The impact of binaries on the determination of the stellar IMF

ESO Garching workshop:
The impact of binaries on stellar evolution

3-7 July 2017

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Charles University in Prague

Issues which affect the determination of the IMF :

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The estimation of this "hilfskonstrukt" is compromised by

the stellar mass-to-light relation
dynamical evolution of birth clusters (very early phase and long-term)
binaries
the most massive star in birth clusters
IMF = probability density distr.function
or an optimally sampled distribution function ?
evidence for top-heavy IMF in extremely massive star burst "clusters"
implications of this for the IMF of whole galaxies.

stellar IMF ≠ IMF of stars in a galaxy

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Without binaries,
slowly evolving late-type
main-sequence stars

The observed stellar luminosity function
and
its relation to the IMF

The distribution of stars

We have $dN = \Psi(M_{\rm V})\, dM_{\rm V}$ = # of stars with $M_{\rm V} \in [M_{\rm V}, M_{\rm V} + dM_{\rm V}]$

$dN = \xi(m)\, dm$ = # of stars with $m \in [m, m + dm]$

since

$$\frac{dN}{dM_{\rm V}} = -\frac{dm}{dM_{\rm V}}\,\frac{dN}{dm}$$

follows

$$\boxed{\Psi(M_{\rm V}) = -\frac{dm}{dM_{\rm V}}\,\xi(m)}$$

the **observable** the **target**

the **obstacle**

Understand ***detailed shape*** of LF from fundamental principles :

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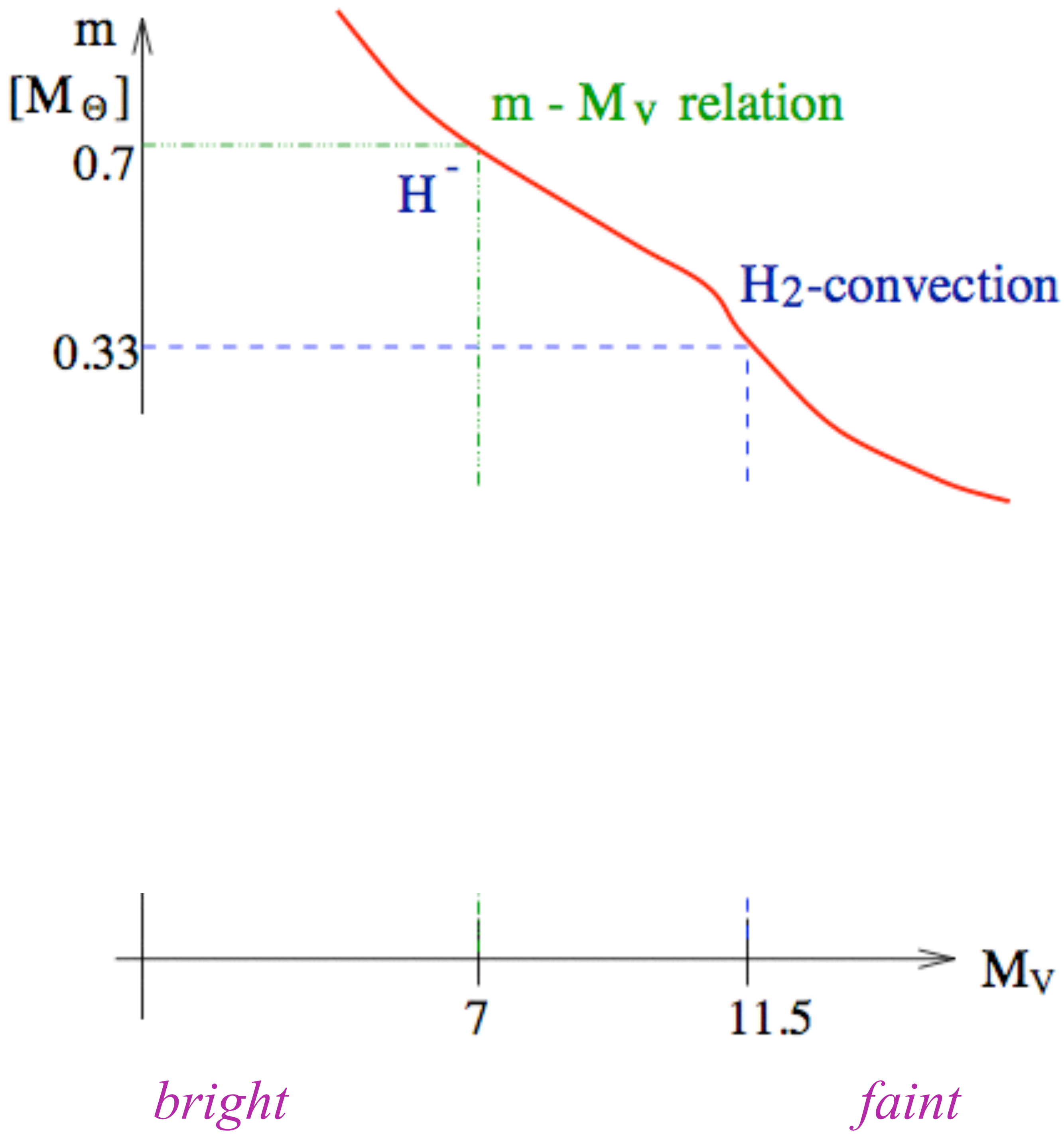

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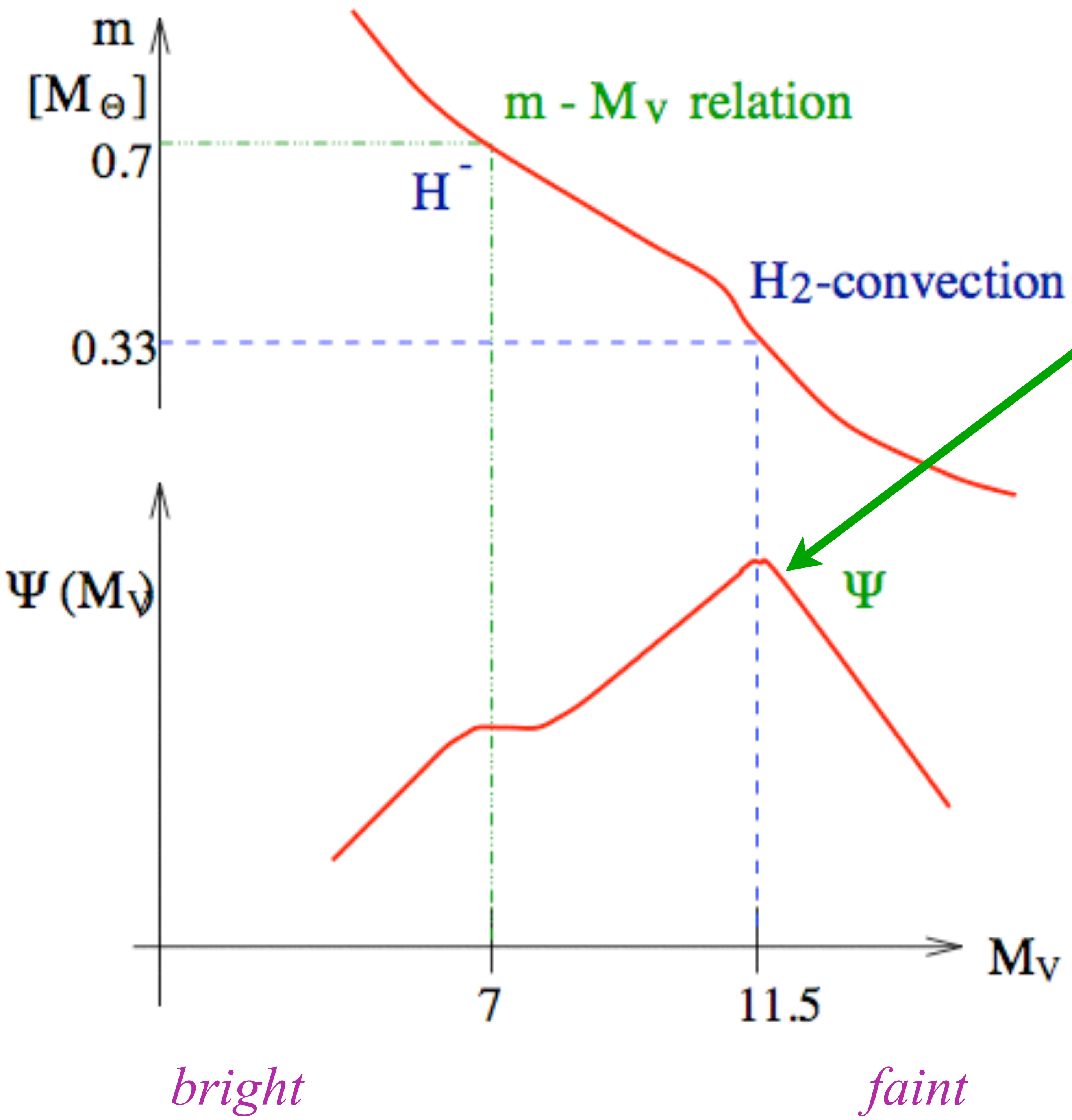
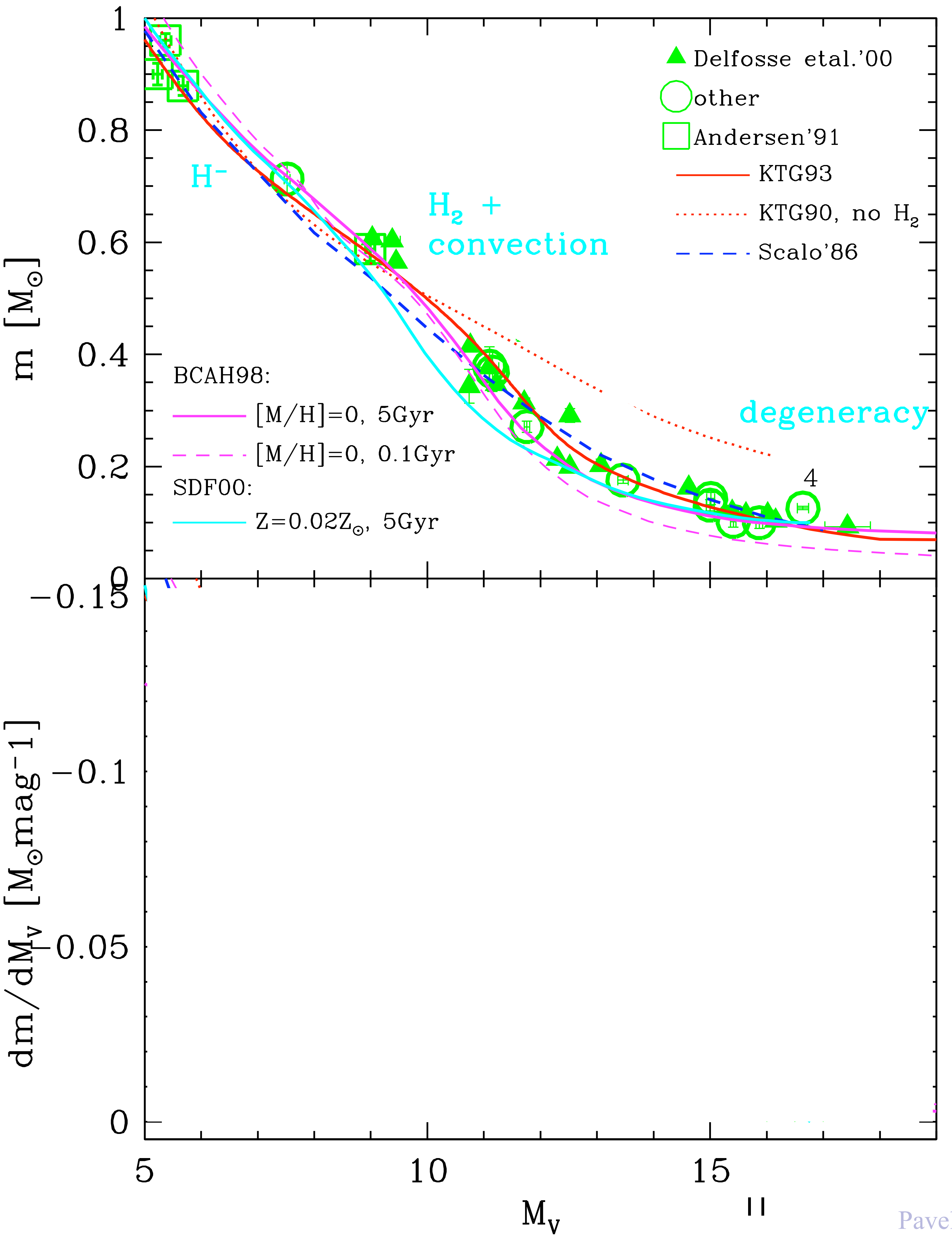


Kroupa, Tout & Gilmore 1991

The mass-luminosity relation of low-mass stars



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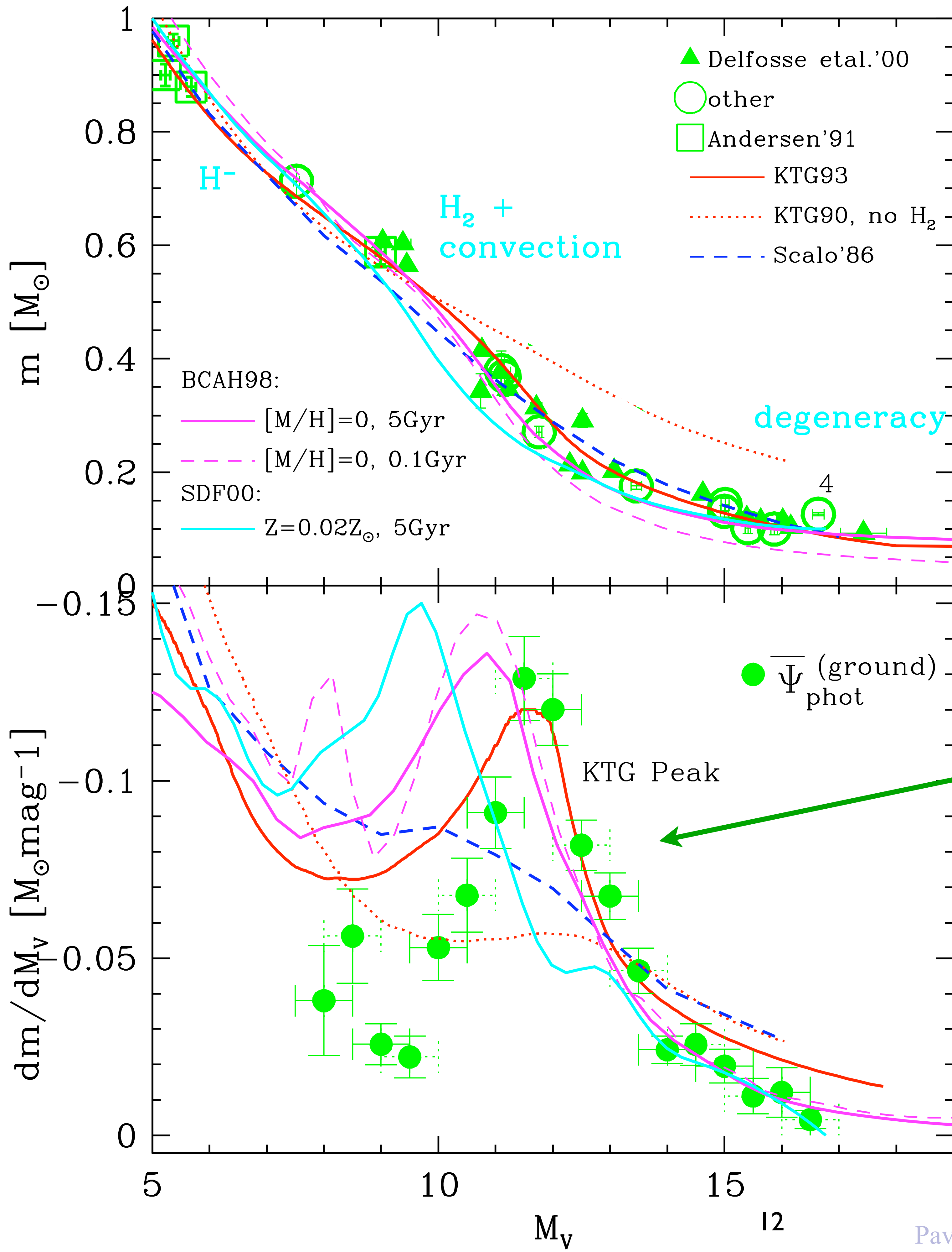

$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$

1. position
2. width
3. amplitude

all agree !

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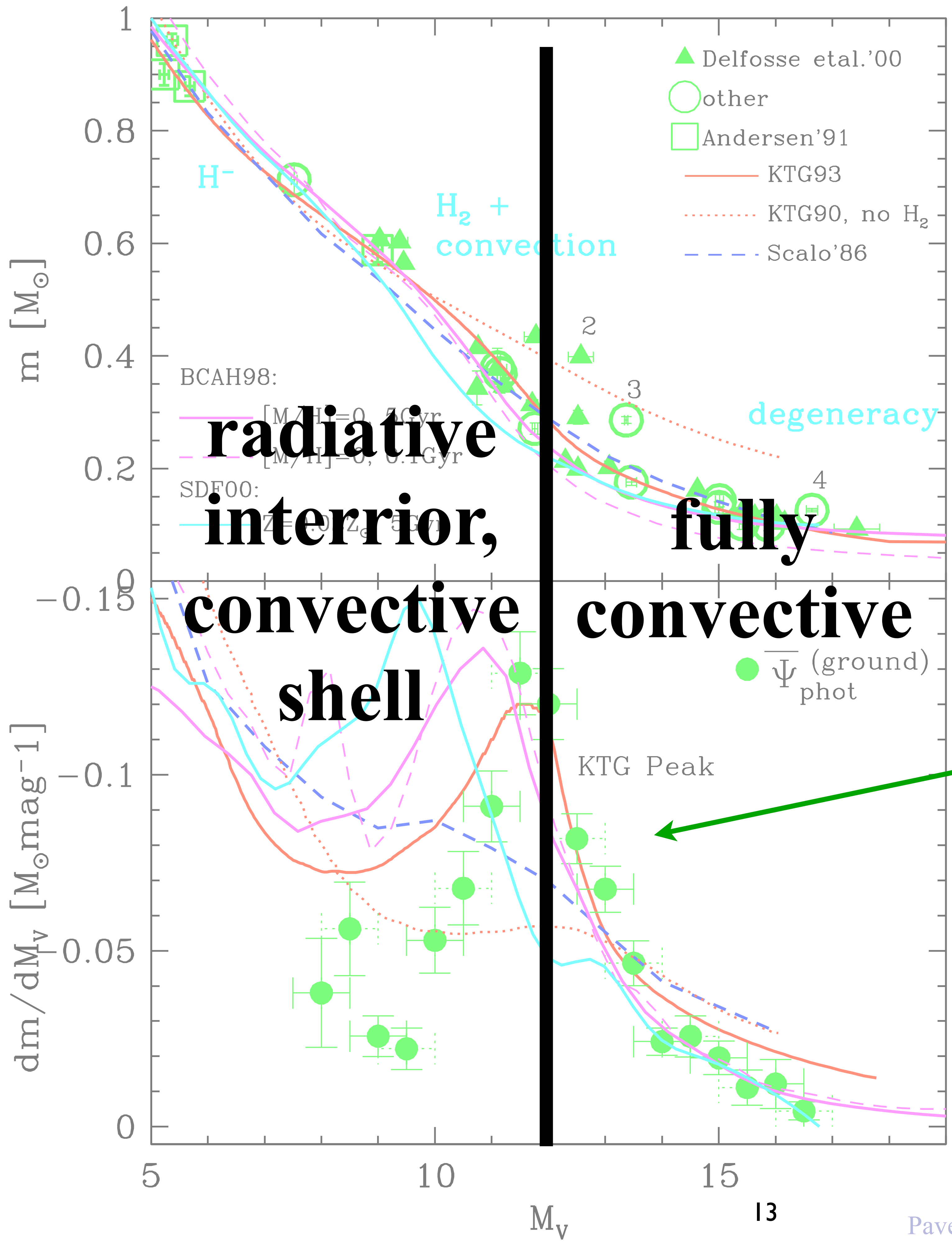

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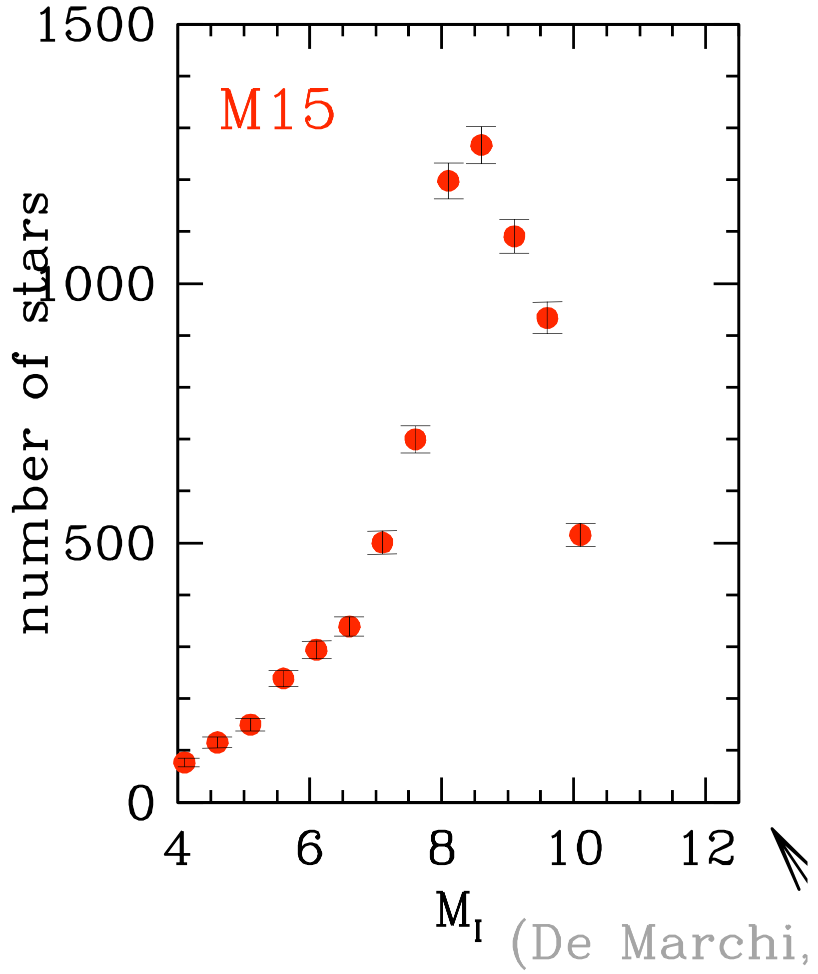

By counting stars we look into their internal constitution !

Peak in LF is not due to MF turnover,

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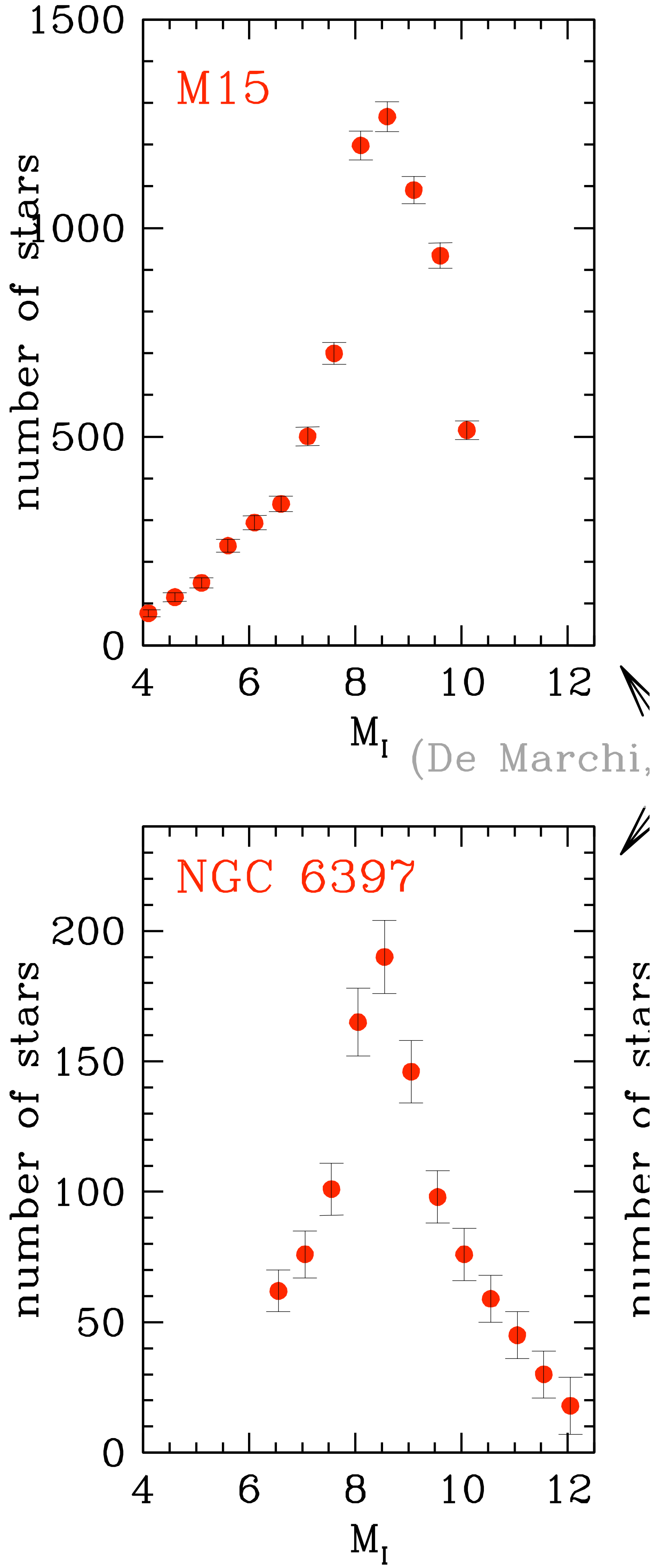

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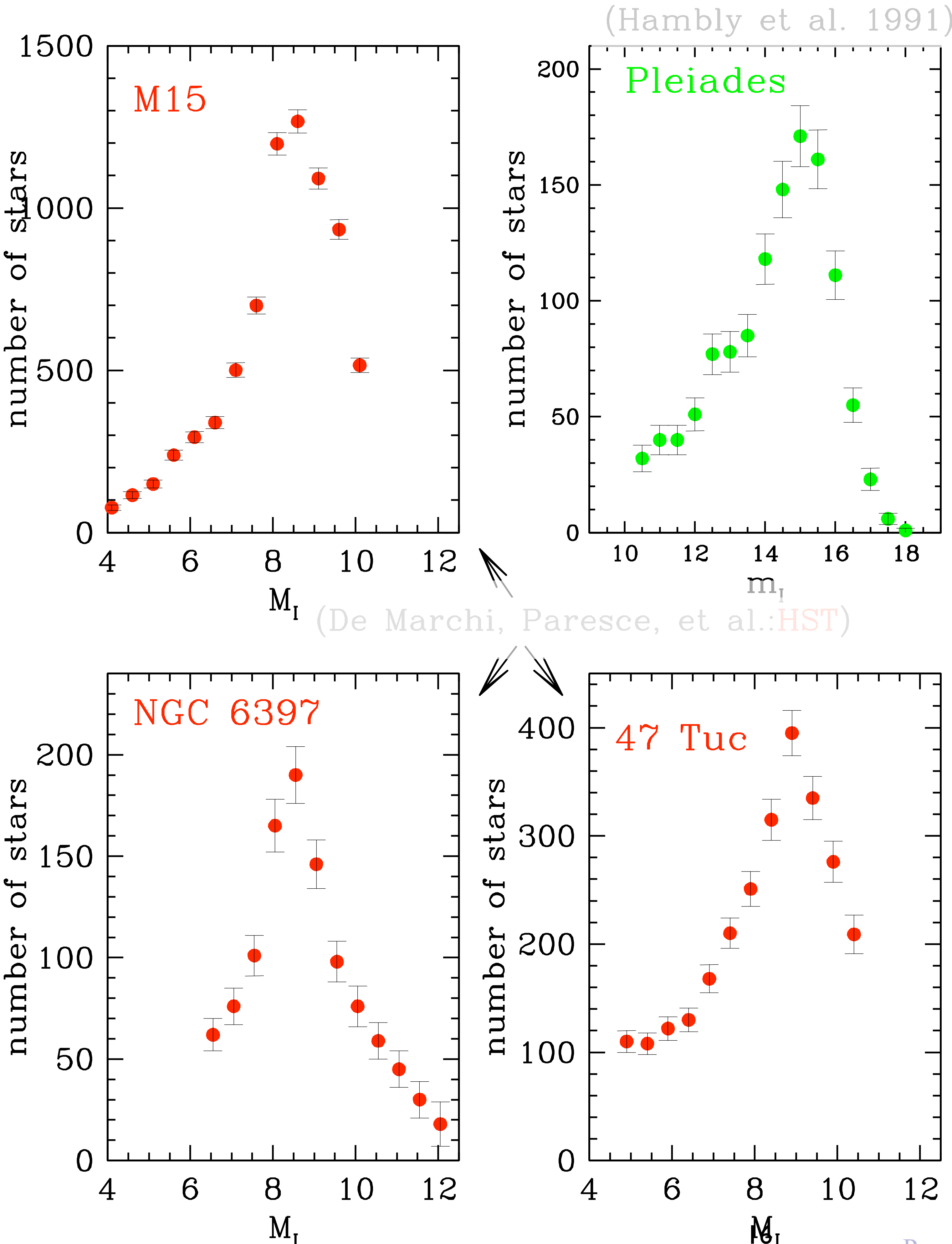

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stellar IMF ≠ IMF of stars in a galaxy

Now take into account binaries,
first for late-type stars

The observed stellar population
in clusters and field
and
its relation to the IMF

What do we know about binary-star distribution functions?

