Spiral structure and gravitational instabilities in protostellar discs

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Gravitational instabilities in protostellar discs

- Conditions for instability
- Dynamics of self-gravitating discs:
  - Self-regulation
  - Local vs global behaviour
- Numerical uncertainties - convergence, fragmentation
- Observations of density waves in protostellar discs
Linear stability criterion

- Linear dispersion relation

\[(\omega - m\Omega)^2 = c_s^2 k^2 - 2\pi G \Sigma |k| + \kappa^2\]

- Well known axisymmetric instability criterion:

\[Q = \frac{c_s k}{\pi G \Sigma} < \bar{Q} \approx 1\]

- Equivalent form of the instability criterion

\[\frac{M_{\text{disc}}(R)}{M_*} \gtrsim \frac{H}{R}\]

- Need the disc to be cold and/or massive

- What are the masses and aspect ratio in actual protostellar discs?
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Are protostellar discs linearly unstable?

• Midplane temperature for irradiated discs (Chiang & Goldreich 1997, Chiang & Youdin 2009) gives:

\[
\frac{H}{R} \simeq 0.02 \left( \frac{R}{\text{AU}} \right)^{2/7}
\]

• Therefore \(H/R\) varies from 0.02 at 1AU to 0.06 at 100 AU

• Need disc masses of order 5% of the stellar mass to be unstable

• Protostellar disc masses difficult to measure (see Hartmann et al 2006)
Are protostellar discs linearly unstable?

- Disc masses in Taurus and Ophiucus by Andrews and Williams (2005, 2007)
- Disc masses might be underestimated significantly (Hartmann et al 2006)
- Uncertainties in dust opacities
- If density profile steep, most of the mass might be hidden in optically thick inner parts (Hartmann 2009)
Are protostellar discs linearly unstable?

Lodato, Dolci, Manara, Ricci, in prep.

Class II (T Tauri) discs are relatively evolved. Can we infer the masses at early stages?

Simple (simplistic?) approach:

Take all objects with measured M and Mdot

Apply similarity solutions (Lynden-Bell & Pringle 1973)

Find “initial’ disc mass and evolutionary timescale

Masses from Andrews & Williams

Mdots from the literature

Not all measurements consistent with similarity solutions (see also Jones, Alexander & Pringle 2012)

\[
M_0 = M_d(t) \left( \frac{t_d}{t_d - t} \right)^{1/2(2-\gamma)}
\]

\[
t_d = \frac{M_d(t)}{2(2 - \gamma) \dot{M}(t)}
\]
Are protostellar discs linearly unstable?

- Very preliminary results
- Limited sample
- Inhomogeneous analysis of Mdot measurements
- Do similarity solutions really apply, at least in an averaged sense?
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Non linear evolution of GI


- Early simulations used an isothermal or polytropic equation of state (Laughlin & Bodenheimer 1994, Mayer et al 2002)

- Starting from Gammie (2001) it has become clear that the evolution is strongly dependent on the cooling time $t_{cool}$

- Introduce a cooling parameter as the ratio of cooling to dynamical timescale

$$\beta = t_{cool} \Omega$$
Thermal self-regulation of GI

- Role of cooling time clear if one thinks at the form of the stability parameter \( Q \)
  \[
  Q = \frac{c_s \kappa}{\pi G \Sigma} \propto T^{1/2}
  \]
- Development of the instability feeds energy back onto the equilibrium and stabilizes the disc
- Works as an effective thermostat for the disc
- Expect the disc to stay close to marginal stability \( Q \sim 1 \) (Paczynski 1977)
- Self-regulated discs models can be constructed (Bertin 1997, Bertin & Lodato 1999)
Long cooling time: self-regulation

Cossins, Lodato & Clarke (2009)

$\beta = 6$
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Self-regulation is established through thermal saturation of the spiral waves.

Amplitude of density perturbation must be related to cooling rate

We find that:

$$\frac{\Delta \Sigma}{\Sigma} \approx \frac{1}{\sqrt{\beta}}$$

Natural if consider that energy content of waves is proportional to the square of the perturbed fields
Spectrum of excited modes

Azimuthal structure: massive discs characterized by small $m$

Radial structure: at all radii, $k$ peaks at roughly $\sim 1/H$
Sonic condition for spiral waves

- We have computed the pattern speed of the underlying spiral structure and its Mach number.
- The Doppler-shifted Mach number is very close to unity, independently on radius, cooling rate, and disc mass.
- Density jump for almost sonic shocks also directly leads to

\[ \frac{\Delta \Sigma}{\Sigma} \approx \frac{1}{\sqrt{\beta}} \]
Local vs global behaviour

- Can the evolution of self-gravitating discs be described within the standard, local, \( \alpha \)-like prescription?
- Can compute gravitational + Reynolds stresses directly from simulations and compare with expectations from standard \( \alpha \)-theory (LR04, see also Boley et al. 2006)
- The disc adjusts so as to deliver the viscosity needed to stay in thermal equilibrium
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Lodato & Rice 2004

Boley et al. 2006
Local vs global behaviour

- Can the evolution of self-gravitating discs be described within the standard, local, $\alpha$-like prescription?
- Described in detail by Balbus & Papaloizou (1999), recently discussed extensively by Cossins et al (2009)
- Relation between energy and angular momentum densities in a density wave
  \[ \mathcal{E} = \Omega_p \mathcal{L} \quad \longrightarrow \quad \dot{\mathcal{E}} = \Omega_p \dot{\mathcal{L}} \]
- Relation between power and stress due to local (viscous) processes
  \[ \dot{\mathcal{E}}_\nu = \Omega \dot{\mathcal{L}}_\nu \]
- If density waves dissipate far from co-rotation, behaviour is non-local
Local vs global behaviour

