleigh reference source is effective in the infrared range only (small values of D/r_0). Use of the sodium LGS allows one to obtain good correction quality in the infrared range and makes it possible to provide nearly diffraction limited angular resolution under the condition of partial compensation in the visible region of spectrum. These results are obtained on the assumption that a sensor and a corrector of a wavefront have the infinite spatial-temporal resolution and neglecting the LGS location fluctuations, i.e., for the case of the complete compensation of the general slope.

REFERENCES

1. A.V. Kudryashov, V.A. Tikhonov, and V.I. Shmal’gauzen, Atm. Opt. **1**, No. 3, 61–65 (1988).
2. D.L. Fried, J. Opt. Soc. Am. **55**, No. 11, 1426–1435 (1965).
3. R.J. Noll, J. Opt. Soc. Am. **66**, No. 3, 207–211 (1976).
4. J.Y. Wang and J.K. Markey, J. Opt. Soc. Am. **68**, No. 1, 78–88 (1978).
5. J. Winocur, Appl. Opt. **21**, No. 3, 433–438 (1982).
6. B.V.Fortes and V.P.Lukin, Proc. SPIE **1668**, 477–488 (1992).
7. R.K. Tyson, Opt. Eng. **29**, No. 10, 1165–1173 (1990).
8. R.B. Shack and B.C. Platt, J. Opt. Soc. Am. **61**, 1586 (1971).
9. D.S. Acton and R.C. Smithson, Appl. Opt. **31**, No. 16, 3161–3169 (1992).
10. V.P. Lukin, N.N. Mayer, and B.V. Fortes, Atmos. Oceanic Opt. **5**, No. 12, 801–807 (1992).