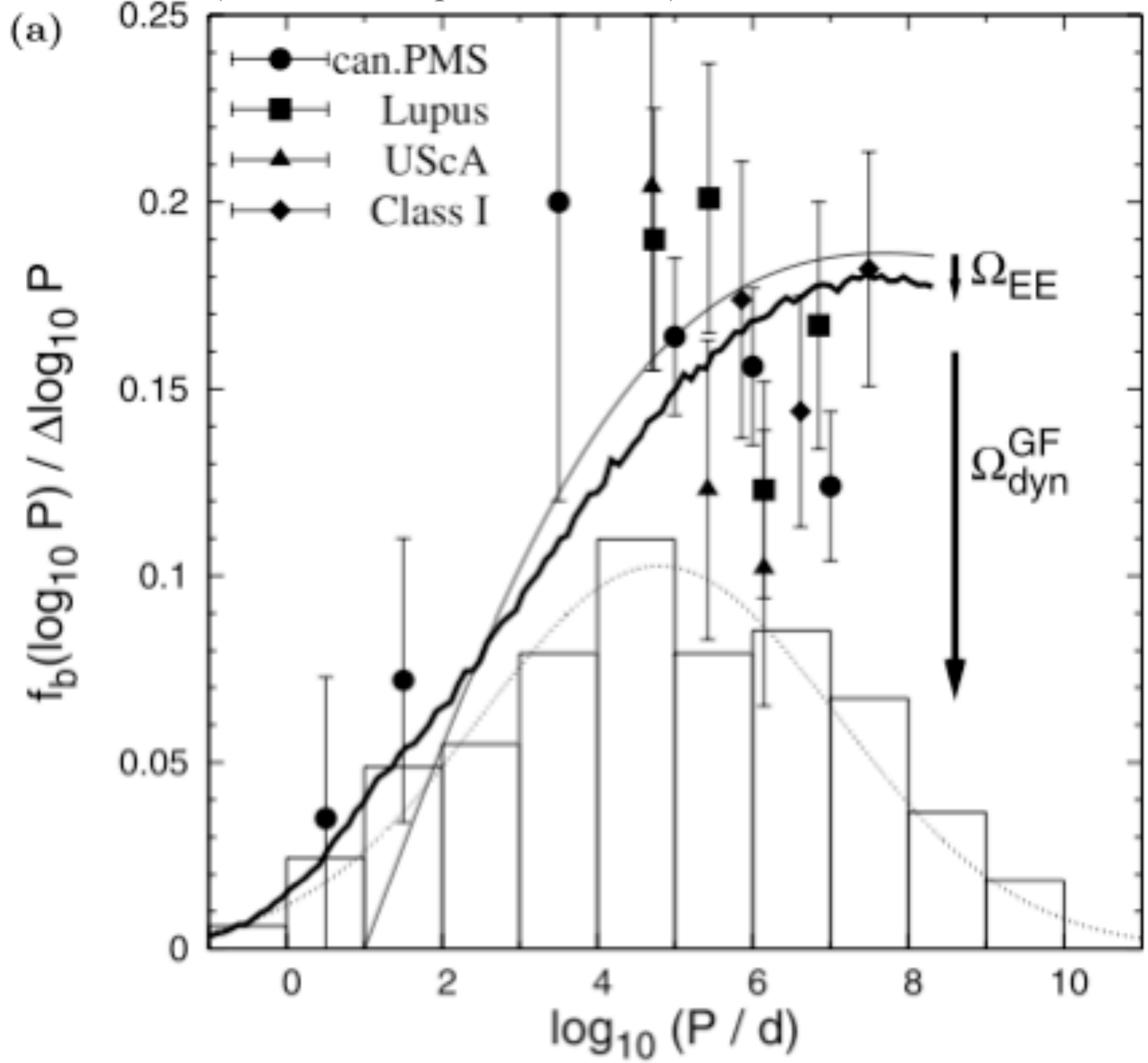
Collapsing pre-stellar molecular cloud core (< 0.1 pc)
==> binary due to angular momentum conservation

(typically not triples, quadruples -- **why ?**)

---> Goodwin & Kroupa (2005, A&A)

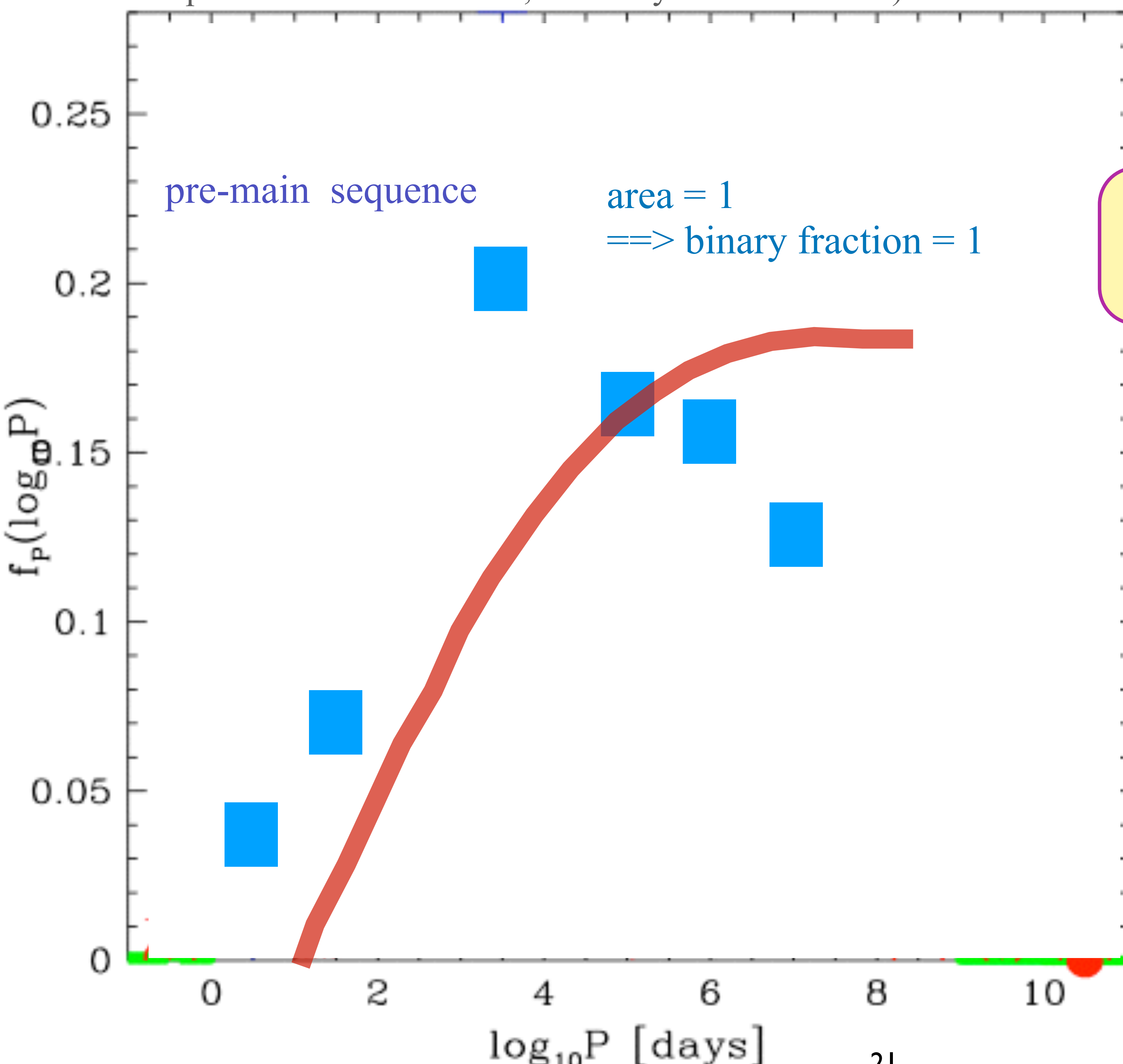
Indeed, the vast majority of
< 5 Msun stars
are observed to form as binary systems **:**

(Marks, Kroupa & Oh 2011)



The birth period / energy -distribution function
for low-density (isolated) <5Msun star formation

Kroupa & Petr-Gotzens 2011; Sadavoy & Stahler 2017)



canonical birth binary population:

$$f_{P,\mathrm{birth}} = 2.5\,\frac{lP-1}{45+(lP-1)^2}$$

with $lP_{\max} = 8.43$

with subsequent pre-main sequence adjustments (*eigenevolution*) for $P < 10^3\,\mathrm{d}$ systems

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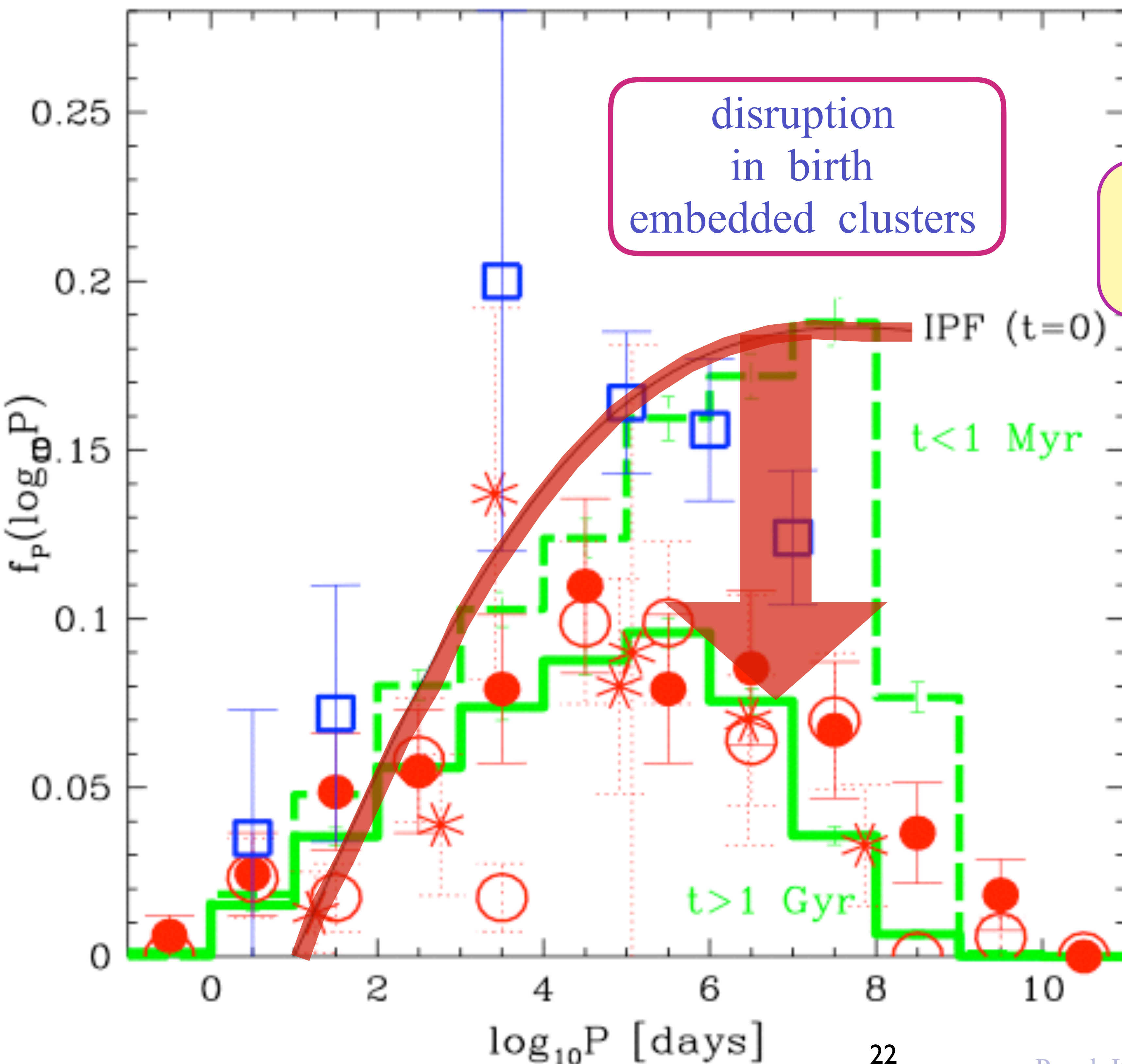
Kroupa 2011, IAU



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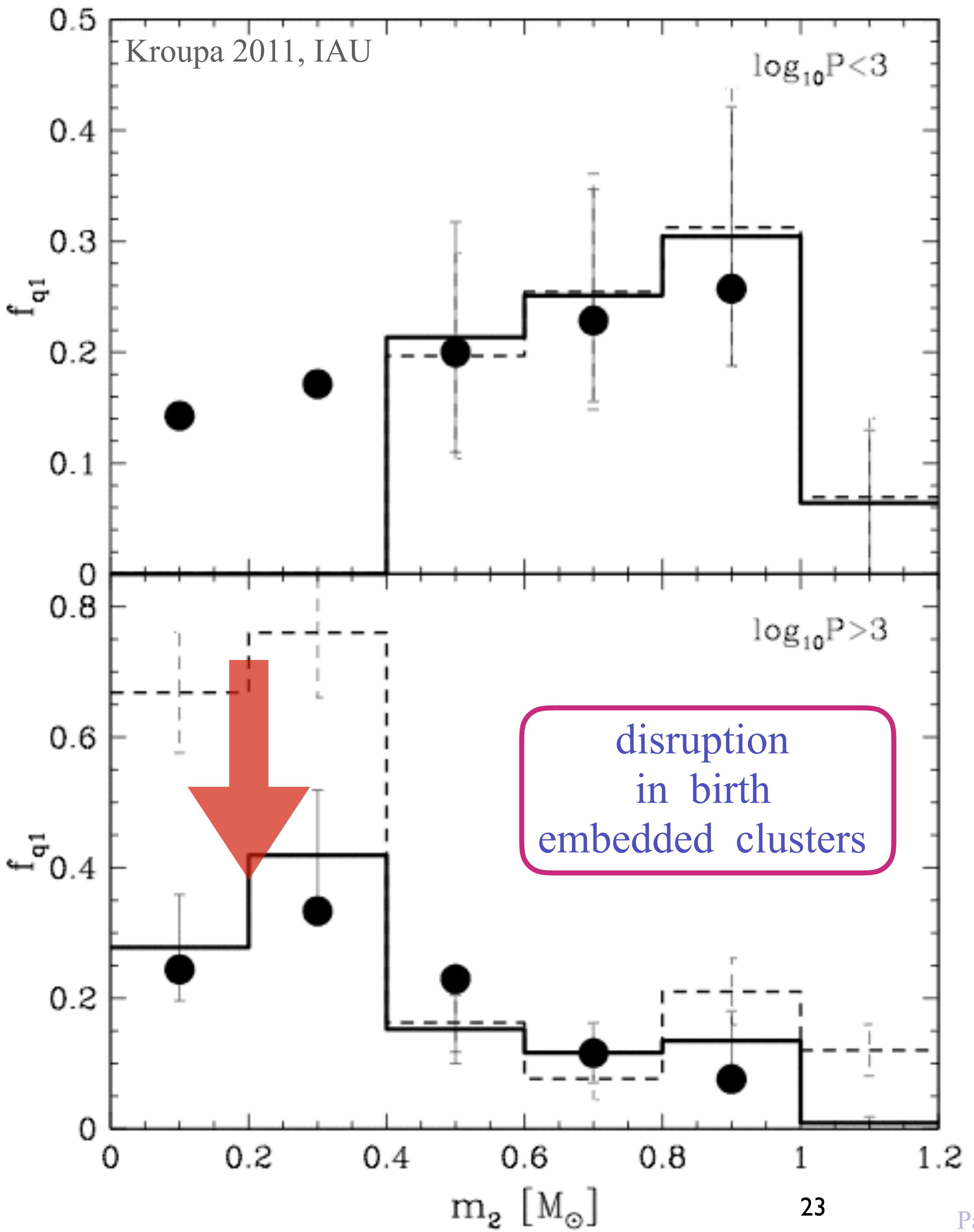


The birth mass-ratio distribution for G-dwarfs

***canonical birth binary population*:**

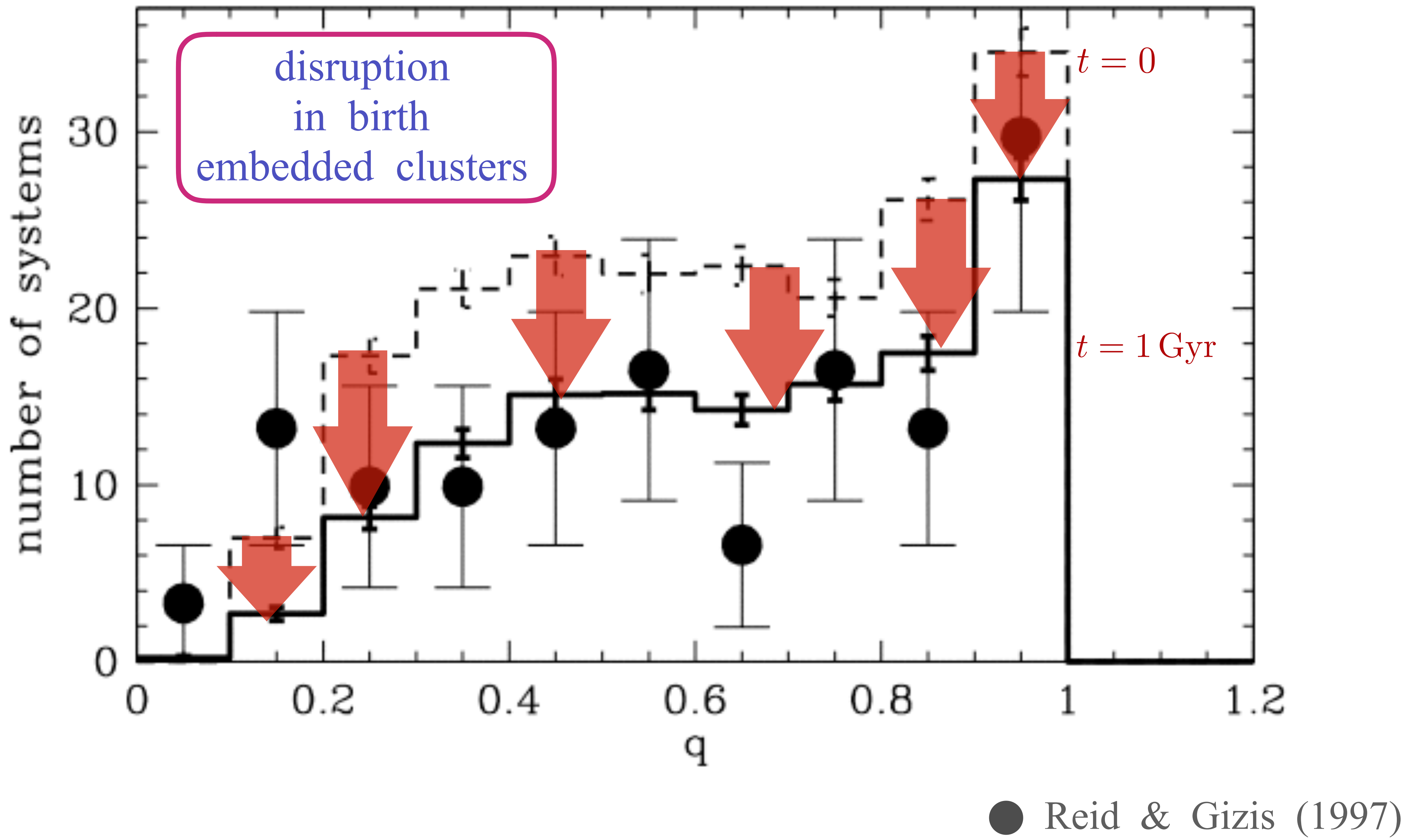
random pairing !

with subsequent pre-main sequence adjustments (*eigenevolution*) for $P < 10^3\,\mathrm{d}$ systems

The initial / final mass-ratio distribution for late-type dwarfs

random pairing !

Kroupa 2011, IAU



From Nbody computations using Aarseth codes we understand the distribution functions of < 5 Msun binaries well

The binary population decreases on a crossing time-scale; disruption and hardening depend on cluster density.

---> major influence on deducing the shape of the stellar mass function (and thus the IMF).

Kroupa 1995 a,b,c; Marks et al. 2011

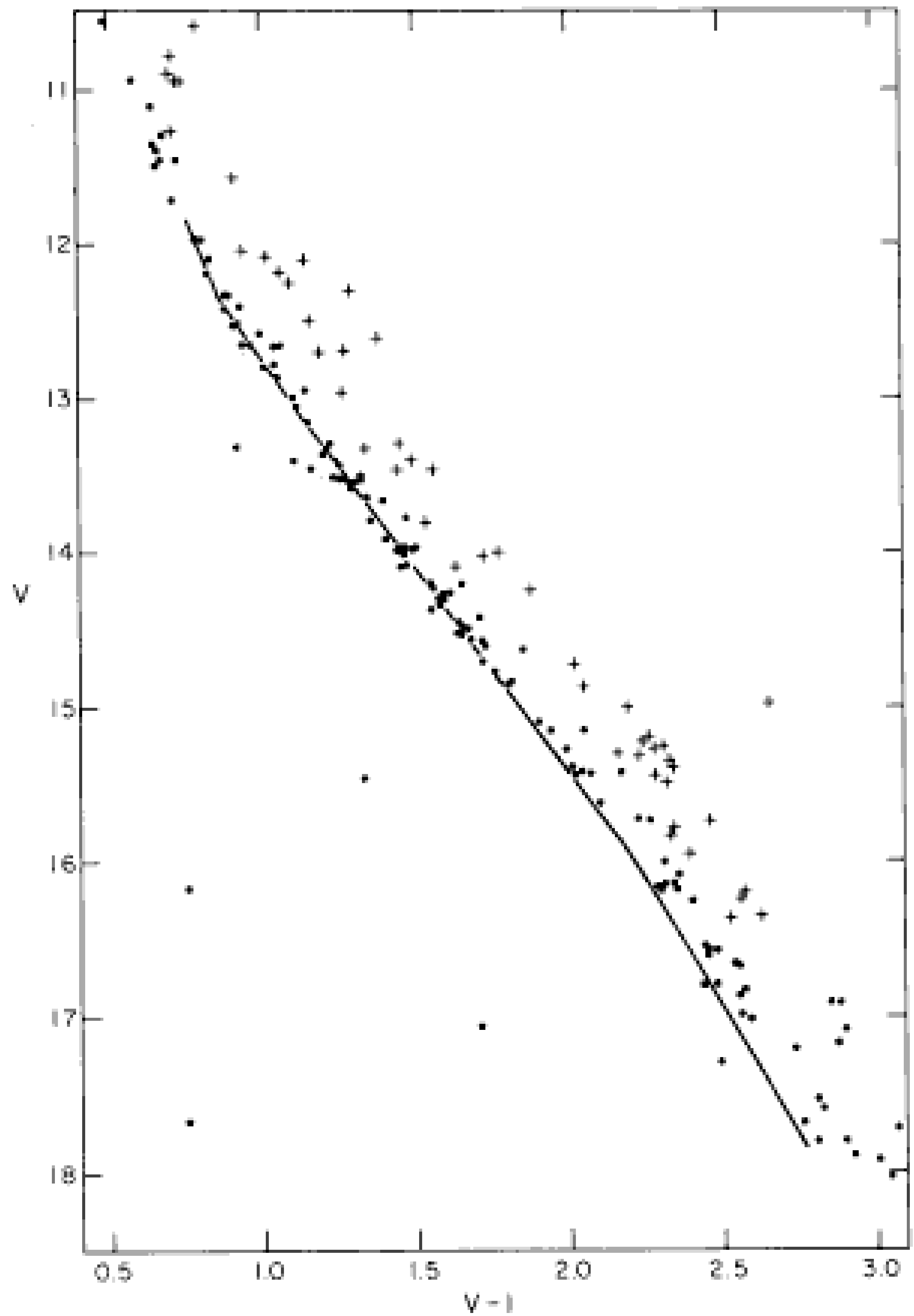


FIG. 7.—Same as Fig. 1, except that the main-sequence line has been shifted downward by 0.25 mag. The larger symbols in this diagram denote stars identified as photometric binaries.

