# Cepheid Pulsations Resolved by the VLTI

*The very high spatial resolution provided by the VLTI makes it possible to measure directly the change in angular diameter of several southern Cepheids over their pulsation cycle. When combined with radial velocity measurements, this allowed us to measure precisely their distances in a quasi-geometrical way. This is essential information to calibrate the Cepheid's Period-Luminosity law, as well as the Period-Radius and Surface Brightness-Colour relations.*

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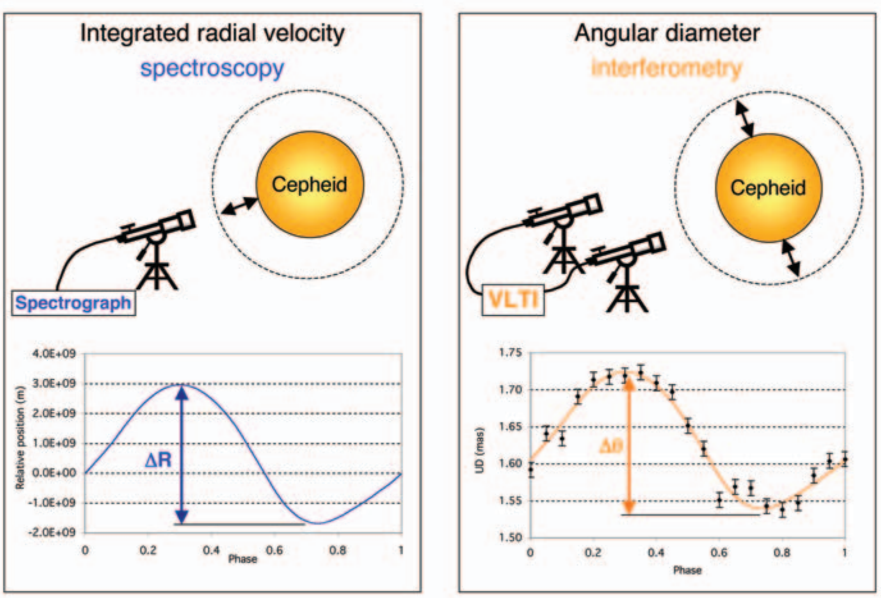
For almost a century, Cepheids have occupied a central role in distance determinations. This is thanks to the existence of the Period-Luminosity relation $M = a \log P + b$ which relates the logarithm of the variability period of a Cepheid to its absolute mean magnitude. These stars became even more important since the HST Key Project on the extragalactic distance scale has totally relied on Cepheids for the calibration of distance indicators to reach cosmologically significant distances. In other words, if the calibration of the Cepheid P-L relation is wrong, the whole extragalactic distance scale is wrong.

There are various ways to calibrate the P-L relation. The avenue chosen by the HST Key Project was to assume a distance to the Large Magellanic Cloud (LMC), thereby adopting a zero point for the distance scale, but the LMC distance is currently the weak link in the extragalactic distance scale ladder. Another avenue is to determine the zero point of the Period-Luminosity relation with Galactic Cepheids, using for instance parallax measurements, Cepheids in clusters, or through the Baade-Wesselink (BW) method (see e.g. Bersier et al. 1997). We proposed in the present work to improve the calibration of the Cepheid Period-Radius (P-R), Period-Luminosity (P-L) and surface brightness-color (SB) relations through the combination of spectroscopic and interferometric observations of bright Galactic Cepheids.



$$d\ [\mathrm{pc}] = 9.305\ \Delta R\ [R_\odot] / \Delta\theta\ [\mathrm{mas}]$$

**Figure 1:** The two observational techniques used for the interferometric version of the BW method are high resolution spectroscopy (left) and interferometry (right). The former provides the radial velocity curve over the pulsation cycle of the star. Once integrated, this provides the linear radius variation of the star (in metres). The interferometric observations give access to the angular radius variation of the star. The ratio of these two quantities gives the distance of the Cepheid.