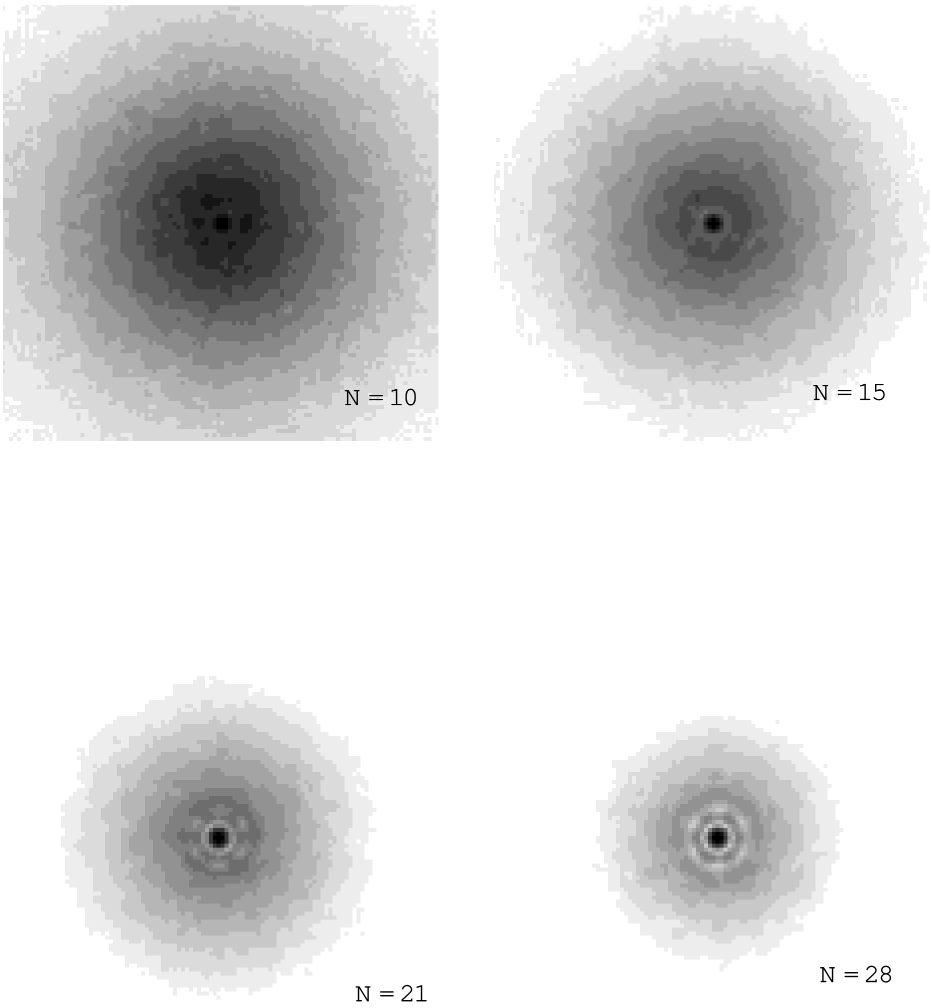


FIG. 1. Two-dimensional intensity distribution of PSF for the modal compensation. Value of the aperture normalized diameter is $D/r_0 = 20$.

