Modelling Galaxies

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Prologue

• Work on surveys of the MW has led to significant advances in model Galaxies
• These advances can now be applied to external galaxies
• Made possible by the advent of techniques for evaluating action-angle coordinates in arbitrary galactic potentials (Binney 10, 12, Sanders 12, Sanders&B 14a, 14b)
• Crucially, we can now assemble dynamical models component by component (incl DM and BH components)
• For now only axisymmetric equilibrium models
• Extension to triaxial & time-dependent models lies ahead (but see S&B14b)
Outline

• Why equilibrium models
• Review of model types
• Models with specified $f(J)$
• Designer $f(J)$ for spheroids & DM
• $f(J)$ for discs
• Worked example: the MW
• Conclusions
Equilibrium models

• Essential because
  – We want distribution of mass \((M/L, \rho_{DM}(x), M_{BH},..)\) & hope to infer these from dynamics of stars
  – Without the assumption of dynamical equilibrium, any phase-space distribution is possible
  – Information limited [in external galaxies \((\alpha, \delta, v_{los})\)] and we hope to complement this by using constraints from dynamics
  – Best way to compensate for defects in data (seeing, spectral resolution, selection effects) is to “observe” a model

• E galaxies quickly settle to equilibrium
• Spiral structure in disc galaxies we plan to model by applying perturbation theory to an equilibrium model
• Use of p-theory brings physical understanding
Model types

• N-body
  – Final model determined by initial conditions through an obscure mapping

• Hence M2M (Syer & Tremaine 96, de Lorenzi + 08, Dehnen 09, Morganti + 13)
  – Adjust weights of particles dynamically to optimise fit to obs

• Schwarzschild modelling (Schwarzschild 79)
  – Build a library of orbits and then choose weights to optimise fit to obs

• Problems with all of above:
  – Discreteness noise (eg McMillan & B 13)
  – Hard to characterise model (non-uniqueness of description)
  – Computational cost
Models with specified DF

• Models with $f(E,L)$ (Michie 63, King 66) fundamental to studies of GCs
  – Can quickly recover $\rho(r)$, $\sigma_r(r)$, etc from $f(E,L)$

• Most galaxies are non-spherical
  – Models with $f(E,L_z)$ studied since Prendergast & Tomer 70
  – Not easy to recover $\rho(R,z)$ from $f(E,L_z)$
  – Relation of observables to $f(E,L_z)$ obscure
  – Models require $\sigma_R = \sigma_z$

• Models from Jeans eqs (Satoh 80, Binney+90) also restricted to $\sigma_R = \sigma_z$
Models with specified $f(J)$

- Most orbits in axisymmetric $\Phi$ are quasiperiodic $\Rightarrow$ admit three integrals of motion (Arnold 78)
  - Integrals best chosen to be actions $J_i$
- In spherical case
  - $J_r$ quantifies radial oscillations
  - $J_\phi = L_z$
  - $J_\theta = L - L_z$ quantifies motion $\perp z=0$
- These choices generalise uniquely to axisymmetric case: various notations for unique objects
  - $J_u = J_r$
  - $J_v = J_\theta = J_z$ quantifies motion $\perp$ equatorial plane
- $J = 0 \Rightarrow$ star at rest at centre
  - $J \to \infty \Rightarrow$ star becomes unbound
  - Any finite $J$ with $J_r > 0, J_z > 0$ corresponds to a bound orbit
Models with specified $f(J)$

- Any non-negative $f(J)$ with $\int d^3J \, f(J) < \infty$ defines a legitimate galaxy model
- $M = (2\pi)^3 \int d^3J \, f(J)$ is the mass
- In any $\Phi(R,z)$ we can evaluate $\rho(R,z), \sigma_R(R,z), \ldots$
- Given 2 DFs $f_1(J), f_2(J)$ with $M_i = (2\pi)^2 \int d^3J \, f_i(J)$ can build a model with 2 stellar populations cohabiting the same $\Phi$
- So we seek $f_i(J)$ for $i = 1,\ldots,N$ stellar populations and $f_{DM}(J)$ for DM
- The self-consistent $\Phi(R,z)$ is easily found by rapidly convergent iteration (Binney14)
Designing f(J)

• f(J) is density of stars in 3d action space \((d^3x \, d^3v = d^3\theta \, d^3J)\), so form of f(J) is readily pictured

• Start in spherical limit when Eddington gives us f(H) that generates given \(\rho(r)\) in given \(\Phi(r)\)
  – In 1 special case (Henon’s isochrone) we have H(J) so can immediate convert to f[H(J)]
    • This case explored by Binney 14
  – Generally don’t know H(J) but good approximations not hard to find (Fermani 13, Williams+14)
  – When \(\Phi(\xi x) = \xi^a(x)\) orbits can be rescaled \((x,v) \rightarrow (x',v')\) with \(x' = \xi x, \, v' = \xi^{a/2}v\) by the virial thm
    • It follows that \(J \sim xv\) scales \(J \rightarrow J' = \xi^{1+a/2}J\)
    • Hence \(H(J) = [h(J)]^{a/(1+a/2)}\) where \(h(\xi J) = \xi h(J)\) is homogeneous of degree one
      • If self-consistent, we require \(\rho \sim r^{a-2}\) and then \(f(J) \sim [h(J)]^{-(4+a)/(2+a)}\)
  – Introduce core by writing \(f(J) \sim [J_0 + h(J)]^{-(4+a)/(2+a)}\)
Example isochrone

