Reducing the Fractal Iterative Method (FRiM) to the cost of half an iteration

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Reminder on the Fractal Iterative Method (FRiM)

- minimum-variance reconstruction algorithm for large Adaptive Optics systems
  
  Thiébaut & Tallon JOSA A, 2010

- performance assessed on Octopus, ESO end-to-end AO simulator, since 2008

- preconditioned conjugate gradients (PCG), as most of the iterative AO reconstructors

- iterative method PROS
  - no full matrix multiplication
  - neither matrix inversion, nor matrix storage in FRiM
  - sparse/fast operators in FRiM
  - easier to update the model

- iterative method CONS?
  - latency increases with the number of PCG iterations
RTC latency requirements in AO

- Sensor exposure $k$
- Sensor exposure $k+1$
- Sensor exposure $k+2$
- Read-out $k$
- Read-out $k+1$
- Reconstruction $k$
- Reconstruction $k+1$
- Control $k$
- Control $k+1$
- Amplitude $k$
- Mirror shape $k$

Time

RTC latency
RTC latency requirements in AO

- most time spent in the reconstruction, and particularly in the PCG
FRiM iterations vs RTC latency requirements

- Projection on DM (proj) and pseudo open-loop control with integrator (\( \int \)) (Gilles, 2003)
- \( r_0 \) and \( r_f \), PCG residuals (initial and final) for iterations
- \( w_0 \) and \( w_f \), starting guess and final estimate
- "warm-start": \( w_0 \leftarrow w_f \) (because slow evolution of the turbulent wavefront)
- projection on DM (proj) and pseudo open-loop control with integrator ($\int$) (Gilles, 2003)
- $r_0$ and $r_f$, PCG residuals (initial and final) for iterations
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- "warm-start": $w_0 \leftarrow w_f$ (because slow evolution of the turbulent wavefront)

- iterations need to be done sequentially
- more iterations means more latency until commands are applicable
- several iterations required to obtain best performance in this classical scheme
Examples of FRiM performance vs #iterations

- **E-ELT**
  - 42-m telescope (central obs.: 0.28)
  - 500 Hz loop frequency
  - Cn2 profile: 9 layers, \( r_0 = 12.9 \text{ cm} \)

- **single-conjugate AO:**
  - DMs: 0 km (85 x 85)
  - 1 NGS, 84x84 subap.
  - \( 10^5 \) photons/frame/subap.
  - \( \tau_0 = 2.8 \text{ ms} \)

- **multi-conjugate AO (MAORY):**
  - DMs: 0 km (85 x 85), 4 km (47 x 47), 12.7 km (53 x 53)
  - 6 LGS, 84 x 84 subap.
    - on a Ø 2 arcmin circle
    - 500 ph/subap.
    - RON 3e-
  - 2 NGS for tip/tilt, 1 NGS for 2 x 2 subap.
    - on a Ø 2.7 arcmin circle
    - 500 ph/subap., H band
    - RON 5e-

3 PCG iterations required for the best performance
back to the classical scheme

PCG iterations

"warm-start": $w_f$

$w_0$

$r_0$

$w_f$

proj

RTC latency

DM commands

WFS data

PCG iterations
back to the classical scheme

- but 3 iterations of FRiM takes too long for the latency specifications
back to the classical scheme

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- solutions?
  - reduce the number of iterations with a better preconditioner. But more computations per iteration
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- reduce the number of iterations with a better preconditionner. But more computations per iteration

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solutions?
  – reduce the number of iterations with a better preconditionner. But more computations per iteration
  – restrict to 1 iteration of FRiM to meet latency requirement and … apply 3 iterations of FRiM to improve the future “warm-start”
A low-latency scheme for FRiM involves iter 1, iter 2, iter 3, and proj. The diagram illustrates the flow of data and commands. The consequences of this scheme are:

- Low-latency path is reduced to the cost of 1 iteration.
- 3 iterations in SCAO < 1 WFS exposure time.
- Best performance is maintained thanks to an optimal "warm-start" (preliminary results by simulations).
Results with low-latency scheme

- simulations E-ELT SCAO
- FRiM + Octopus
- 500Hz

1) vs measurement noise (flux)

- no impact at low flux
- good “warm-start” = key point to afford 1 iteration at high SNR
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2) vs wind speed
   (atmosphere coherence time $\tau_0$)
   - high flux conditions
     - again good “warm-start”= key point
     - also helps for high wind speeds
reducing to the cost of half an iteration

- computations for the low-latency branch (only 1 iter.) can be revisited
- only the first iteration to be applied
  - PCG 1rst iteration = steepest descent
  - simplified computations (no need to update the residuals)

<table>
<thead>
<tr>
<th>classical FRiM 1rst iter.</th>
<th>low-latency FRiM 1 iter.</th>
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half of the heavy computations of 1 iteration
no longer an iterative reconstructor
Conclusions

- For iterative methods, RTC latency is constrained by the # of iterations
  - sequential iterations
  - although only 3 are required for best performance of FRiM

- We developed a new low-latency application of FRiM, based on:
  - 1 iteration for the commands computation
  - 3 iterations to improve the warm-start of the next reconstruction

- First results from simulations demonstrate the efficiency of the improved “warm-start”

- The computational cost of 1 iteration only is half the cost of a classical iteration

- With only 1 iteration, the reconstruction is no longer iterative. This may be applied to any iterative method…