Observed binaries in clusters

Pleiades (Stauffer 1984; Kaehler 1999)

$d = 126\text{pc}$, age $= 100\text{Myr}$

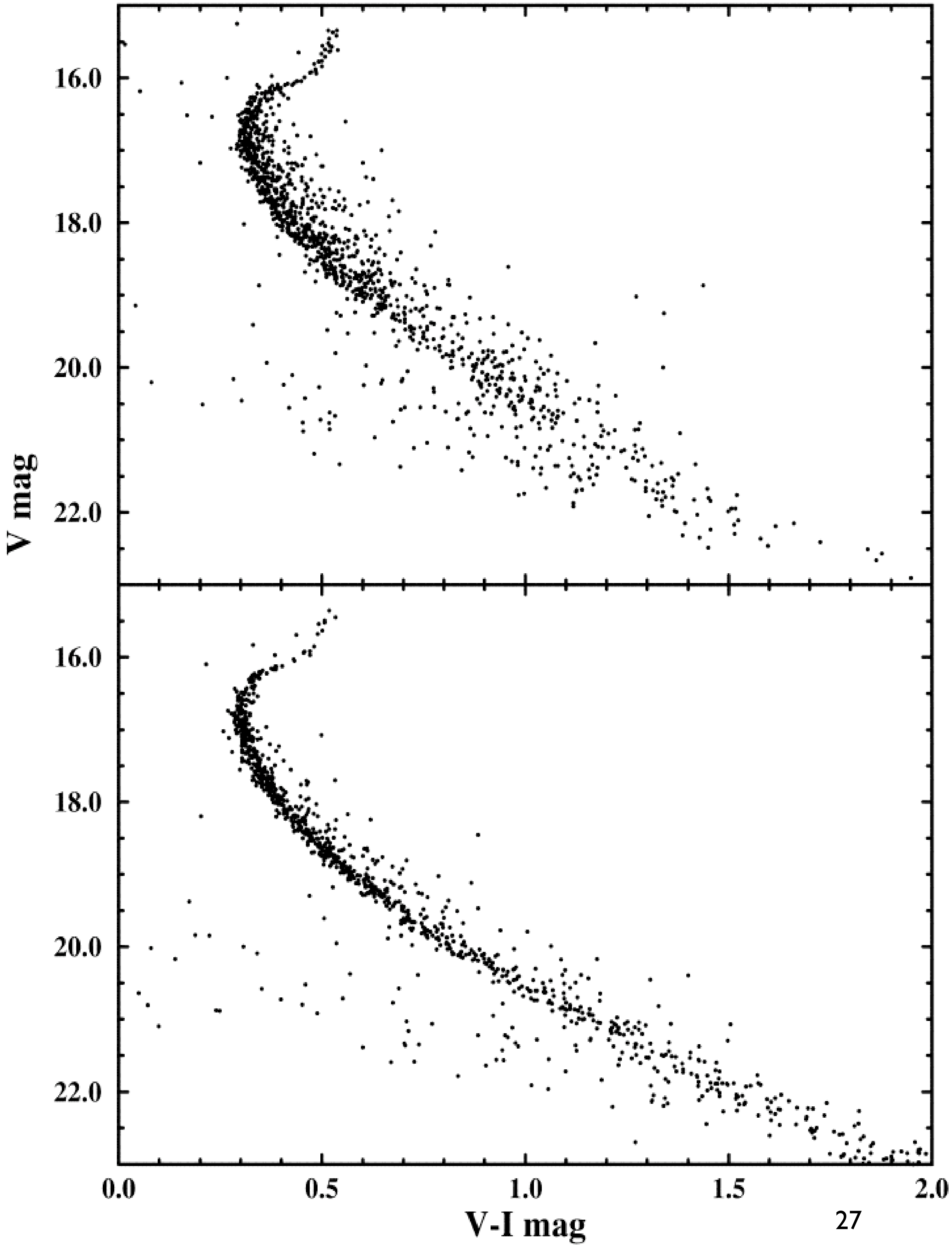
Stauffer $f_{\text{phot}} \approx 0.26$

Kaehler $f_{\text{phot}} \approx 0.6 - 0.7$ possible

The binary fraction ***f*** evolves with time :

$$f = \text{fn}\left(\frac{t}{t_{\text{cross}}}\right)$$

but $t_{\text{cross}} = \text{fn}(t)$



GC *NGC6752*

(Rubenstein & Bailyn 1997)

inner core: $0.15 \lesssim f_{\rm phot} \lesssim 0.4$

outer region: $f_{\rm phot} \lesssim 0.16$

Generally:

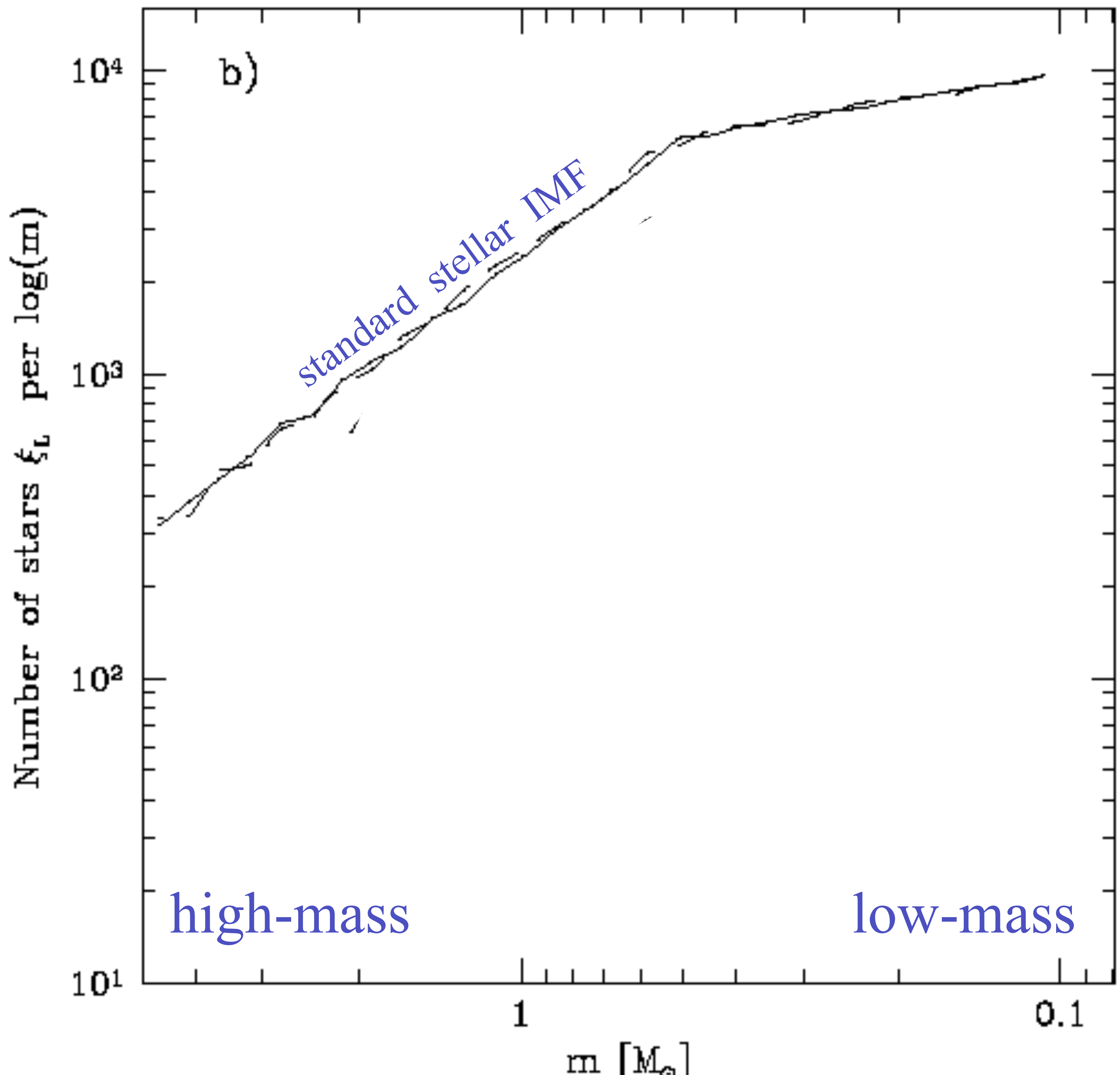
$f_{\rm GC} < f_{\rm popI}$

Clusters evaporate (loose stars) and
their substantial binary population evolves dynamically

***stellar MF in star clusters
evolves due to cluster evolution***

MF(t) due to cluster evolution: Clusters evaporate

(Baumgardt & Makino 2003)



$f = 0$

—— $N = 1.28 \times 10^5$

- - - - $4 \times (N = 8000)$

t = 0
t = 0.3 Tdiss
t = 0.6 Tdiss
t = 0.9 Tdiss

MF(t) due to cluster evolution: Clusters evaporate

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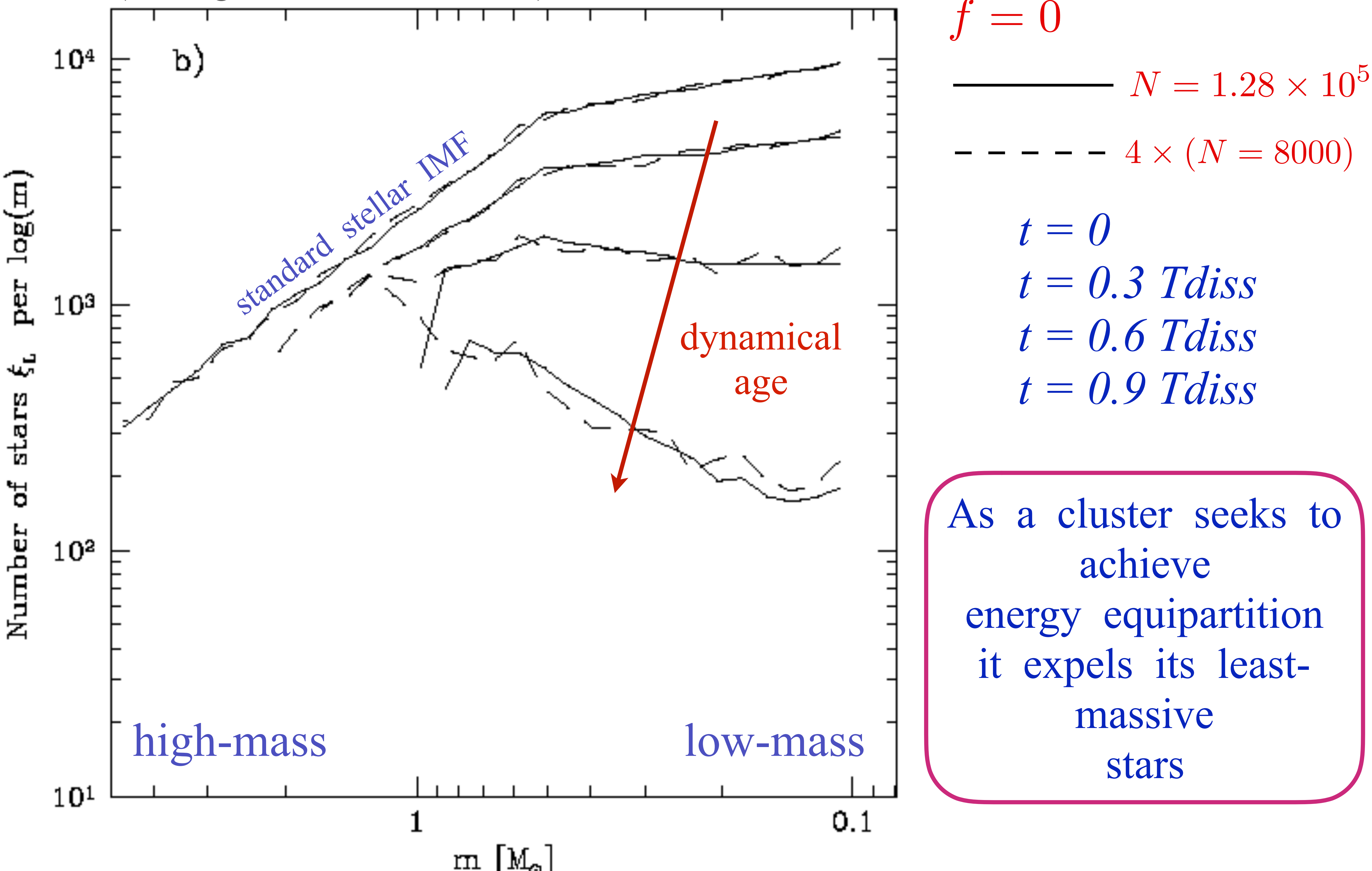

t = 0
t = 0.3 Tdiss
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As a cluster seeks to achieve energy equipartition it expels its least-massive stars

stellar MF(t) in clusters

Clusters evaporate (loose stars) and
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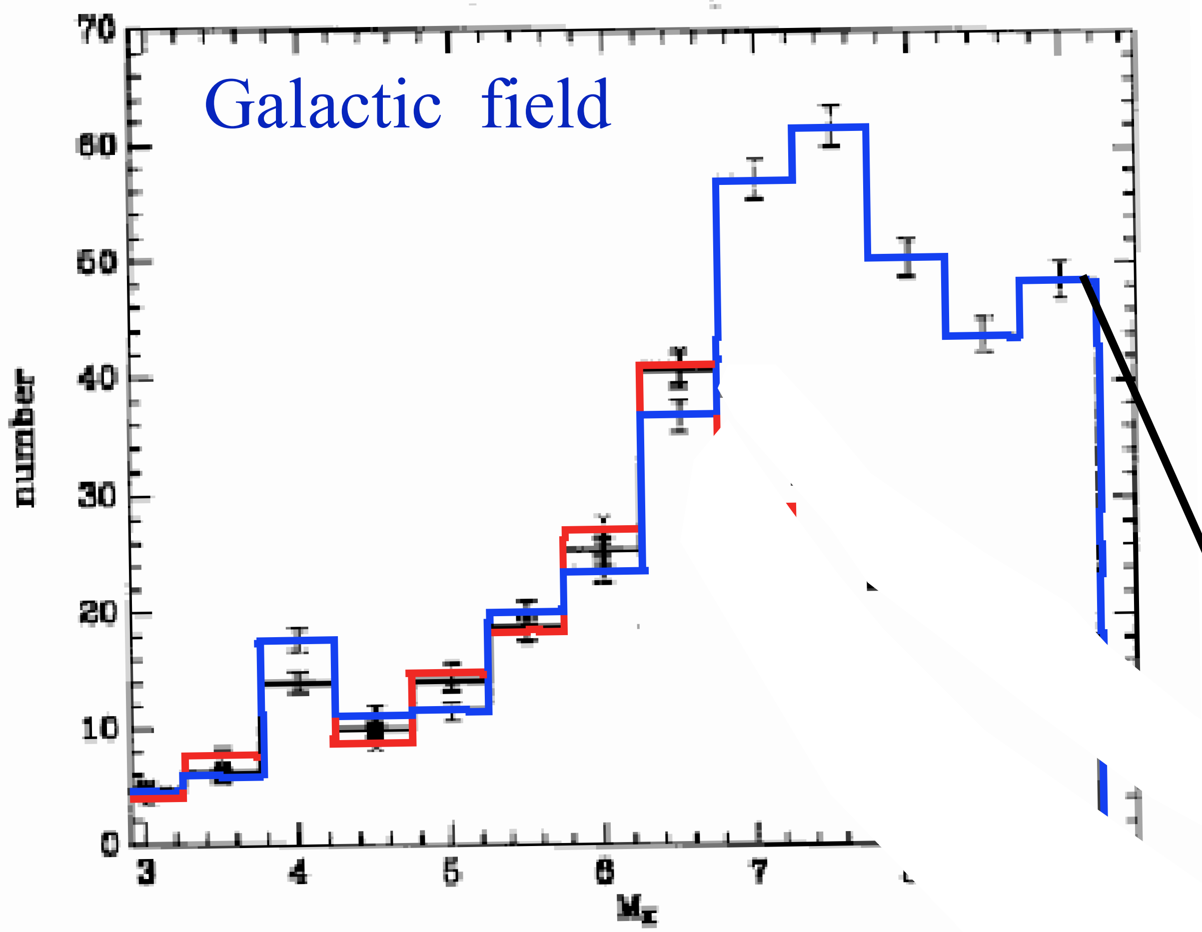

N-body Models of Binary-Rich Clusters

(Kroupa 1995)

$f = 1$ $20 \times (N = 400\,\mathrm{stars})$

individual-star LF $t = 0$

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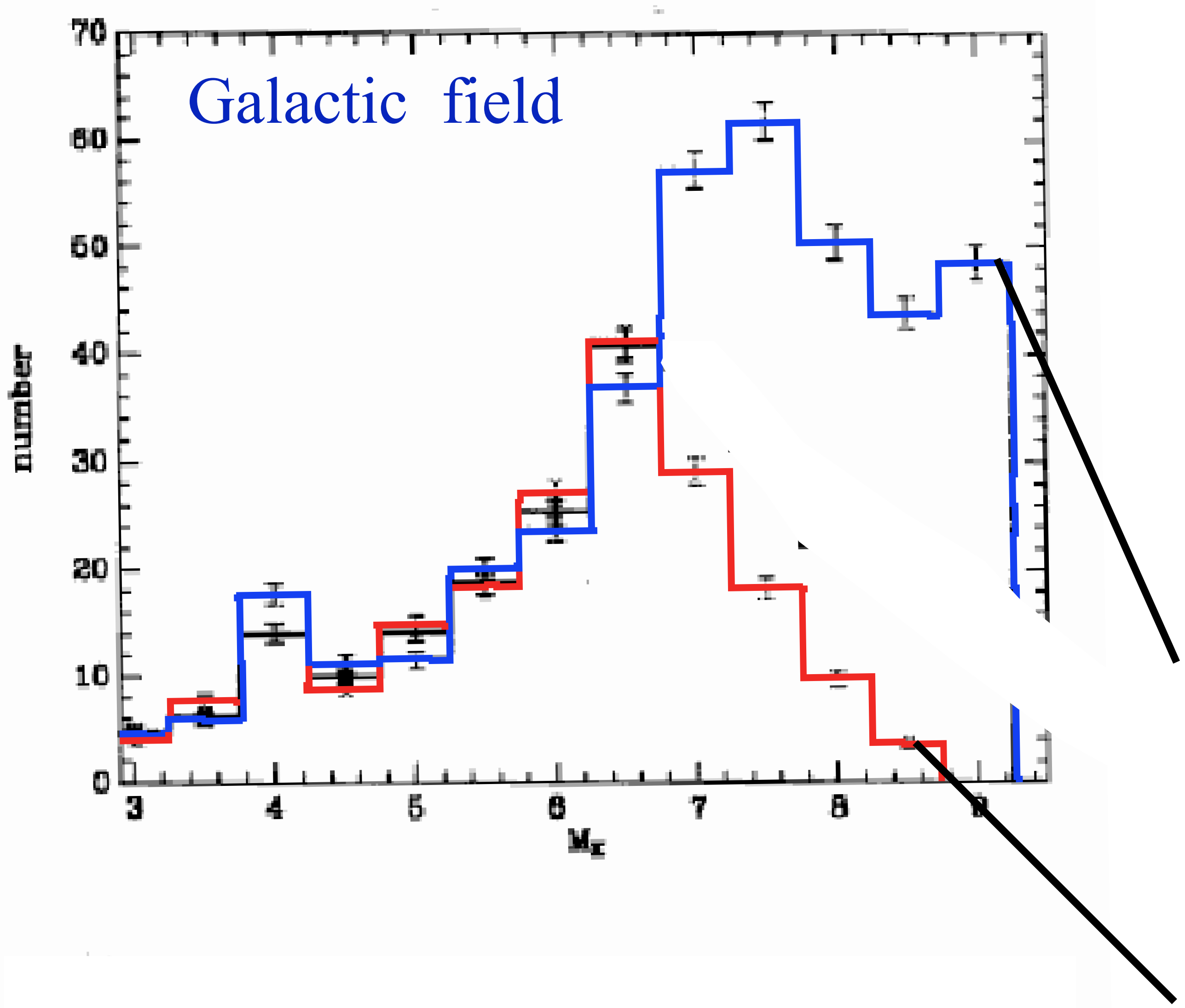
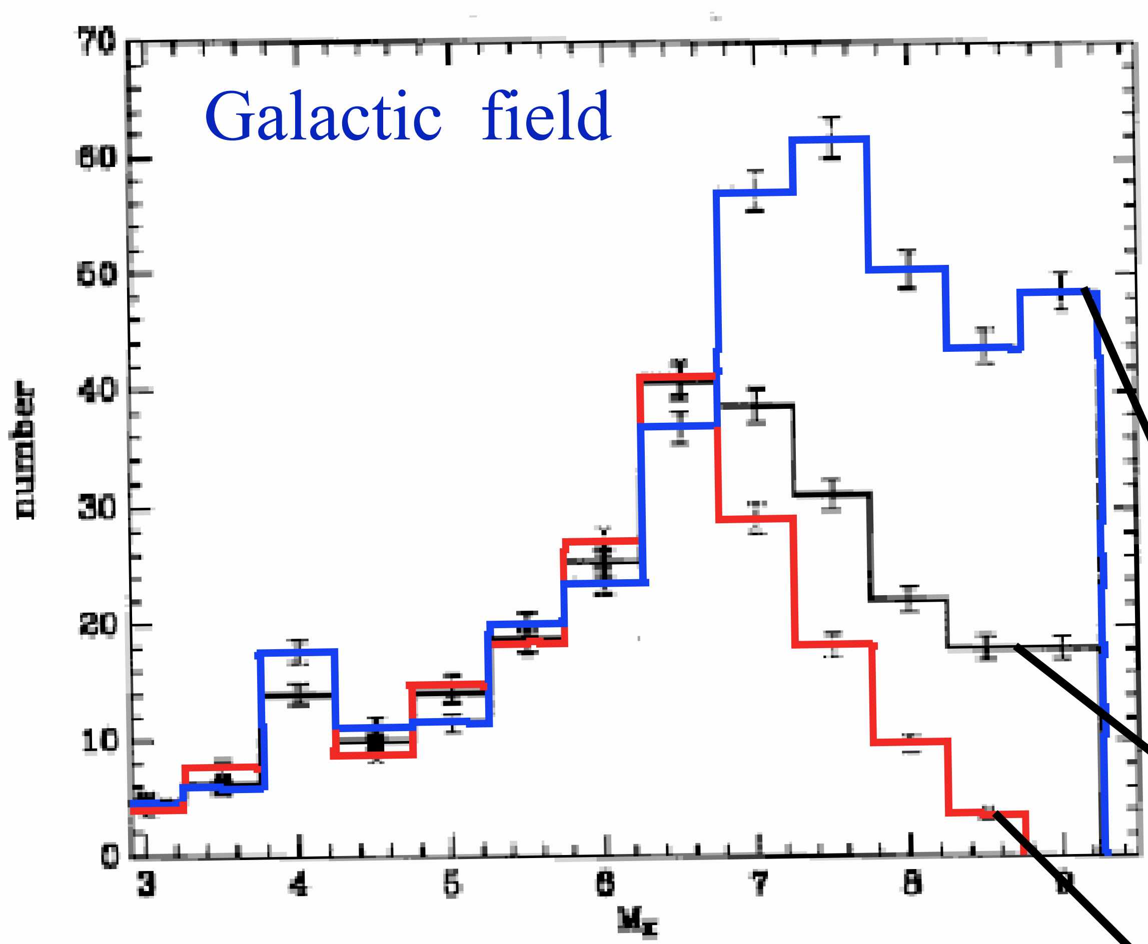


individual-star LF $t = 0$

system LF $t = 0$
$f = 1$



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$f = 1$ $\quad 20 \times (N = 400\,\mathrm{stars})$

individual-star LF $\quad t = 0$

system LF $\quad t = 10^9$ yr
$f = 0.48$

system LF $\quad t = 0$
$f = 1$

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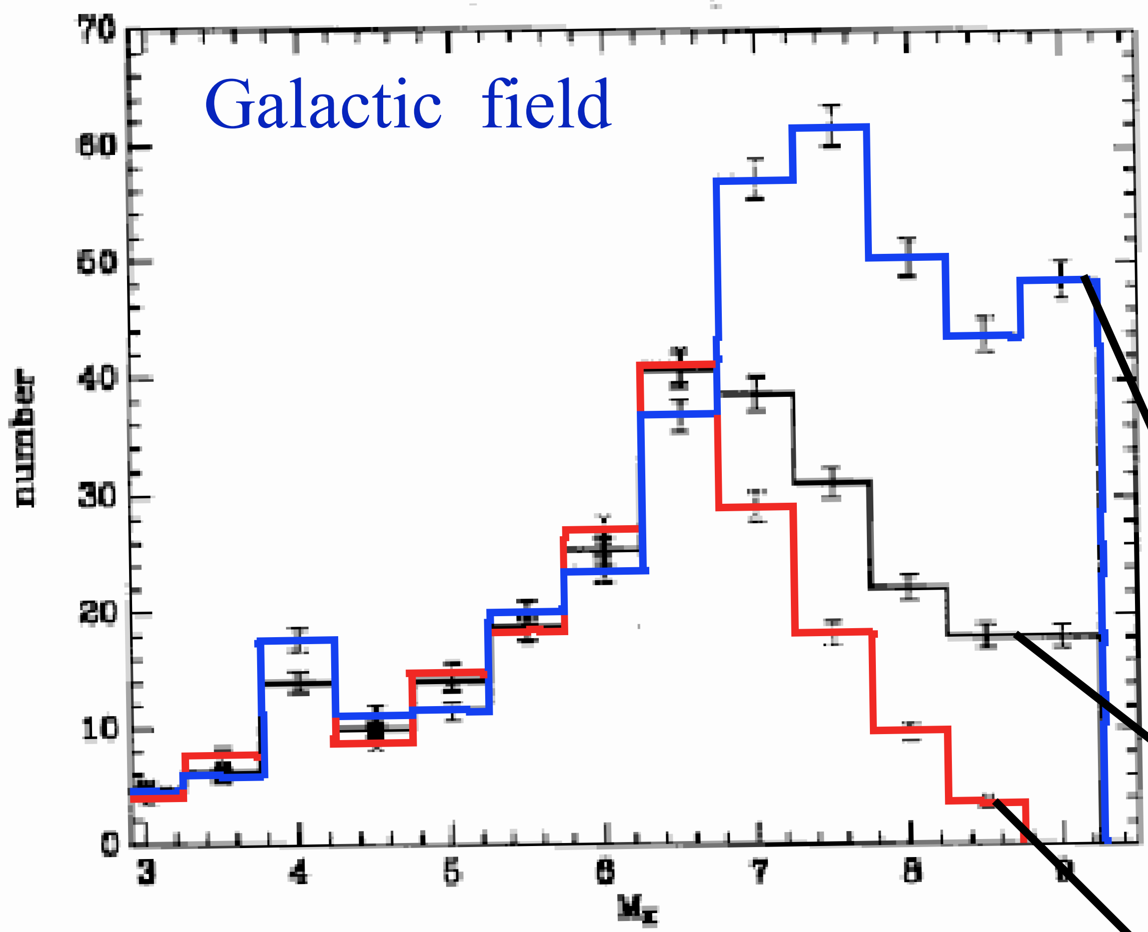

individual-star LF $\quad t = 0$

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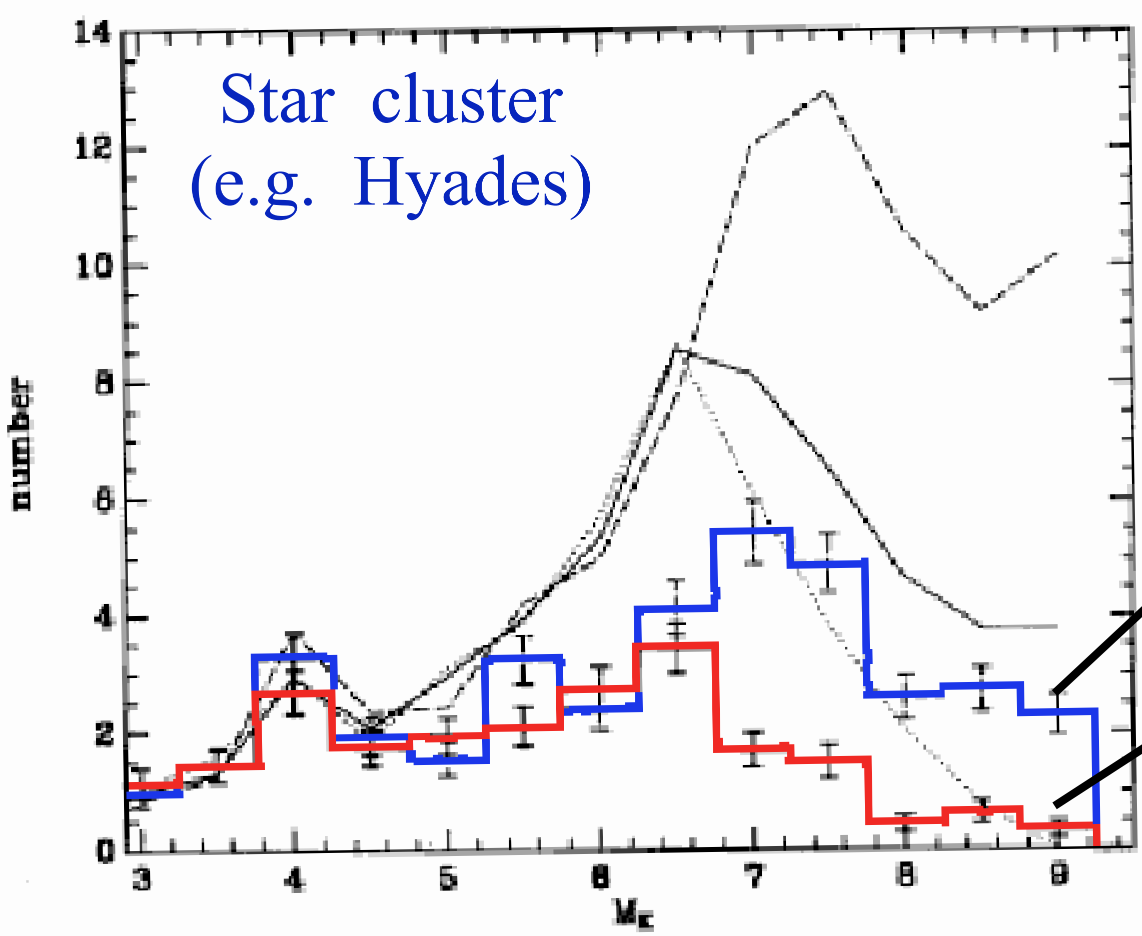


In the cluster :

individual-star LF

system LF $\longrightarrow$ $\xi_{\rm obs}(m) \neq \xi_{\rm true}(m)$

$t = 44\, t_{\rm cross} = 5 \times 10^8$ yr

Thus, in order to *constrain the IMF*
for an observed population
need to statistically correct
for *unresolved companions* and
for *dynamical evolution*.