## The interferometric Baade-Wesselink method

The basic principle of the BW method is to compare the linear and angular size variation of a pulsating star, in order to derive its distance through a simple division. This method is a well-established way to determine the luminosity and radius of a pulsating star. Figure 1 illustrates the two quantities measured for this technique: the radius variation curve (left part), that is integrated from the radial velocity curve, and the angular diameter variation, measured by interferometry.

On one hand, the linear size variation can be obtained by high resolution spectroscopy, through the integration of the radial velocity curve obtained by monitoring the Doppler shift of the spectral lines present in the spectrum. A difficulty in this process is that the measured wavelength shifts are inte-

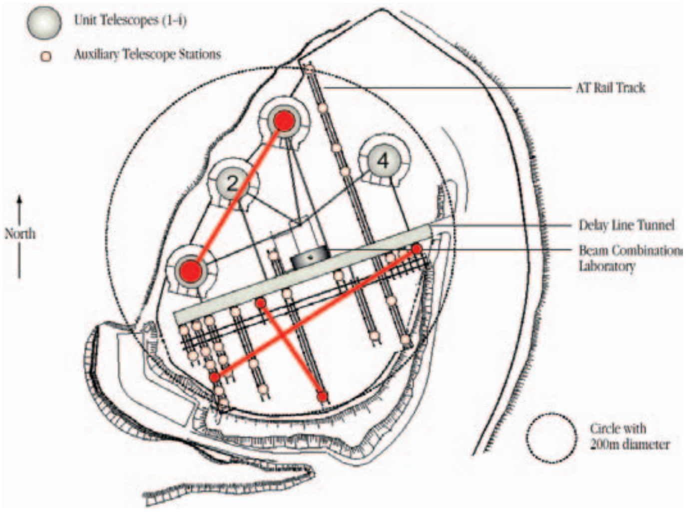


**Figure 2:** View of the Paranal platform with the three baselines used for the VLTI observations of Cepheids (in red).

grated values over the full stellar disc. To convert them into a pulsation velocity, i.e. a physical displacement of the photosphere at the center of the disc, we have to multiply it by a projection factor $p$ that encompasses the sphericity of the star and the structure of its atmosphere (limb darkening,…). Unfortunately, the $p$-factor is still uncertain at a level of a few percent but classical high resolution spectroscopy coupled with the dispersed fringes mode of AMBER are expected to bring strong constraints on this important factor. We have also started a theoretical approach in order to strongly constraint this parameter and to improve our distance determinations (Nardetto et al. 2004).

On the other hand, the angular size is difficult to estimate directly. Until recently, the only method to estimate the angular size was through the surface brightness of the star. With the advent of powerful infrared long baseline interferometers, it is now possible to resolve spatially the star itself, and thus measure directly its photospheric angular diameter. An uncertainty at a level of about 1% remains on the limb darkening of these stars, that is currently taken from static atmosphere models. In the near future, the gain in spatial resolution brought by the short wavelength modes of AMBER ($J$ and $H$ bands) will allow us to measure directly the intensity profile of several Cepheids.

## Observations with VINCI and the VLTI Test Siderostats

For our observations, the beams from the two VLTI Test Siderostats (0.35m aperture) or the two Unit Telescopes UT1 and UT3 were recombined coherently in VINCI, the VLTI Commissioning Instrument. We used a regular $K$ band filter ($\lambda$ = 2.0–2.4 µm) that gives an effective observation wavelength of 2.18 µm for the effective temperature of typical Cepheids. Three VLTI baselines were used for this program: E0-G1, B3-M0 and UT1-UT3 respectively 66, 140 and 102.5m in ground length. Figure 2 shows their positions on the VLTI platform.

In total, we obtained 69 individual angular diameter measurements, for a total of more than 100 hours of telescope time (2 hours with the UTs), spread over 68 nights (Kervella et al. 2004a). One of the key advantages of VINCI is to use single-mode fibres to filter out the perturbations induced by the turbulent atmosphere. The resulting interferograms are practically free of atmospheric corruption, except the piston mode (differential longitudinal delay of the wavefront between the two apertures) that tends to smear the fringes and affect their visibility. But this residual can be brought down to a very low level by using a fast acquisition rate. As a result, the visibility measurements from VINCI are currently the most precise ever obtained, with a relative statistical dispersion that can be as low as 0.05% on bright stars.

