SOME CONSIDERATIONS CONCERNING THE SPECKLE STRUCTURE OF ASTRONOMICAL IMAGES

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Characteristics of intensity spikes in astronomic images have been analyzed theoretically under conditions of the speckle-structure developed due to large-scale turbulent inhomogeneities in the atmosphere. The following parameters were computed: mean area of a spike, density of a spike succession, distance between successive spikes and their volume. It was shown that under such conditions an astronomical image is formed of random diffraction limited images. Intensities of these images are determined by mean intensity of the image as a whole.

The image of an extraterrestrial object observed through a turbulent atmosphere by means of a large telescope is usually a random pattern of dark and light speckles (speckle-structure). To improve the quality of images distorted by atmospheric turbulence the adaptive technique[1] or postdetector processing of images[2] is usually used. In both methods information is obtained from the whole image or from some part of it. Naturally, the information may be extracted only from light spots of the image. So it is interesting to assess characteristic parameters of these regions. Thorough analysis is performed in this paper taking light speckles as random spikes of an astronomical image intensity.

A theory describing random spikes of homogeneous and isotropic field was developed by Bunkin and Gochelashvily.[3] They obtained analytical equations for the following parameters of random field spikes: a mean area of spikes, density of spikes, distance between successive spikes, and volume of spikes. Analysis of random spikes of laser radiation intensity in a turbulent atmosphere was performed by these authors based on this theory.[4] The regime of weak fluctuations, i.e., log-normal distribution of fluctuations was assumed. An alternative approach to description of local maximum statistics of light wave intensity fluctuations in the turbulent atmosphere was proposed in Refs. 5-7. Analysis of the problem was continued in Refs. 8 and 9 where characteristics of the intensity spikes distributed according to normal law were considered.

The authors of Refs. 8 and 9 showed that average characteristics of intensity spikes exceeding some fixed level I_{sp} which are several times greater than mean intensity $\langle I_g \rangle$ can be calculated using the following formulas:

mean area of a single spike is

$$\langle S(I_{sp}) \rangle \cong \frac{\pi}{2} \frac{\langle I_g \rangle^2}{2R_g''(0)\, I_{sp}},$$

mean density of spikes (mean number of spikes in a unit square) is

$$\langle N(I_{sp})\rangle \cong \frac{2}{\pi}\frac{R_g''(0)\, I_{sp}}{\langle I_g\rangle^2}\exp\left(-\frac{I_{sp}}{\langle I_g\rangle}\right),$$

mean distance between spikes is

$$\langle l(I_{sp})\rangle \cong \sqrt{\frac{\pi}{8}}\frac{\langle I_g\rangle}{\sqrt{R_g''(0)\, I_{sp}}}\exp\left(\frac{I_{sp}}{2\langle I_g\rangle}\right),$$

and mean volume of a spike (mean energy flux through a single spike) is

$$\langle V(I_{sp})\rangle \cong \frac{\pi}{2}\frac{\langle I_g\rangle^2}{R_g''(0)}.$$

The angular brackets mean here averaging over an ensemble of random samples and

$$R_g''(0) = \frac{1}{2}\mathrm{Re}\left\{\frac{\partial^2}{\partial y_1 \partial y_2}\langle U_g(\vec{\rho}_1)U_g^*(\vec{\rho}_2)\rangle\right\}\Bigg|_{\vec{\rho}_1=\vec{\rho}_2=0}$$

is a real part of second-order derivative with respect to transverse coordinates at $\rho_1 = \rho_2 = 0$ taken from a second mutual coherence function $U_g(\rho_1)U_g^*(\rho_2)$ of an optical wave $U_g(\rho)$.

In the plane of the best image behind a receiving focusing lens the field of optical wave is usually written in the form of Huggins-Kirchhoff integral[1, 2]:

$$U_g(\rho) = \frac{k\exp\left(ikF_t + \frac{ik}{2F}\rho_2\right)}{2\pi i F_t}\times$$

$$\times\iint_{-\infty}^{\infty} d\rho' U(\rho')\exp\left(-\frac{ik}{2F_t}\rho\rho'\right),$$

where $U_g(\rho')$ is the field of an incident optical wave, $\grave{O}(\rho)$ is the transmittance of the system, F_t is a focal length of the receiving lens, $k = 2\pi/\lambda$ and λ is the wavelength in vacuum, ρ' and ρ are transverse coordinates in the plane of the receiving aperture and in the best image plane correspondingly. Under conditions of developed speckle-structure the field of an optical wave $U_g(\rho)$ can be considered normal.

If can easily be shown that the mean value of the optical field behind the receiving lens is almost zero, i.e.,

$$\left\langle U_g(\rho)\right\rangle \cong 0.$$

In isoplanatic region behind the lens the distribution of the mean intensity is uniform $(\tilde{A}_2(\rho_1,\rho_2) = \tilde{A}_2(\rho_1-\rho_2))$ so it can be written in the following form:

$$\left\langle I_g(\rho)\right\rangle = \frac{k^2}{4\pi^2 F_t^2}\iint_{-\infty}^{\infty} d\eta\, \tilde{A}_2(\eta)\times$$

$$\times \exp\left[-\frac{ik}{F_t}\rho\eta\right]\iint_{-\infty}^{\infty} d\xi T(\xi)T^*(\xi-\eta),$$

where $\tilde{A}_2(\rho_1,\rho_2) = \left\langle U(\rho_1)U^*(\rho_2)\right\rangle$ is a second-order mutual coherence function of an optical wave incident on the input aperture.