Both corrections can be significant.

Put a birth binary population into embedded cluster models, use Aarseth Nbody code to synthesize a Galactic field

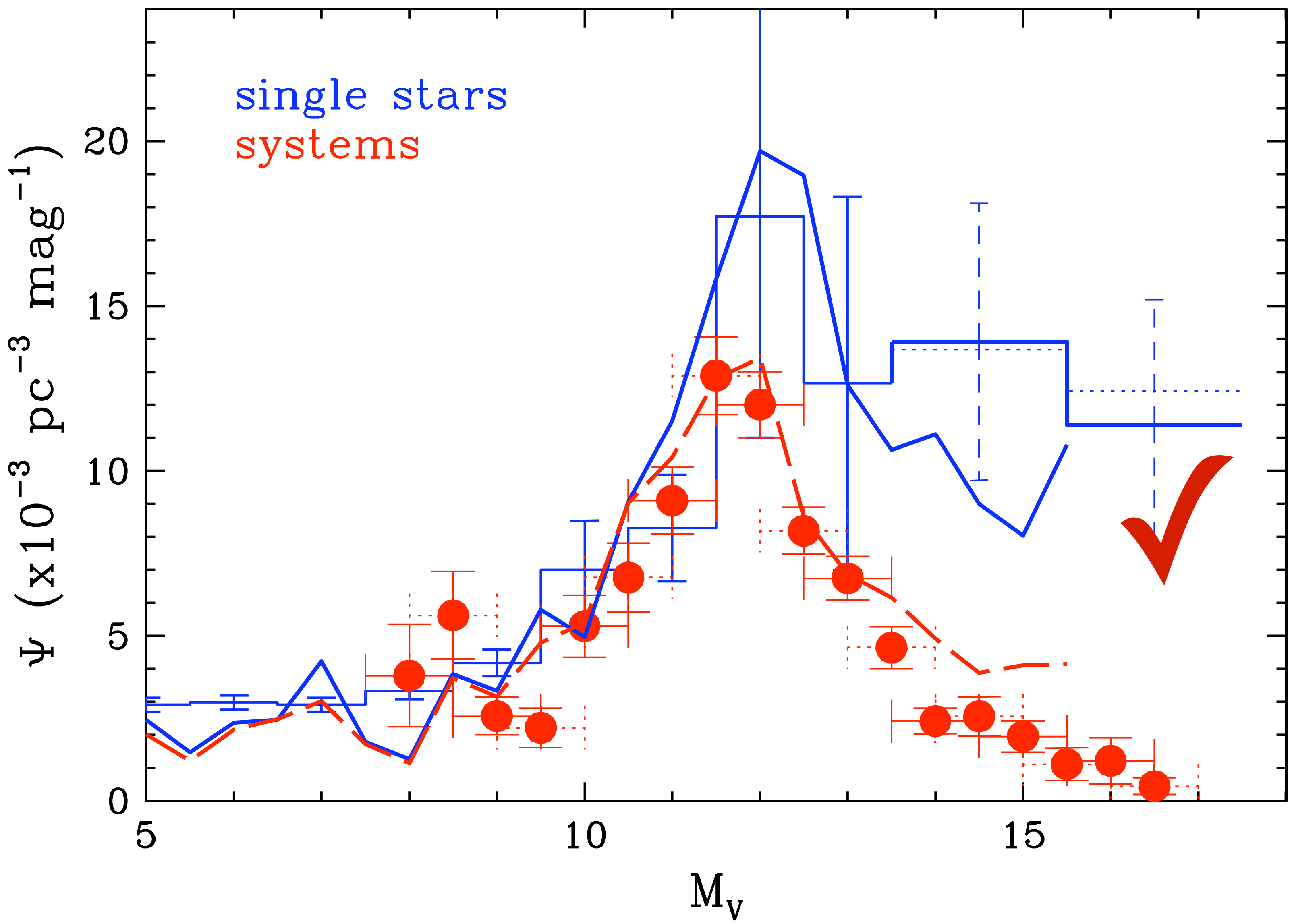
Marks & Kroupa 2011

The Galactic field MF

Dynamical Population Synthesis:

$$\Psi(M_V) = -\frac{dm}{dM_V}\,\xi(m)$$

(Kroupa 1995)



Assume all stars form as binaries in typical open clusters:

$R \approx 0.8$ pc, $N \approx 800$ stars

(Using KTG93 MLR)

---> a good understanding
of the
stellar IMF (deduced form the PDMF)
in embedded / very young clusters
and in the field

(Kroupa et al. 2013)

With binaries, early-type stars

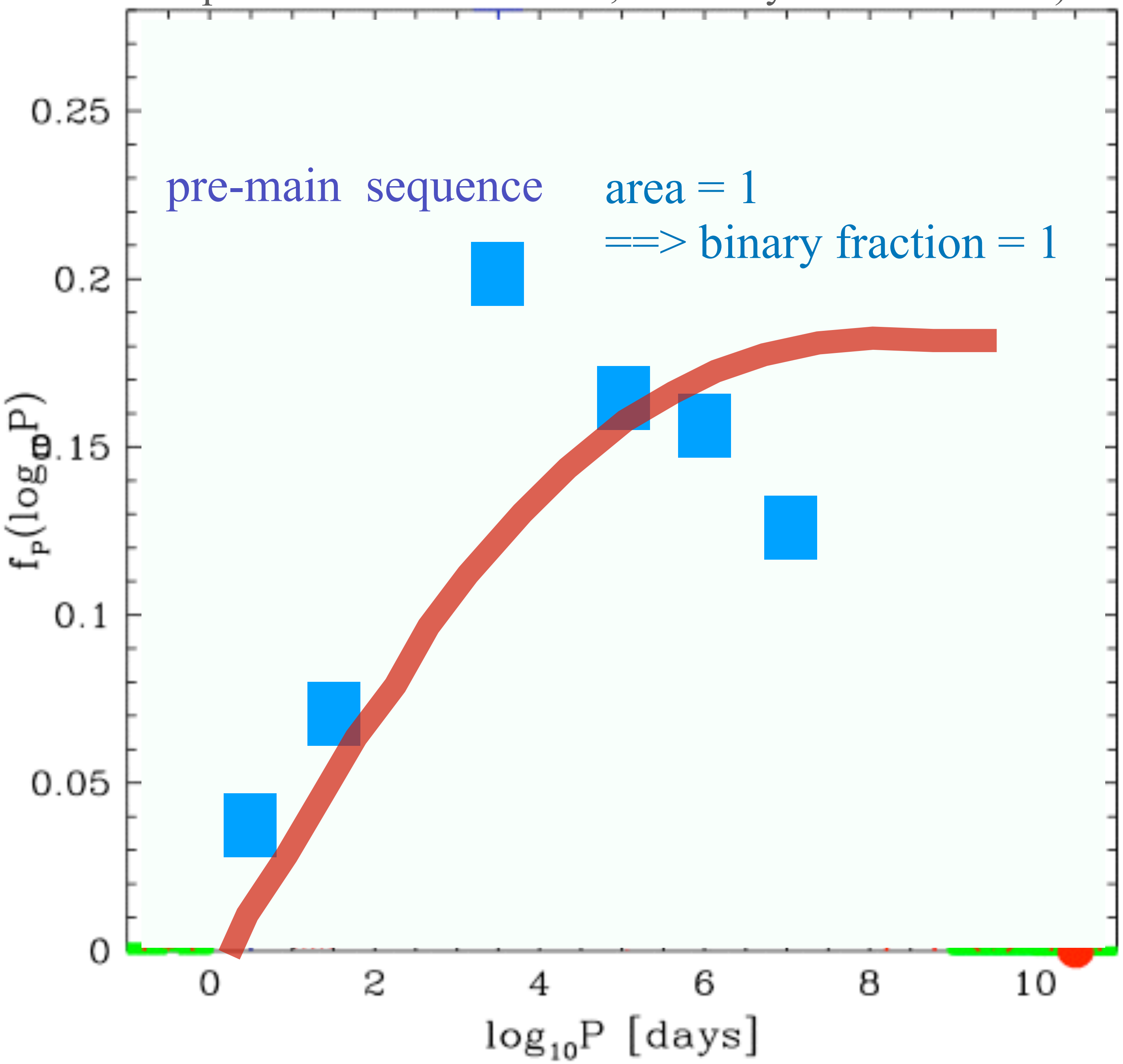
What do we know about binary-star distribution functions?

The vast majority of
> 5 Msun stars
are observed to form as binary systems,
perhaps as triples/quadruples

Sana et al. 2012
Chini et al. 2013
Moe & Di Stefano 2017

Most < 5 Msun binaries are born wide

Kroupa & Petr-Gotzens 2011; Sadavoy & Stahler 2017)



Most < 5 Msun binaries are born wide

> 5 Msun binaries are tighter

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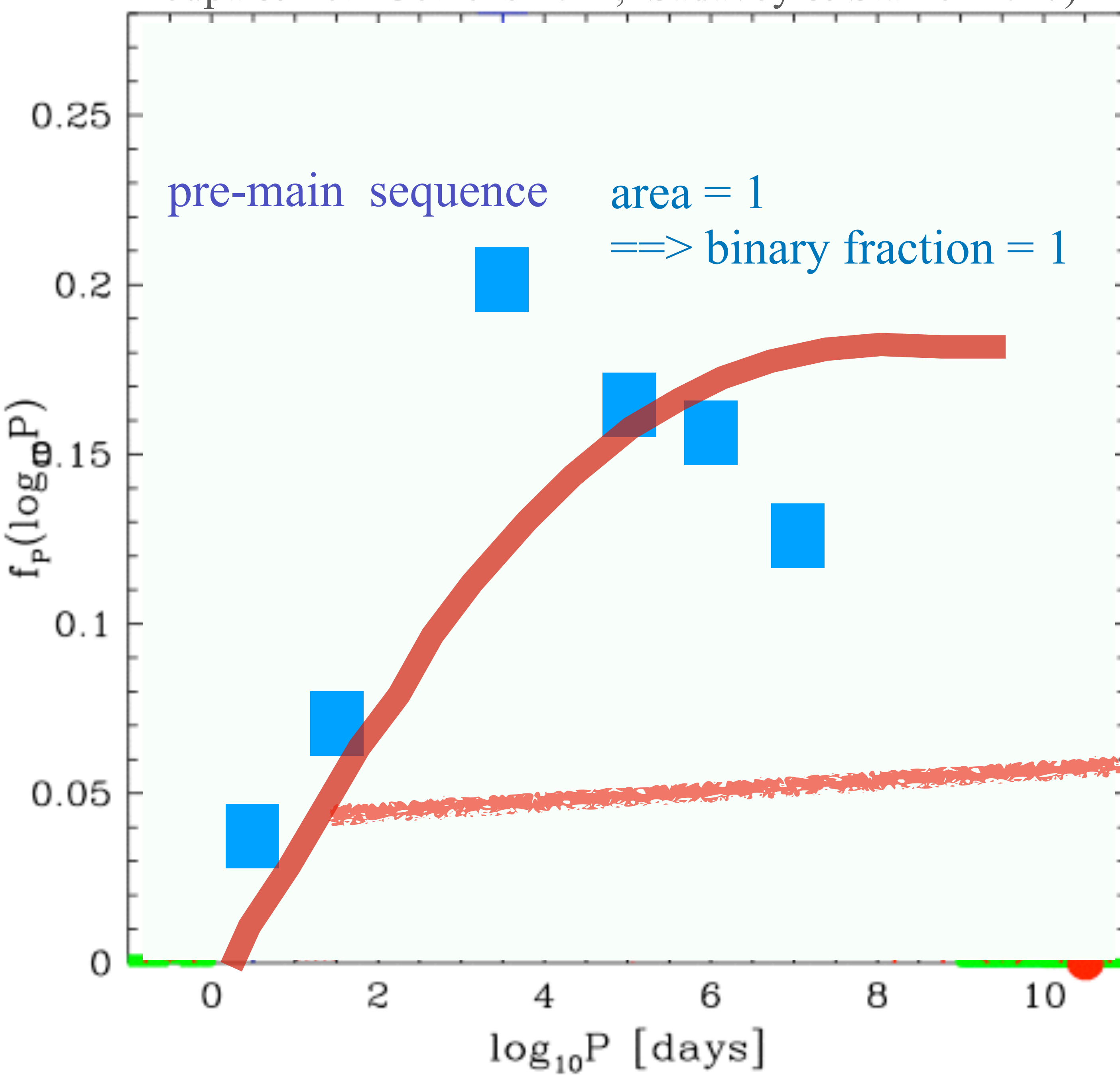
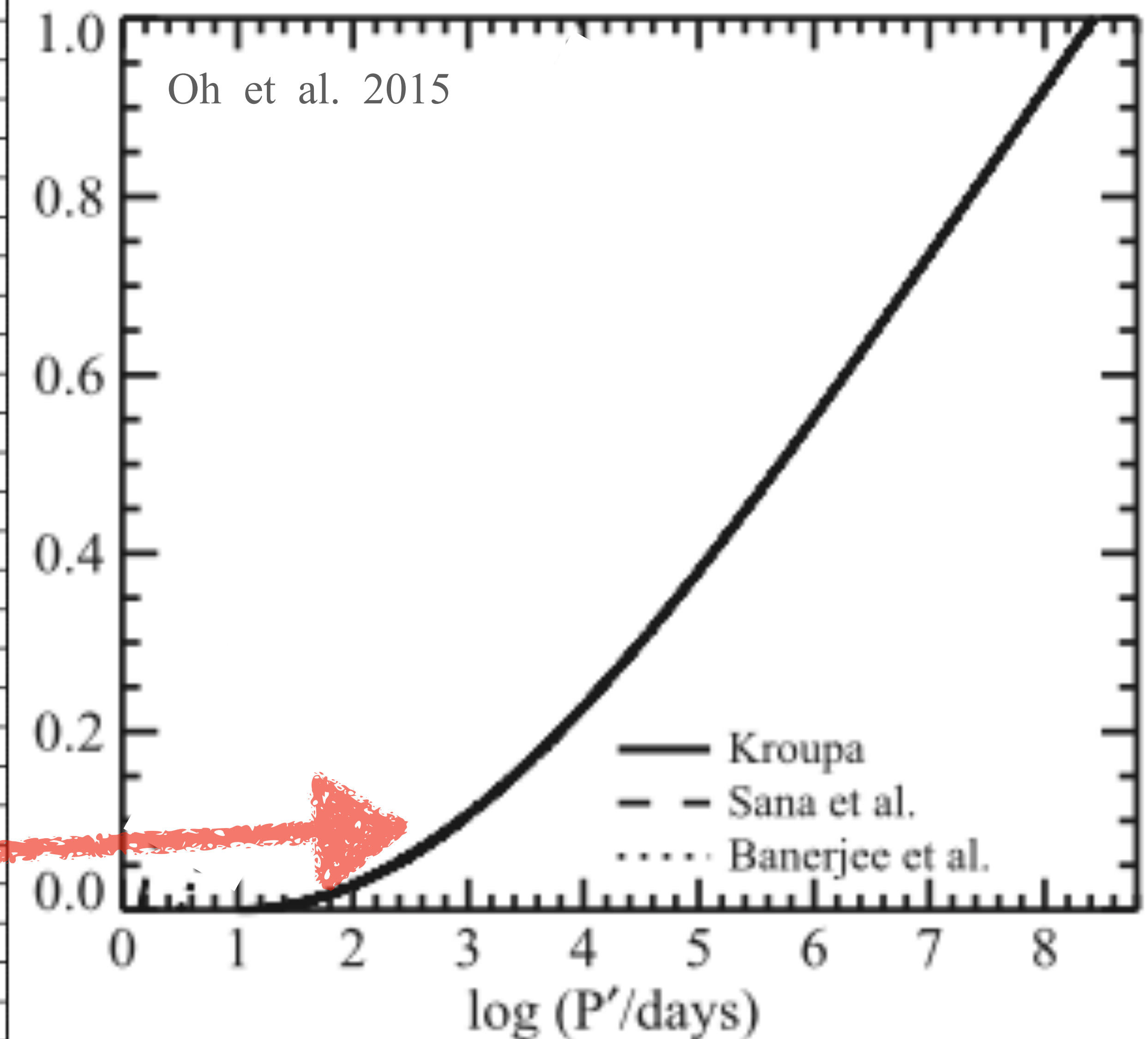


ASTROPHYSICAL JOURNAL, 805:92 (18pp), 2015 June 1



e 1. Cumulative binary fraction for $\log_{10} P < \log_{10} P'$ among all O stars. and dashed lines are calculated from the Kroupa (1995b, Equation (2)) ana et al. (2012, Equation (3)) period distributions, respectively. The l line is a uniform period distribution with a period range of $0.5 < \log_{10}(P/\mathrm{days}) < 4$ used in Banerjee et al. (2012) for O star binaries. For example, 49% of all O stars are binaries with period shorter than 100 days according to the Sana et al. distribution.

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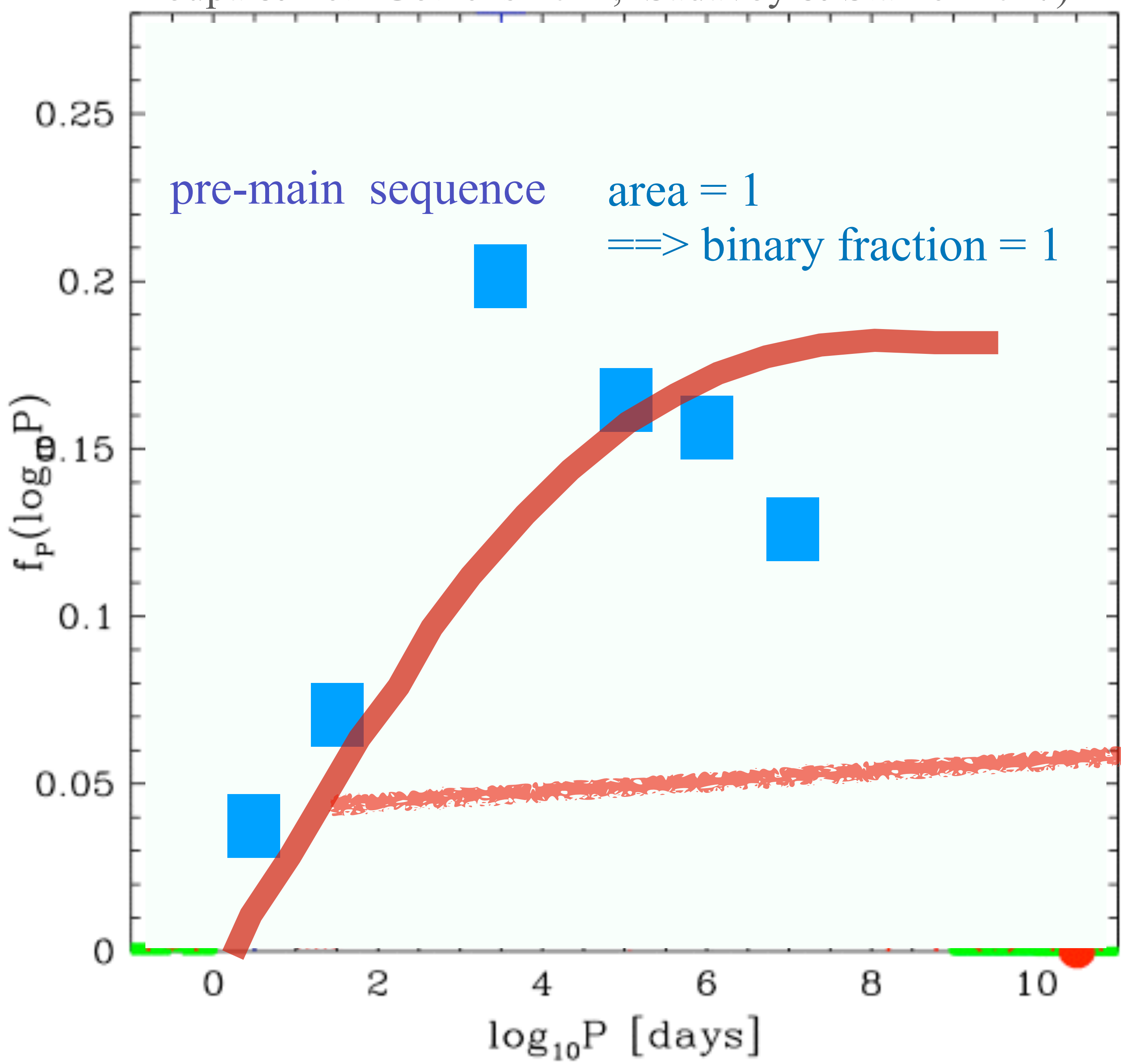

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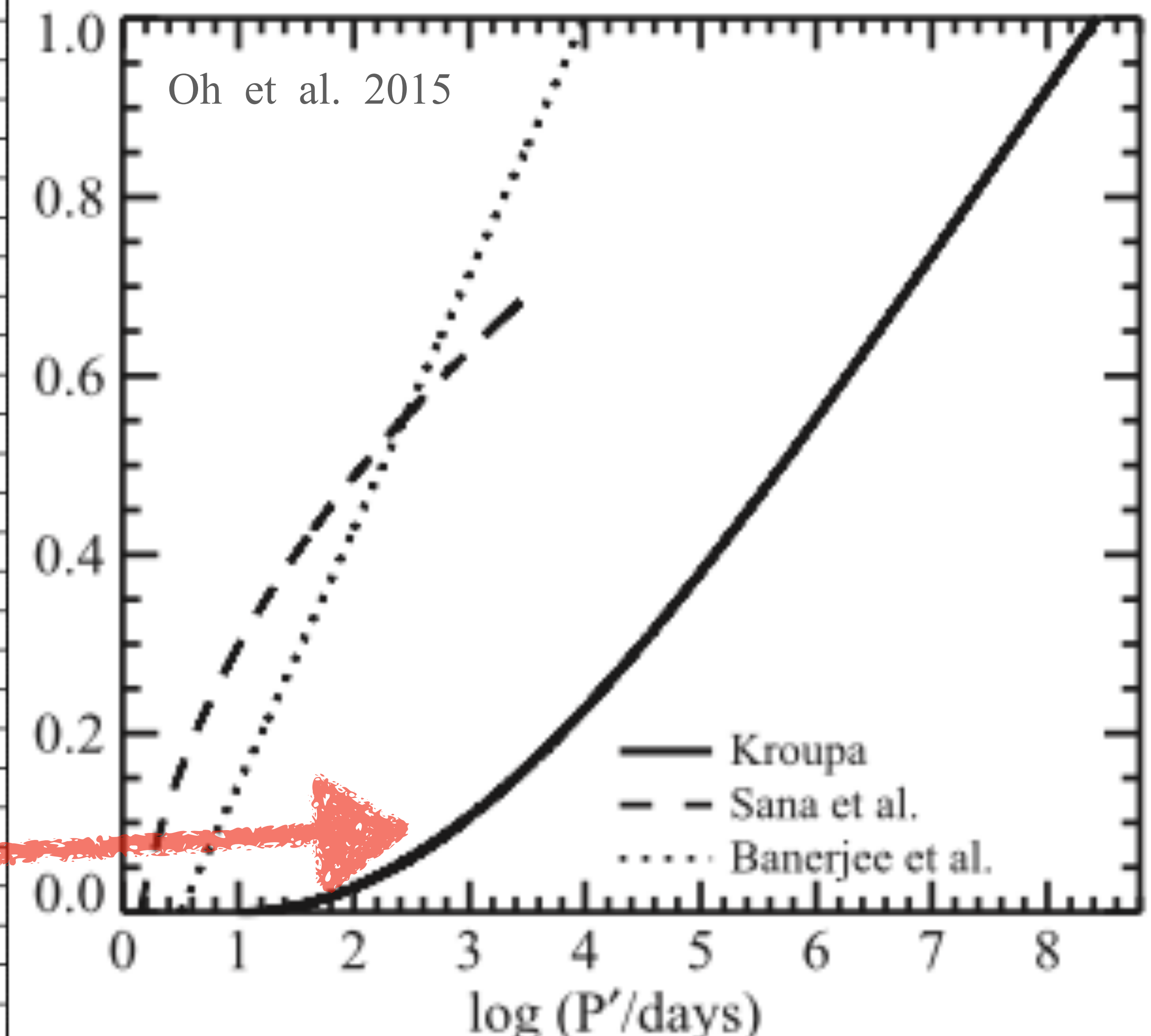