## Selected sample of Cepheids

Despite their apparent brightness, Cepheids are located at large distances, and ESA's Hipparcos satellite could only obtain a limited number of Cepheid distances with a relatively poor precision. If we exclude the peculiar first overtone pulsator Polaris, the closest Cepheid is $\delta$ Cep, located at approximately 250 pc. Using the interferometric BW technique, it is possible to derive directly the distance to the Cepheids for which we can measure the amplitude of the angular diameter variation. Even for the nearby Cepheids, this requires an extremely high resolving power, as the largest Cepheid in the sky, L Car, is only 0.003″ in average angular diameter. Long baseline interferometry is therefore the only technique that allows us to resolve these objects.

The capabilities of the VLTI for the observation of nearby Cepheids are outstanding, as it provides long baselines (up to 202m) and thus a high resolving power. Though they are supergiant stars, the Cepheids are generally very small objects in terms of angular size. A consequence is that

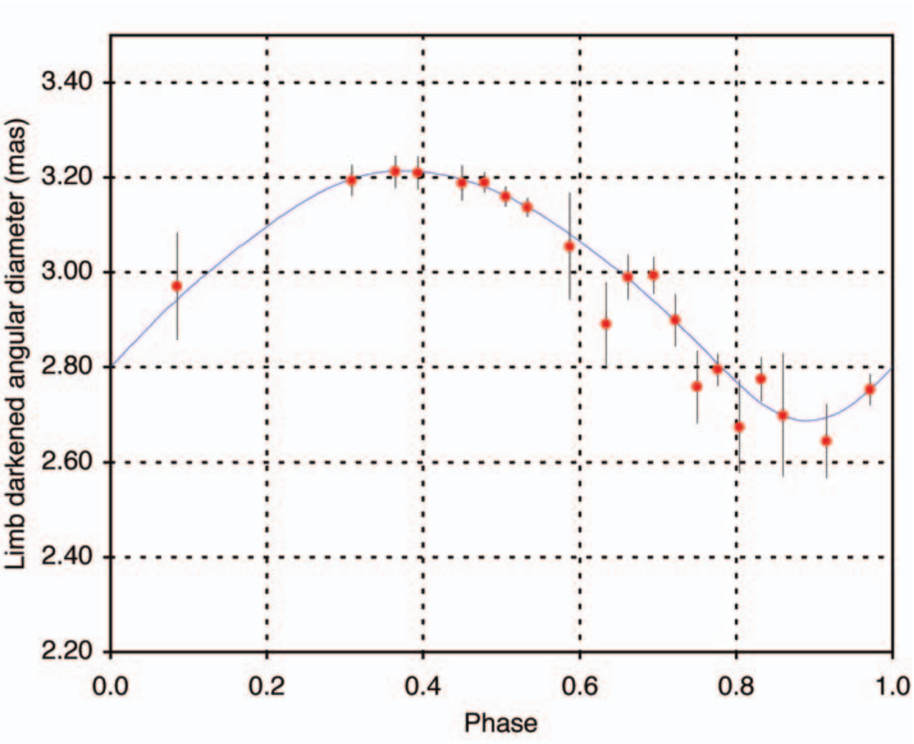


**Figure 3:** VINCI observations of the pulsation of L Car ($P$ = 35.5 days, red dots) and the adjusted radius curve (green line), as deduced from the integration of the radial velocity measured on this star over its pulsation.

the limit on the number of interferometrically resolvable Cepheids is not set by the size of the light collectors, but by the baseline length. From photometry only, several hundred Cepheids can produce interferometric fringes using the VLTI Auxiliary Telescopes (1.8m aperture). However, in order to measure accurately their size, one needs to resolve their disc to a sufficient level, and this reduces the total number of accessible Cepheids to about 30.