Cossins, Lodato & Clarke 2009

- Degree of non-locality can be measured by

\[ \xi = \left| \frac{\Omega - \Omega_p}{\Omega} \right| \]

- Sonic condition for wave dissipation also tells us something about this:

\[ \xi \approx \frac{c_s}{v_\phi} = \frac{H}{R} \]

- To the extent that the disc is thin \((H \ll R)\), global behaviour should be negligible

- Possible to construct local, viscous models of disc evolution (Clarke 2009, Rafikov 2009)
Convergence of numerical results

- It is well known that for short cooling times the disc is subject to fragmentation.
- Meru and Bate (2011) show that such simulations are not converged.
- As resolution increases, fragmentation appears to become effective for longer cooling times.

Simulation by Peter Cossins

$$\beta = 4$$
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Convergence of numerical results

- Non-convergence most likely due to the effects of artificial viscosity in SPH simulations: additional viscosity (or any heating terms) weakens the gravitational instability (Lodato & Clarke 2011)

- Meru and Bate (2012): result converge at extremely high resolution. Fragmentation for $\beta < 20$

- Paardekooper (2012): non-convergence observed in 2D grid-based simulations. Fragmentation seen as a stochastic process
  - Essential to compute the likelihood of fragmentation in realistic protostellar discs (Hopkins and Christiansen 2013)

Work in progress by Young and Clarke
Convergence of numerical results

- What about convergence of results in the non-fragmenting limit?
- Is the stress and effective alpha converged?
  - At low resolution alpha appears to be overestimated (more power in large scale structures \(\rightarrow\) potentially more global transport)
Observing gravitational instabilities in Herbig Ae/Be stars

- Several discs with spiral structures observed in scattered light
- Most of these are relatively evolved systems (transitional discs): most likely the origin of the spiral is not due to self-gravity

Muto et al (2012)  
HD 135344B

Grady et al (2013)  
MWC 758
Observing gravitational instabilities in Herbig Ae/Be stars

- The case of HD 142527
- Christiaens et al (2014) estimate $Q \sim 2$, possibly marginally unstable

H-band: Fukagawa et al 2006

ALMA line emission: Christiaens 2014
Self-gravitating discs with ALMA

Dipierro, Lodato & Testi (in preparation)

• Is it possible to resolve the spiral structure from GI and derive some system parameters?
• Extend the work of Cossins, Lodato & Testi (2010)
• Consider some simulated discs with a variety of different parameters
  • Disc masses ($M_{\text{disc}}/M^* = 0.1, 0.25$)
  • Stellar mass ($M^* = 0.3, 1, 3$Msun)
  • Distance
  • Inclination
  • Size (Outer radius at either 25AU, or 100AU)
• Assume a “standard” opacity law (maximum grain size + 1cm)
• Build an “atlas” of mock ALMA images in the dust continuum
Self-gravitating discs with ALMA

Assume a 25AU disc in TW Hya, at 220GHz
Self-gravitating discs with ALMA

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A 100AU massive disc in Taurus or Orion
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Completely off-topic: warped discs


• Observations of warped disc can tell us a lot on disc internal physics (King et al. 2013)

• Interaction of a circumbinary disc with the binary can produce warps (and breaks for large misalignments)
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- Observations of warped disc can tell us a lot on disc internal physics (King et al. 2013)
- Interaction of a circumbinary disc with the binary can produce warps (and breaks for large misalignments) (Facchini & Lodato 2013, Lodato & Facchini 2013)
KH 15D: a peculiar binary system \citep{LodatoFacchini2013}

See Windemuth & Herbst (2013), arXiv: 1310.8126, for most recent observations.

Evolution of light curve explained by occultation due to a precessing narrow ring, confirmed by IR excess.
KH 15D: a peculiar binary system (Lodato and Facchini, 2013)

Condition: alignment timescale

\[ t_{\text{align}} > \frac{1}{\Omega_p} \]

We can set an upper limit for alpha!

cfr. King et al. (2013)
TW Hya

- $d = 54$ pc
- $R_{\text{hole}} = 4$ AU in submm
- Almost face-on, $i = 7^\circ$
- Kinematics from $^{12}\text{CO } J=2-1$ and 3-2 emission lines (Rosenfeld et al., 2012)
- Projected velocity too high in inner regions, $\Delta \beta \approx 4^\circ$
A planet in TW Hya?

\[ R_{\text{hole}} = R_{\text{Hill}} + a = \sqrt[3]{\frac{M_p}{3M_*}} a + a \]

3 unknown quantities: \( M_p, \alpha, \beta_\infty \)

\[ q=0.26, \ p=0.93 \quad \text{Facchini, Ricci & Lodato (2014, submitted)} \]

\[ q=0.53, \ p=0.93 \]
Conclusions

- Young protostellar discs are likely to be gravitationally unstable.
- Self-regulated evolution of GI leads to sustained angular momentum transport for ~ 1 Myr, bringing the disc into the T Tauri phase.
- Density waves dissipate when they become sonic.
- Induced transport is local IF disc is sufficiently thin.
- GI could lead to fragmentation: exact fragmentation conditions unfortunately strongly affected by numerical resolution.
- ALMA will be very important not only to detect, but also to characterize the gravitational instability in young discs.