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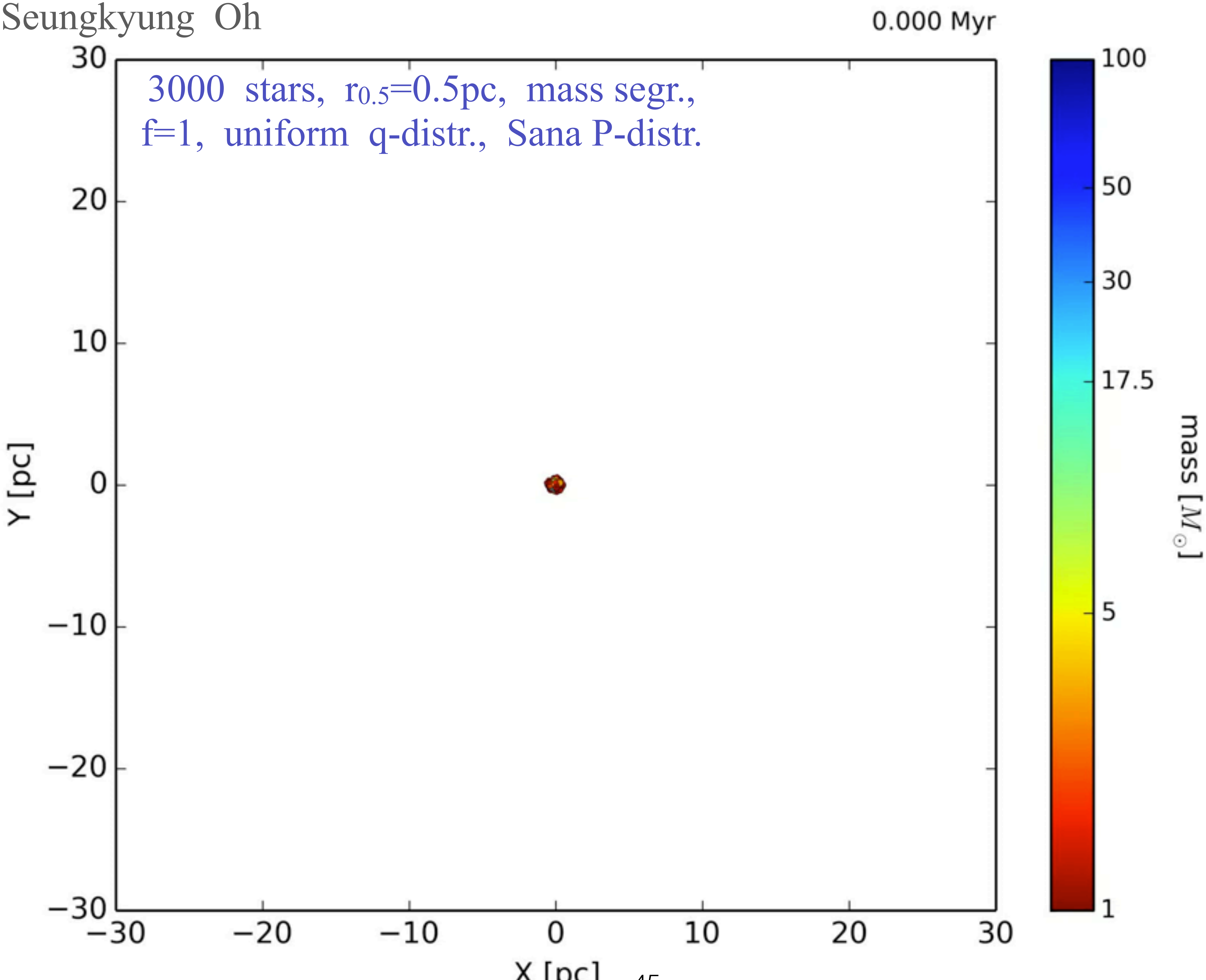
Massive stars are either born near cluster centres or segregate there on a short (<1 Myr) time-scale

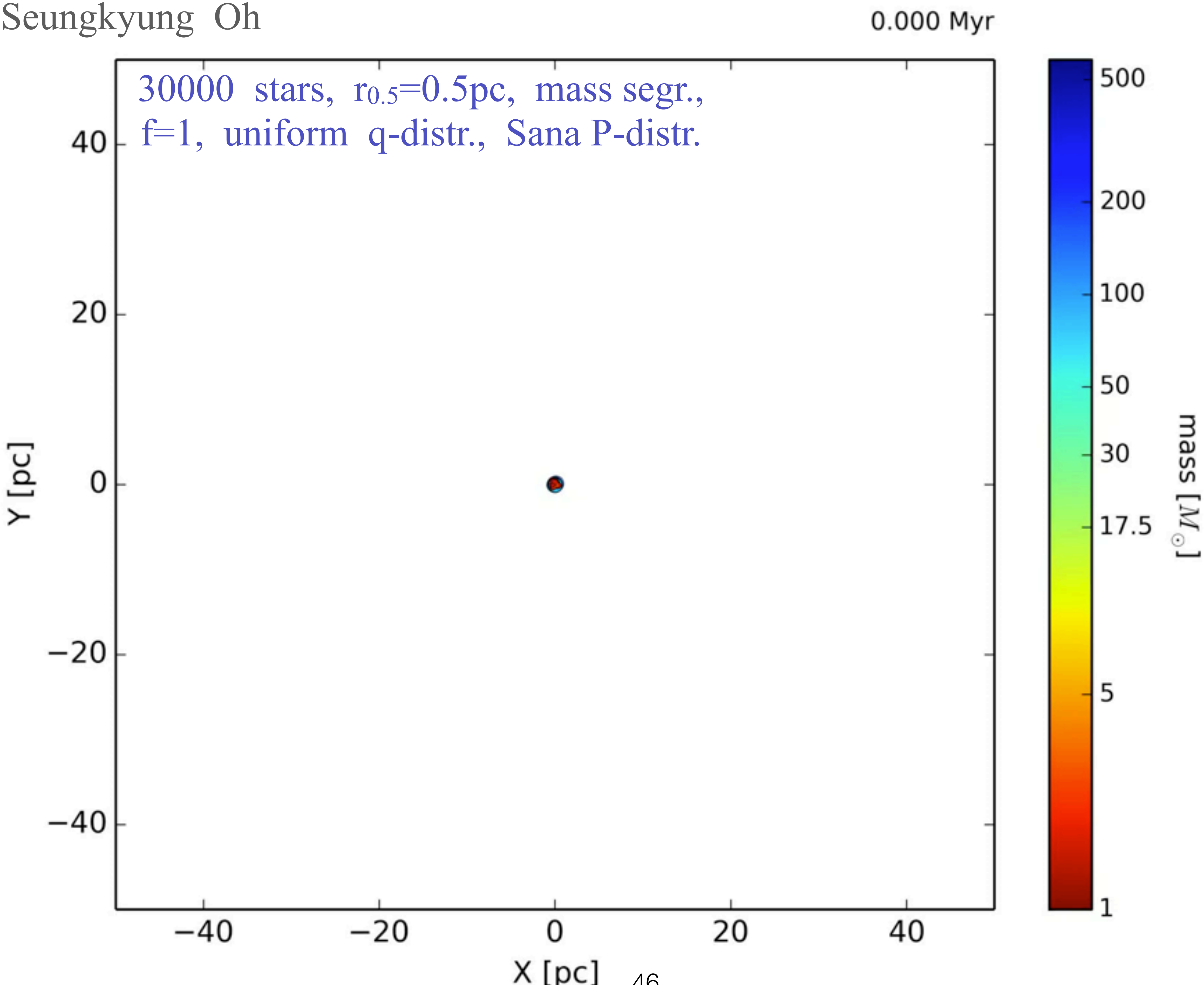
---> significant dynamical activity in the cores of young clusters

---> papers by Seungkyung Oh ; Sambaran Banerjee & PK

Seungkyung Oh

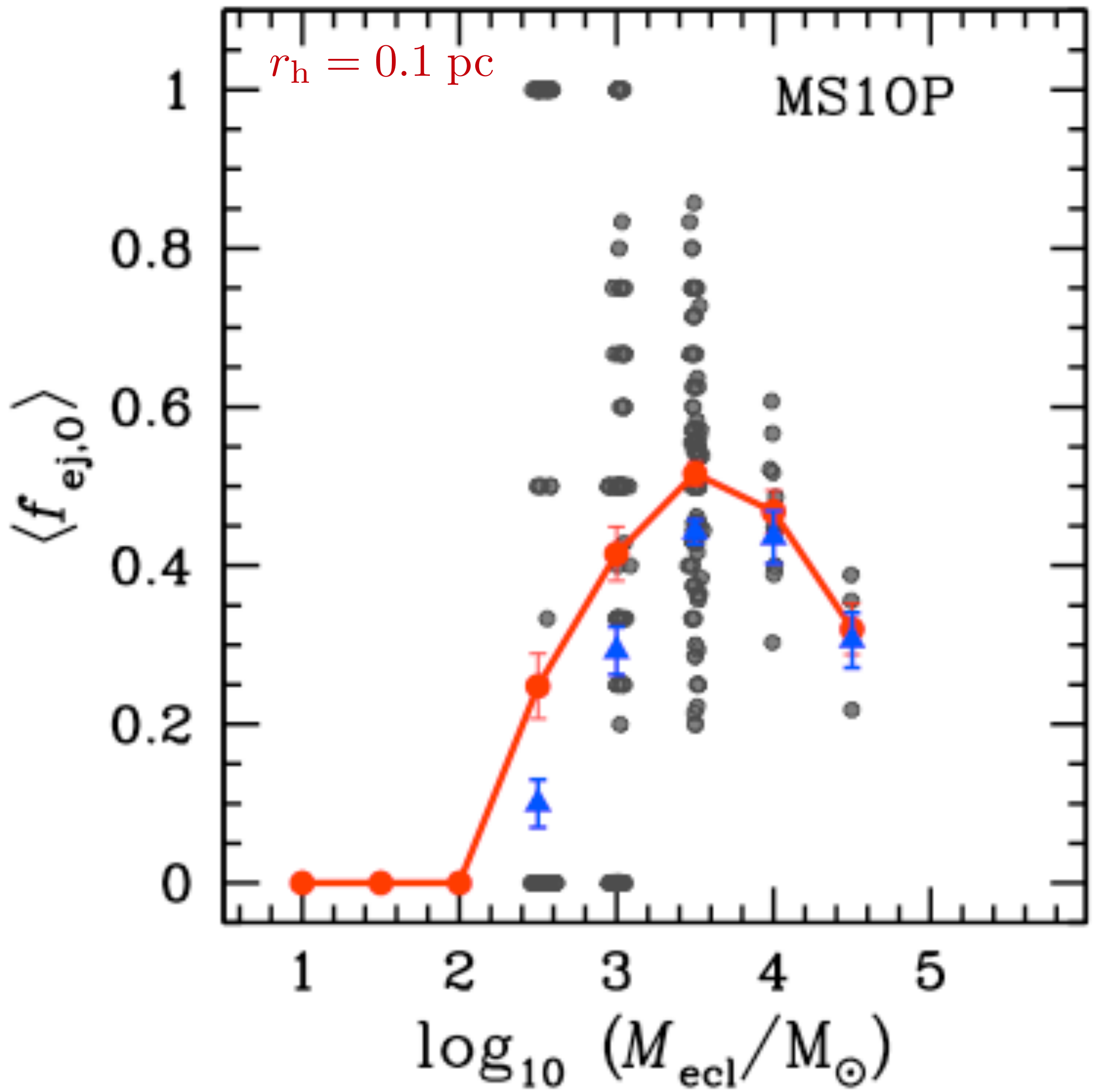


Seungkyung Oh



Efficient ejections of O stars (within 3Myr)

100 % initial binarity



(Oh, Kroupa & Pflamm-Altenburg 2015)

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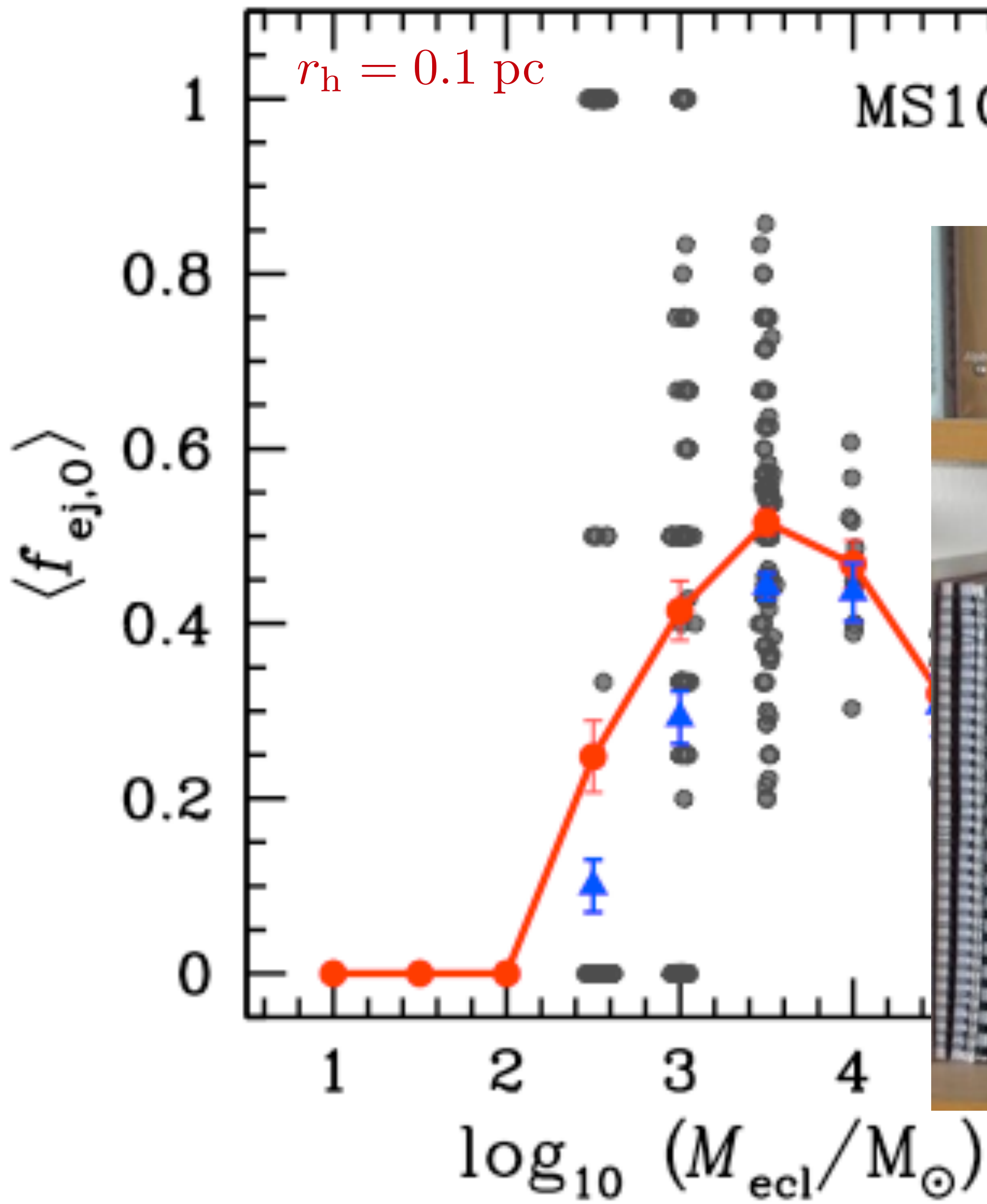

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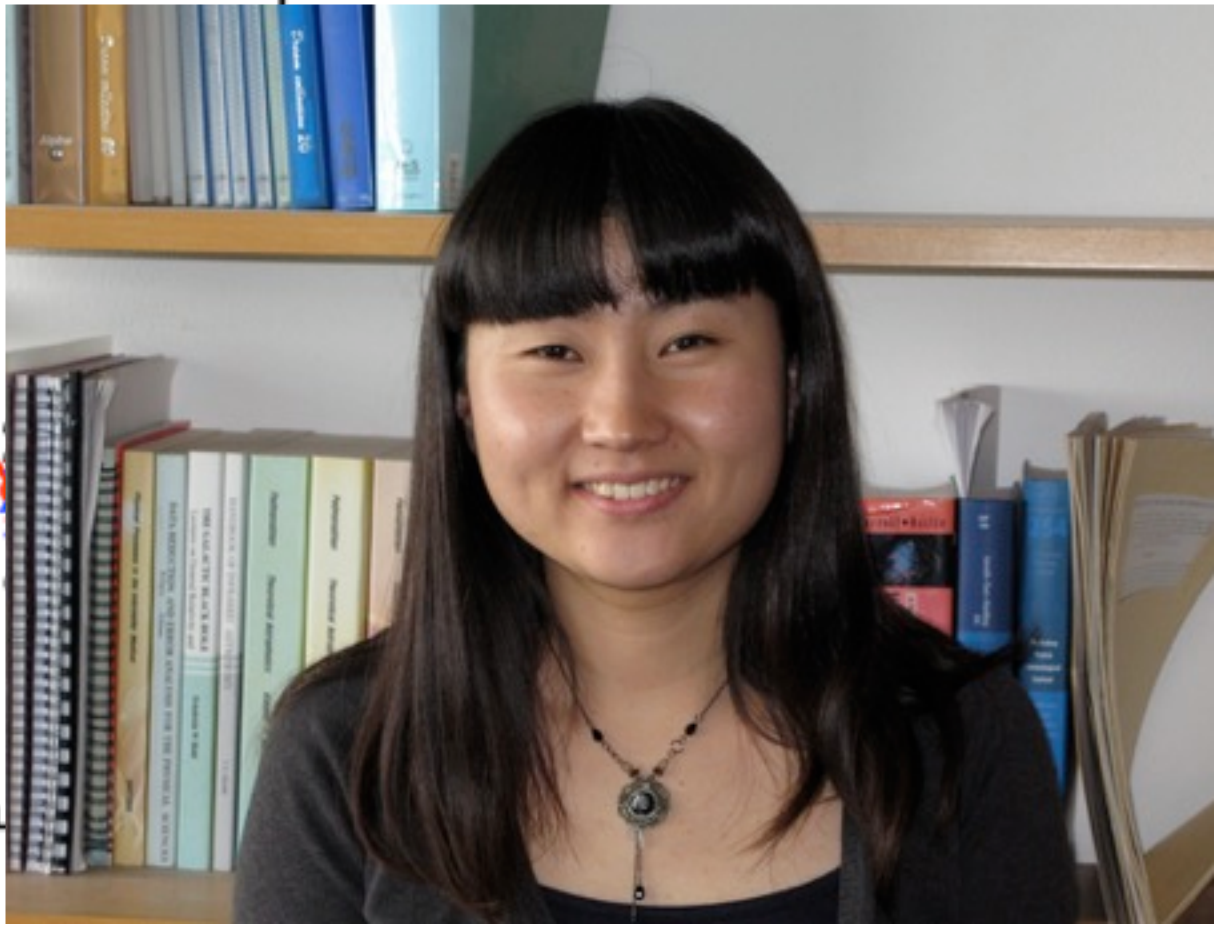

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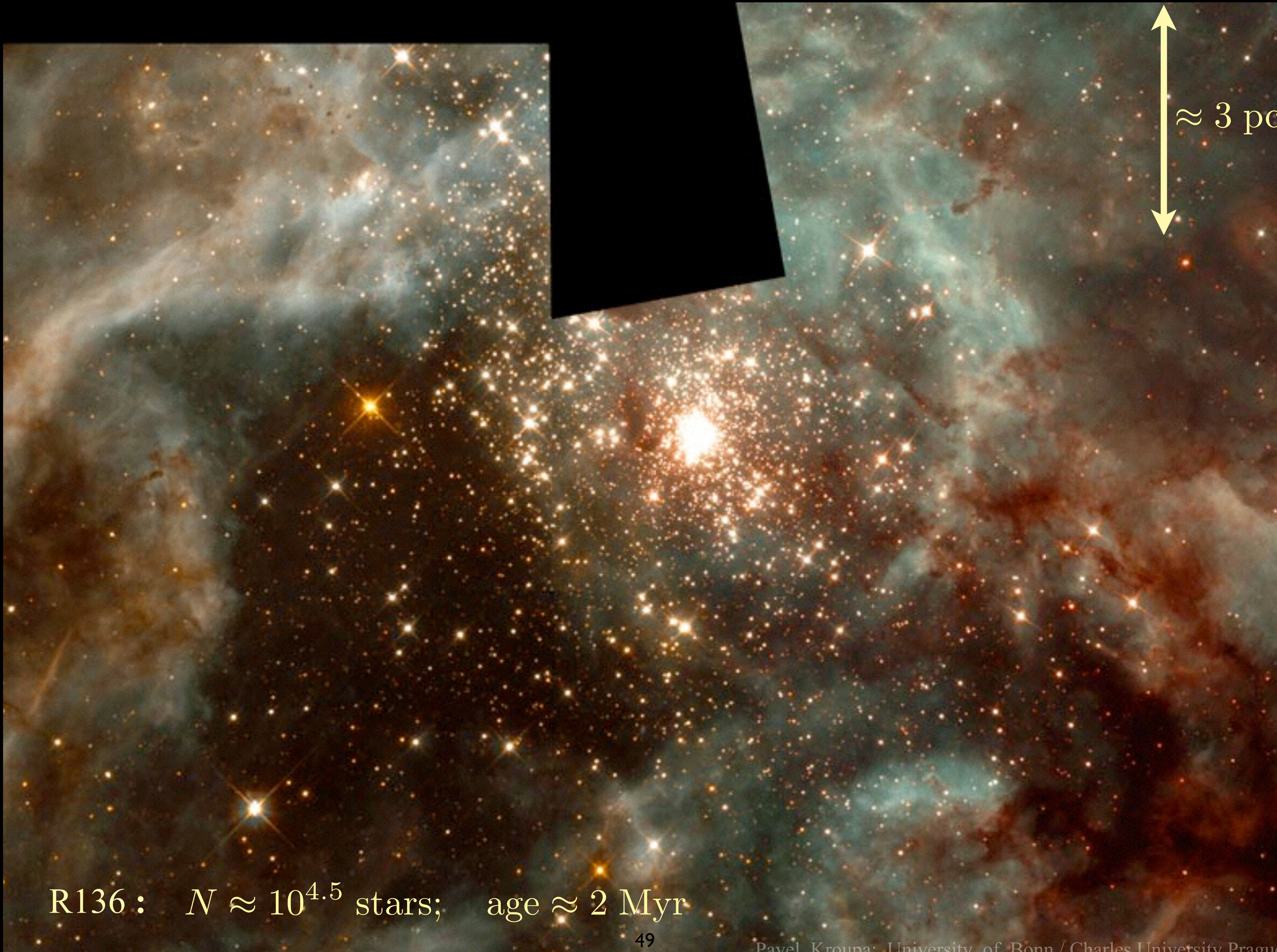
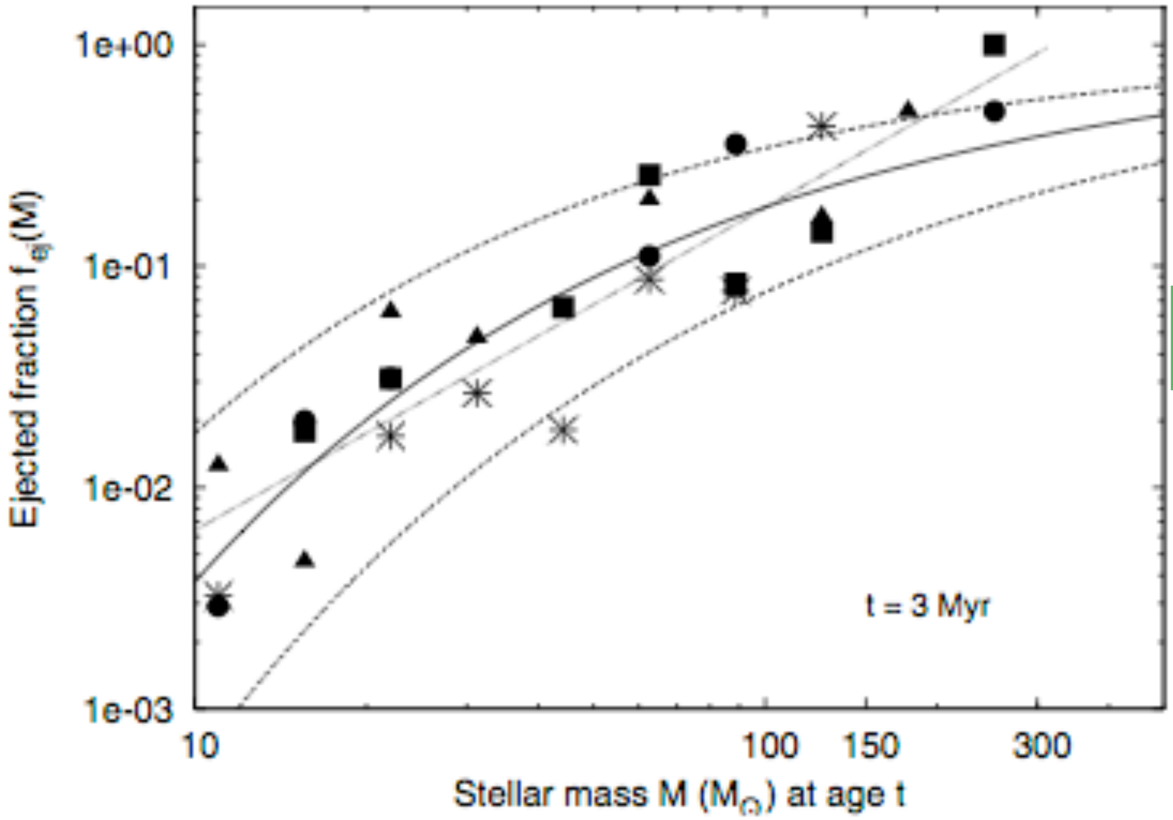

≈ 3 pc
R136 : $N \approx 10^{4.5}$ stars; age ≈ 2 Myr

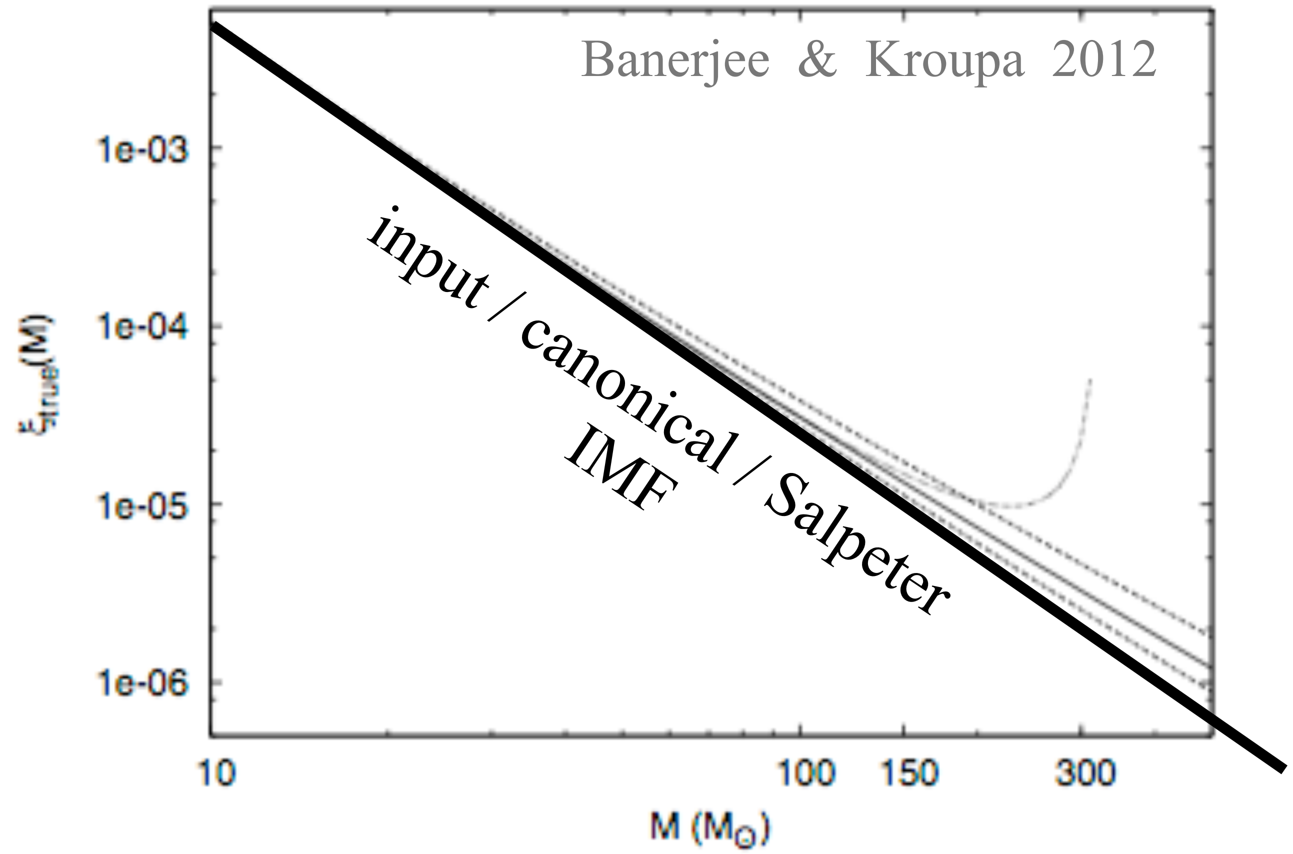
Statistically add ejected massive stars back into the cluster
---> improved IMF estimate

Nbody model of R136

Ejection fraction = fn(stellar mass)



Corrected IMF

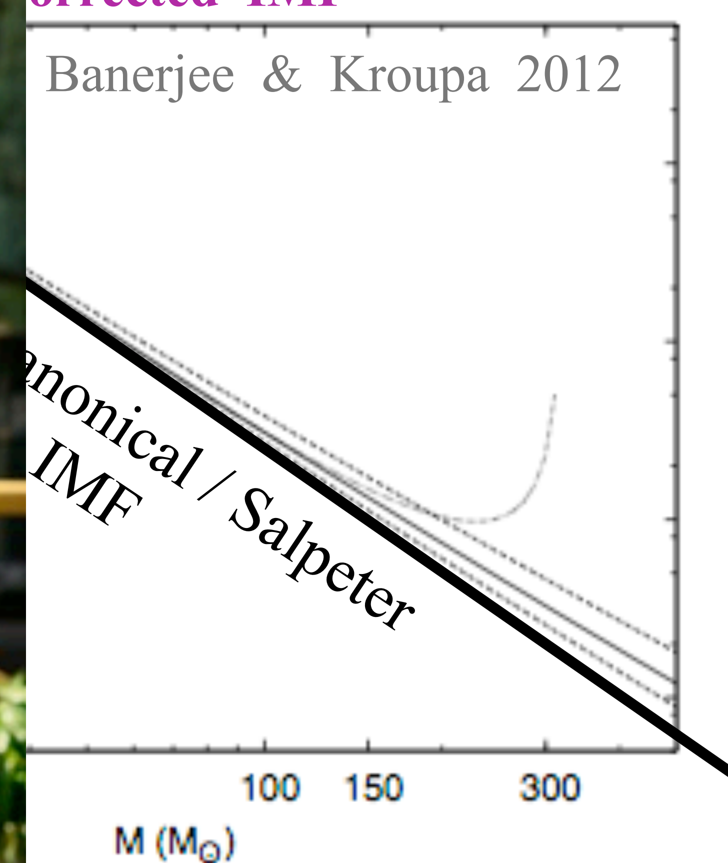


IMF becomes top heavy with increasing density

Statistically add ejected massive stars back into the cluster ---> improved IMF estimate

IMF becomes top heavy with increasing density :

Sambaran Banerjee + Sverre's code



Random sampling in mass ranges :

0.1 -- 5 Msun and separately
5 - 150 Msun

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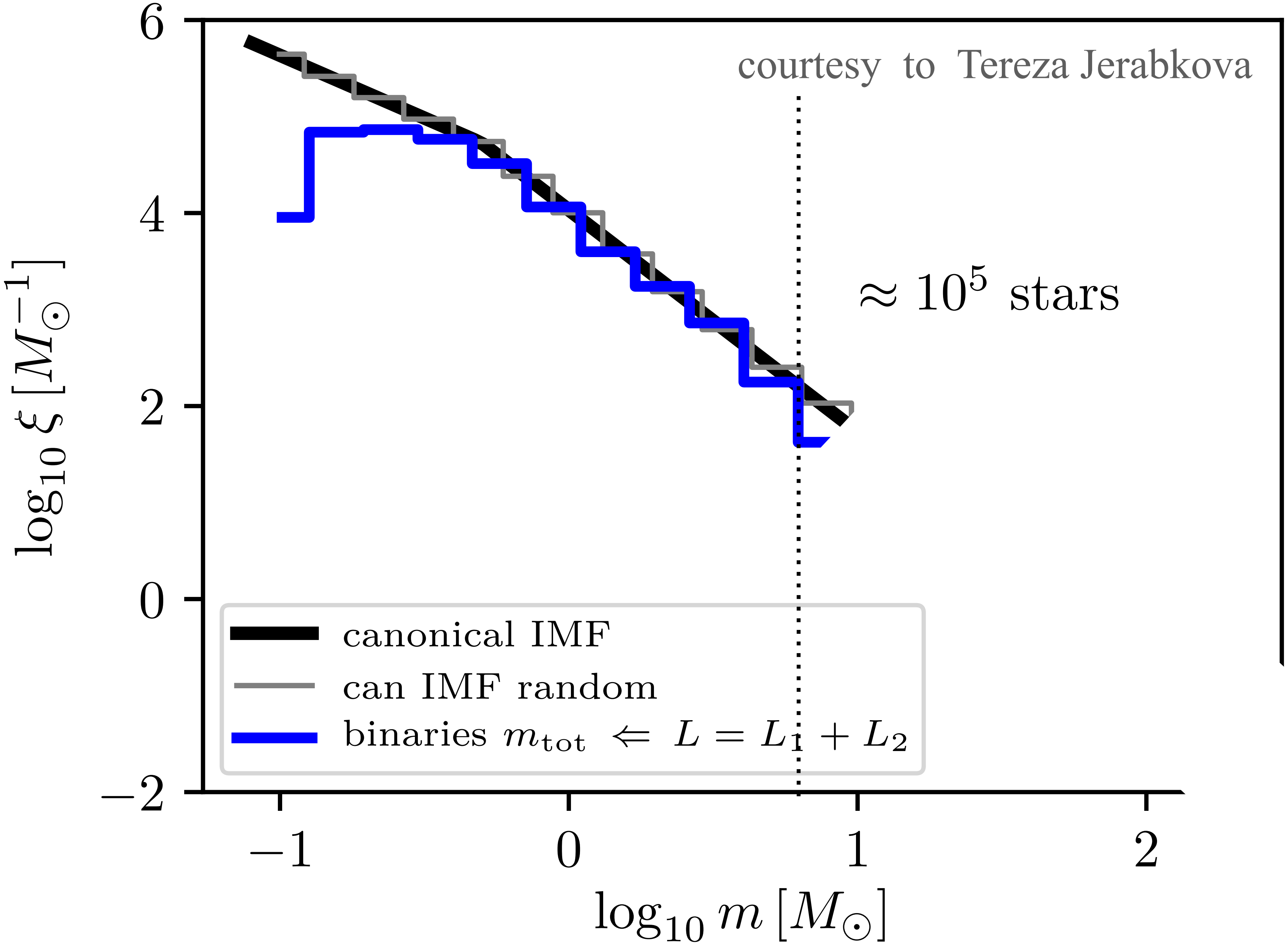
Salaris, Mauricio & Casssi (2005)
Duric & Nebojsa (2004)

$$\frac{L}{L_\odot} \approx .23\left(\frac{M}{M_\odot}\right)^{2.3} \qquad (M < .43M_\odot)$$

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{4} \qquad (.43M_\odot < M < 2M_\odot)$$

$$\frac{L}{L_\odot} \approx 1.5\left(\frac{M}{M_\odot}\right)^{3.5} \qquad (2M_\odot < M < 20M_\odot)$$

$$\frac{L}{L_\odot} \approx 3200\frac{M}{M_\odot} \qquad (M > 20M_\odot)$$



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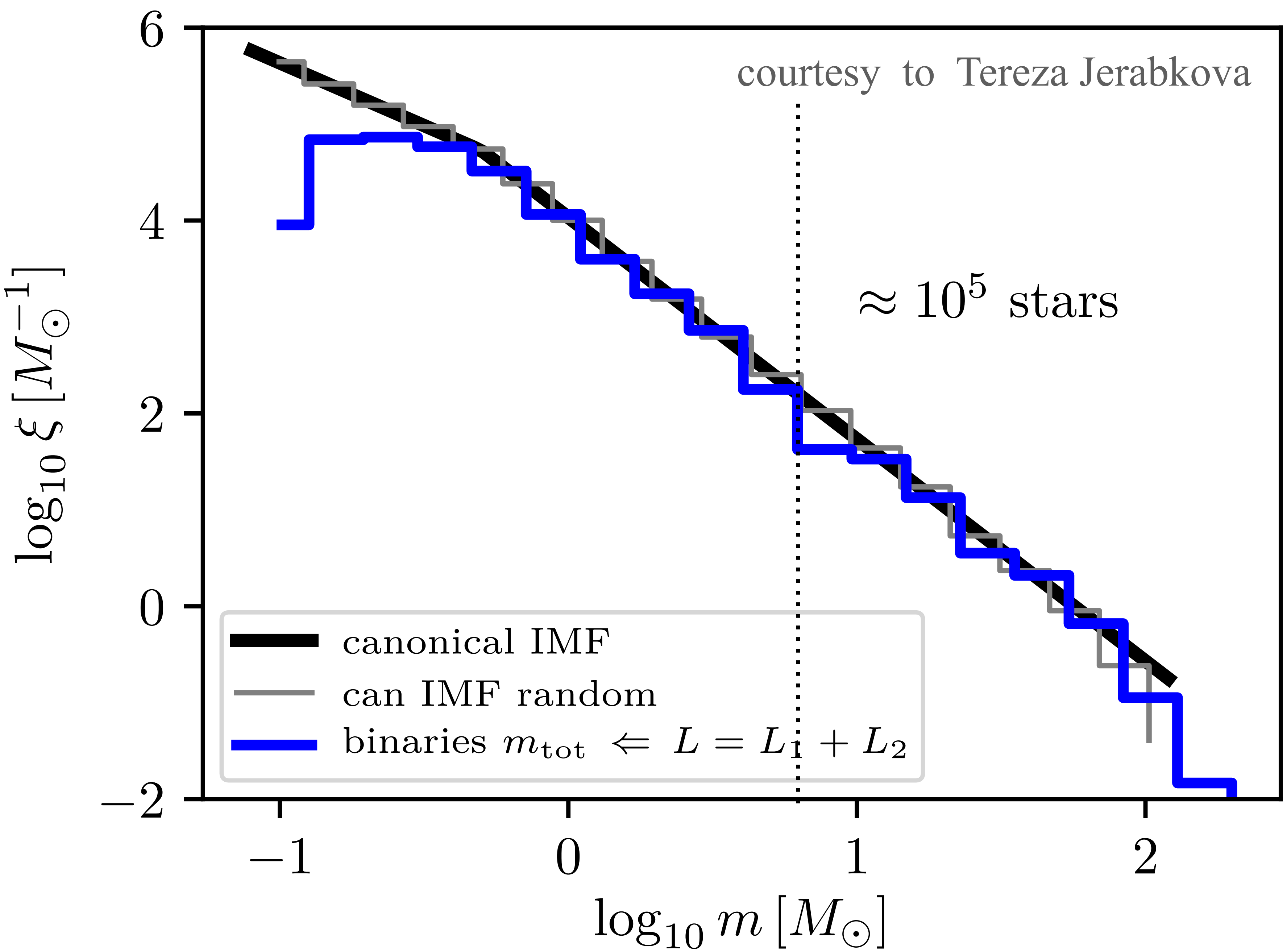
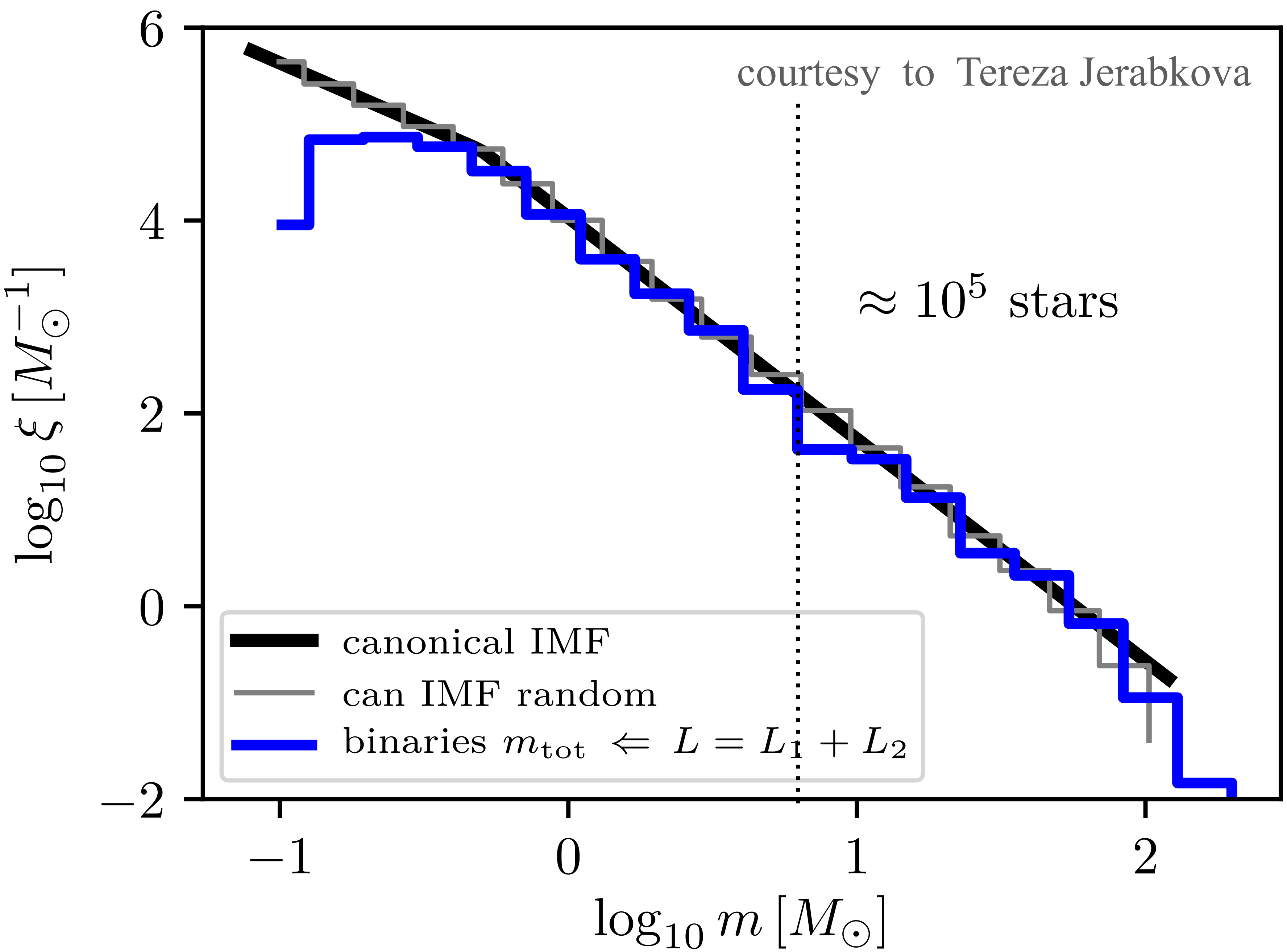
$$\frac{L}{L_\odot} \approx .23\left(\frac{M}{M_\odot}\right)^{2.3} \qquad (M < .43M_\odot)$$
$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{4} \qquad (.43M_\odot < M < 2M_\odot)$$
$$\frac{L}{L_\odot} \approx 1.5\left(\frac{M}{M_\odot}\right)^{3.5} \qquad (2M_\odot < M < 20M_\odot)$$
$$\frac{L}{L_\odot} \approx 3200\frac{M}{M_\odot} \qquad (M > 20M_\odot)$$



Random sampling in mass ranges :

0.1 -- 5 Msun and separately
5 - 150 Msun



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Unresolved binaries have a negligible effect in mass range 1-80 Msun

significantly flatter unresolved IMF below 1 Msun

significantly steeper unresolved IMF above 80 Msun

Stellar mass loss moves stars in the present-day MF towards smaller masses;

mass gain (mergers, overflow from companion) move stars to larger masses;

unresolved binaries remove stars (unseen companions) moving the system to larger mass;

dynamical ejections remove stars from population

---> high-mass end of the IMF is difficult

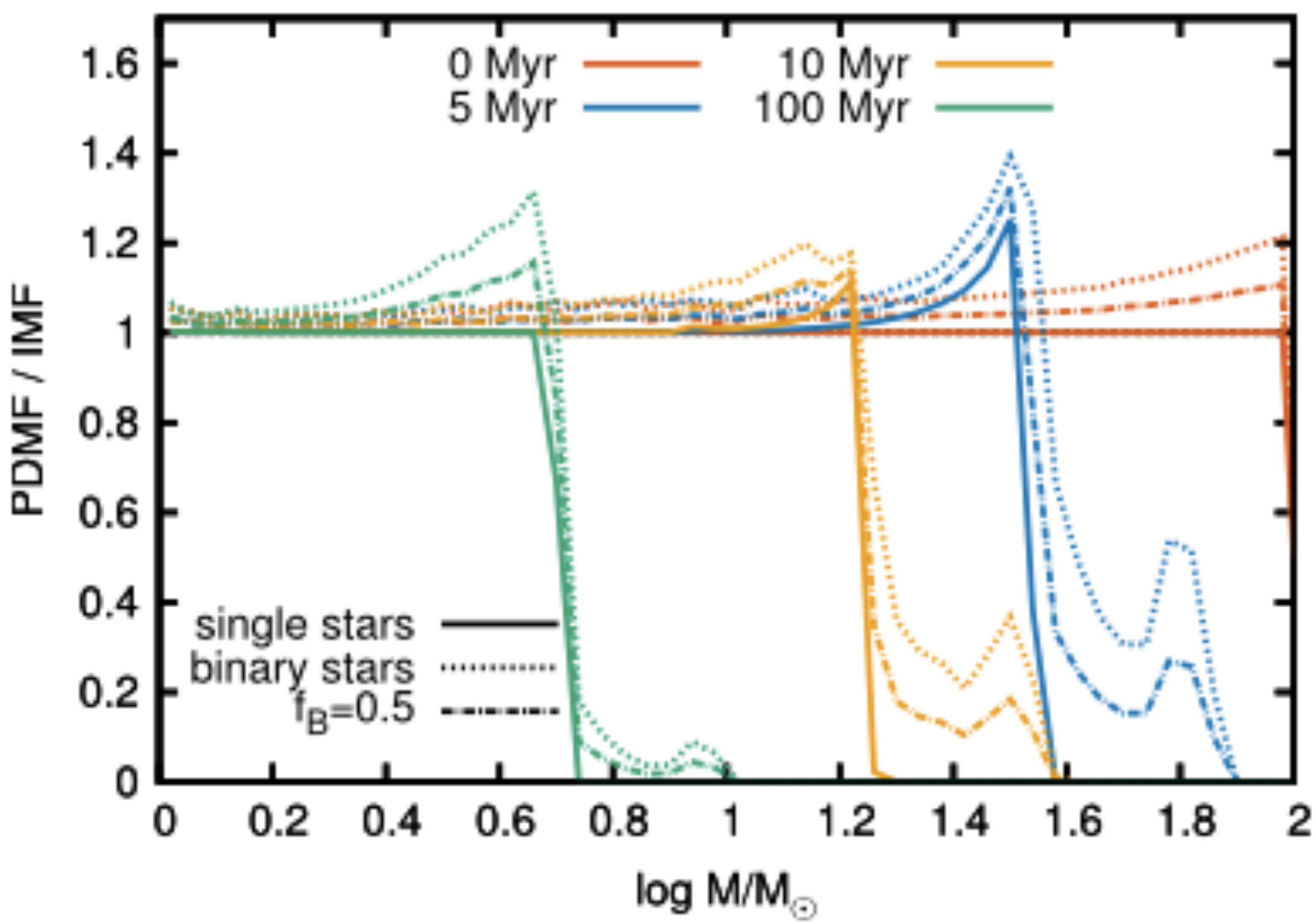
see also e.g. De Donder & VanBeveren (2003) and Zapartas, de Mink et al. (2017)

Weidner et al. (2009) : detailed study of massive unresolved binaries / triples / quadruples, the IMF and stellar evolution

---> apparent age spreads in young clusters and IMF shape not much affected.

Schneider, Izzard et al. (2015)

The Astrophysical Journal, 805:20 (20pp), 2015 May 20



"We have shown above that stellar wind-mass loss, binary products, and unresolved binaries re-shape the high-mass end of mass functions, which may complicate IMF determinations."

see also seminal work by
Scalo (1986);
Massey (2003, ARA&A)

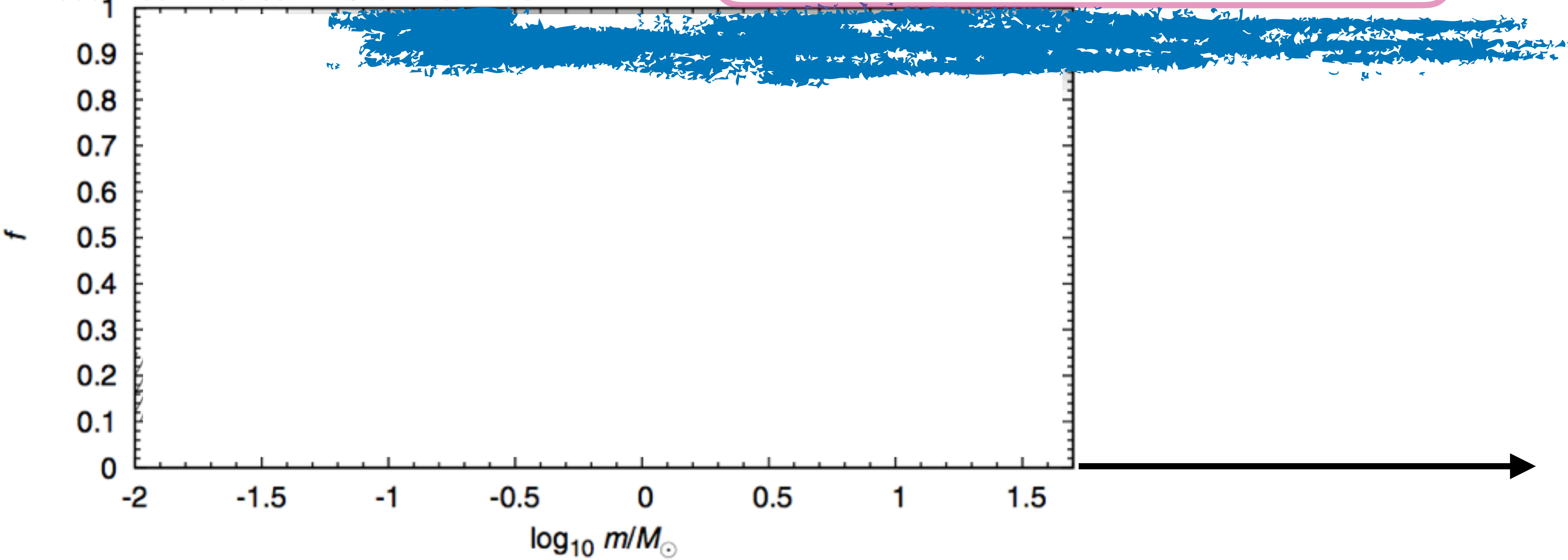
Brown Dwarfs

Binaries provide the clue to their origin

The binary fraction in dependence of primary mass

initial (<1Myr) binary fraction is *independent of primary mass* !!

Thies et al. 2015, ApJ
see also Moe & Di Stefano 2017



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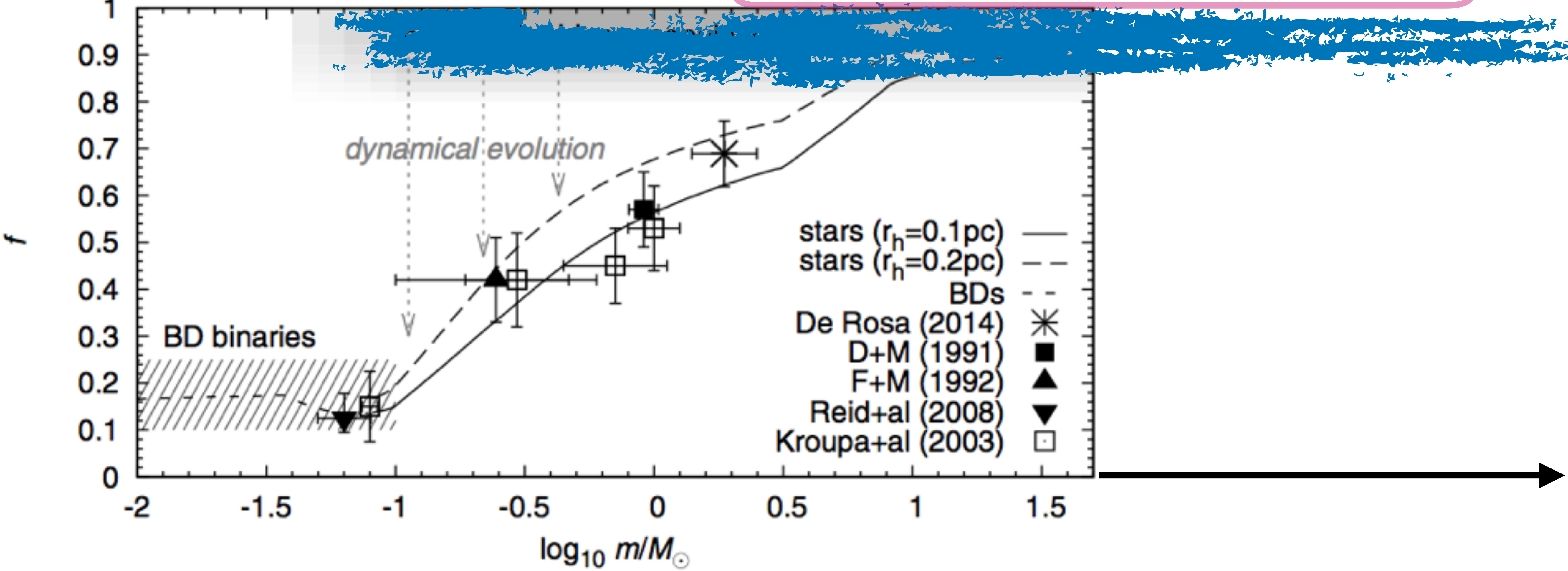
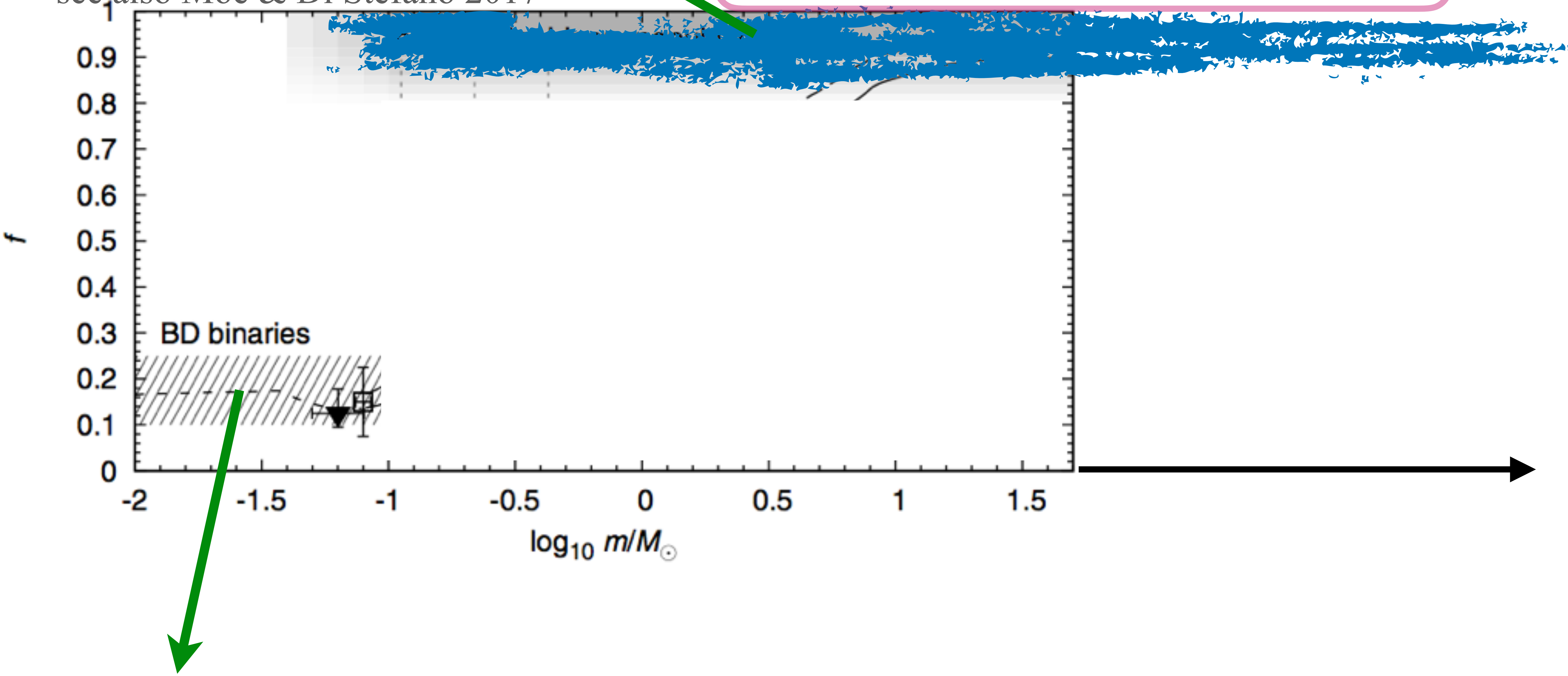


Primary fragmentation => high binary fraction due to angular momentum conservation

initial (<1Myr) binary fraction is *independent of primary mass* !!

