Data modeling and interpretation of interferometric data

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Tone of the talk

- Oriented toward instrumental aspects with important effects on the science data
- Not exhaustive
- Through different examples, gives general methods
Bandwidth smearing

- The effects of the wavelength bandpass on the interferometric observables
- Most beam combiners have crude spectral resolution to favor sensitivity
- e.g. whole K band, $R = \frac{\lambda}{\Delta \lambda} = \frac{2.2}{(2.4-2.0)} = 4.4$
our setting

• single baseline interferometer

• we measure only the visibility (amplitude of the fringes)

• we want to measure stellar angular diameters using the first null of the visibility curve

\[ V = 2 \frac{J_1(\pi B \theta / \lambda)}{\pi B \theta / \lambda} \]

in broad band

• our beam combiner has $R = \frac{\lambda}{\Delta\lambda} = 5$

• What is the observed visibility in broad band?
Reasoning

- Beam combiner sees the **sum** of fringes for each wavelengths inside the band.

- The observed visibility must be the **average** of the visibility in the band.
What is going on ?!?
There is information in the V(B) curve

\[
V_{UD}(B, \lambda) = \left| 2 \frac{J_1(x = \pi B \theta / \lambda)}{x} \right|
\]

centro-symmetric: Hankel transform

\[
V(B, \lambda) = \frac{\int I(r, \lambda) J_0(rB/\lambda) r dr}{\int I(r, \lambda) dr}
\]
• Measuring diameters == inverting $V(B, \Theta, \lambda)$

• True stars are NOT uniform disks

• limb darkening

• lowers the visibility lobes

• bias the diameter measurements
General considerations

• The instrument does not observe visibility, it observes fringes

• An estimator is used to derived the visibility from the fringes using a data reduction software (DRS)
The correct approach

The numerical model should match

- the estimator
- the instrumental characteristics
- the object characteristics
we were on the right direction...

- we **synthesized** a signal using an instrumental characteristic: bandwidth smearing

- we used an **estimator** of the visibility:

  \[ V \sim \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]
better yet, do it analytically
Let’s start all over again…

- What is the visibility estimator?
- What instrument’s characteristics should I take into account?
- What object’s characteristics should I take into account?
Visibility estimator

- visibility has additive noise: $V + n$
- we measure fringe’s contrast $\mu = |V+n|$
- averaging:
  - $\langle \mu \rangle = \langle |V+n| \rangle$
  - $\langle \mu \rangle$ is biased
- what about $\langle \mu^2 \rangle$?
  - $\langle \mu^2 \rangle = \langle |V+n|^2 \rangle$
  - $\langle \mu^2 \rangle = \langle |V|^2 \rangle + \langle 2\text{Re}\{Vn\} \rangle + \langle |n|^2 \rangle$
  - assuming $V$ and $n$ are uncorrelated:
    - $\langle \text{Re}\{Vn\} \rangle = 0$
  - $\langle \mu^2 \rangle = \langle |V|^2 \rangle + \langle |n|^2 \rangle$
  - $\langle \mu^2 \rangle$ is biased but can be unbiased if $\langle |n|^2 \rangle$ is estimated
Fourier estimator

• Remember Parseval’s identity?

\[ \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |TF(f)(\sigma)|^2 d\sigma \]

• The average squared amplitude of the signal == The average PSD
Analytical fringe signal

Fringes signal $F(\delta)$: function of monochromatic fringes $f(\delta, \lambda)$, function of OPD ($\delta$) and wavelength ($\lambda$)

$$F(\delta) = \frac{\int_0^{\infty} B(\lambda)T(\lambda)f(\delta, \lambda)\lambda d\lambda}{\int_0^{\infty} B(\lambda)T(\lambda)\lambda d\lambda}$$