Considering the usual constraints in terms of sky coverage, limiting magnitude and accessible resolution, we selected seven bright Cepheids observable from Paranal Observatory (latitude –24 degrees): X Sgr, η Aql, W Sgr, β Dor, ζ Gem, Y Oph and L Car. The periods of these stars cover a wide range, from 7 to 35.5 days, an important advantage to properly constrain the P-R and P-L relations. Using the interferometric BW method, we derived the distances to η Aql, W Sgr, β Dor and L Car. For the remaining three objects of our sample, X Sgr, ζ Gem and Y Oph, we obtained average values of their angular diameters, and we applied a hybrid method to derive their distances, based on published values of their linear diameters. Figure 3 shows the angular diameter curve and the fitted radius curve of L Car ($P$ = 35.5 days), that constrains its distance to a relative precision better than 5%. A discussion of these data can be found in Kervella et al. (2004d).

## PERIOD-RADIUS RELATION

The Period-Radius relation (P-R) is an important constraint to the Cepheid models (see e.g. Bono et al. 1998). It takes the form of the linear expression $\log R = a \log P + b$. In order to calibrate this relation, we need to estimate directly the linear radii of a set of Cepheids. To complement the VINCI sample of seven Cepheids, we added the measurements of δ Cep, ζ Gem and η Aql obtained previously by other interferometers. We have applied two methods to determine the radii of the Cepheids of our sample: the interferometric BW method, and a combination of the average angular diameter and trigonometric parallax. While the first provides directly the average linear radius and distance, we need to use trigonometric parallaxes to derive the radii of the Cepheids for which the pulsation is not detected. For these stars, we applied the Hipparcos distance, except for δ Cep, for which we considered the recent parallax measurement by Benedict et al. (2002).

Figure 4 shows the distribution of the measured diameters on the P-R diagram. When we choose to consider a constant slope of $a=0.750\pm0.024$, as found by Gieren, Fouqué & Gómez (1998, hereafter GFG98), we derive a zero point of $b=1.105\pm0.029$ (Kervella et al. 2004b). As a comparison, GFG98 obtained a value of $b=1.075\pm0.007$,

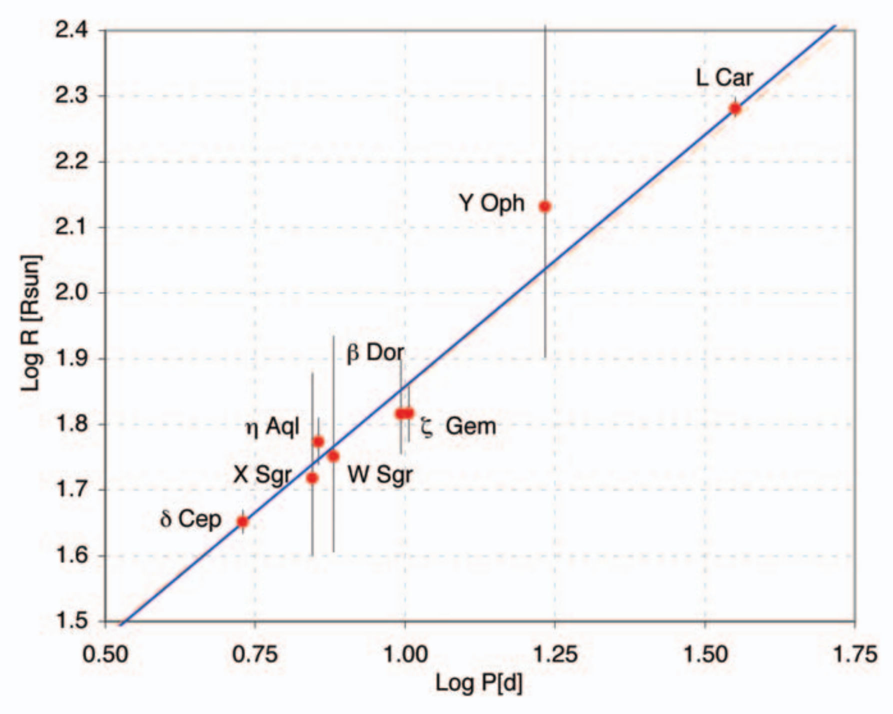


**Figure 4:** Period-Radius relation deduced from the interferometric measurements of Cepheids. The blue line corresponds to the simultaneous fit of both the zero point and slope of the line. The orange dashed line assumes the slope from GFG98, fitting only the zero point.

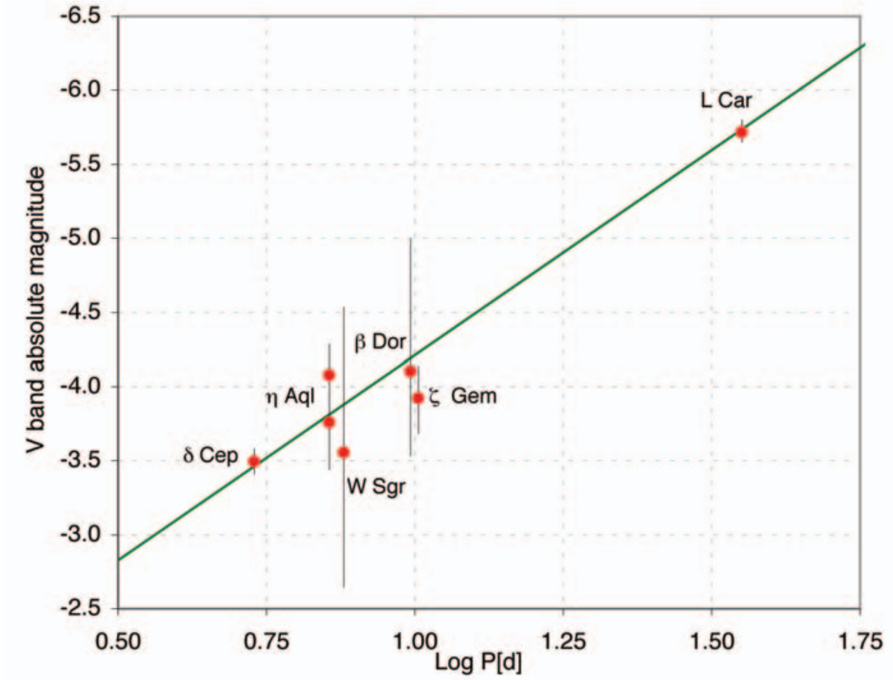


**Figure 5:** Period-Luminosity relation in the *V* band, as deduced from the interferometric observations of Cepheids and the HST parallax measurement of δ Cep. The green line is the fitted *P-L* relation, assuming the slope from GFG98. The agreement between the model and the measurements is excellent, in particular for the high precision measurements of δ Cep and L Car.