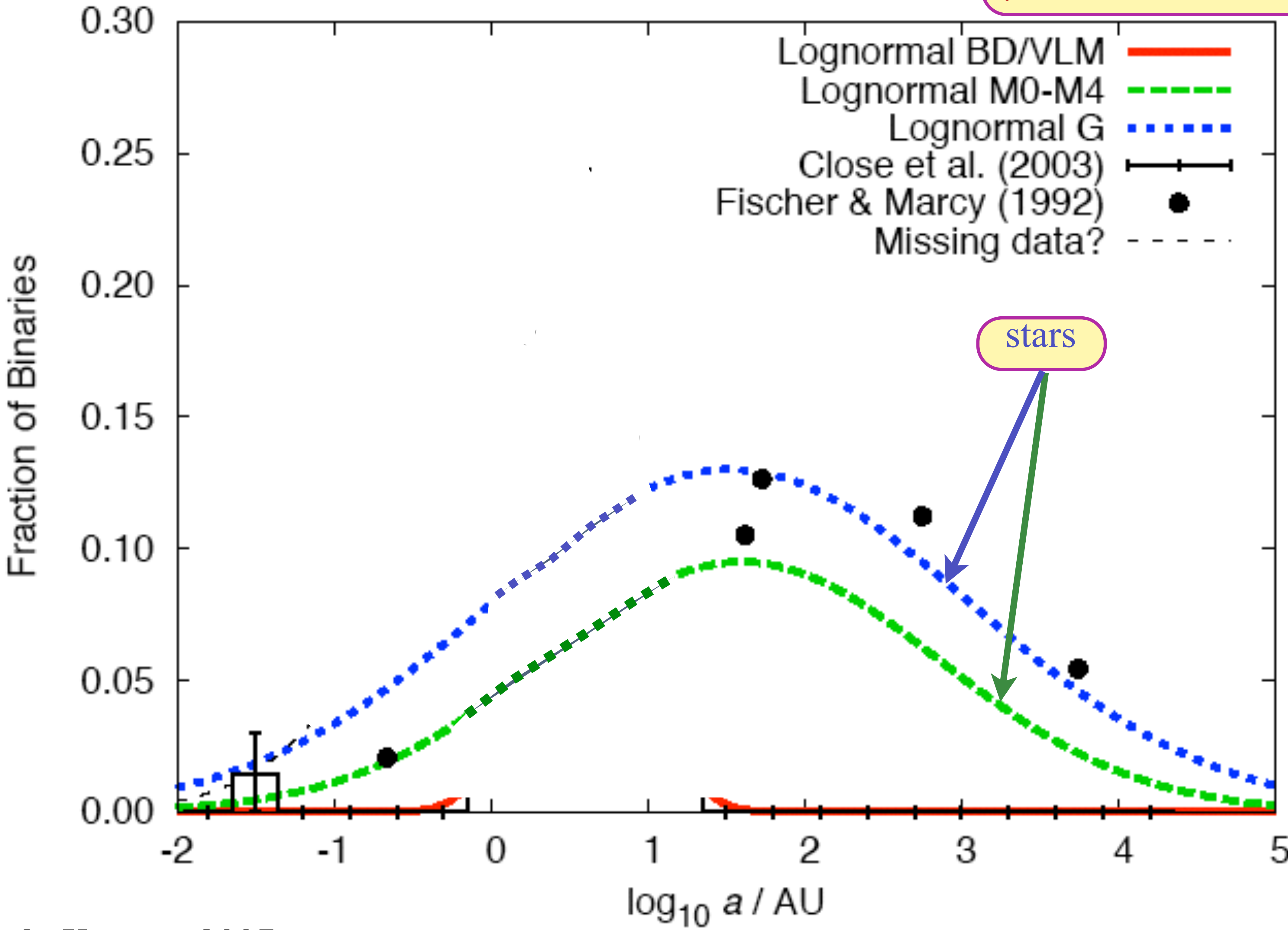
Thies et al. 2015, ApJ
see also Moe & Di Stefano 2017



Peripheral fragmentation => low binary fraction due to chance pairing in circumstellar accretion discs

The semi-major axis distributions of BDs and stars differ :

$f \approx 0.55 \pm 0.10$



Thies & Kroupa 2007

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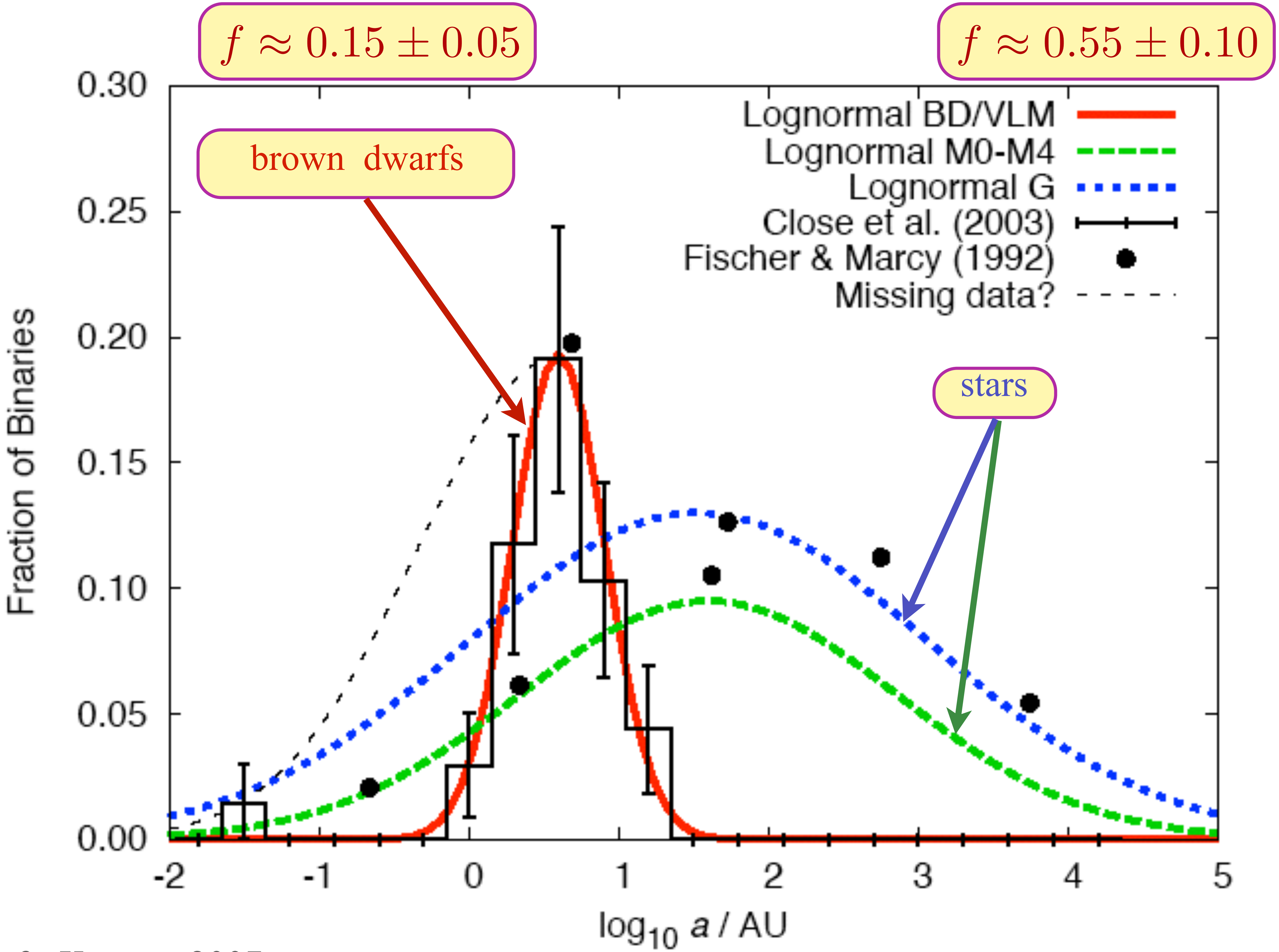
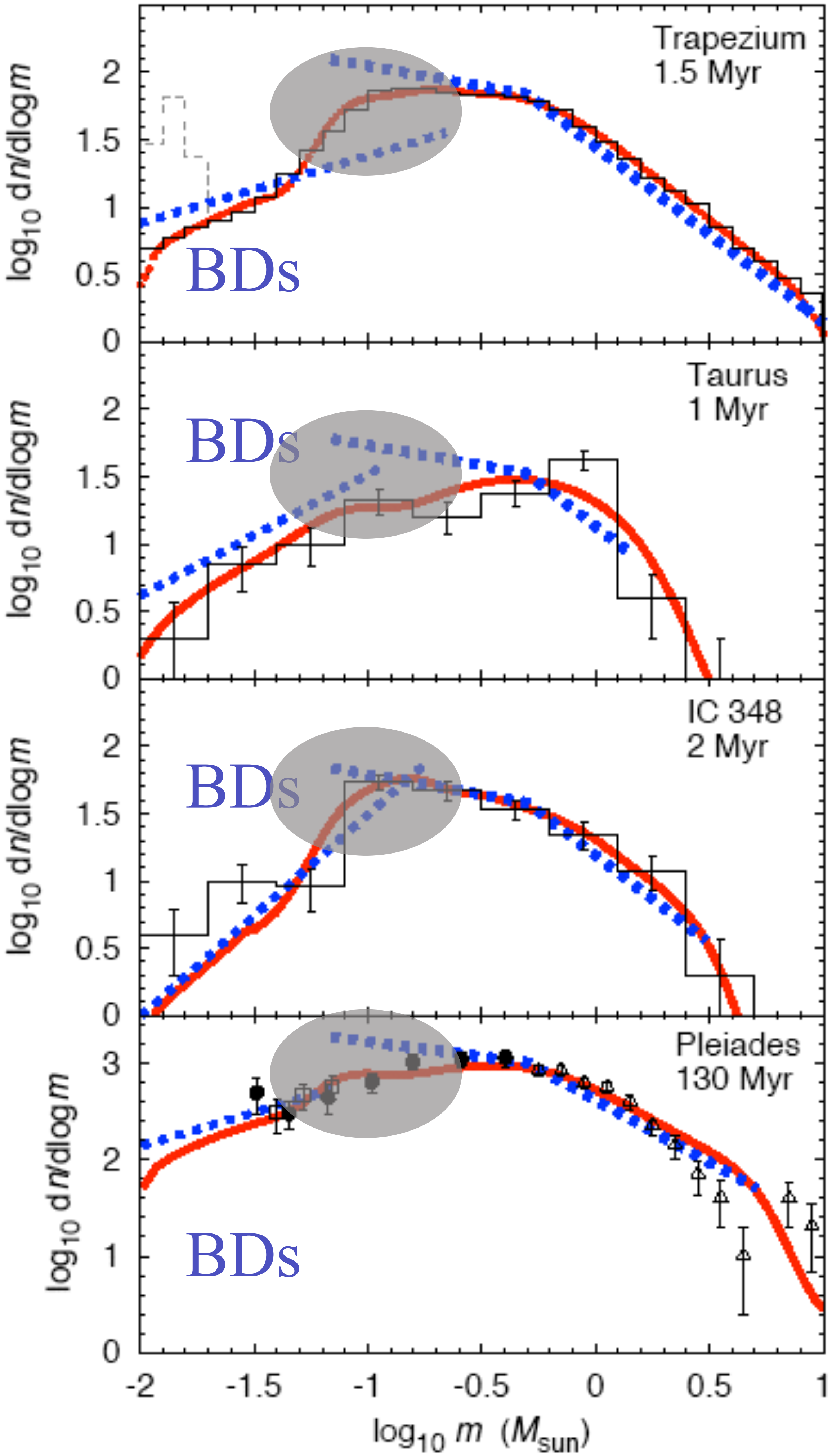


Thies & Kroupa 2007

... this implies that BDs and stars
are
different populations;
they have a
***different dynamical history* !**

see also Riaz et al. (2012);
Dieterich et al. (2012)
Thies et al (2015)
Marks et al. (2015, 2017)



Correcting the observed IMF for unresolved components

(Thies & Kroupa 2007, 2008)

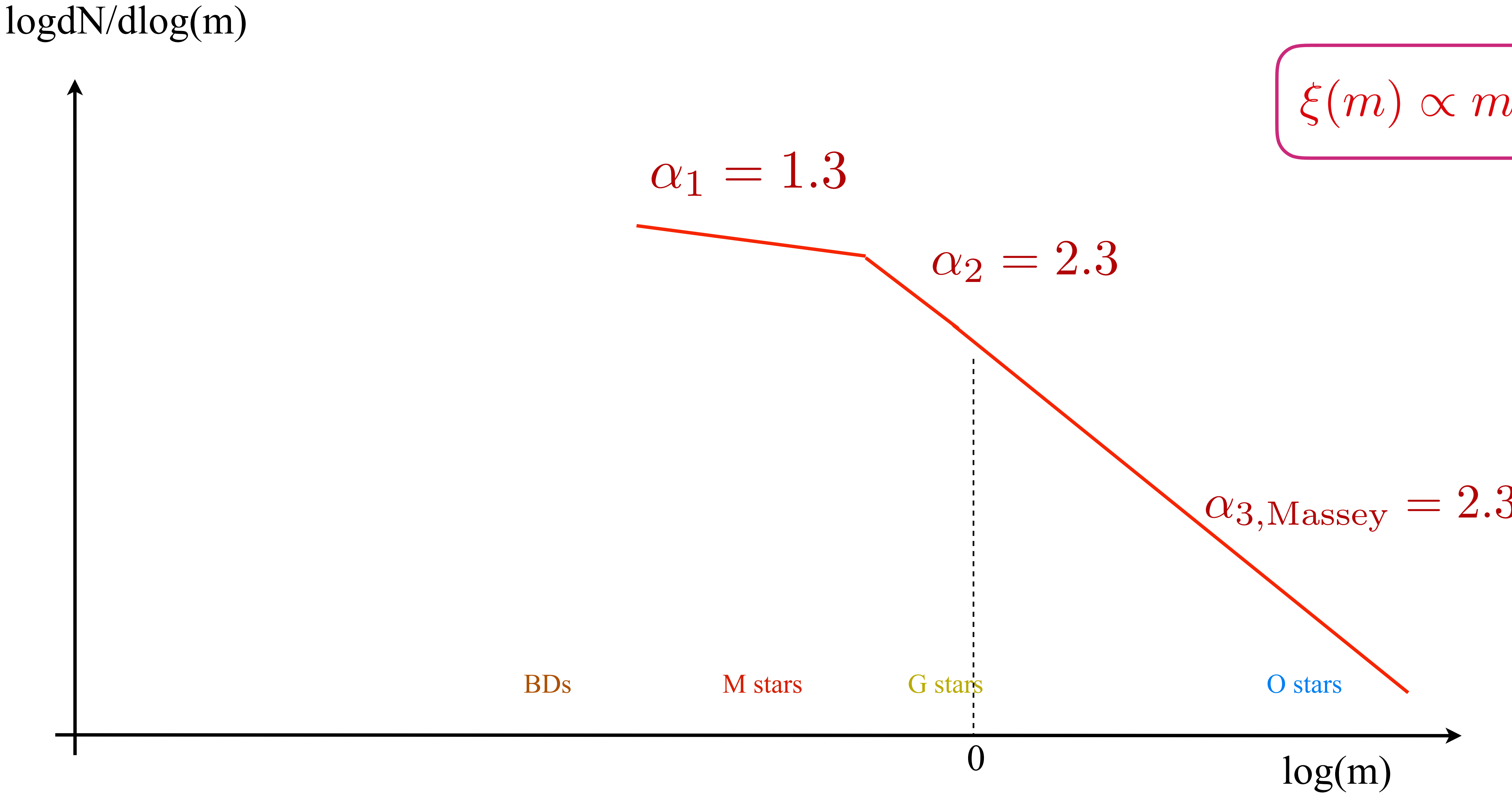
Conclusions

---> The data suggest **:**

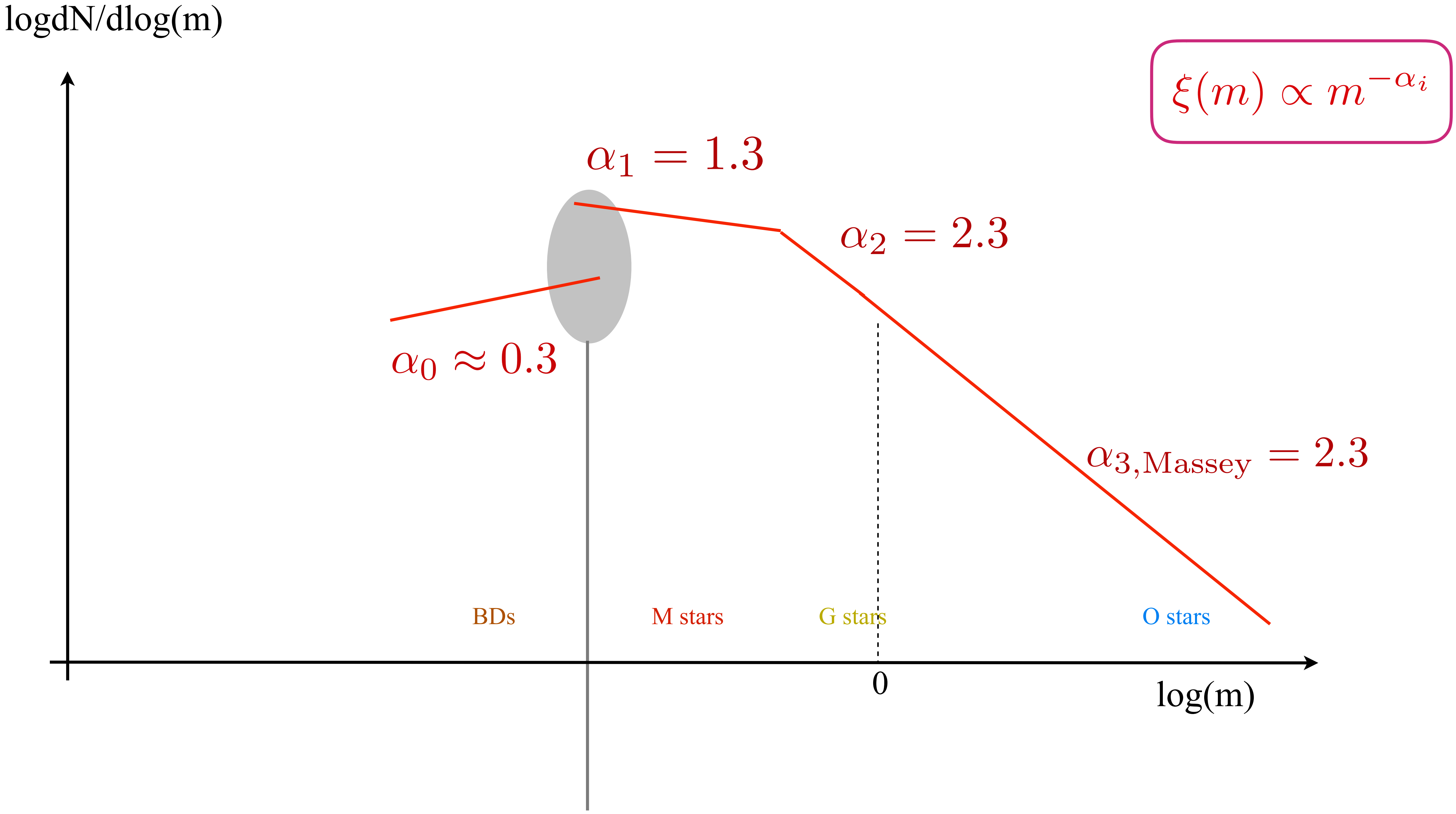
2-part power-law canonical stellar IMF

+ separate BD IMF + separate Planet IMF

$$\xi(m) \propto m^{-\alpha_i}$$



2-part power-law canonical stellar IMF
+ separate BD IMF + separate Planet IMF



BD IMF : Thies & Kroupa (2007, 2008), Parker & Goodwin (2010)

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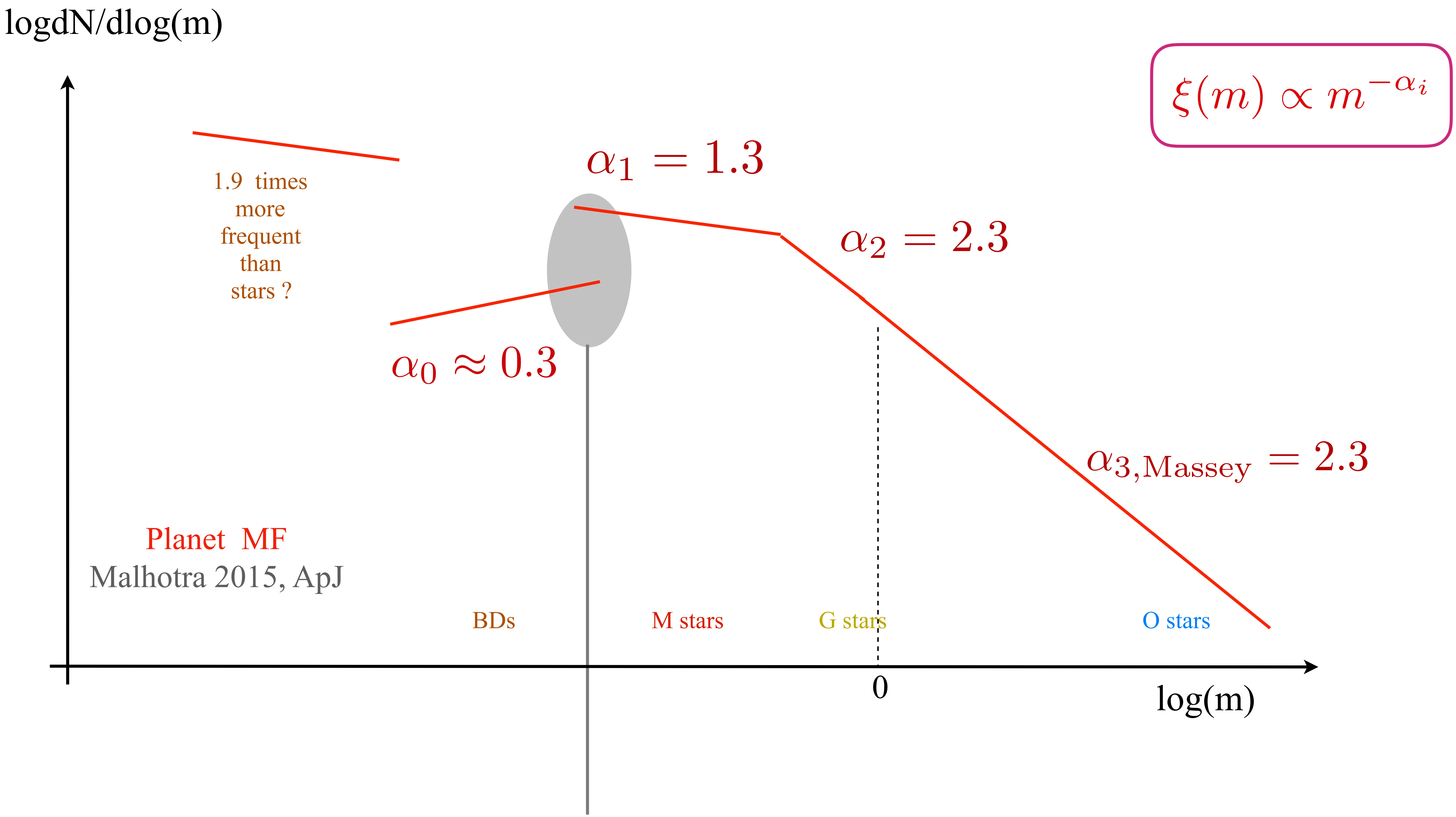
+ separate BD IMF + separate Planet IMF



BD IMF : Thies & Kroupa (2007, 2008), Parker & Goodwin (2010)

What the observations suggest :

Globular clusters : deficit of low-mass stars increases with decreasing concentration

→ disagrees with dynamical evolution

UCDs : higher dynamical M/L ratios

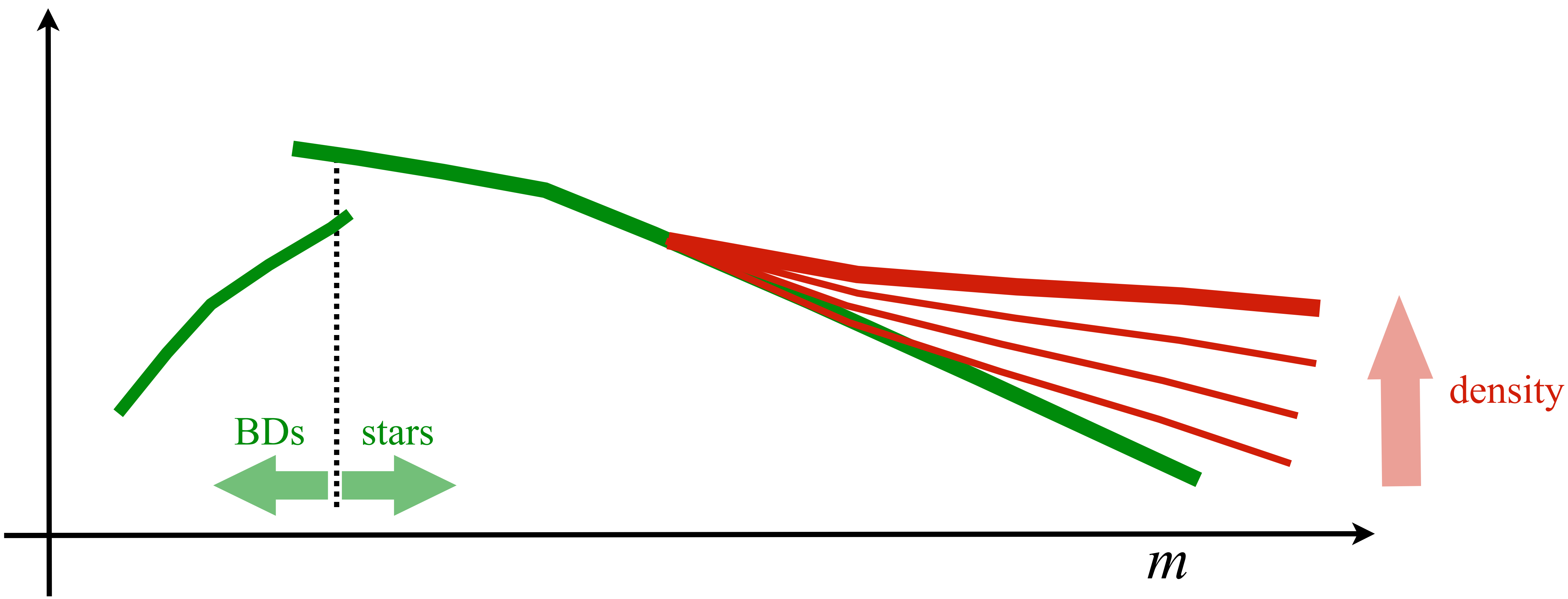
→ cannot be exotic dark matter

UCDs : larger fraction of X-ray sources than expected

→ no explanation other than many remnants

Dabringhausen et al., Marks et al. 2012

What this implies :



Using for $m < 1\,M_\odot$

$$\alpha_{1,2} = \alpha_{1/2c} + 0.5\left[\frac{\mathrm{Fe}}{\mathrm{H}}\right]$$

estimated from resolved MW populations
(Kroupa 2001)

where the canonical
solar-abundance values are

$\alpha_{1c} = 1.3 \quad 0.07 < m/M_\odot < 0.5$

$\alpha_{2c} = 2.3 \quad 0.5 < m/M_\odot < 1.3$

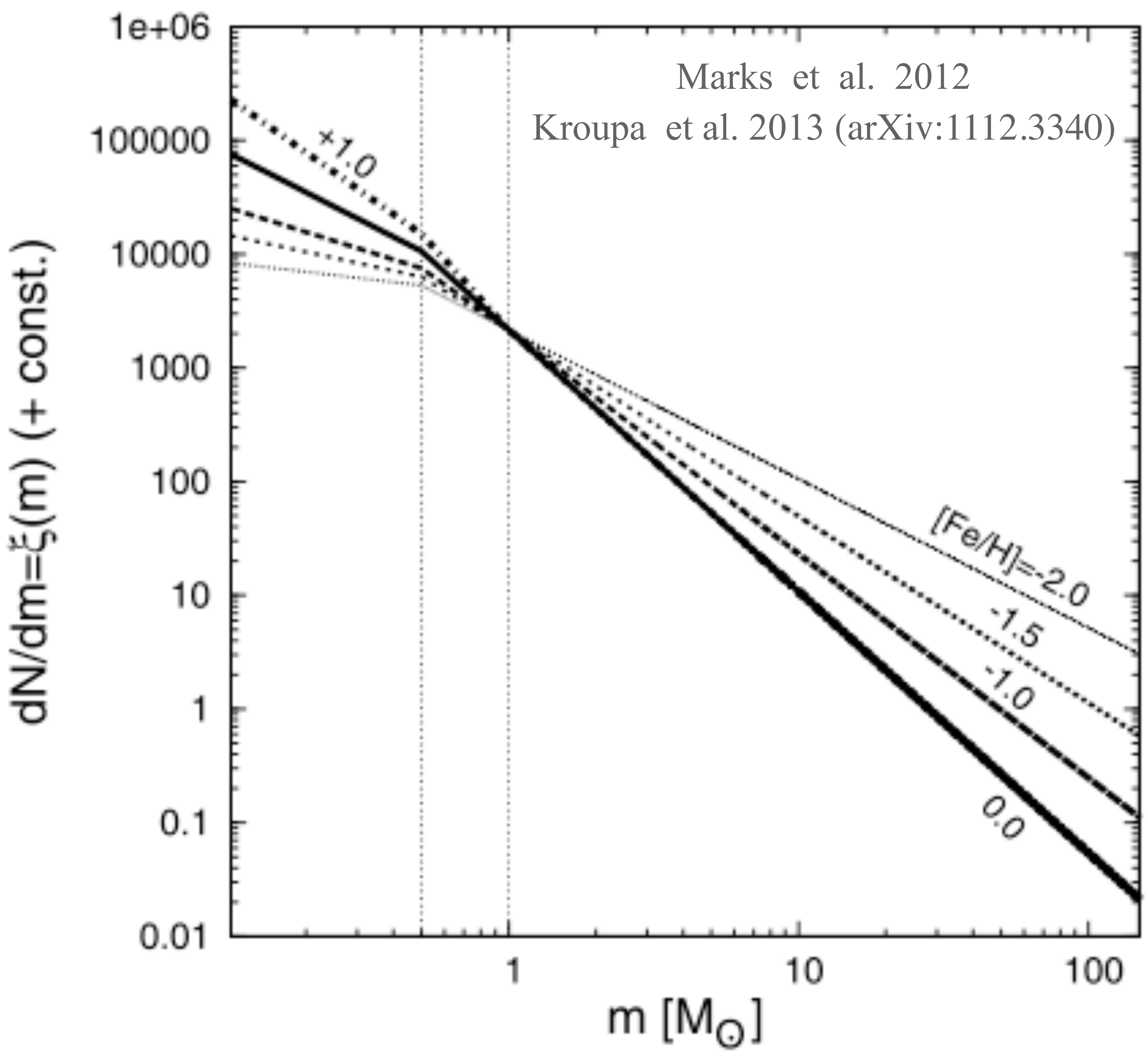


Figure 5. Suggested shape of the stellar IMF for different metallicities, [Fe/H] (not taking into account the density dependence of the IMF). The IMFs are scaled such that their values agree at $m = 1\,\mathrm{M}_\odot$. Above $1\,\mathrm{M}_\odot$ the IMF slope is determined by the present work (Fig. 4, equation 11). Below $1\,\mathrm{M}_\odot$ the parametrization is determined by Kroupa (2001, equation 12), whose results suggest tentative evidence that more metal-rich environments produce relatively more low-mass stars. Note that only the metallicity dependence is shown, but not the dependence on mass (Fig. 2) or density (Fig. 3).

Conclusions

The *stellar IMF* is a computational *hilfskonstrukt*

Very significant progress has been made in constraining the shape of this hilfskonstrukt :

1) surveys (counts of stars)
2) stellar & binary-star evolution
3) binary star population
4) most stars born in embedded clusters
5) stellar-dynamical processes
6) stellar-dynamical evolution

---> The *stellar IMF does change with metallicity and density* and BDs and planets have their own IMFs

stellar IMF ≠ IMF of stars in a galaxy

Thank You !

The *birth binary distribution functions* are equally computational *hilfskonstrukts*

END

Summary

We have seen that

- A birth binary population exists which unifies the observed field population and the pre-main sequence isolated population (e.g. Taurus-Auriga); the *canonical birth binary population* (**BBP**).

- The *field* population derives from the *birth* population because stars form in *clusters*.

- A compact pre-main sequence *eigenevolution theory* exists which creates the observed e, P and q, P correlations for short-P systems.

- A cluster with $N_{\rm bin} = 200$ and $R_{0.5} = 0.8\,{\rm pc}$ gives the right evolution for long-P systems - the typical ("*dominant-mode*") star-forming dynamical system.

- This is the $\Omega^{N,R_{0.5}} = \Omega^{200,0.8}$ operator on the initial binary population.

Summary

For stars in the mass range $0.1 < m/M_\odot <$ few

the **canonical birth binary population** can be described by

$$f_{P,\mathrm{birth}} = 2.5 \, \frac{lP - 1}{45 + (lP - 1)^2}$$

+ **random pairing !**

with $lP_{\max} = 8.43$.

with subsequent pre-main sequence adjustments (*eigenevolution*) for $P < 10^3\,\mathrm{d}$ systems

Issues which affect the determination of the IMF :

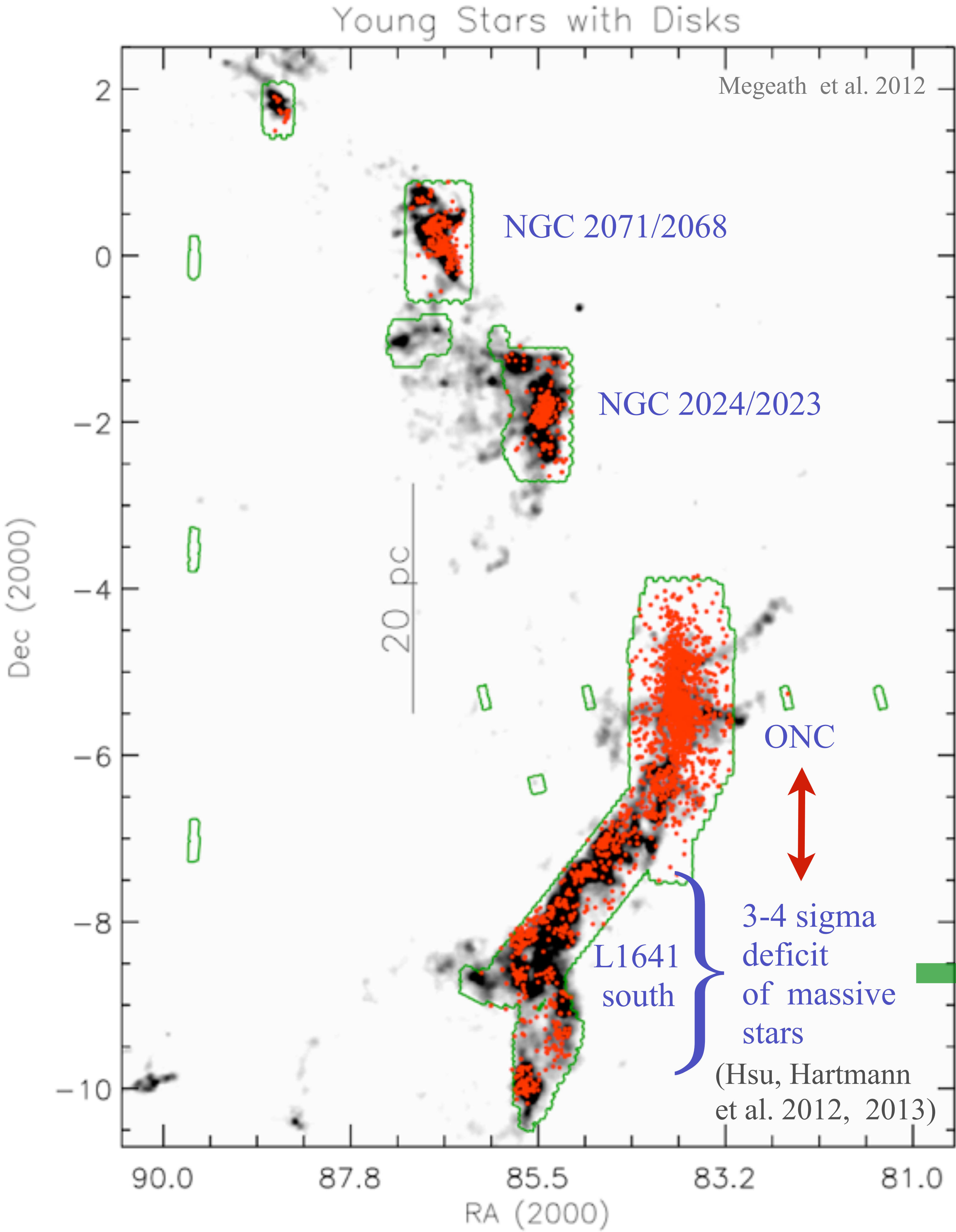
1) the IMF does not exist !

2) What we call the IMF is a mathematical "hilfskonstrukt"

The estimation of this "hilfskonstrukt" is compromised by

the stellar mass-to-light relation
dynamical evolution of birth clusters (very early phase and long-term)
binaries
the most massive star in birth clusters
IMF = probability density distr.function
or an optimally sampled distribution function ?
evidence for top-heavy IMF in extremely massive star burst "clusters"
implications of this for the IMF of whole galaxies.

Lack of O stars



Many small / low-mass groups or clusters do not yield the same IMF as one massive cluster

stochastic IMF in each group is ruled out.

What is an optimally sampled distribution function ?

Given the *mass reservoir* in stellar mass M_{ecl},
starting with the most massive star,
select the next most massive such that
M_{ecl} is distributed over the distribution function
without Poisson scatter.

Kroupa et al. 2013
Schulz, Pflamm-Altenburg & Kroupa 2015

The above ansatz can be extended to a discretized optimal distribution of stellar masses: Given the mass, M_{ecl}, of the population, the following sequence of individual stellar masses yields a distribution function which exactly follows $\xi(m)$,

$$m_{i+1} = \int_{m_{i+1}}^{m_i} m\,\xi(m)dm, \quad m_L \leq m_{i+1} < m_i, \quad m_1 \equiv m_{max}. \tag{4.9}$$

The normalization and the most massive star in the sequence are set by the following two equations:

$$1 = \int_{m_{max}}^{m_{max*}} \xi(m)\,dm, \tag{4.10}$$

with

$$M_{ecl}(m_{max}) - m_{max} = \int_{m_L}^{m_{max}} m\,\xi(m)\,dm \tag{4.11}$$

as the closing condition. These two equations need to be solved iteratively. An excellent approx-

What is an optimally sampled distribution function ?

$$N_{\rm tot} = \int_{M_{\rm min}}^{m_{N_{\rm tot}}} \xi(M)\,{\rm d}M + \int_{m_{N_{\rm tot}}}^{m_{N_{\rm tot}-1}} \xi(M)\,{\rm d}M + \ldots + \int_{m_{i+1}}^{m_i} \xi(M)\,{\rm d}M + \ldots + \int_{m_3}^{m_2} \xi(M)\,{\rm d}M + \int_{m_2}^{M_{\rm max}} \xi(M)\,{\rm d}M, \quad (4)$$

Schulz, Pflamm-Altenburg & Kroupa 2015

$$M_{\rm tot} = \int_{M_{\rm min}}^{m_{N_{\rm tot}}} M\xi(M)\,{\rm d}M + \int_{m_{N_{\rm tot}}}^{m_{N_{\rm tot}-1}} M\xi(M)\,{\rm d}M + \ldots + \int_{m_{i+1}}^{m_i} M\xi(M)\,{\rm d}M + \ldots + \int_{m_3}^{m_2} M\xi(M)\,{\rm d}M + \int_{m_2}^{M_{\rm max}} M\xi(M)\,{\rm d}M, \quad (5)$$

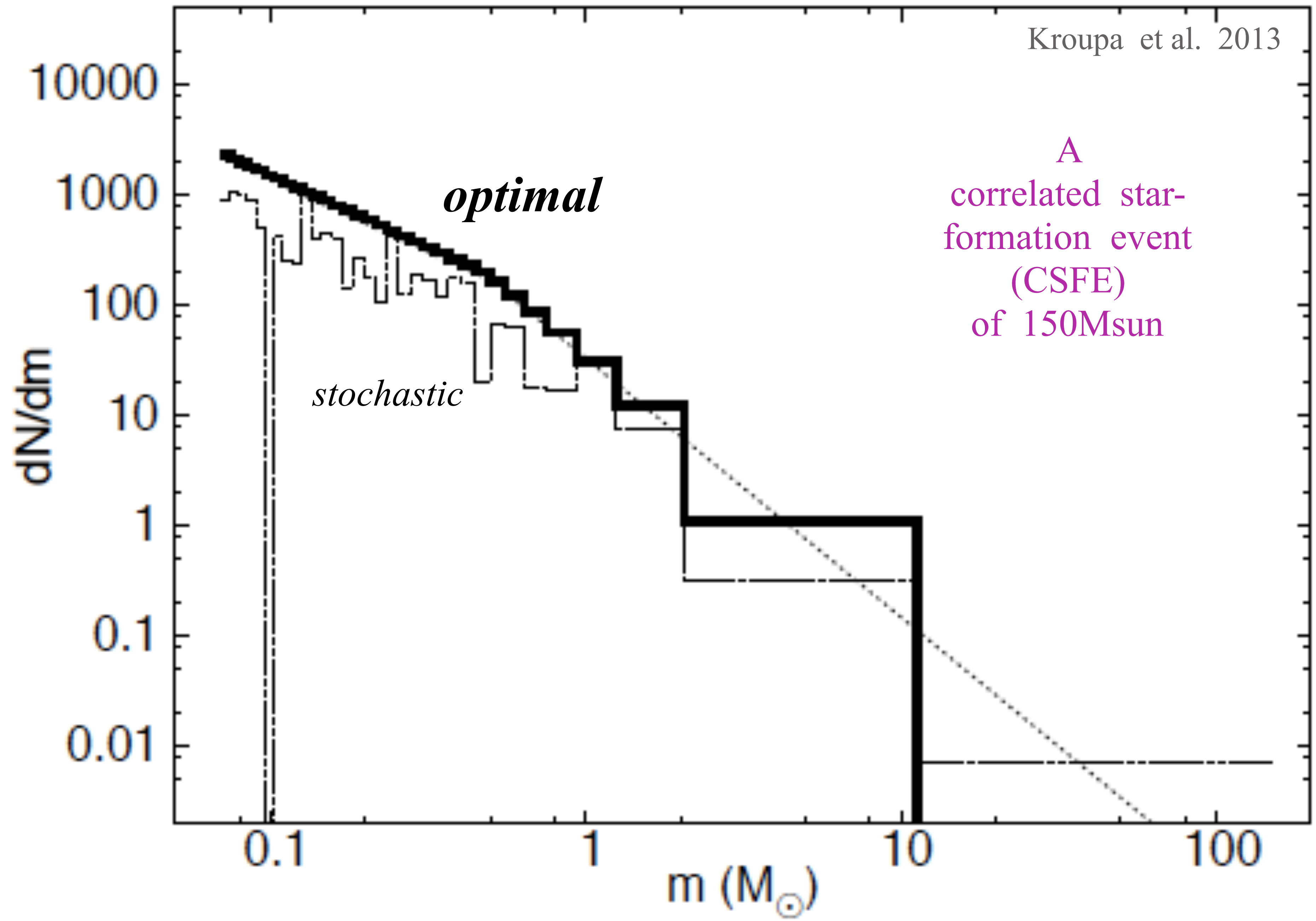
1. Each integral must give one object, i.e. integrating $\xi(M)$ within the limits m_i and m_{i+1} yields exactly unity:

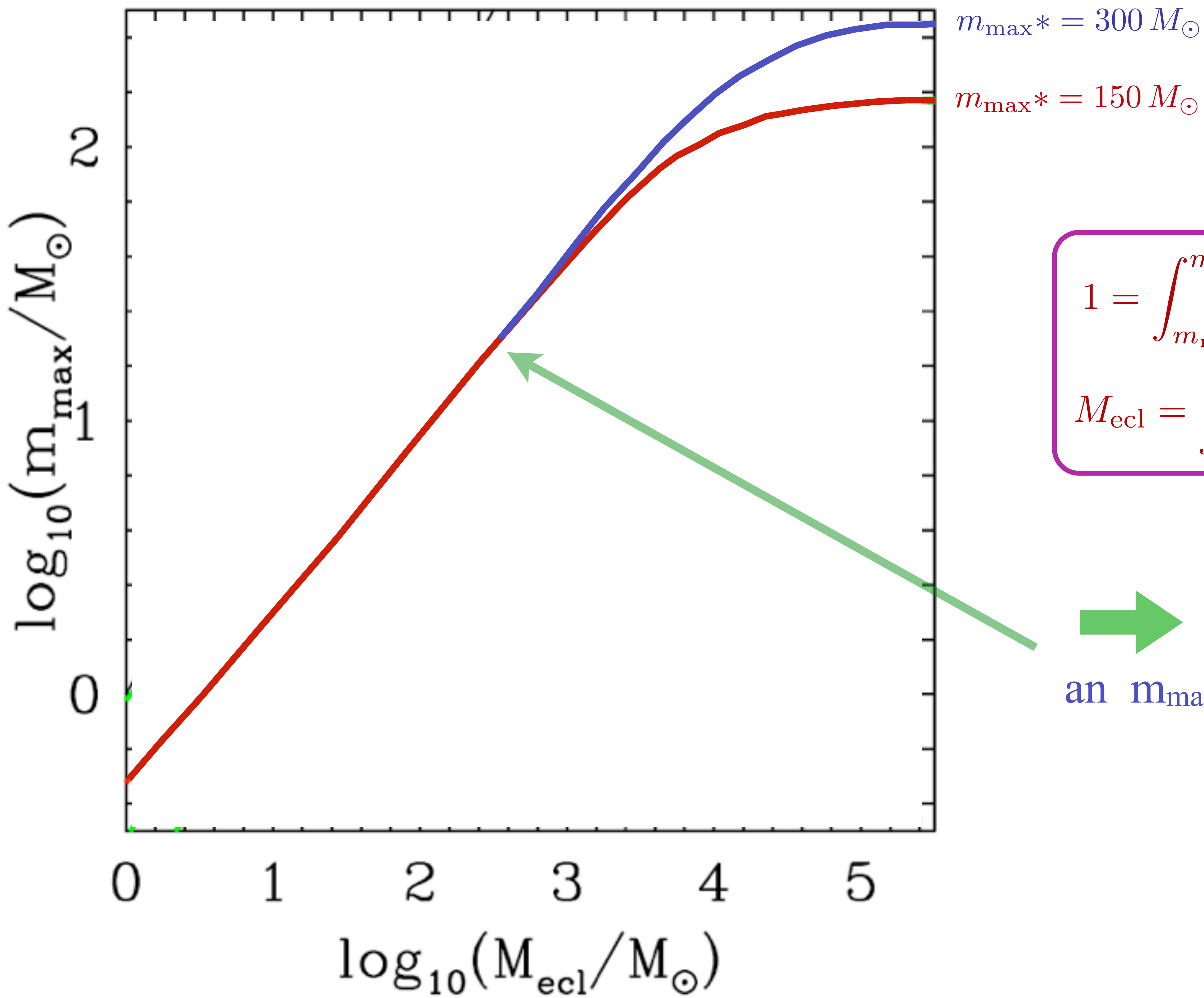
$$1 = \int_{m_{i+1}}^{m_i} \xi(M)\,{\rm d}M. \quad (6)$$

2. Then the mass of this i-th object, M_i, is determined by

$$M_i = \int_{m_{i+1}}^{m_i} M\,\xi(M)\,{\rm d}M, \quad (7)$$

where the limits m_i and m_{i+1} have to be equal to those in eq. 6.


Kroupa et al. 2013
10000
1000
100
10
1
0.1
0.01
dN/dm
optimal
stochastic
A
correlated star-
formation event
(CSFE)
of 150Msun
0.1
1
10
100
m ($M_\odot$)



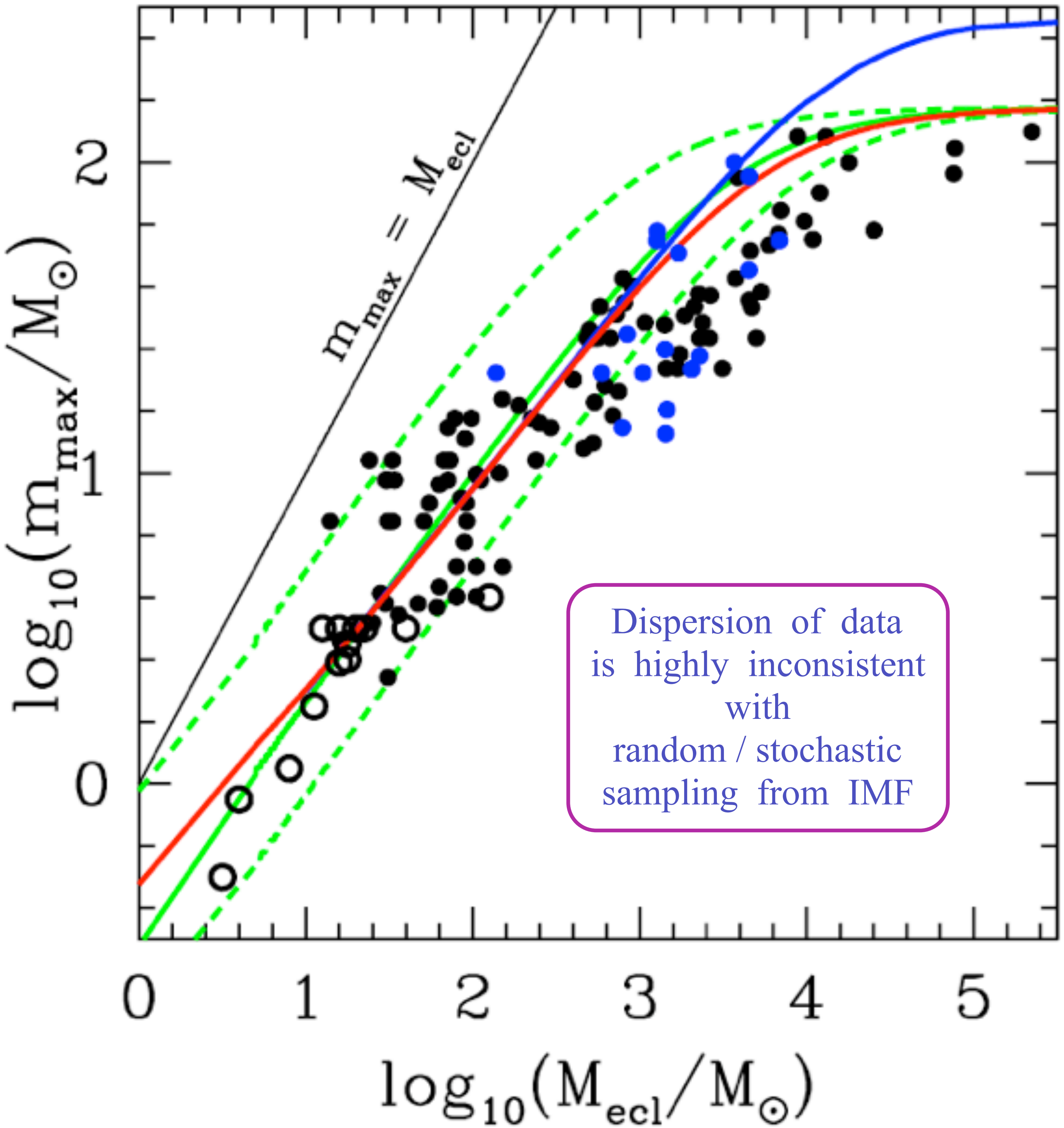
$$1 = \int_{m_{\max}}^{m_{\max *}} \xi(m)\, dm$$

$$M_{\rm ecl} = \int_{m_{\rm l}}^{m_{\max}} m\, \xi(m)\, dm$$

$\Rightarrow$ $m_{\max} = {\rm fn}(M_{\rm ecl})$

an m_{max} -- Mecl relation

Weidner & Kroupa 2005, 2006; Weidner et al. 2010, 2013; Kroupa et al. 2013; Kirk & Myers, 2010, 2012; Hsu, Hartmann et al. 2012, 2013; Ramirez Alegria, Borossiva et al. 2016



$m_{max*} = 300\, M_\odot$

$m_{max*} = 150\, M_\odot$

$$1 = \int_{m_{max}}^{m_{max*}} \xi(m)\, dm$$

$$M_{ecl} = \int_{m_l}^{m_{max}} m\, \xi(m)\, dm$$

$$\Rightarrow \quad m_{max} = \mathrm{fn}(M_{ecl})$$

an mmax -- Mecl relation

Real young populations
are *not stochastic ensembles*
from invariant distr. functions

the IMF / star cluster MF **an invariant probability density distribution function**

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