Statistics of Binary & Multiple Stars: Implications for Formation & Evolution

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The Impact of Binaries on Stellar Evolution
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Topics:

- Detection methods and selection effects
- Corrected joint pdf \( f(M_1, P, q, e) \neq f(M_1) \times f(P) \times f(q) \times f(e) \)
  for systems near ZAMS (\( \tau \sim 5 \text{ Myr} \)) and \( Z = Z_{\odot} \)
- Variations as a function of \( \tau, Z, \) and environment
- Implications for binary star formation and evolution

Main Resources:

- Review by Duchene & Kraus (2013)
- Meta-analysis by Moe & Di Stefano (2017)
Mind your Ps and Qs: $f(P, q) \neq f(P) \times f(q)$

Detection techniques for companions to solar-type primaries (Raghavan et al. 2010)

Relatively complete across $q = M_{\text{comp}}/M_1 = 0.1 - 1.0$ and $\log P (\text{days}) = 0 - 8$
Detection Techniques for Companions to OB-type MS Primaries ($M_1 > 3 \, M_\odot$)

Insensitive across small $q$ & intermediate $P$ – must correct for incompleteness!

**Cannot** directly measure multiplicity fractions $F_{\text{bin}}$, $F_{\text{trip}}$, or $F_{\text{quad}}$.

But can measure **multiplicity frequency** $f_{\text{mult}} = F_{\text{bin}} + 2F_{\text{trip}} + 3F_{\text{quad}}$.  

Moe & Di Stefano (2017)
Wide Companions: log P (days) = 5 - 9; a = 100 - 30,000 AU; 
Core Fragmentation

- $f_{\text{wide}} = 0.5$, initially independent of $M_1$
- $f(q)$ initially consistent with random pairings drawn from IMF
- Subsequent dynamical ejections: systems with smaller $M_1$ and $q$ are preferentially disrupted by ZAMS
Multiplicity Statistics: Diagnostics for Binary Star Formation

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Intermediate-Period Companions:
log P (days) = 1 - 5;
a = 0.1 - 100 AU;
Disk Fragmentation
• \( f_{\text{mid}} = 0.4 \ (M_1 = 1M_\odot) - 1.5 \ (30M_\odot) \)
• \( f(q) \) correlated due to co-evolution / shared accretion in the disk
• \( M_1 = 1M_\odot \): uniform \( f(q) \)
• \( M_1 > 5M_\odot \): weighted toward \( q = 0.2 \)
**Multiplicity Statistics: Diagnostics for Binary Star Formation**

(Abt+ 90; Kroupa 95a,b; Bate+ 95,02; Tokovinin 00,06; Tohline 02; Goodwin & Kroupa 05; Sana+ 12; Kratter+ 06,10; Raghavan+ 10; Offner+ 12; Duchene & Krause 13; Tobin+ 16a,b; Moe & Di Stefano 17)

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**Very Close Binaries**
- \( \log P \) (days) < 1;
- \( a < 0.1 \) AU;

*Dynamical Hardening in Triples during Pre-MS*
- \( f_{\text{close}} = 0.02 \) (\( M_1 = 1M_\odot \)) - 0.2 (\( 30M_\odot \))
- Most have outer tertiaries
- Uniform \( f(q) \) with excess twin fraction
Corrected frequency of companions with $q > 0.1$ and $\log P (\text{days}) < 8$ ($a < 10,000 \text{ AU}$) per primary near ZAMS

Solar-type MS ($M_1 = 1M_\odot$): $f_{\text{mult}} = 0.50 \pm 0.04$;  
$F_{\text{single}} = 60\%$, $F_{\text{bin}} = 30\%$, $F_{\text{trip/quad}} = 10\%$

O-type MS ($M_1 = 28M_\odot$): $f_{\text{mult}} = 2.1 \pm 0.3$;  
$F_{\text{single}} < 5\%$, $F_{\text{bin}} = 20\%$, $F_{\text{trip/quad}} = 75\%$
For $M_1 = 1 M_\odot$, frequency of **wide** companions ($a > 100 \text{ AU}$) is 2 - 3 times larger during the early pre-MS phase ($\tau < 3 \text{ Myr}$) (Ghez et al. 1993; Duchene et al. 2007; Connelley et al. 2008; Tobin et al. 2016) 