$B(\lambda) \rightarrow$ stellar spectrum

$T(\lambda) \rightarrow$ instrumental transmission

$f(\delta, \lambda) = 1 + Re \left( V(\lambda)e^{-2\pi \delta/\lambda} \right)$

$\lambda \rightarrow$ photon detection
\[ PSD(\sigma) = |FT_\delta[F(\delta)]|^2 \]

\[
= \left| \frac{\int_0^\infty B(\lambda)T(\lambda)f(\delta, \lambda)\lambda d\lambda}{\int_0^\infty B(\lambda)T(\lambda)\lambda d\lambda} \right|^2
\]

\[
= \frac{\left| \int_0^\infty B(\lambda)T(\lambda)\tilde{f}(\delta, \lambda)\lambda d\lambda \right|^2}{\left| \int_0^\infty B(\lambda)T(\lambda)\lambda d\lambda \right|^2}
\]

\[
= \frac{\left| \int_0^\infty B(\lambda)T(\lambda)V(\lambda)\delta_{1/\sigma}(\lambda)\lambda d\lambda \right|^2}{\left| \int_0^\infty B(\lambda)T(\lambda)\lambda d\lambda \right|^2}
\]

\[
= \left[ \frac{B(1/\sigma)T(1/\sigma)V(1/\sigma)}{\int_0^\infty B(\lambda)T(\lambda)\lambda d\lambda} \right]^2
\]

- Linearity of FT
- Frequency selection
- Normalised, frequency averaged object's visibility
Real signal

Power = \mid \text{FFT} \mid^2

Photometric Variations

Fringes Power

floor noise (white): read–out + photon shot

Wavenumber
Real signal

\[ \langle \mu^2 \rangle = \langle |V|^2 \rangle + \langle |n|^2 \rangle \]
Fourier estimator

\[ V_{\text{measured}}^2 \propto \int_{\sigma_{\text{min}}}^{\sigma_{\text{max}}} B(1/\sigma)^2 T(1/\sigma)^2 V(1/\sigma)^2 1/\sigma^2 d\sigma \]

- weighted average of the squared visibilities
- function of the object spectrum
- function of the instrumental transmission
$$V_{\text{measured}}^2(\theta, B) \propto \int_{\sigma_{\text{min}}}^{\sigma_{\text{max}}} B(1/\sigma)^2 T(1/\sigma)^2 V_{UD}(B, \theta, 1/\sigma)^2 1/\sigma^2 d\sigma$$
Fast Rotating stars

- Aufdenberg+ 2006
- Observations of the star Vega with FLUOR@CHARA
- Accurate modelling allowed to prove that the star is a rapid rotator seen pole-on
- astrophysical effect ~ bandwidth smearing effects
Why it is important

• Interferometric observations lead to visibilities, closure phases (+ differential quantities)

• **Images** can be reconstructed…

• … But the astrophysical **quantitative** results will always be derived from visibilities
When instrumental effects mimic astrophysical signal

- We have seen “obvious” effects: model disagree with the observations
- Some effects are more difficult to spot!
- Some signal:
  - Astrophysical phenomenon?
  - Instrumental effect?
Atmospheric Dispersion

The refractive index of air is chromatic

Zheng+13
The OPD is in **vacuum**, delay lines are in **air**.
longitudinal dispersion

- The OPD in air is $l \times n(\lambda)$
- The OPD chromatic
Effect on differential phase

**Ideal fringes**

\[ F(x, \lambda) = 1 + \cos\left(\frac{2\pi x}{\lambda} + \phi_\lambda\right) \]

**OPD modulation**

\[ x = L_{\text{vac.}} - [\Delta l_{i,j} = (l_j - l_i)]n_{\text{air}} = 0 \]

**Polynomial expansion**

\[ F(x, \lambda) = 1 + \cos \left[ 2\pi (L_{\text{vac.}} - \Delta l_{i,j}n_0 + \delta x)/\lambda + \phi_\lambda + 2\pi \Delta l_{i,j}n_1 + 2\pi \Delta l_{i,j}(n_2\lambda + n_3\lambda^2 + ...) \right] \]

**Actual fringes**

\[ F(x, \lambda) = 1 + \cos \left[ 2\pi \delta x n_0/\lambda + \phi_\lambda + 2\pi \Delta l_{i,j}(n_2\lambda + n_3\lambda^2 + ...) \right] \]

**Chromatic phase**

\[ \phi_c(\lambda) = 2\pi \Delta l_{i,j} \left( n_2\lambda + n_3\lambda^2 + ... \right) \]

\[ = 2\pi \Delta l_{i,j} \frac{n_{\text{air}}(\lambda) - n_0 - n_1\lambda}{\lambda} \]
Example: AMBER

\[
\phi_c(\lambda) = 2\pi \Delta l_{i,j} \left( n_2 \lambda + n_3 \lambda^2 + \ldots \right) \\
= 2\pi \Delta l_{i,j} \frac{n_{\text{air}}(\lambda) - n_0 - n_1 \lambda}{\lambda}
\]
Observed differential phase and model based on DL positions
There is information in the differential phase!
Use of differential phase?