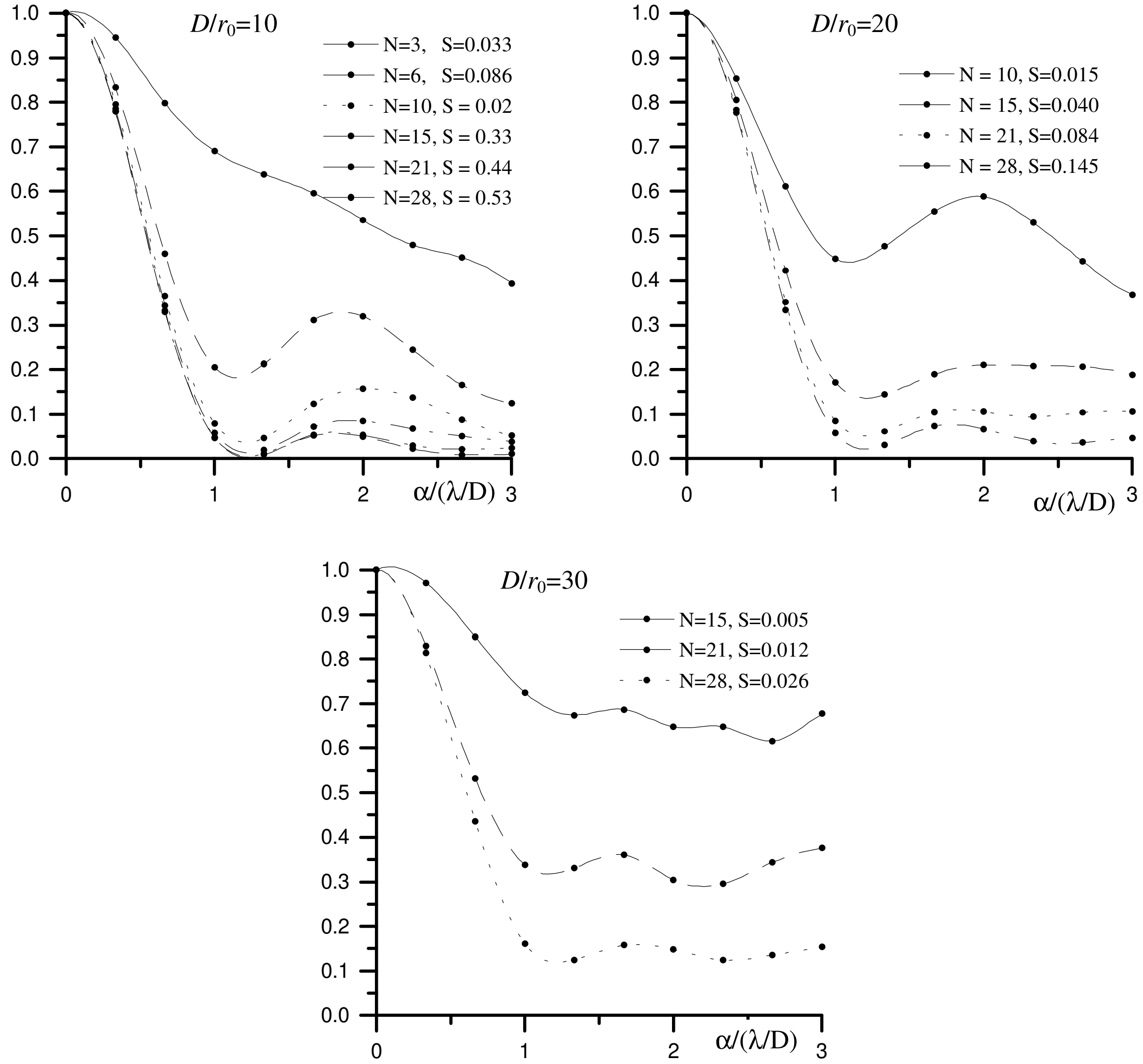


FIG. 2. PSFs, when a modal corrector is used. Parameter N corresponds to the number of Zernike polynomials. PSF is normalized to the axial value. S is the Strehl ratio.