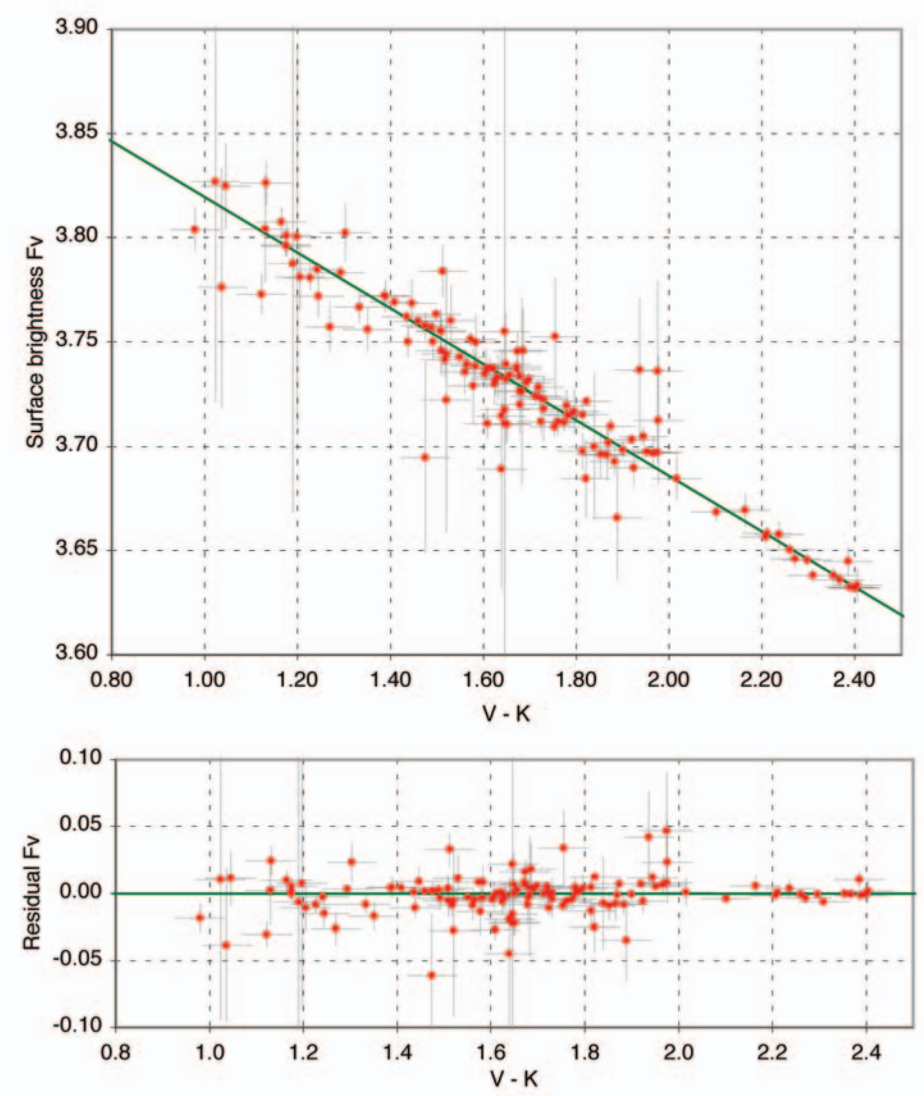


**Figure 6:** Surface brightness-colour relation $F_V(V-K)$ of Cepheids (upper part) and the residuals of the fit (lower part). The 145 individual interferometric measurements are displayed as red dots, and the fitted linear model as a green line.

only –1.6σ away from our result. These relations are compatible with our calibration within their error bars. Fitting simultaneously both the slope and the zero point to our data set, we obtain $a$=0.767±0.009 and $b$=1.091±0.011. These values are only $\Delta a$ = +0.7σ and $\Delta b$= +1.2σ away from the GFG98 calibration. Considering the limited size of our sample, the agreement is very satisfactory. On the other hand, the slopes derived from numerical models of Cepheids are significantly different, such as the slope $a$=0.661±0.006 found by Bono et al. (1998).

## Period-Luminosity relation

The Cepheid P-L relation is the basis of the extragalactic distance scale, but its calibration is still uncertain at a ΔM = ±0.10 mag level. Moreover, it is not excluded that a significant bias of the same order of magnitude affects our current calibration of this relation. Until now, the classical BW method (where one combines photometry and radial velocity data) was used to obtain the distance and radius of a Cepheid. A recent application of this method to individual stars can be found for instance in Taylor & Booth (1998). A requirement of this method is a very accurate measurement of the Cepheid's effective temperature at all observed phases, in order to determine the angular diameter. Interferometry allows us to bypass this step and its associated uncertainties by measuring directly the variation of angular diameter during the pulsation cycle. The latest generation of long baseline visible and infrared interferometers has the potential to provide precise distances to Cepheids up to about 1 kpc, using the interferometric BW method.

