Bias correction for high precision science

David Buscher

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Outline

The problem

Existing bias-removal methods

New method

Examples
Acknowledgements

- Hrobjarthur Thorsteinsson
- James Gordon
High-precision measurements yield new science

Van Belle et al. 2001

Baseline PA ~ 195°
Baseline PA ~ 245°
3.32 mas UD
3.13 mas UD
3.40 mas UD

Normalized $V^2$

Van Belle et al. 2001
Closure phases are a high-precision observable.
Detection noise affects all interferometry data

- Mean photon rate $\{\Lambda_p, \quad p = 1 \ldots N_{\text{pix}}\}$
- Detected flux $\{i_p, \quad p = 1 \ldots N_{\text{pix}}\}$
Zero-mean noise can lead to a non-zero bias

\[ C = \sum_p \exp(2\pi iux_p) \Lambda_p \]
\[ c = \sum_p \exp(2\pi iux_p) i_p = C + n \]

\[ \langle |C + n|^2 \rangle = \langle |C|^2 \rangle + 2 \langle \Re [Cn^*] \rangle + \langle |n|^2 \rangle \]
\[ = \langle |C|^2 \rangle + \langle |n|^2 \rangle \]
Photon-noise bias on the bispectrum is well-known

\[
\langle C^{ij} C^{jk} C^{ki} \rangle = \langle c^{ij} c^{jk} c^{ki} \rangle - \langle |c^{ij}|^2 \rangle - \langle |c^{jk}|^2 \rangle - \langle |c^{ki}|^2 \rangle + 2 \langle N_{\text{phot}} \rangle
\]
Bias correction is only possible under restricted conditions

- “DFT conditions”
  - Integer number of fringe cycles in a scan
  - No “tapering” allowed
- No read noise allowed
We generalise the estimator for the complex amplitude

DFT estimator

\[ c_{ij} = \sum_p \exp(2\pi i u_{ij} x_p) i_p \]

Generalised to

\[ c_{ij} = \sum_p H_{ij}^p i_p \]

For example, a tapered DFT:

\[ H_{ij}^p = W(x_p) \exp(2\pi i u_{ij} x_p) \]

Can represent any linear estimator, e.g. P2VM, ABCD
We allow any combination of read noise and photon noise

\[ P \text{Poisson} \left( N_p | \Lambda_p \right) = \sum_{n}^{\infty} \delta (N_p - n) \frac{\Lambda_p^N_p}{N_p!} \exp \left[ -\Lambda_p \right]. \]

\[ P \text{Gaussian} \left( \epsilon_p | \sigma_p, N_p \right) = \frac{1}{\sqrt{2\pi\sigma_p^2}} \exp \left[ -\frac{(\epsilon_p)^2}{2\sigma_p^2} \right]. \]

\[ P \left( i_p | \Lambda_p, \sigma_p \right) = \int_{0}^{i_p} P \text{Poisson} \left( N_p | \Lambda_p \right) P \text{Gaussian} \left( i_p - N_p | \sigma_p, N_p \right) dN_p. \]
We derive a bias-free bispectrum estimator

$$B_{0}^{ijk} = c^{ij} c^{jk} c^{ki}$$

$$- c^{ij} \sum_{p} (i_{p} + \sigma_{p}^2) H_{p}^{jk} H_{p}^{ki}$$

$$- c^{jk} \sum_{p} (i_{p} + \sigma_{p}^2) H_{p}^{ij} H_{p}^{ki}$$

$$- c^{ki} \sum_{p} (i_{p} + \sigma_{p}^2) H_{p}^{ij} H_{p}^{jk}$$

$$+ \sum_{p} (2i_{p} + 3\sigma_{p}^2) H_{p}^{ij} H_{p}^{jk} H_{p}^{ki}.$$
The closure phase bias typically increases for non-zero closure phase.
A practical example is closure-phase nulling.
Systematic errors can arise from detection noise

We have presented a general bias-free bispectrum estimator

A small step towards higher-precision science
Spare slides
The power spectrum variance contains coupled terms

- Photon terms
- Read terms
- Coupled terms

Graph showing the relationship between the number of source photons detected and the variance of the power spectrum.