In Kolmogorov approximation of turbulence the second-order mutual coherence function which forms an astronomical image can be written as

$$\tilde{A}_2(\rho) = U_0^2 \exp\left\{-\left(\frac{\rho}{\rho_0}\right)^{5/3}\right\},$$

where U_0 is the amplitude of incident optical wave, ρ_0 is a coherence length of a plane wave in the turbulent atmosphere. In most cases the radius R of the receiving aperture of an astronomical system is much greater than the coherence length ρ_0 of the incident optical wave $\rho_0(R >> \rho_0)$, so with developed speckle-structure and disappearance of Airy pattern the mean intensity of an optical wave behind the lens can be approximated by a Gaussian function[8]

$$\left\langle I_g(\rho)\right\rangle \cong I_i \exp\left\{-\left(\frac{\rho}{\rho_i}\right)^2\right\},$$

where $I_i = I_0\dfrac{R^2}{\rho_i^2}$ is the intensity of the astronomical image on the optical axis of the system, $I_0 = U_0^2 T_0^2$ is the intensity of optical wave passed through the telescope, $\grave{O}_0$ is the amplitude transmittance of the telescope on its axis, $\rho_i = 2F_t/(k\rho_0)$ is the linear size of a point source observed through the turbulent atmosphere. Naturally, in this case the mean turbulent size of an object is greater than the diffraction limited one

$$\rho_i >> \rho_d,$$

where $\rho_d = 2F_t/(kR)$ is the liner size of the image of a point object observed through a homogeneous medium (diffraction astronomical image intensity and its derivative is absent only in the vicinity of an optical axis of a telescopic system so quantitative estimations of intensity spike parameters are valid only in the central part of the image.

For an optical field behind the receiving lens an integral expression for second-order derivative of a mutual coherence function of the second-order can be written as

$$R_g''(0) = \frac{k^4}{16\pi^2 F_t^4} \iint\limits_{-\infty}^{\infty} d\eta\,\eta \iint\limits_{-\infty}^{\infty} d\xi\xi \tilde{A}_2(\eta - \xi) T(\eta) T^*(\xi).$$

After calculations of integrals entering into this equation for $R >> \rho_0$ we obtain the following resulting equation for the second-order derivative of the mutual coherence function of the second order:

$$R_g''(0) = I_i \frac{1}{\rho_d^2}.$$

So, for an astronomical image the parameters of intensity spikes can be written as follows:

mean area of an individual spike is

$$\left\langle S\left(I_{sp}\right)\right\rangle \cong \frac{1}{2}\pi\rho_d^2 \frac{I_i}{I_{sp}},$$

mean density of spikes is

$$\left\langle N\left(I_{sp}\right)\right\rangle \cong \frac{2}{\pi\rho_d^2} \frac{I_{sp}}{I_i} \exp\left(-\frac{I_{sp}}{I_i}\right),$$

mean distance between spikes is

$$\left\langle l\left(I_{sp}\right)\right\rangle \cong \sqrt{\frac{\pi}{8}}\rho_d \sqrt{\frac{I_i}{I_{sp}}} \exp\left(\frac{I_{sp}}{2I_i}\right),$$

and mean volume of a spike

$$\left\langle V\left(I_{sp}\right)\right\rangle \cong \frac{1}{2}\pi\rho_d^2 I_i.$$

It is seen that for an astronomical image parameters of intensity spikes are determined by its diffraction limited size ρ_d and ratio of a given level I_{sp} to intensity I_i of an image at the optical axis of a telescope.

On the basis of the relations obtained the following conclusions can be drawn.

1. Under conditions of the developed speckle-structure an astronomical image is a pattern of light speckles. Characteristic size of speckles is determined by the diffraction limited size of an image (the size is proportional to $\approx \rho_d$).
2. The distance between speckles is proportional to diffraction limited size of the image. Mean energy flux of speckles is determined by intensity of the mean image an by diffraction limited size of the image. The flux is proportional to $\approx \pi\rho_d^2 I_i$.
3. Speckles-structure of the astronomical image is formed of randomly disposed diffraction limited images with intensities determined by the intensity I_i of the mean image. This intensity is considerably less than the intensity of diffraction limited image $\left(I_i << I_d = I_o\left(R^2/\rho_d^2\right)\right)$. The number of diffraction images increases proportionally to their intensity reduction (the number of speckles is proportional to $\approx I_d/I_i$).
4. In an astronomical image the number of speckles increases at their constant density, i.e., the region occupied by speckles increases. Increase of the astronomical image (broadening of the image) due to atmospheric disturbance is from the diffraction limited size ρ_d and up to turbulent size ρ_i.

The influence of temporal variations of atmospheric parameters was not allowed for in the analysis so the obtained results are valid only for short observation intervals with characteristic time $\tau \le 10^{-3}\, s$, i.e., only for short exposure images of astronomical objects.

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