Our sample is currently too limited to allow a robust determination of the P-L relation, defined as $M_\lambda = \alpha_\lambda (\log P - 1) + \beta_\lambda$ that would include both the slope and the log $P$ = 1 reference point $\beta_\lambda$. However, if we suppose that the slope is known *a priori* from the literature, we can still derive a precise calibration. We have considered for our fit the P-L slope measured on LMC Cepheids. This is a reasonable assumption, as it was checked successfully on the Magellanic Clouds Cepheids, and in addition our sample is currently too limited to derive both the slope and the reference point simultaneously.

For the $V$ band, we obtain $\beta_V$= –4.209±0.075 (Kervella et al. 2004b). The positions of the Cepheids on the P-L diagram are shown on Fig. 5. Our calibrations differ from GFG98 by $\Delta\beta_V$ = +0.14 mag, corresponding to +1.8σ. The sample is dominated by the high precision L Car and δ Cep measurements. When these two stars are removed from the fit, the difference with GFG98 is slightly increased, up to +0.30 mag, though the distance in σ units is reduced (+1.5). From this agreement, L Car and δ Cep do not appear to be systematically different from the other Cepheids of our sample. It is difficult to conclude firmly that there is a significant discrepancy between GFG98 and our results, as our sample is currently too limited to exclude a small-statistics bias. However, if we assume an intrinsic dispersion of the P-L relation $\sigma_{PL}\sim$ 0.1 mag, as suggested by GFG98, then our results point toward a slight underestimation of the absolute magnitudes of Cepheids by these authors. On the other hand, we obtain precisely the same log $P$ = 1 reference point value in $V$ as Lanoix et al. (1999) did, using parallaxes from Hipparcos. The excellent agreement between these two fully independent, geometrical calibrations of the P-L relation is remarkable.

Averaging our $K$ and $V$ band zero point values, we obtain a final LMC distance modulus of $\mu_0$ = 18.55 ± 0.10. This value is statistically identical to the LMC distance used for the HST Key Project, $\mu_0$ = 18.50 ± 0.10. We would like to emphasize that this result is based on six stars only, and our sample needs to be extended in order to exclude a small-number statistics bias. In this sense, the P-L calibration presented here should be considered as an intermediate step toward a final and robust determination of this important relation by interferometry.

## Surface brightness relations

The surface brightness (hereafter SB) relations link the emerging flux per solid angle unit of a light-emitting body to its colour, or effective temperature. Intuitively, the conservation of the SB can easily be understood as both the solid angle subtended by a star and its apparent brightness are decreasing with the square of its distance. In theory, for a perfect blackbody emission, their ratio is therefore a constant for a given effective temperature. In practice, the stars are not perfect blackbodies, and we have to rely on

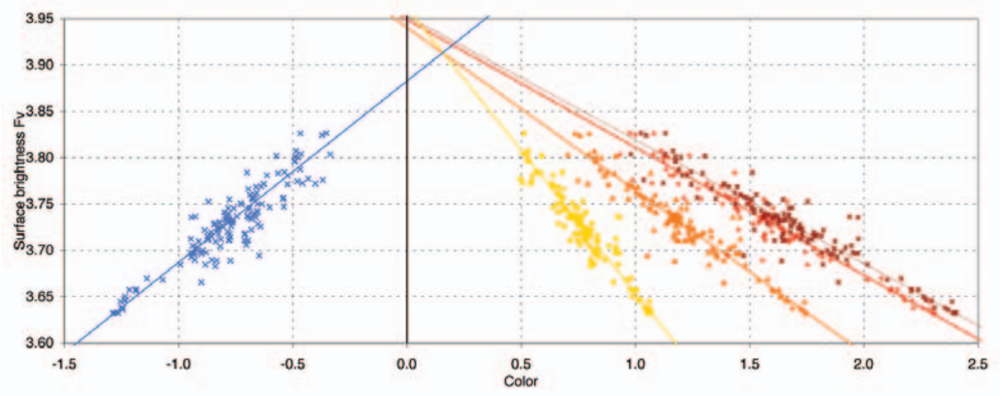


**Figure 7:** Overview of the surface brightness-colour relations in the *V* band, based on the *V-B* (blue), *V-I* (yellow), *V-J* (orange), *V-H* (red) and *V-K* (dark red) colors.

a colour index (i.e. the difference between magnitudes in two photometric bands) as a tracer of the effective temperature.