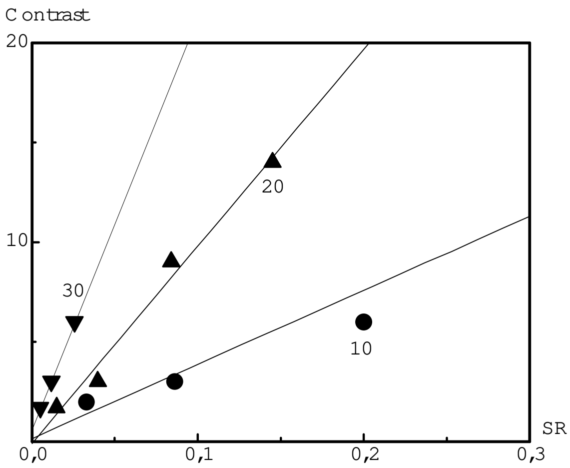


FIG. 3. The dependence of the contrast on Strehl ratio for modal correction

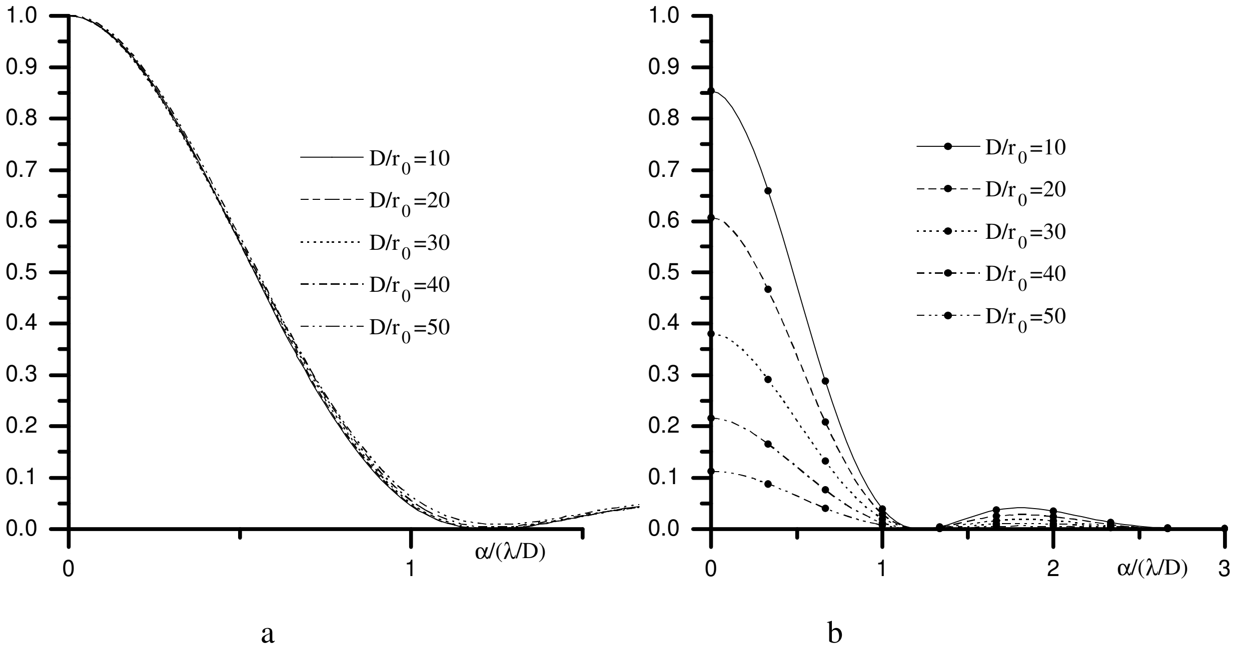


FIG. 4. PSF, when the segmented adaptive mirror with 84 elements of hexagonal form is used. Every segment is controlled by the position and slopes. a – PSF is normalized by its axial value. b – PSF is normalized by the diffraction maximum.