- Has Kepler $\Phi$ as $r \to \infty$ and Eddington $f(H) \sim H^{-5/2}$
- Suggests $f(J) = A[J_0 + h(J)]^{-5}$
- We choose $h(J) = J_r + (\Omega_\phi/\Omega_r)J_\phi + (\Omega_z/\Omega_r)J_z$ which would be homogeneous if frequency ratios were invariant under $J \to \zeta J \sim \text{energy-independent}$
- Even with $h(J)$ inhomogeneous, we have a valid DF
Designing $f(J) – 2$ power models

- Many popular models (Jaffe, Hernquist, NFW,..) are 2-power models
- Apply reasoning above to regions $r \to 0 \ (\alpha=1, \beta=-1)$ and $r \to \infty$
- In each region $\Phi(r) \sim r^\alpha \rho(r) \sim r^\beta$ implies by Eddington $f \sim E^{(\beta/\alpha - 3/2)}$ and therefore
- $f(J) \sim [h(J)]^{-(3\alpha - 2\beta)/(\alpha + 2)}$
- Example: Hernquist model

$$f(J) = \frac{A}{[h(J)]^{5/3} [J_0 + h(J)]^{5-5/3}}$$
\[ f(J) = \frac{A}{[h(J)]^{5/3} [J_0 + h(J)]^{3-5/3}} - F_M \text{ truncation} \]
Flattening a DM halo with a disc
Velocity anisotropy

- Isotropic model has \( f[H(J)] \) so \( \frac{\partial f}{\partial J_r} = \frac{\partial H}{\partial J_r} = \frac{\Omega_r}{\Omega_\phi} \)

- Using \( h(J) = J_r + \frac{\Omega_\phi}{\Omega_r} J_\phi + \frac{\Omega_z}{\Omega_r} J_z \) ensures near isotropy because \( \Omega_i/\Omega_j \sim \text{const} \)

- \( \Omega_i/\Omega_j \) depends mostly on \( E \), i.e. \( |J| \)

- Anisotropic mode has \( f_{\alpha}(J) \equiv (\alpha_r \alpha_\phi \alpha_z) f_1(\alpha_r J_r, \alpha_\phi J_\phi, \alpha_z J_z) \) with \( \alpha_i \neq 1 \)

- If \( \alpha_z > \alpha_\phi \) the model becomes oblate

Isochrone model with \( \alpha_\phi = 1, \alpha_z = 1.5 \) (Binney 2014)
Rotation

- So far models non-rotating; flattened by velocity anisotropy
- Generate rotation by adding to DF part odd in $J_\phi$
  \[ f(J) = (1 - k)f_+(J) + kf_-(J) \]
- A natural choice is
  \[ f_-(J) = g(J_\phi)f_+(J) \]
- We adopt
  \[ g(J_\phi) = \tanh\left( \frac{\chi J_\phi}{\sqrt{GMb}} \right) \]

Projected density & mean velocity of maximally rotating ($k=0.5$) isochrone $\alpha_\phi=0.7$, $\alpha_z=1.4$ (Binney 2014)
Maximal rotation (k=0.5)

- Emsellem+ 07 defined

\[ \lambda_R = \frac{\langle R\nu \rangle}{\langle R\sqrt{\sigma^2 + \overline{v}^2} \rangle} \]

Graph showing:
- Tangential bias
  - \( i = 15, 30, 45, \ldots \)
- Radial bias
Galactic discs

- Built up from “quasi isothermal” components (B10 B12)

\[
f_{\sigma_0, \sigma_z}(L_z, J_r, J_z) = \frac{\Omega \Sigma}{\pi \sigma_r^2 \kappa} \left[ 1 + \tanh(L_z/L_0) \right] e^{-\kappa J_r / \sigma_r^2} \times \frac{\nu}{2\pi \sigma_z^2} e^{-\nu J_z / \sigma_z^2}.
\]

\[
\Sigma(L_z) = \Sigma_0 e^{-R_c/R_d}
\]

\[
\sigma_i(R_c) = \sigma_0 e^{(R_0 - R_c)/R_\sigma}
\]

Quasi isothermal $\sigma_{r0}=40, \sigma_{z0}=20$ km/s (Binney 12)
Model from Geneva-Copenhagen survey (Binney 12, Binney+14)

- Thick disc a single quasi-isothermal
- Each cohort of coeval stars age $\tau$ a quasi-isothermal

$$\sigma_{0i}(\tau) = \left( \frac{\tau + \tau_1}{\tau + \tau_T} \right)^{\beta_i} \sigma_{0i}.$$ 

- Exponentially decaying SFR
- Fit model to GCS
- Predict velocity distributions for RAVE

![Diagram showing comparisons between data and prediction for cool dwarfs with T<6000K.](image)
Secular evolution

- Equilibrium models just the start
- Given $f(J)$ one can solve for evolution of population as stars diffuse through action space
- Diffusion coefficients $\langle J_i^2 \rangle$ computed from 1st-order perturbation theory based on fluctuations in $\Phi$
- If you have $(J, \theta)$, you can also use method of “perturbation particles”
- N-body in which particles used only to represent fluctuations
Conclusions

- Equilibrium dynamical models are key tools
- Advent of techniques for computing $J(x,v)$ in general potentials gives new possibilities for galaxy modelling
- Start from $f(J)$ for each population
- Population defined by age, metallicity, place of birth,...
- $f$ describes star density in 3d action space
- Transparent relation between $f(J)$ and structure of component
- Simple functional forms for $f(J)$ yield models similar to familiar spheroids and exponential discs
- Can inspect observables for component in any $\Phi$
- Can readily find $\Phi$ self-consistently generated by a sum of components (incl DM component)