The SB relations are of considerable astrophysical interest for Cepheids, as a well-defined relation between a particular colour index and the surface brightness can provide accurate predictions of their angular diameters. When combined with the radius curve, integrated from spectroscopic radial velocity measurements, they give access to the distance of the Cepheid through the classical (non interferometric) BW method. This method has been applied recently to Cepheids in the SMC (Storm et al. 2004), i.e. at a far greater distance than what can be achieved by the interferometric BW method. But the accuracy that can be achieved on the distance estimate is conditioned for a large part by our knowledge of the SB relations.

When considering a perfect blackbody curve, any colour can in principle be used to obtain the SB, but in practice, the linearity of the correspondance between log Teff and color depends on the chosen wavelength bands. The surface brightness $F_\lambda$ is given by the following expression (Fouqué & Gieren 1997) : $F_\lambda = 4.2207 - 0.1\ m_\lambda - 0.5 \log q_{LD}$ where $q_{LD}$ is the limb darkened angular diameter, i.e. the angular size of the stellar photosphere. To fit the individual measurements, we used a linear function of the stellar color indices, expressed in magnitudes (logarithmic scale), and SB relations can be fitted using for example the following expression: $F_V(V\text{-}K) = a\ (V\text{-}K)_0 + b$. The *a* and *b* coefficients represent respectively the slope and zero point of the SB relation.

We assembled a list of 145 individual interferometric measurements of Cepheids, related to nine stars. For each measurement epoch, we estimated the *BVRIJHK* magnitudes based on Fourier interpolations of the available photometric data from the literature and corrected them for interstellar extinction.

The resulting $F_V(V\text{-}K)$ relation fit is presented in Fig. 6. The other relations based on the *V* band surface brightness $F_V$ and *BIJHK* colours are plotted on Fig. 7. The smallest residual dispersions are obtained for the infrared based colors, for instance: $F_V = -0.1336{\pm}0.0008(V{-}K){+}3.9530{\pm}0.0006$ (Kervella et al. 2004c). The intrinsic dispersion is undetectable at the current level of precision of our measurements, and could be as low as 1%.

## PROSPECTS WITH AMBER

While our present results are very encouraging, the calibration of the P-R and P-L relations as described here may still be affected by small systematic errors. In particular the method relies on the fact that the displacements measured through interferometry and through spectroscopy (integration of the radial velocity curve) are in different units (milli-arcseconds and kilometers respectively) but are the same physical quantity. This may not be exactly the case, as the regions of a Cepheid's atmosphere where the lines are formed do not necessarily move homologously with the region where the *K*-band continuum is formed. The limb darkening could also play a role at a level of ~1%. An exploration of these refined physical effects will soon be possible with the high spectral resolution mode of the AMBER instrument.

The upcoming availability of 1.8m Auxiliary Telescopes on the VLTI platform in 2004, to replace the 0.35m Test Siderostats, will allow us to observe many Cepheids with a precision at least as good as our observations of L Car (Fig. 3). In addition, the AMBER instrument will extend the VLTI capabilities toward shorter wavelengths (*J* and *H* bands), thus providing higher spatial resolution than VINCI (*K* band). The combination of these two improvements will extend significantly the accessible sample of Cepheids, and we expect that the distances to more than 30 Cepheids will be measurable with a precision better than 5%. This will provide a high precision calibration of both the reference point (down to ±0.01 mag) and the slope of the Galactic Cepheid P-L. As the galaxies hosting the Cepheids used in the HST Key Project are close to solar metallicity on average, this Galactic calibration will allow us to bypass the LMC step in the extragalactic distance scale. Its associated uncertainty of ±0.06 due to the metallicity correction of the LMC Cepheids will therefore become irrelevant for the measurement of the Hubble constant $H_0$.

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