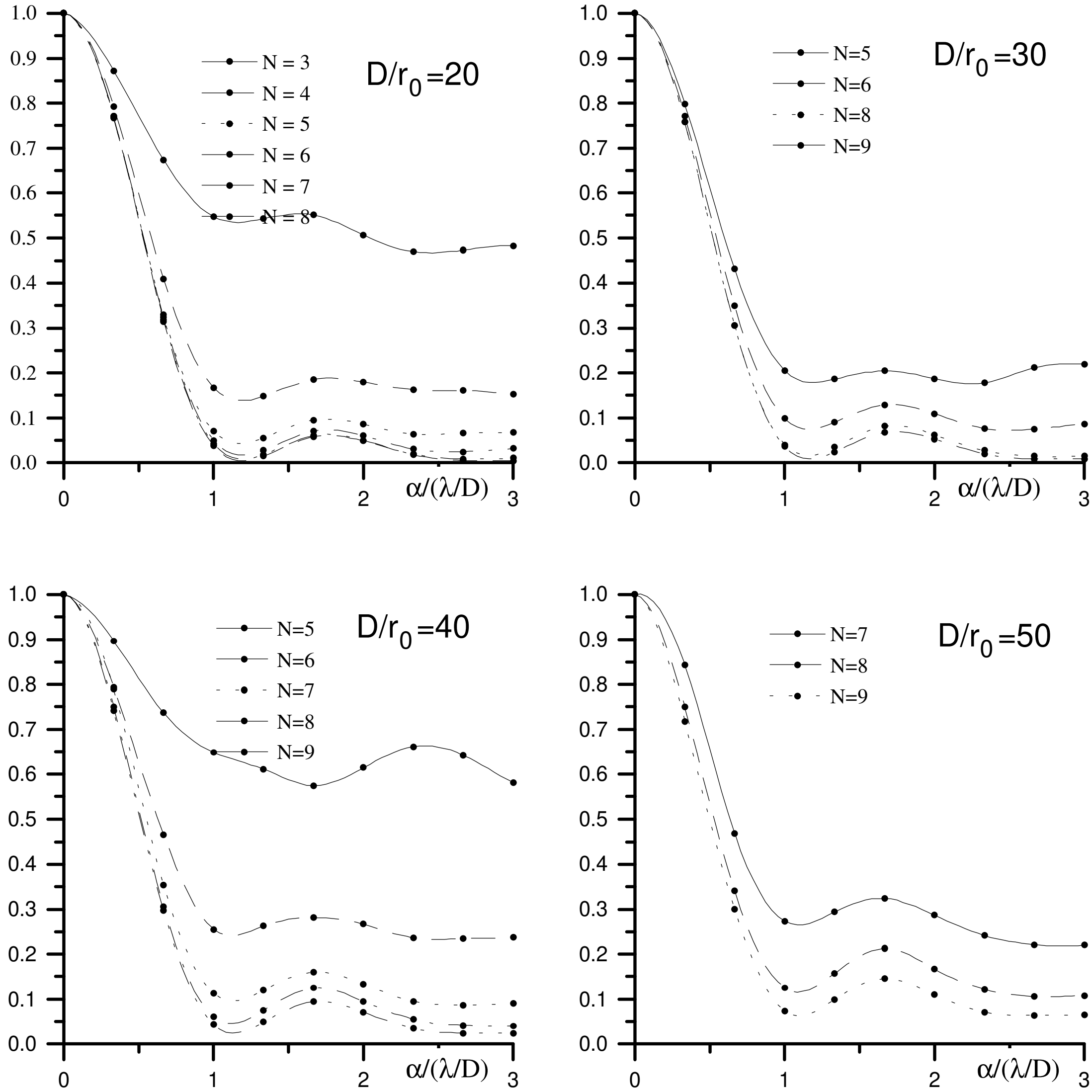


FIG. 5. PSF, when a flexible adaptive mirror with the Gaussian reference function is used. Parameter N corresponds to a number of the control points at the aperture diameter. PSF is normalized by its axial value.

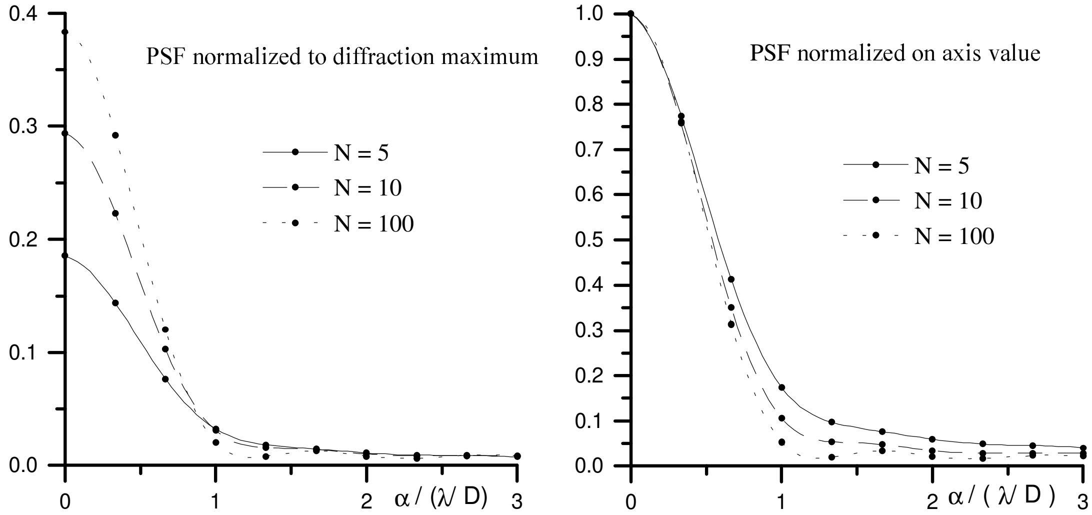
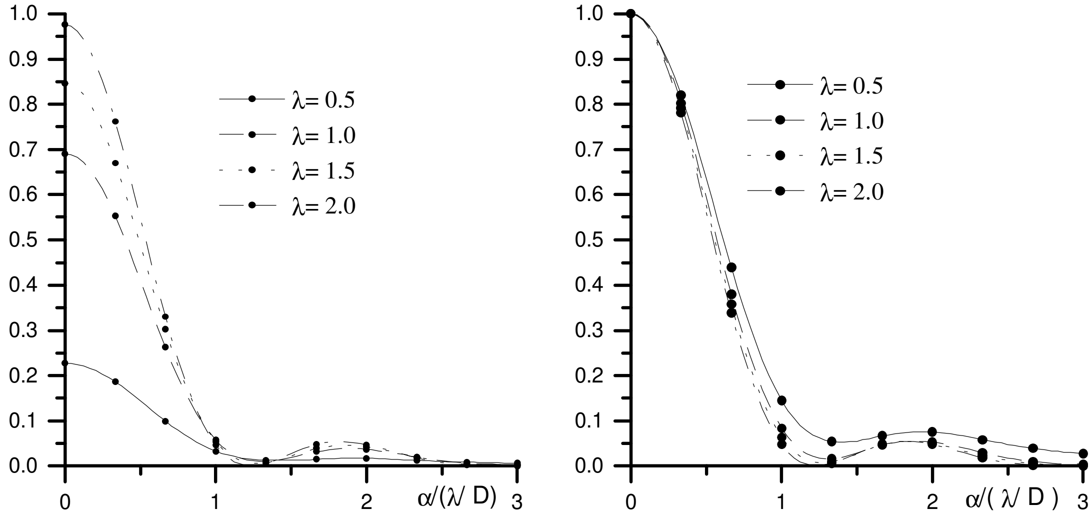


