Most of the models that have been put forward to explain the acceleration can be assigned to one of two categories. The first class of models assumes that the cosmological constant is indeed the correct theory of gravity, and accounts for the observed acceleration by postulating that, in the latter half of the Universe’s history, its mass-energy was dominated by an unusual form of energy – unusual in that it has negative pressure. In its simplest incarnation this so-called dark energy is the cosmological constant \( \Lambda \), but numerous other – some quite exotic – possibilities have been suggested (e.g. quintessence, phantom energy, Chaplygin gas), which all differ in the details of their equation of state and evolution. A feature that is common to all of these variants, however, is that it has so far proven very difficult to underpin any form of dark energy with a viable physical theory, i.e. to understand its origin and nature within the standard model of (particle) physics.

Instead of introducing a new mass-energy component, the models of the second type seek to explain the acceleration by replacing General Relativity with a different theory of gravity. Again, there are many ways in which the field equations can be modified in order to reproduce the late-time accelerated expansion, without spoiling the standard theory’s success in explaining early structure formation. In this case the challenge is to physically motivate the more complicated structure of the field equations.

Whatever the correct explanation for the acceleration will turn out to be, it is clear that it will have far-reaching implications. That is why cosmologists have taken such an intense interest in exploring different ways of measuring the expansion history of the Universe.

Observing the expansion history

Observables that depend on the expansion history include distances and the linear growth of density perturbations; so SN Ia surveys, weak lensing (Heavens, 2003) and baryon acoustic oscillations (BAO) in the galaxy power spectrum (Seo & Eisenstein, 2003) are all generally considered to be excellent probes of the acceleration.

In practice, however, extracting information on the expansion history from weak lensing and BAO requires a prior on the spatial curvature, a detailed understanding of the linear growth of density perturbations and hence a specific cosmological model. Given the uncertain state of affairs regarding the source of the acceleration, these are conceptually undesirable features and the importance of taking a cosmographic, model-independent approach to determining the expansion history is evident. Using SN Ia to measure luminosity distances as a function of redshift is Conceptually the simplest experiment and hence appears to be the most useful in this respect. The caveats are that distance is ‘only’ related to the expansion history through an integral over redshift and that one still requires a prior on spatial curvature.

The redshifts of all cosmologically distant sources are expected to experience a small, systematic drift as a function of time due to the evolution of the Universe’s expansion rate. Here, we briefly review the motivation for measuring this effect and summarise our reasons for believing that the E-ELT will be the first telescope to detect it.

Accelerated expansion

1998 was a remarkable year for astronomy. Not only did the VLT see first light, but it was also the year in which two research groups independently announced a result that would profoundly change cosmology (again), if not all of physics: the measured distances to remote type Ia supernovae seemed to indicate that the expansion of the Universe was accelerating (Riess et al., 1998; Perlmutter et al., 1999)

Since its discovery by Hubble in 1929 it had been assumed – more or less as a matter of course – that the universal expansion was forever being slowed down by the gravitational pull exerted by all of the matter in the Universe. Without any proof to the contrary, this was indeed a rather sensible assumption, because an accelerating expansion has quite fundamental consequences: it requires new physics.

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Sandage (1962) first discussed an effect that suggests an extremely direct measurement of the expansion history. He showed that the evolution of the Hubble expansion causes the redshifts of distant objects partaking in the Hubble flow to change slowly with time. Just as the redshift, z, is in itself evidence of the expansion, so is the change in redshift \((z = (1 + z)H_0 - H(z))\), evidence of its deceleration between the epoch \(z\) and today, where \(H\) is the Hubble parameter and \(H_0\) its present-day value. This equation implies that it is remarkably simple (at least in principle) to determine the expansion history: one simply has to monitor the redshifts of a number of cosmologically distant sources over several years.

This simple equation has two remarkable features. The first is the stunning simplicity of its derivation. For this equation to be valid all one needs to assume is that the Universe is homogeneous and isotropic on large scales, and that gravity can be described by a metric theory. That’s it. One does not need to know or assume anything about the geometry of the Universe or the growth of structure. One does not even need to assume a specific theory of gravity. The redshift drift is an entirely direct and model-independent measure of the expansion history of the Universe which does not require any cosmological assumptions or priors whatsoever.

