• Output of an interferometer after calibration:

\[ V(u, v, w) = \int \int \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1 - l^2 - m^2} - 1))} \, dl \, dm \]

• \((u,v,w)\) : interferometer's geometrical vector
• \((l,m)\) : position on the sky
• \(I\) : sky brightness ("image")

Imaging : Calculating \(I(l,m)\) from \(V(u,v,w)\)
- Full visibility function:

\[
V(u, v, w) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1-l^2-m^2}-1))} \, dl \, dm
\]

- For small field of view \((l \sim 0, m \sim 0)\) or \(w \sim 0\):

\[
V(u, v, w) \approx \iint I(l, m) e^{-2\pi i (ul + vm)} \, dl \, dm
\]

- \((u, v, w)\) : interferometer's geometrical vector
- \((l, m)\) : position on the sky
- \(I\) : sky brightness ("image")
LOFAR dirty image (3c196)

The dirty image
• Högbom CLEAN algorithm (1974):
  – Find largest peak in image
  – Scale PSF to fraction of peak and subtract
  – Repeat until peak < threshold or nIter > limit
  – Finally: restore subtracted components
Högbom CLEAN

LOFAR undeconvolved ("dirty") image

Deconvolved with Högbom CLEAN
Deconvolving diffuse structures

Undeconvolved “dirty” image  Deconvolved image with Högbom CLEAN
Deconvolving diffuse structures

Deconvolved image (Högbom CLEAN)    Actual model
Improved algorithm by Cornwell (2008):

- “Multi-scale clean”
- Fits small smooth kernels (and delta functions) during a Högbom CLEAN iteration
Multi-scale CLEAN

Normal Högbom CLEAN

Multi-scale CLEAN
(implementation in WSClean)
Normal Högbom CLEAN Output model

Multi-scale CLEAN (as implementation in WSClean)

Multi-scale CLEAN
• 2D FT does not hold for new arrays: $l,m,w >> 0$

Correcting w-terms

Without correcting w-terms

The w-term
• 2D FT relationship does not hold for new arrays: $l,m,w >> 0$

• Have to use full function:

$$V(u, v, w) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1 - l^2 - m^2} - 1))} dldm$$

• Easy solution: facetting
  - But: slow, stitching artefacts

• Better & most used solution: 'w-projection'
\[ V(u, v, w) = \int \int \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1 - l^2 - m^2} - 1))} \, dldm \]

- **Visibility function:**

- **W-projection:** (Cornwell et al, 2008)

\[ V(u, v, w) \ast \mathcal{F}(e^{-2\pi i w(\sqrt{1 - l^2 - m^2} - 1)}) = \int \int \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm)} \, dldm \]

  This convolution turns out to have a “limited” support

- **Performance very dependent on zenith angle, coplanarity of array, field of view and resolution.**

**w-projection**
Another problem; convolution theorem no longer works when w-terms present in

\[ V(u, v, w) = \int \int \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1-l^2-m^2}-1))} \, dl \, dm \]

- Högbom CLEAN assumes constant PSF
- But PSF changes (slightly) over the image
- Solved with Cotton-Schwab algorithm (Schwab 1984)
- Normal CASA imaging mode will automatically use CS

w-projection
Another problem; convolution theorem no longer works when w-terms present in

\[ V(u, v, w) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1 - l^2 - m^2} - 1))} \, dl \, dm \]

Högboom CLEAN assumes constant PSF

But PSF changes (slightly) over the image

Solved with Cotton-Schwab algorithm (schwab 1984)

Normal CASA imaging mode will automatically use CS
(i.e., Cotton-Schwab, not Compressed Sensing)

w-projection
The Cotton-Schwab + w-projection algorithm:

- Make initial dirty image & central PSF
- Perform minor iterations:
  - Find peak
  - Subtract scaled PSF at peak with small gain
  - Repeat until highest peak ~ 80-90% decreased
- Major iteration: “Correct” residual
  - Predict visibility for current model
  - Subtract predicted contribution and re-image
• W-projection is the standard way to solve w-terms in radio astronomy

• W-term convolution can be slow...
  – Imaging 2 minutes of data of the MWA telescope (30 degree FOV) costs hours

• New imager with new algorithm implemented: WSClean\(^1\) (“w-stacking clean”).
  – Offringa et al, 2014

\(^1\)see [http://wsclean.sourceforge.net/](http://wsclean.sourceforge.net/)
w-projection:

Visibilities

w-stacking:

uv plane

Convolve w-projection kernel

image plane

Multiply w-projection term

...
Multi-frequency synthesis (MFS) means gridding different frequencies on the same uv grid:

- This is the standard for modern telescopes
Related, but not the same:

- **Multi-frequency deconvolution** *(see Rau and Cornwell, 2011)*
  sometimes called **multi-term deconvolution**
  
  *Selected by setting nterms in CASA's clean task*

- Takes spectral variation into account during deconvolution

- Useful for wideband, sensitive imaging
Frequency-dependent deconvolution

- Right image: fit for flux over frequency to improve deconvolution (Sault & Wieringa, 1994)
Recent focus on deconvolution using 'compressed sensing' (abbrev. CS – but CS can mean “Cotton-Schwab” too)

CS methods assume the sky is 'sparse' (“solution matrix is sparse in some basis”)

Minimizes “L1-norm” (= abs sum of CLEAN components)

Högbom clean is actually (almost) a compressed sensing method called “Matching Pursuit”

CS considers MP to be non-ideal… but radio data is not the perfect CS case: Calibration errors, w-terms
Model created by Höggbom clean
Model created by a CS method (“non-linear conjugate gradient using IUWT”)
Model created by multi-scale clean
• Compressed sensing does not work well with calibration artefacts
• Multi-scale is more robust
• On well-calibrated data:
  – CS gives more accurate model
  – But residuals don't improve much
• Clean components can be used as calibration model
• Often applied as:
  
  Phase cal
  →
  Shallow clean
  →
  Phase cal
  →
  Deep clean
  →
  Phase & amplitude cal
  →
  Deep clean

Self-cal & CLEAN
• Clean components can be used as calibration model
• Often applied as:

  Phase cal
  ▼
  Shallow clean
  ▼
  Phase cal
  ▼
  Why only phase?
  ▼
  Deep clean
  ▼
  Phase & amplitude cal
  ▼
  Deep clean

Self-cal & CLEAN
• Clean components can be used as calibration model

• Often applied as:
  
  Phase cal
  ↓
  Shallow clean
  ↓
  Phase cal
  ↓
  Deep clean
  ↓
  Phase & amplitude cal
  ↓
  Deep clean

Why only phase?
→ Avoid clean bias
Self-calibration using CLEAN

After initial calibration

After self-cal on clean components

Image credit: N. Hurley walker (using the MWA)
· Result of imaging – is this how the sky looks like? (and I don't mean the orange colour)
Result of imaging – is this how the sky looks like? (and I don't mean the orange colour)
• Correction is required for the antenna response
• This is called “primary beam” correction (as opposed to the synthesized beam / psf)

• For dishes, the primary beam is ~constant
• To correct for: multiply final image with the inverse beam
• **Scalar** for total brightness, **matrix** for polarized
What if...

This is our field of interest →

(In practice, actual galaxies look different)

... and

this is our primary beam →

Mosaicing
What if…

This is our field of interest →

(In practice, actual galaxies look different)
What if…

This is our field of interest

- This is called mosaicing
- Should we average the 3 primary-beam-corrected images together?
This is called mosaicing

Should we average the 3 primary-beam-corrected images together?

No → Weight with $1/\sigma^2 = (\text{primary beam})^2$
• Primary beam of tiled arrays varies in time, per station
• Primary beam of tiled arrays varies in time
  – Or even per station
• Has to be accounted for during cleaning
• Algorithm to do this is “aw-projection”
  – similar to w-projection
  – Specialized software package for LOFAR ("AWImager")
• Homogeneous arrays can also use snapshot imaging

Variable primary beam
• **Direction-dependent effects** might require further correction during imaging:

![Diagram showing star positions and arrows indicating movement]

• Positions of 'calibrators' (red) are known
• Apparent position has moved due to ionosphere

More variable effects...
• **Direction-dependent effects** might be time-variable (e.g. ionosphere)

• Besides position, **DD effects** can also affect polarization angle and brightness

• Not a fully solved problem, but possible solutions:
  - image in small “facets” where DDE's are constant
  - or interpolation – AWImager can do this.
  - Peeling

**Direction-dependent effects**
Discussed topics:

- CLEAN
- When to use Multi-scale or other deconvolution methods
- The effect of and solution to w-terms
- Multi-term deconvolution
- Self-cal using CLEAN components
- Primary beam correction
- Mosaicing
- Direction-dependent effects during imaging
Thank you for your attention!