FIG. 6. Results of the simulation of an adaptive telescope with the Shack-Hartmann sensor. Normalized diameter of the aperture is $D/r_0 = 10$. Dimensions of the sensor lens diaphragm are 10×10. Estimation of the wavefront aberrations was performed by the modal algorithm (28 Zernike polynomials). Parameter N corresponds to the average-statistical number of photons at the subaperture during one exposure.



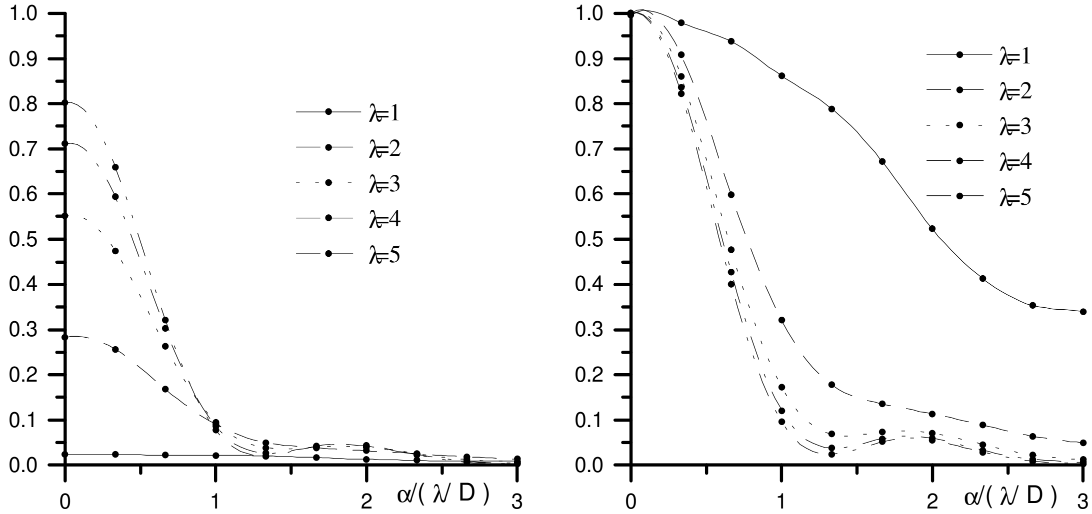


FIG. 7. PSFs of the 10-meter telescope at different wavelengths for the adaptation using a laser guide star. Upper plots are for the sodium LGS (H = 100 km), *the lower plots are for the Rayleigh LGS* (H = 10 km). *At the left plots the PSF is normalized to the diffraction maximum, at the right plots the PSF is normalized to its axial value. The wavelength* λ *is given in* μm.