- Longitudinal air dispersion needs to be **accurately modeled** to extract the astrophysical signal.

- $\text{L}_{\text{air}}$ is easy to estimate (function of zenithal distance).

- $n_{\text{air}}$ is a also function of temperature, pressure, water content etc... **never perfect correction** ($\neq$ accurate).
Closure Phase

- Measure phase sum in a close triangle

- $CP = (\phi_{12} + \phi_a) + (\phi_{23}) + (\phi_{31} - \phi_a) = \phi_{12} + \phi_{23} + \phi_{31}$

- $CP$ is insensitive to longitudinal dispersion!
Differential Phase

- Differential phase has a strong instrumental bias (air dispersion)

- Bias is very large (many 10°)

- We have seen 2 solutions:
  1. model the effect
  2. use a robust estimator (closure phase)

- Alternate solution: correct with glass with refractive chromaticism inverse to air (hard to get accurate)
Phase jitter correction

1m bubble of air with 1.5°K difference produces a 1μm OPD difference

same for 10m bubble of air with 0.15°K difference

index of air: Mathar 2007
How long can we integrate?

- Reminder: for photon shot and readout noises, the longer integration the better

- How about the turbulent piston?

- Loss of contrast:

\[ V_{\text{loss}}^2 = e^{-\phi^2} = e^{-\left(2\pi \frac{\sigma_{\text{OPD}}}{\lambda}\right)^2} \]
Interferometric SNR

- signal: coherent flux \( \sim N_{\text{phot}} \times V_{\text{obs}} \)
- Noises: read-out and photon noises

\[
SNR \sim \frac{N_{\text{phot}}(t)e^{-\pi \frac{\sigma_{opd}^2(t)}{\lambda}}}{\sqrt{N_{\text{phot}}(t) + ron^2}}
\]
Turbulent PSD

Typical of cascading energy phenomena

• energy injected at a low frequency (wind, gravity waves)

• breaks down in smaller and smaller scales, losing each time more energy

→ + heat
variance?

Parseval identity:

\[ \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |TF(f)(\sigma)|^2 d\sigma \]

Variance \( \sim \) integral of PSD

\[ \int_0^T |f(t)|^2 dt \sim \int_{1/T}^{\infty} |TF(f)(\sigma)|^2 d\sigma \]

Variance grows as \( T^{(-p-1)} \)

Kolmogorov \( p \sim -11/3 \) \( \sigma_{\text{OPD}}^2(T) \sim T^{8/3} \)
under turbulent atmosphere

\[ SNR \sim \frac{N_{\text{phot}}(T)e^{-\pi \frac{\sigma_{\text{opd}}^2(T)}{\lambda}}}{\sqrt{N_{\text{phot}}(T) + ron^2}} \sim \frac{Te^{-T^{8/3}}}{\sqrt{T + ron^2}} \]
Fringe Tracking goal

Case of spectrally dispersed interferometer:

- Lack of sensitivity is a lack of photons, requiring long DIT
- everything being equal, low spectral resolution would have better SNR.
- FT measures fringes every 0.001s
- SC integrates for ~10s
- FT has a tremendous amount of phase information during the SC integration
- post processing can assess the visibility loss due to FT residuals: Gravity’s V-Factor
FINITO+AMBER

Fringe contrast per frame

AMBER $\mu^2$ frame by frame

FINITO V factor
• MIDI observed at 10μm, PRIMA FSU tracked at 2.2μm

• additional **post-processing correction** 2.2->10μm assumes to estimate water vapor (Koresko+ 2006)

• **Gain in sensitivity of MIDI 2.5mag:**
  - Observing mode unchanged
  - FT telemetry data recorded
  - Post processing
  - Paves the way for “Gravity for MATISSE”
Recipe for Accuracy

Post processing and data modeling: