

The Monte Carlo Method in Astrophysics *(Part 4)*

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2nd Example

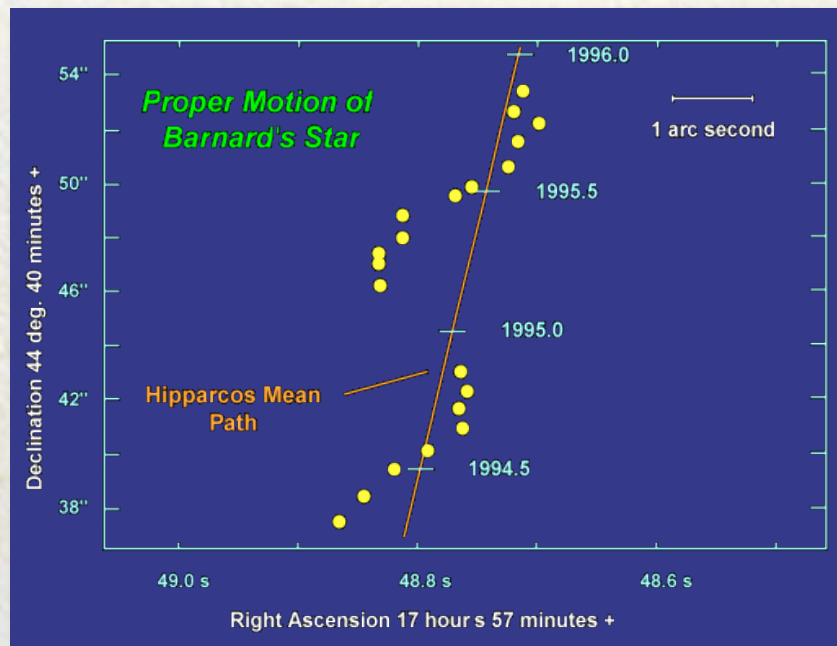
Simulation of a proper motion survey



Our 2nd exercise is the Monte Carlo simulation of a proper motion survey.

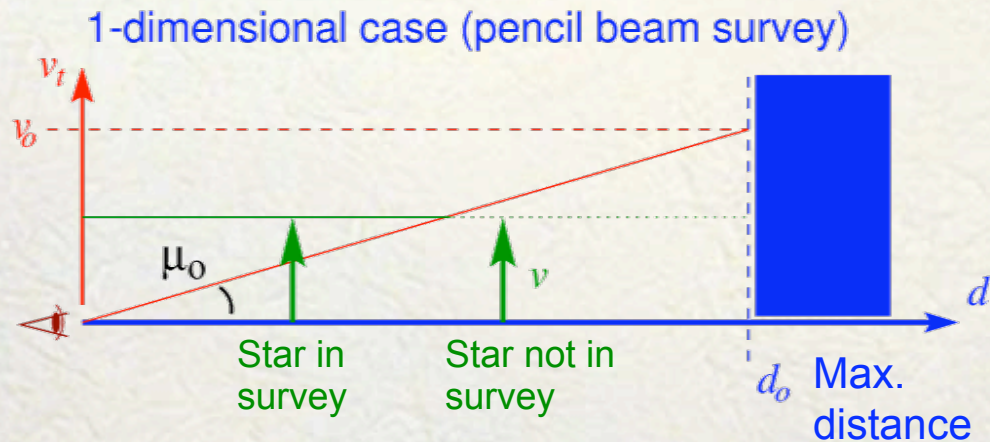
Proper motion surveys

A proper motion survey is good to pick nearby stars, and so, they are ideally suited to study high-velocity stars in the solar neighborhood



Proper motion surveys

Kinematical Bias in Proper Motion Surveys



The problem, is that they introduce a kinematical bias against low-velocity stars.

Proper motion survey limits: m_o apparent magnitude
 μ_o proper motion.

Maximum distance: $d_o = 10^{0.20(m_o - M_v) + 1}$

Minimum tangential velocity at d_o : $v_o = \mu_o d_o$

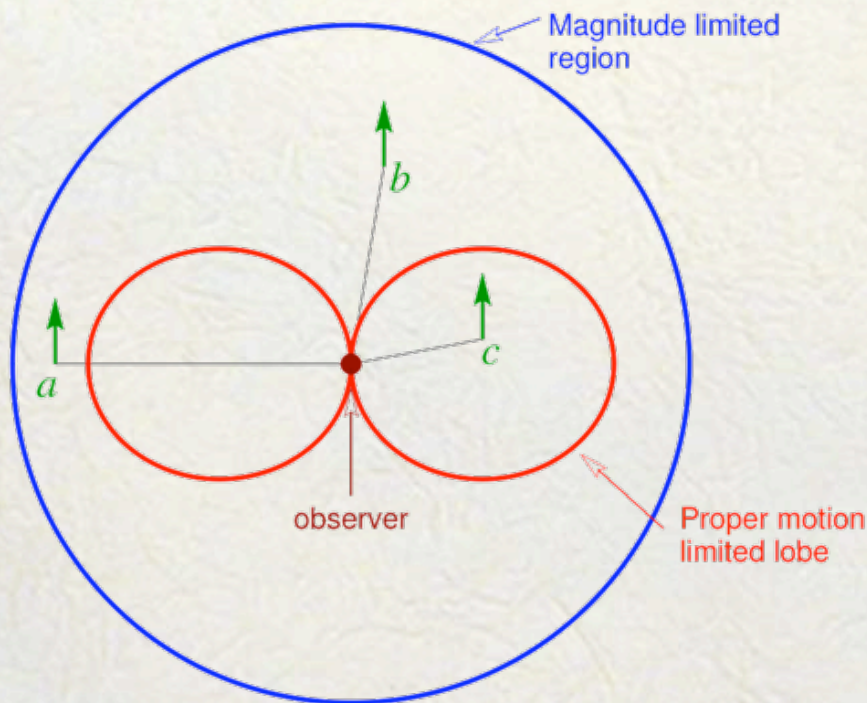
Proper motion constraint: $v_t > \mu_o d$

1-D kinematical bias

Proper motion surveys

Kinematical Bias in Proper Motion Surveys

2-dimensional case



Case a: Although all the motion is transverse to the line of sight, the star is too far to show up in the proper motion survey.

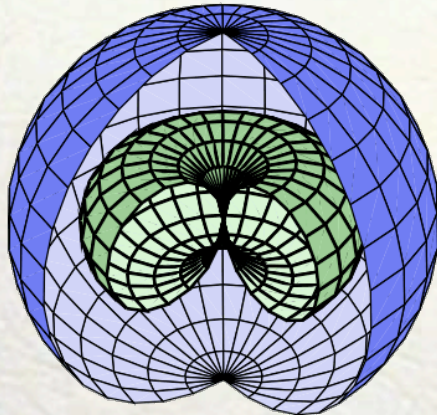
Case b: Because of its position in the sky relative to its motion, the proper motion of the star is not large enough to appear in the survey.

Case c: The star will appear in the survey.

2-D kinematical bias

Proper motion surveys

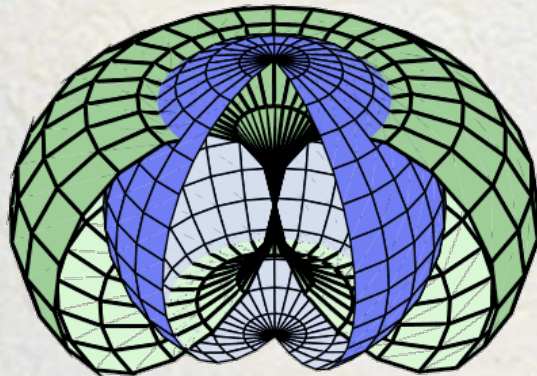
3-dimensional case with full sky coverage



Proper motion limited survey

Proper motion torus with magnitude limited sphere.
The star's velocity is aligned with the torus axis.

In the upper figure, the proper motion is so severe, that no star within the survey can reach the magnitude limited sphere.



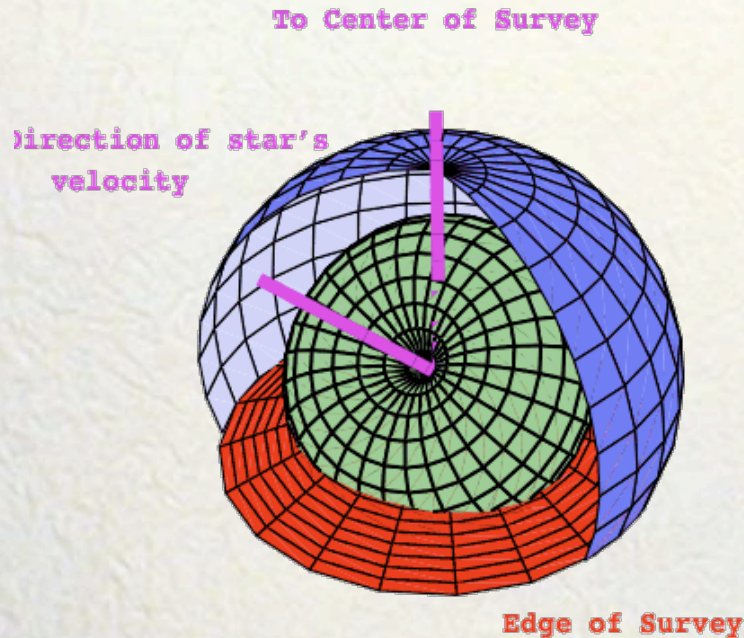
Apparent magnitude limited survey

In the lower figure, the survey is magnitude limited near the torus equator (the star gets too faint before its proper motion falls below detection), while it is proper motion limited near the torus poles (the star's proper motion falls below detection while still bright enough to be seen).

3-D kinematical bias

Proper motion surveys

3-dimensional case with partial sky coverage

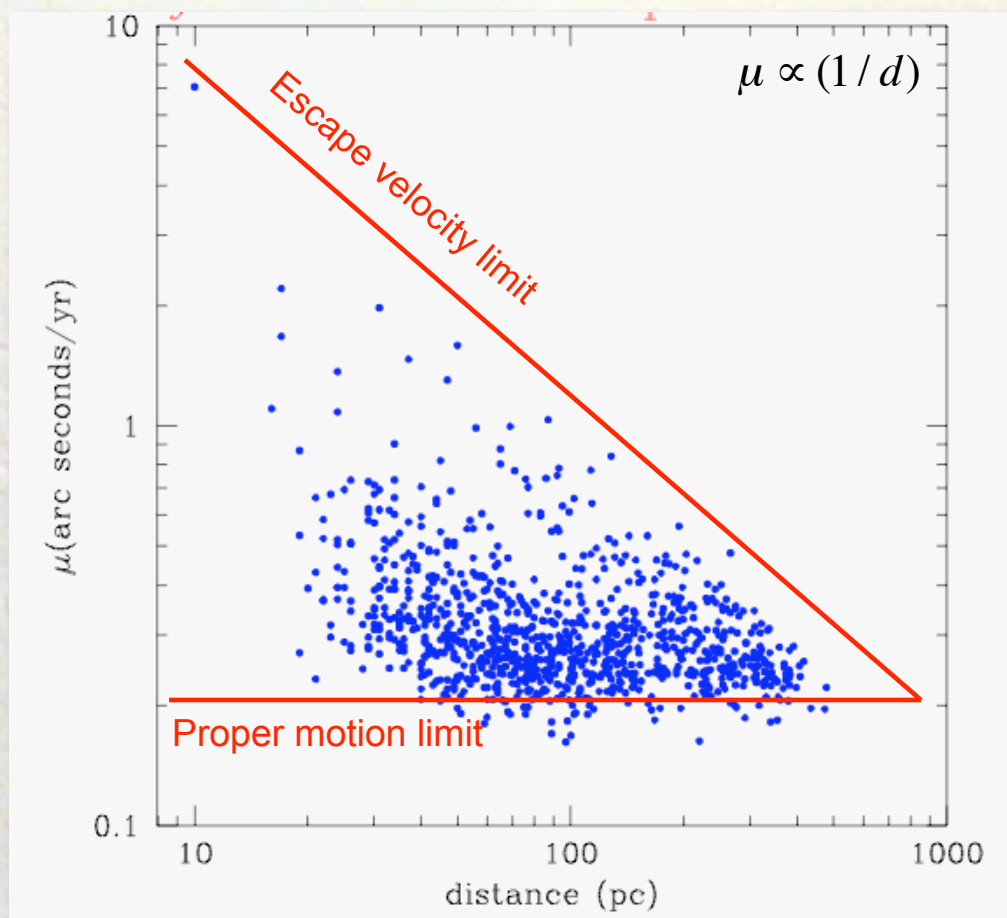


Proper motion limited torus within a truncated magnitude limited sample sphere, for a star in a survey with incomplete sky coverage. The star's velocity is aligned with the torus axis.

The incompleteness of the sky coverage makes the kinematical bias dependent on the angle between the velocity vector of the star and the direction to the center of the survey.

Apparent magnitude limited survey with partial sky coverage

Proper motion surveys

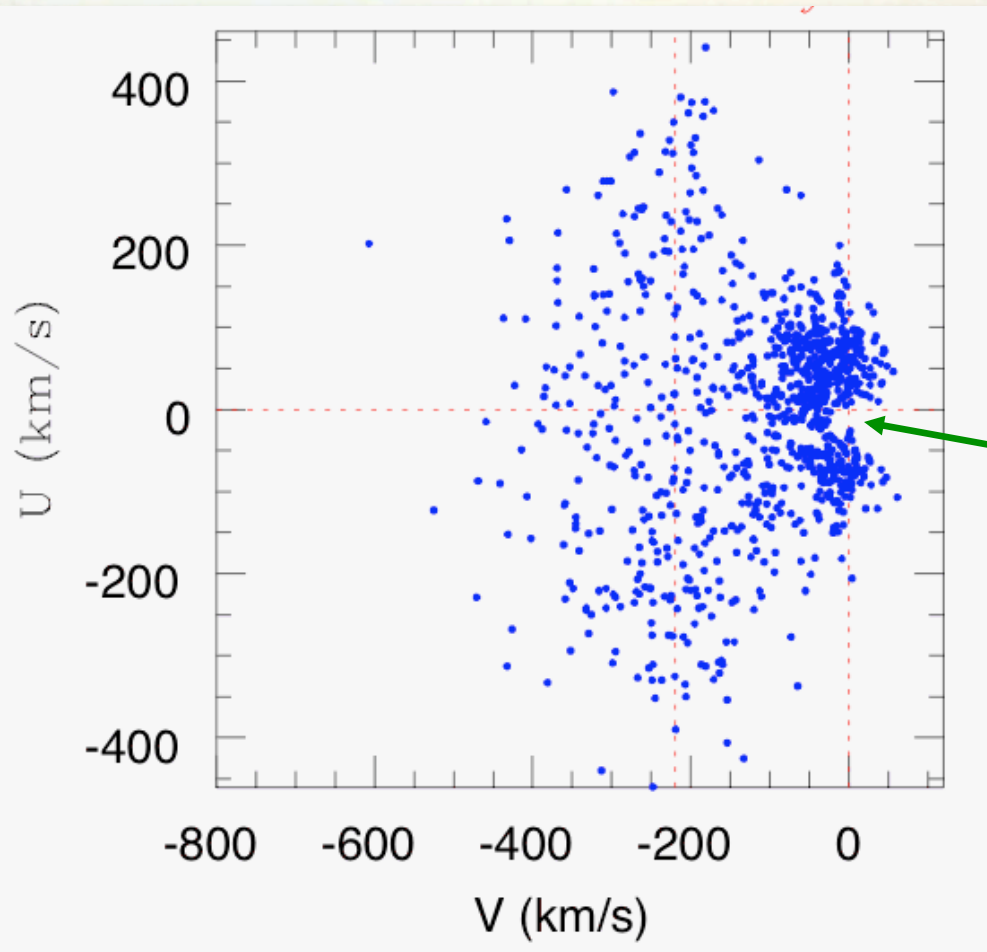


**An example:
The CLLA sample**

The CLLA sample is taken from the Lowell proper motion survey, which has a limit of $\sim 0.27''/\text{yr}$.

Carney, B., Latham, D., Laird, J.,
Aguilar, L., (1994) *AJ*, **107**, 2240

Proper motion surveys

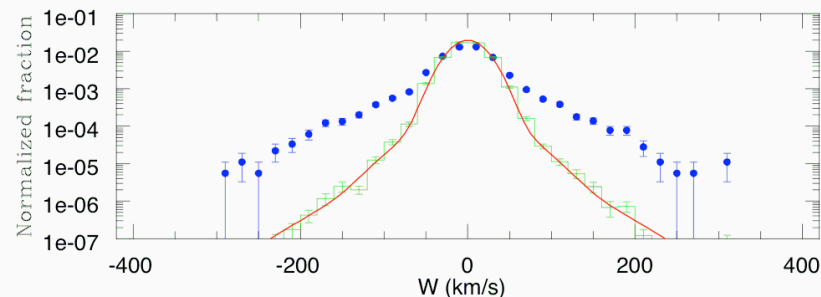
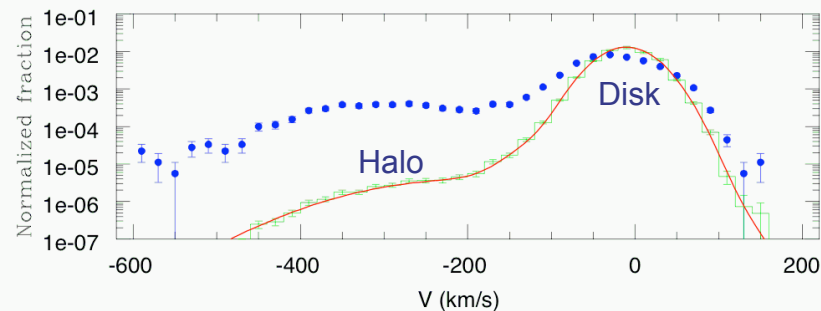
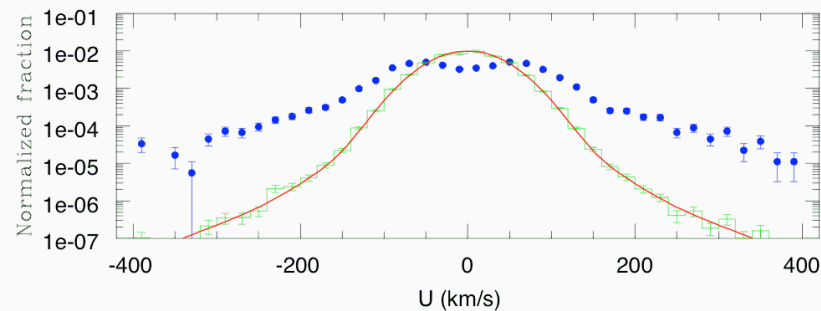


The kinematical bias is readily apparent in a plot of U (toward galactic center) vs. V (toward galactic rotation) velocities.

LSR

Notice the lack of stars near the LSR, where we know the velocity distribution must peak.

Proper motion surveys



A Monte Carlo experiment allows us to assess the magnitude and effect of the kinematical bias

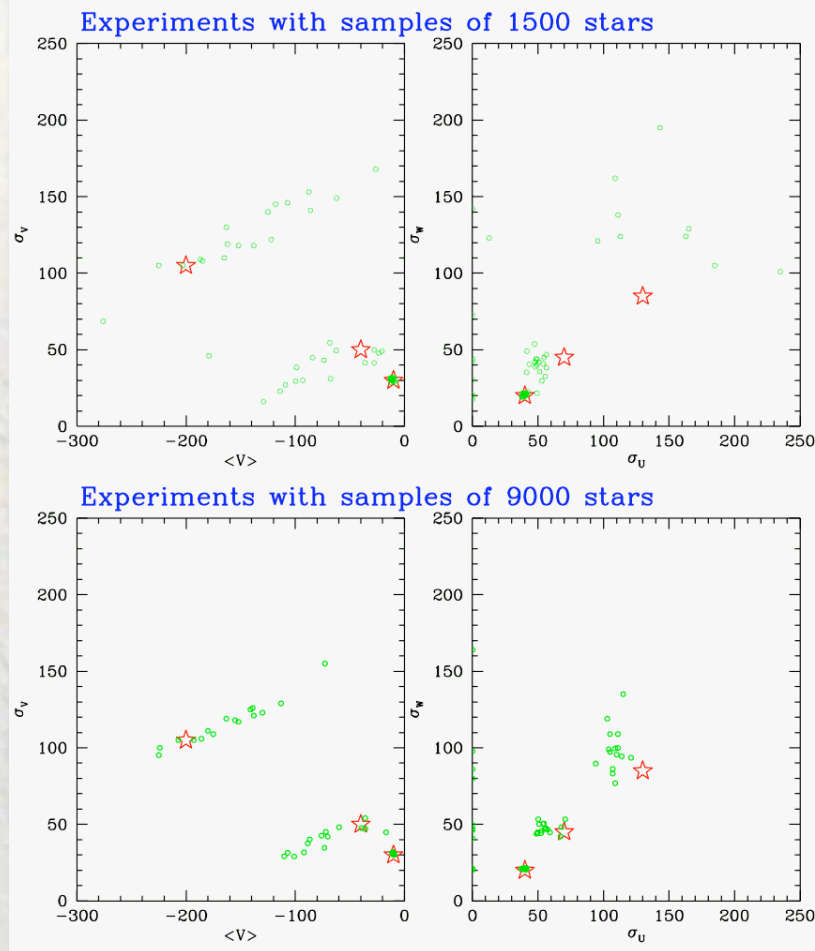
Simulated survey with proper motion cut-off.

Simulated survey without proper motion cut-off.

Theoretical model.

All histograms have been normalized to unit area.

Proper motion surveys



We can quantify (and correct) the bias by doing χ^2 fits and comparing the derived values for the population parameters vs. the theoretical values.

Theoretical values for bulge, disk and halo components.

Fitted parameters.

We can even play with the sample size to see how large should the sample grow to achieve a given confidence level.

A simulated proper motion survey

To simulate a proper motion survey, we need:

1. A luminosity function.
2. A model for the spatial distribution.
3. A kinematical model.
4. A model for the survey.

A simulated proper motion survey

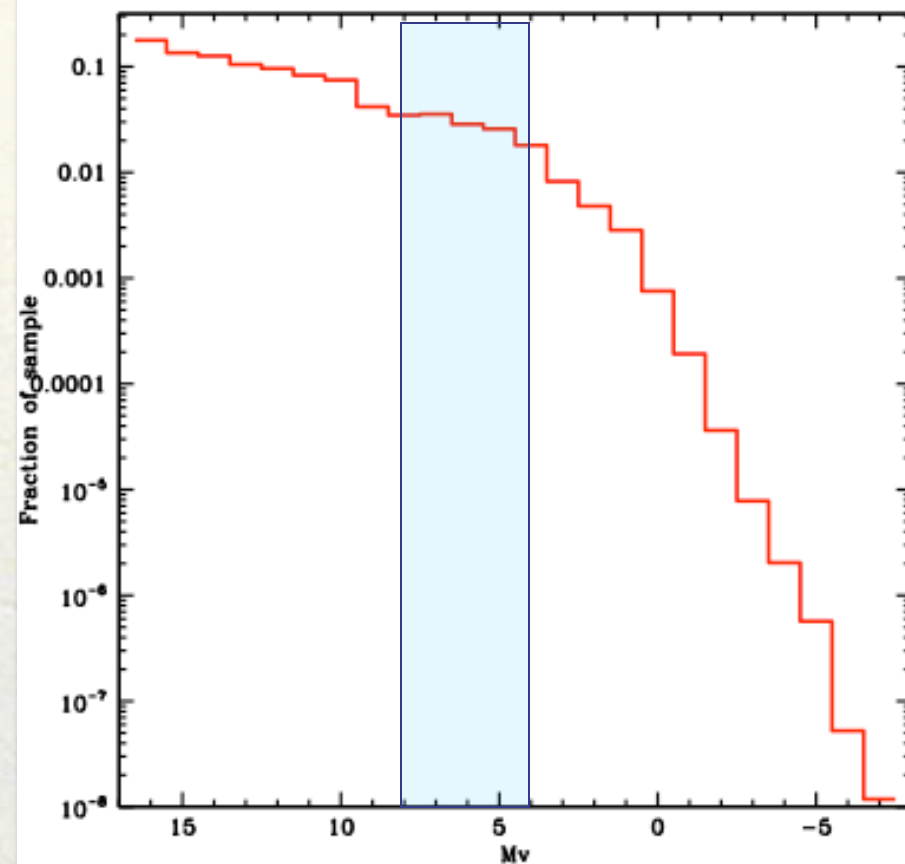
We begin with a luminosity function: The Luyten Luminosity Function (table 4.5 in *Galactic Astronomy*, Mihalas & Binney).

M_V	Stars/ 10^6pc^3	Fraction
-7	0.00133	1.1822E-08
-6	0.00590	5.2444E-08
-5	0.06400	5.6888E-07
-4	0.22900	2.0355E-06
-3	0.88000	7.8221E-06
-2	4.09000	3.6355E-05
-1	21.6100	1.9209E-04
0	85.0010	7.5555E-04
1	320.000	2.8444E-03
2	540.000	4.7999E-03
3	1040.00	8.2243E-03
4	2030.00	1.8044E-02
5	2900.00	2.5777E-02
6	3210.00	2.8533E-02
7	4000.00	3.5555E-02
8	3910.00	3.4755E-02
9	4700.00	4.1777E-02
10	8410.00	7.4754E-02
11	9330.00	8.2932E-02
12	10800.00	9.5998E-02
13	11800.00	1.0489E-01
14	14200.00	1.2622E-01
15	15200.0	1.3511E-01
16	20000.0	1.7777E-01

A simulated proper motion survey

We now select a range in M_V that we desire to simulate and produce the normalized histogram of that range.

We use this histogram to draw a random absolute magnitude for our star.



M_V

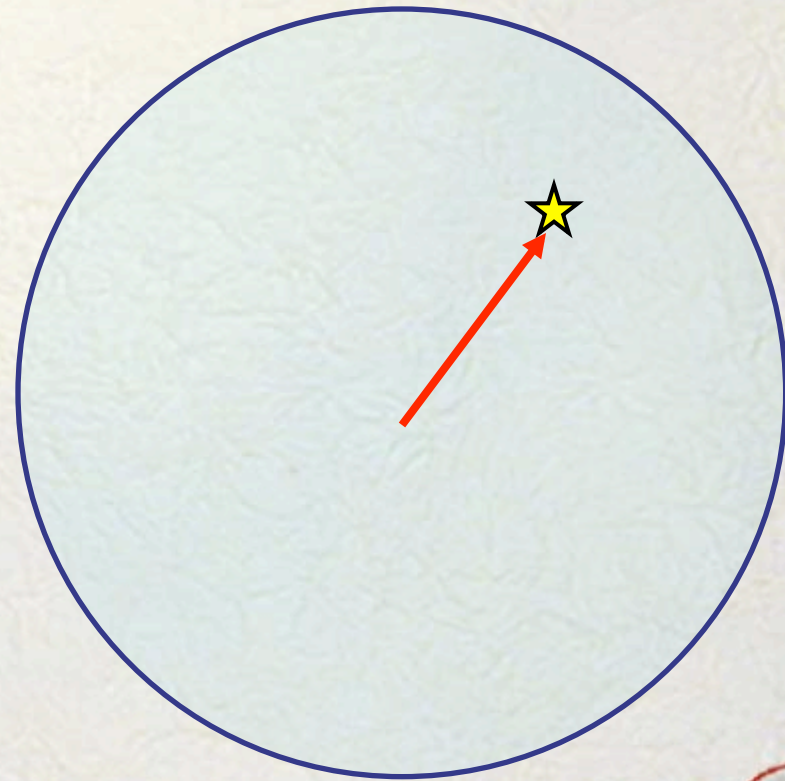
A simulated proper motion survey

Our next step is to draw a random position for the star.

Since we are assuming a constant density model within the surveyed volume, we compute the maximum distance d_{max} at which a star of this absolute magnitude M_V can be seen, given the apparent magnitude limit m_o of the survey, and then draw a random position within the apparent magnitude limited sphere centered at the Sun.

$$d_{max} = 10^{0.20(m_o - M_V) + 1}$$

$$M_V \rightarrow d_{max} \rightarrow (x, y, z)$$



A simulated proper motion survey

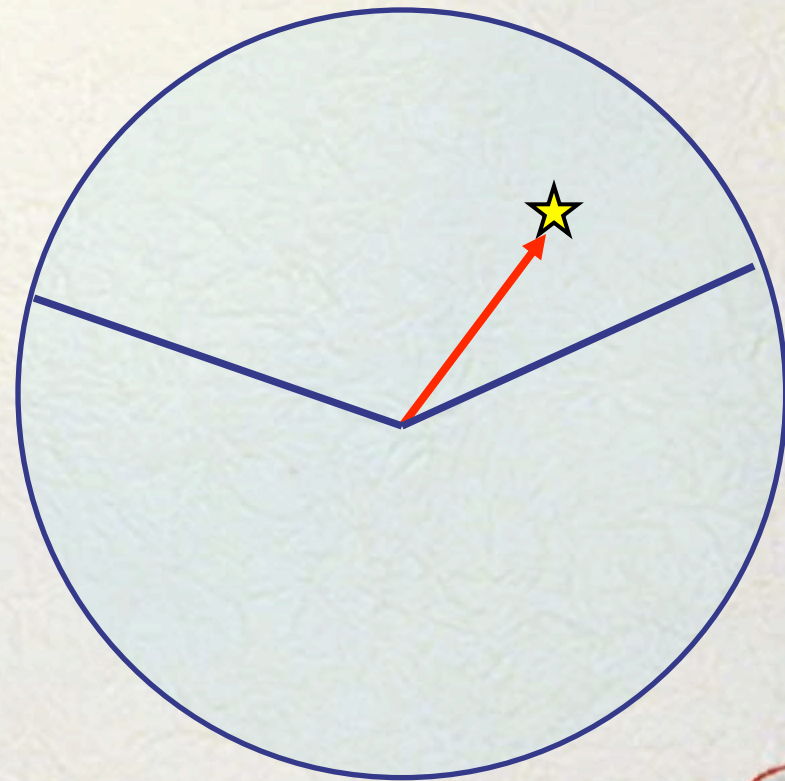
Our next step is to draw a random position for the star.

If the survey does not cover the whole sky, we accept only stars within the covered region.

If the restriction is in say, galactic latitude, we can simply convert cartesian to spherical coordinates and throw away stars outside the surveyed region.

It is very important to start all over from step 1, if a star is thrown away!

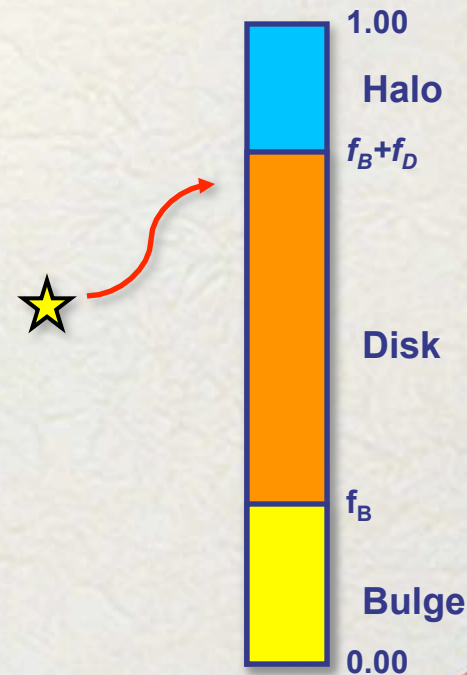
$$M_V \rightarrow d_{\max} \rightarrow (x, y, z)$$



A simulated proper motion survey

If our model includes several galactic components, say bulge, disk and stellar halo, that overlap within the surveyed region, we must now assign our star to one of these components. This is done by using the normalized fractions of each component: f_B , f_D , f_H .

A simple draw of a random number between 0 and 1 does the trick.



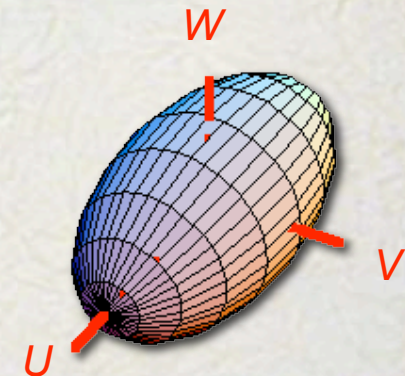
$M_V \rightarrow d_{max} \rightarrow (x, y, z) \rightarrow \text{Component}$

A simulated proper motion survey

Once we know to what galactic component the star belongs to, we can draw a random velocity for it from the corresponding velocity distribution.

$$f(U, V, W) = \frac{1}{\sqrt{2\pi} \sigma_U \sigma_V \sigma_W} \exp \left[-\frac{U^2 + (V - V_{Asym})^2 + W^2}{2 \sigma_U^2 \sigma_V^2 \sigma_W^2} \right]$$

A popular function is a triaxial gaussian with an offset (asymmetric drift) in the V component. We can draw each component independently and then add the offset to the V component.



$M_V \rightarrow d_{max} \rightarrow (x, y, z) \rightarrow \text{Component} \rightarrow (U, V, W)$

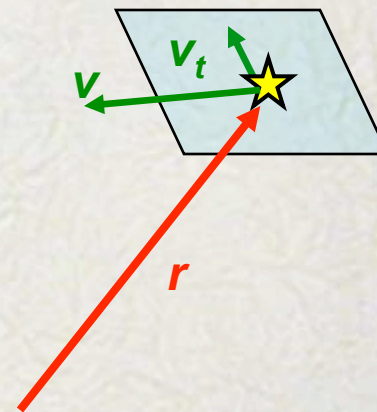
A simulated proper motion survey

The last step is to compute the proper motion of the star μ_* and see if it is larger than the survey cutoff μ_o . If it is not, we throw away the star and start all over again, from step 1.

The 1st step is to compute the tangential velocity:

$$v_t = |\vec{v} - \vec{v}_r| = |\vec{v} - (\vec{v} \cdot \hat{r})\hat{r}|$$

Then, we obtain the proper motion: $\mu = v_T / (4.74 d)$, where v_t is in km/s and μ in $"/year$



$M_V \rightarrow d_{max} \rightarrow (x,y,z) \rightarrow \text{Component} \rightarrow (U,V,W) \rightarrow \mu$

A simulated proper motion survey

The last step is to compute the proper motion of the star μ_* and see if it is larger than the survey cutoff μ_o . If it is not, we throw away the star and start all over again, from step 1.

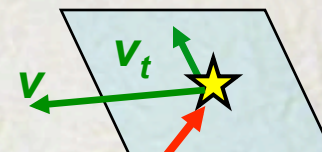
The 1st step is to compute the tangential velocity:

$$v_t = v - v_r = v - v \cdot \hat{r}$$

Then, we obtain the proper motion: $\mu = v_t / (4.74 \text{ d})$, where v_t is in km/s and μ in $"/\text{year}$

If $\mu_* > \mu_o$, we incorporate the star in our simulated survey, otherwise, we throw it away.

$M_V \rightarrow d_{\text{max}} \rightarrow (x, y, z) \rightarrow \text{Component} \rightarrow (U, V, W) \rightarrow \mu_*$



A simulated proper motion survey

We then keep iterating until we get a simulated proper motion survey of the desired size.

Once you have found the trick of it, you can add as many effect as you wish:

- ✓ Photometric information
- ✓ Varying spatial density
- ✓ Absolute magnitude - kinematics relations
- ✓ More complicated selection criteria
- ✓ ... etc.



Homework

4th Assignment

- Write a program that allows us to conduct Monte Carlo simulations of a proper motion survey.
- This survey should have the following characteristics:
 - Assume uniform spatial density.
 - The survey should be restricted to the northern galactic hemisphere ($b > 20^\circ$).
 - The survey should be restricted to stars with $4 \leq M_V \leq 8$; assume the Luyten Luminosity Function:

M_V	4	5	6	7	8
Fraction	0.106	0.155	0.211	0.239	0.289

- Use a cut off in apparent visual magnitude of $m_V = 16$.

Homework

- For the kinematics, assume 2 populations, a thin disk and a halo; the normalizations in spatial density, asymmetric drifts and dispersions in U, V and W, velocity components, where these are taken along the line to the galactic center, direction of rotation, and north galactic pole, respectively, are given by:

Population	Normalization	Asymetric drift	σ_U	σ_V	σ_W
Halo	0.02	200	130	105	85
Thin disk	0.98	10	40	30	20

$$f(U,V,W) = \frac{1}{\sqrt{2\pi} \sigma_U \sigma_V \sigma_W} \exp \left[-\frac{U^2 + (V - V_{Asym})^2 + W^2}{2 \sigma_U^2 \sigma_V^2 \sigma_W^2} \right]$$

Homework

- The survey should be limited to stars with proper motions larger than $\mu=0.27''/\text{year}$ (a star at d parsecs and with v_T km/s of tangential velocity, has a proper motion of $\mu = v_T / (4.74 d)$ $''/\text{year}$).

The parameters given above correspond roughly to the Lowell Proper Motion Catalog, and the kinematic parameters of the thin disk and stellar halo of our Galaxy at the solar neighborhood.

Homework

The suggested algorithm should proceed as follows:

For each star:

1. Choose a random absolute magnitude M_V , from the Luyten luminosity function and within the specified range.
3. Compute the corresponding maximum distance d_{max} , at which a star of such absolute magnitude would still be visible within the apparent magnitude limit m_o of the catalog: $d_{max} = 10^{0.20(m_o - M_V) + 1}$.
5. Choose a random radial distance r_{star} within the interval $0 < r_{star} < r_o$ and a random direction in the celestial sphere constrained to be within $b > 20^\circ$.

Homework

4. Choose now a random velocity vector for the star as follows:
 - a) Pick at random one of the two galactic components using their relative normalizations.
 - b) Then pick a random velocity vector from the corresponding velocity distribution.

6. The last step is to decide whether the generated star is visible within the proper motion limit of the survey. For this, compute the tangential velocity and then its proper motion; if it is equal or larger than the survey proper motion cut-off, keep the star, otherwise, reject it.

Homework

The program should store the absolute magnitudes and velocity components of the survey stars. It should also store the ratio of accepted to total number of generated stars (accepted plus rejected) for each galactic component.

Generate a survey of 5,000 stars, report the resulting ratios of accepted to total number of stars per population, and make the following plots:

- I. Histograms of the U , V and W velocity components and the corresponding theoretical velocity distributions. It is quite likely that these histograms should have a logarithmic ordinate axis, due to the kinematical bias.
- II. A histogram of the luminosity distribution function of the survey stars. Compare with the Luyten Luminosity Function.
- III. A plot of distance vs apparent magnitude for survey stars.

Homework

1. What conclusions can you derive from the results obtained?
2. What is the effect of the proper motion constraint on the “observed” kinematics?
3. Is it possible to say something about the true ratio of halo to disk stars in the solar neighborhood?
4. Is it possible, based in the observed kinematics, to notice that the sample consists of two distinct galactic components?
5. If this is the case, is it possible to recover their respective kinematical parameters (velocity dispersions and asymmetric drift)?

End of fourth lecture