Overall multiplicity frequency: $f_{\text{mult}} = 0.5 \rightarrow 0.8$
Corrected frequency of companions with $q > 0.1$ and $\log P \text{ (days)} < 8$ ($a < 10,000 \text{ AU}$) per primary near ZAMS

$f_{\text{mult}} \approx 1.0$ for all $M_1 < 1M_\odot$ during pre-MS, but disruption of wide binaries with smaller binding energies (smaller $M_1$ and $q$) reduces $f_{\text{mult}}$ by ZAMS (Goodwin & Kroupa 2005; Marks & Kroupa 2011; Thies et al. 2015)
For $M_1 > 1M_\odot$, disk fragmentation is progressively more likely with increasing primary mass (Kratter et al. 2006, 2011).
Period distribution $f_{\log P; q>0.3}(M_1, P)$ from Moe & Di Stefano (2017)

~2% of solar-type MS primaries have companions with $q > 0.3$ and $P = 1 - 10$ days.

Integral under dotted lines yields the MS multiplicity frequency $f_{\text{mult}; q>0.3}(M_1)$. 
Companions to solar-type MS stars: log-normal period distribution as found by Duquennoy & Mayor (1991) and Raghavan et al. (2010)
Period distribution $f_{\log P; q>0.3}(M_1, P)$ from Moe & Di Stefano (2017)

Very close binary fraction increases dramatically with $M_1$ (Abt et al. 1990; Sana et al. 2012; Chini et al. 2012; Kobulnicky et al. 2014)
Early-type MS stars also have a large companion frequency at intermediate \( P \); Rizzuto+2013, LBI, early-B; Sana+2014, LBI, O-type; Evans+2015, SB2s, Cepheids

Disks around massive protostars are more prone to fragmentation (Kratter et al. 2006, 2011)
periods.

This effect may be non-monotonic with respect to mass for all orbital periods. In this case, our model for large $q > 0.1$ is completely described by the analytic functions that fit the data as well. Not all surveys we have examined in this study are sensitive to binaries with primaries, the companion frequency is corrected binary frequency of $0.22 \pm 0.03$ for $\log P > 4$ orbital periods. For mid-B and early-B bright stars, we find $0.06 \pm 0.01$ for the combined SB sample of companions with size $< 50$ AU). For early-type primaries, the companion frequencies are sensitive to binaries with early-B primaries, the companion frequency is $0.1 - 0.2$ are substantially larger at short $\log P$ than $\log P > 0.1$, which is why $\log P(q > 0.1)$ / $N$ is consistent with theories of core fragmentation (Goodwin & Kroupa 2005; Offner et al. 2012; Thies et al. 2015).
Distribution $f_q(M_1,P)$ of mass ratios $q = M_{\text{comp}}/M_1$

A single-component power-law model $f_q \propto q^{\gamma}$ does NOT adequately describe the data.

Need 3 parameters:

1. $\gamma_{\text{small}}(M_1,P)$: power-law slope across $q = 0.1 - 0.3$
2. $\gamma_{\text{large}}(M_1,P)$: power-law slope across $q = 0.3 - 1.0$
3. $F_{\text{twin}}(M_1,P)$: excess fraction of twins with $q \approx 1.0$

Let the data be your guide!

$\gamma_{\text{small}} = 0.5$
$\gamma_{\text{large}} = -0.5$
$F_{\text{twin}} = 25\%$

(Halbwachs et al. 2003)
Excess Twin Fraction $F_{\text{twin}}(M_1,P)$

$M_1 = 1 - 3 \, \text{M}_\odot$ \quad $M_1 = 3 - 30 \, \text{M}_\odot$

(adapted from Moe & Di Stefano 2017)

Solar-type binaries: larger $F_{\text{twin}}$ due to pre-MS RLOF and/or shared accretion in longer-lived disks (Kroupa 1995; Tokovinin 2000; Halbwachs et al. 2003)

Massive protostars have rapid contraction timescales and shorter disk lifetimes
Mass-ratio distribution $f_q(P)$ for solar-type primaries $M_1 = 1M_\odot$

**a < 100 AU:**
uniform $f_q$ with excess twins (fragmentation & co-evolution in disk)

**a > 100 AU:**
weighted toward small $q = 0.2 - 0.4$ (core fragmentation + dynamical ejections)

Data (Raghavan et al. 2010); corrections for selection biases & missing stellar companions $\diamondsuit$ and $\triangle$ (Moe & Di Stefano 2017)
Mass-ratio distribution $f_q(P)$ for OB-type MS primaries $M_1 > 3M_\odot$

For $P < 20$ days:

$\gamma_{\text{large}q} = -0.4$; 
$F_{\text{twin}} = 10 - 20$

For $P = 20 - 500$ days:

$\gamma_{\text{large}q} = -1.6$; 
$F_{\text{twin}} < 5$

(Moe & Di Stefano 2017)
Power-law component $\gamma_{\text{largeq}}(M_1,P)$ of mass-ratio distribution $f_q$

Solar-type MS binaries (Raghavan et al. 2010);
SB2 companions to O / early-B MS primaries (Abt+90; Sana+12; Kobulnicky+14)
Power-law component $\gamma_{\text{largeq}}(M_1, P)$ of mass-ratio distribution $f_q$

Eclipsing binaries with early-B primaries
(Moe+ 2013; 2015)
Power-law component $\gamma_{\text{largeq}}(M_1, P)$ of mass-ratio distribution $f_q$

Radial velocity companions to Cepheid primaries (Evans+ 2015)
Power-law component $\gamma_{\text{largeq}}(M_1,P)$ of mass-ratio distribution $f_q$

Long-baseline interferometry of early-B (Rizzuto+ 2013) and O-type (Sana+ 2014) MS primaries
Power-law component $\gamma_{\text{largeq}}(M_1, P)$ of mass-ratio distribution $f_q$

Visual companions to early-type MS primaries, including CPM (Abt+ 1990), AO (Duchene+ 2011; Sana+ 2014), lucky imaging (Peter+ 2012), and HST imaging (Aldoretta+ 2015)
Power-law component $\gamma_{\text{largeq}}(M_1,P)$ of mass-ratio distribution $f_q$

Very close binaries ($a < 0.1$ AU): uniform $f_q$ with excess twin fraction.
Wide binaries ($a > 100$ AU): initially consistent with random pairings from IMF.
Transition occurs at shorter separations for more massive binaries.
Power-law component $\gamma_{\text{largeq}}(M_1, P)$ of mass-ratio distribution $f_q$

The mass-ratio distribution $f_q$ of binaries that will interact ($a < 10$ AU; left of dotted line) depends critically on $M_1$ and $P$
The tidal circularization period increases from $P_{\text{circ}} = 6$ days (pre-MS) to $P_{\text{circ}} = 15$ days (halo) beyond $P > 100$ days, $f_e \propto e^\eta$ with $\eta = 0.4$. 

Eccentricity distribution for solar-type binaries (Meibom & Mathieu 2005)
Eccentricity distribution for early-type binaries

\[ P_{\text{circ}} = 2 \text{ days} \]
(tides less efficient in early-type stars)

\[ e_{\text{max}}(P) : \text{maximum eccentricity without filling Roche-lobe at periastron} \]

Let the data be your guide!
Eccentricity distribution $f_e (M_1, P)$

$$f_e \propto e^n$$ across domain $0 < e < e_{\text{max}}(P)$

Tidal circularization dominates at $P < 20$ days

For $P > 20$ days, early-type binaries are consistent with a thermal distribution ($\eta = 1$; Ambartsumian 1937), indicating dynamical interactions play a role in their formation.
Period distribution $f_{\log P; q > 0.1}(M_1, P, \tau)$ from Moe & Di Stefano (2017)

**Frequency of companions with $q > 0.1$ per decade of period**

Frequency of wide companions ($a > 100 \text{ AU}$) to $M_1 = 1M_\odot$ pre-MS primaries is 2 - 3 times larger than that measured for $M_1 = 1M_\odot$ MS stars (Duchene+07, Connelley+08) and consistent with that measured for O-type MS primaries.
For pre-MS (Mathieu 94; Melo 03) and solar-type MS primaries in open clusters (Patience+03; Geller+12; Leiner+15), the companion frequency across a < 100 AU matches that for field solar-type MS stars, which is substantially smaller than that measured for O-type MS stars.
Very close binaries derive from dynamical interactions in triples

\[ F_{\text{close}} = 0.16 F_{\text{triple/quadruple}} \]

~80% of solar-type binaries with \( P_{\text{inner}} < 7 \) days have tertiary companions, while only ~30% of slightly wider binaries with \( P_{\text{inner}} > 20 \) days have such tertiary components (Tokovinin+ 2006)

