The dynamics of solids in self-gravitating protostellar discs

Giuseppe Lodato - Università degli Studi di Milano
Peter Cossins - University of Leicester, UK

5 November 2009 - From circumstellar disks to planetary systems
Gravitational instabilities in protostellar discs

- Conditions for instability
- Dynamics of self-gravitating discs:
  - Conditions for fragmentation/self-regulation
- Planetesimal formation and evolution in spiral arms
- Self-regulated disc models and their application to planetesimal formation
Linear stability criterion

• Well known axisymmetric instability criterion:
  \[ Q = \frac{c_s \Omega}{\pi G \Sigma} < \bar{Q} \approx 1 \]

• Equivalent form of the instability criterion
  \[ \frac{M_{\text{disc}}(R)}{M_\star} \geq \frac{H}{R} \]

• Need the disc to be cold and/or massive

• What are the masses and aspect ratio in actual protostellar discs?
Are protostellar discs linearly unstable?

- Midplane temperature for irradiated discs (Chiang & Goldreich 1997, Chiang & Youdin 2009) gives:
  \[
  \frac{H}{R} \approx 0.02 \left( \frac{R}{\text{AU}} \right)^{2/7}
  \]

- Therefore $H/R$ varies from 0.02 at 1 AU to 0.06 at 100 AU

- Need disc masses of order 5% of the stellar mass to be unstable

- Protostellar disc masses difficult to measure (see Hartmann et al 2006)
Are protostellar discs linearly unstable?

- Disc masses in Taurus and Ophiucus by Andrews and Williams (2005, 2007)
- Clear trend to have smaller masses at later stages of evolution
- A substantial fraction of Class I (and even some Class II) objects expected to be unstable
- Disc masses might be underestimated significantly (Hartmann et al 2006)
- Uncertainties in dust opacities
Are protostellar discs linearly unstable?

- Disc masses in Taurus and Ophiucus by Andrews and Williams (2005, 2007)
- Clear trend to have smaller masses at later stages of evolution
- A substantial fraction of Class I (and even some Class II) objects expected to be unstable
- Disc masses might be underestimated significantly (Hartmann et al 2006)
- Uncertainties in dust opacities
Are protostellar discs linearly unstable?

- Disc masses in Taurus and Ophiucus by Andrews and Williams (2005, 2007)
- Clear trend to have smaller masses at later stages of evolution
- A substantial fraction of Class I (and even some Class II) objects expected to be unstable
- Disc masses might be underestimated significantly (Hartmann et al 2006)
- Uncertainties in dust opacities
Are protostellar discs linearly unstable?

- Disc masses in Taurus and Ophiucus by Andrews and Williams (2005, 2007)
- Clear trend to have smaller masses at later stages of evolution
- A substantial fraction of Class I (and even some Class II) objects expected to be unstable
- Disc masses might be underestimated significantly (Hartmann et al 2006)
- Uncertainties in dust opacities
Non linear evolution of GI


Disc thickness: for $H/R << 1$, transport induced by spiral can be described by standard, local accretion disc models (Lodato & Rice 2004, 2005, Cossins et al. 2009a)
Thermal saturation of GI

Cossins, Lodato & Clarke 2009a

- Self-regulation is established through thermal saturation of the spiral waves.

- **IMPORTANT**: Amplitude of density perturbation must be related to cooling rate.

- We find that:
  \[
  \frac{\Delta \Sigma}{\Sigma} \approx \frac{1}{\sqrt{\Omega t_{\text{cool}}}}
  \]

- Naturally predicts a radially varying value of \( \alpha \)
Evolution of solids in self-gravitating discs

- Effects of gas drag on solid particles is to induce fast migration towards pressure maxima.
- In a laminar disc this produces a fast inward migration of meter-sized particles (Weidenschilling 1977)

\[
\Delta v \approx \frac{c_s^2}{\nu_K} \frac{\partial \ln \rho}{\partial \ln R}
\]

\[
v_r = \frac{\Delta v}{\Omega t_s + 1/\Omega t_s}
\]
Evolution of solids in self-gravitating discs

- Pressure maxima in spiral structure efficient trap for meter sized objects (see also Haghighipour & Boss 2003, Durisen et al 2005).

- Run SPH simulations of a two component system (gas + solids)
Solid agglomeration in pressure maxima

- Density of meter sized objects enhanced by up to two orders of magnitude
- Density becomes high enough to become comparable to Roche density
- Gravitational collapse of solids is possible
- Confirmed through additional simulations including the solids self-gravity (Rice et al. 2005)
- Resulting planetesimals mass expected to be high (but difficult to measure from simulations)

(Rice, Lodato et al 2004, 2006)
Planetesimals in self-gravitating discs

- Particle traps in spiral arms are an effective way of producing large solid bodies in the disc:
  - Resulting planetesimal mass quite large
  - *Dynamically stirred population of planetesimals* (Britsch, Lodato & Clarke 2008)
  - Expected to occur in early phases of star formation (<~ 1Myr)
  - **Is this process limited to some specific radial range in the disc?**
    - *Note:* Rice et al. used an idealized cooling function leading to a rather large amplitude spiral $\Delta \Sigma / \Sigma \approx 0.1$
    - Need a detailed model of self-gravitating discs with realistic cooling
Local models of self-regulated protostellar discs


- If transport is local (cf. Cossins et al 2009), then in thermal equilibrium (and absent other sources of heating, e.g. irradiation):

\[ \alpha = \frac{4}{9} \frac{1}{\gamma (\gamma - 1)} \frac{1}{\Omega t_{\text{cool}}} \]

- Possible to construct models of self-regulated discs ($Q \sim 1$), where viscosity is related to cooling time (Clarke 2009, Rafikov 2009)

- Identify various possible regimes for self-gravitating protostellar discs
Where do planetesimals form?

Planetary formation through this process occurs at \(30 \text{AU} < R < 50 \text{AU}\)

Roughly coincident with the location of the Kuiper belt

Some evidence for a large inner hole in debris disc systems (Currie et al. 2008), based on the apparent increase of debris disc brightness at late ages ~ 10 Myrs (Meyer’s talk)

Rapid production of large bodies in the outer disc may preserve sub-mm emission in the T Tauri phase (Takeuchi, Clarke & Lin 2005)
Will we be able to observe a spiral structure at ~ 50 AU with ALMA?

\[ \frac{M_{\text{disc}}}{M^*} = 0.1 \]
\[ M^* = 2M_{\text{Sun}} \]
\[ R_{\text{disc}} = 50 \text{AU} \]
\[ D = 140 \text{pc} \]

10h integration
Conclusions

- Class I discs are likely to be gravitationally unstable
- Self-regulated evolution of GI leads to sustained angular momentum transport for ~ 1 Myr, bringing the disc into the T Tauri phase
- Spiral arms where the first sites to be identified as optimal particle traps for the formation of planetesimals
- Such process works only in the outer disc, between 30 AU and 50 AU
- Leads to the rapid formation of solid in an annular region at large distances: possibly consistent with observations of debris discs and the Kuiper belt