The very close binary fraction (P < 7 days) is directly proportional to the overall triple/quadruple star fraction, independent of \( M_1 \) (Moe & Di Stefano 2017)
Very close binaries derive from dynamical interactions in triples
**BUT mostly during the early pre-MS phase**

Two mechanisms:  

1) Kozai-Lidov oscillations in stable triples coupled with tidal friction (Kiseleva+ 98; Fabrycky & Tremaine 07)

Moe & Kratter 2017 (on arXiv today)

20%  
MS

20%  
pre-MS

2) Dynamical unfolding of unstable triples combined with significant energy dissipation in disk (Bate+ 02, 09)

Inclined tertiaries with
\(a_{\text{outer}} = 500 - 5,000 \text{ AU}\)

Inclined tertiaries with
\(a_{\text{outer}} = 20 - 1,000 \text{ AU}\)

Coplanar tertiaries with
\(a_{\text{outer}} = 0.5 - 50 \text{ AU}\)

20%  
pre-MS

60%  
pre-MS
Frequency of companions with 
log P (days) < 3.7 
(a < 10 AU) 
and q > 0.1

- Only 15% of $M_1 = 1 \, M_\odot$ primaries will interact via RLOF
- 80 - 90% of O-type primaries will experience RLOF (consistent with Sana+ 2012)
- 10 - 20% of O-type primaries are in compact triples with $a_{\text{outer}} < 10$ AU
Even for $a < 10$ AU, the distribution of ZAMS binary properties are highly correlated (see also Abt+ 1990 and Duchene & Kraus 2013):

$$f(M_1, P, q, e) \neq f(M_1) \times f(P) \times f(q) \times f(e)$$

The density of binaries in certain pockets of the $f(M_1, P, q, e)$ parameter space differs by up to a factor of $\sim 50$ compared to canonical initial conditions adopted in many binary population synthesis studies.

Separately adjusting the individual distributions $f(P)$, $f(q)$, and $f(e)$ to the extremes will still not encompass the true nature of the binary population.

Monte Carlo code that generates population of binaries based on observed $f(M_1, P, q, e)$ is available.
Binary evolution affects your multiplicity statistics (Moe & Di Stefano 2017)

For a volume-limited sample:

30% ± 10% of massive stars are the products of binary evolution (de Mink+ 2014)

20% ± 10% of early-type “primaries” are actually the secondaries in which the true primaries have already evolved into compact remnants

11% ± 4% of solar-type “primaries” have WD companions

30% ± 10% of SB1s contain compact remnant companions

Solar-type SB1s:
1) Sirius-like binaries with hot WDs
2) Barium stars

Early-type SB1s:
1) EBs vs. SB1s
2) N(SB1s)/N(SB2s) increases with age
Regulus (α Leonis - the heart of the lion):
- SB1 with $P = 40$ days
- Rapidly rotating B-type star
- Companion either K-dwarf or WD

$30 \pm 10\%$ of SB1s have compact remnant companions
(Moe & Di Stefano 2017)
How the close binary fraction changes with decreasing metallicity $Z$

<table>
<thead>
<tr>
<th>Reference</th>
<th>Spectral Type</th>
<th>Minimum log($Z/Z_{\odot}$)</th>
<th>As $Z\downarrow$, $\Delta F/F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carney+ 2005</td>
<td>G</td>
<td>$-2.4$</td>
<td>$&lt;30%$</td>
</tr>
<tr>
<td>Gao+ 2014</td>
<td>G</td>
<td>$-1.5$</td>
<td>$+50%$</td>
</tr>
<tr>
<td>Hettinger+ 2015</td>
<td>F</td>
<td>$-1.7$</td>
<td>$-25%$</td>
</tr>
<tr>
<td>Moe+ 2013</td>
<td>B</td>
<td>$-0.7$</td>
<td>$&lt;20%$</td>
</tr>
<tr>
<td>Dunstall+ 2015</td>
<td>B</td>
<td>$-0.4$</td>
<td>$&lt;30%$</td>
</tr>
</tbody>
</table>

Variations with respect to $Z$ are small and possibly due to sensitivity and selection biases, e.g., lower-metallicity stars are systematically older and more likely to contain WD companions.
Multiplicity Statistics: Diagnostics for Binary Star Formation and Initial Conditions for Binary Star Evolution (Moe & Di Stefano 2017)

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