The other remarkable feature of the equation is that it involves observations of the same objects at different epochs (albeit separated only by a few years or decades). Other cosmological observations, such as those of SN Ia, weak lensing and BAO also probe different epochs but are separated only by a few years or decades. These H\(_\alpha\) absorption lines are seen in the spectra of all QSOs and arise in the intervening intergalactic medium. Using hydrodynamical simulations we have explicitly shown that the peculiar motions of the gas responsible for the absorption are far too small to interfere with a redshift drift measurement (Liske et al., 2008). Similarly, other gas properties, such as the density, temperature or ionisation state, also evolve too slowly to cause any headaches. Furthermore, QSOs exist over a wide redshift range, they are the brightest objects at any redshift, and each QSO spectrum displays hundreds of lines. These are all very desirable features.

### Measuring the redshift drift with E-ELT

The trouble with the redshift drift is that it is exceedingly small. From Figure 1 we see that at \(z = 4\) the redshift drift is of the order of \(10^{-3}\) or 6 cm/s per decade! Putting meaningful data points onto Figure 1 will clearly require an extremely stable and well-calibrated spectrograph as well as a lot of photons. Let us assume that the first requirement has been met, i.e. that we are in possession of a spectrograph capable of delivering radial velocity measurements that are only limited by photon-noise down to the cm/s level. In this best possible (but by no means unrealistic) scenario, how well can we expect the E-ELT to measure the redshift drift, and hence constrain the cosmic expansion history?

First of all, we need to define where we want to measure the redshift drift. There are several reasons to believe that the so-called Lyman-\(\alpha\) forest is the most suitable target, as first suggested by Loeb (1998). These H\(_\alpha\) absorption lines are seen in the spectra of all QSOs and arise in the intervening intergalactic medium. Using hydrodynamical simulations we have explicitly shown that the peculiar motions of the gas responsible for the absorption are far too small to interfere with a redshift drift measurement (Liske et al., 2008).

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The next question is how the properties of the Lyman-\(\alpha\) forest (the number and sharpness of the absorption features), and the signal-to-noise (S/N) at which it is recorded, translate to the accuracy, \(\sigma\), with which one can determine a radial velocity shift. In order to obtain this translation we have performed extensive Monte Carlo simulations of Lyman-\(\alpha\) forest spectra. Mindful of the forest’s evolution with redshift, we have derived a quantitative relation between the \(\sigma\), of the Lyman-\(\alpha\) forest on the one hand, and the spectral S/N and the background QSO’s redshift on the other hand (Liske et al., 2008).

Now in a photon-noise limited experiment the S/N only depends on the flux density of the source, the size of the telescope (\(D\)), the total combined telescope/instrument throughput (\(\epsilon\)) and the integration time \(t_{\text{int}}\). Unfortunately, the photon flux from QSOs is not a free parameter that can be varied at will. In Figure 2 we show the fluxes and redshifts of all known high-\(z\) QSOs. Assuming values for \(\Delta y_{\text{det}}\) we can calculate the expected S/N for any given \(N_{\text{phot}}\). Combining this with a given \(\Delta_{\text{QSO}}\) and using the relation derived above, we can calculate the value of \(\sigma\), that would be achieved if \(all\) of the time \(t_{\text{int}}\) were invested into observing a single QSO with the given values of \(N_{\text{phot}}\) and \(\Delta_{\text{QSO}}\). The background colour image and solid contours in Figure 2 show the result of this calculation, where we have assumed \(D = 42\) m, \(\epsilon = 0.25\), and \(t_{\text{int}} = 2.000\) h. Note that \(t_{\text{int}}\) denotes the total integration time, summed over all epochs.

![Figure 1](image_url)

**Figure 1.** The solid lines and left axis show the redshift drift \(\dot{z}\) as a function of redshift for standard relativistic cosmology and various combinations of \(\Omega_m\) and \(\Omega_{\Lambda}\) as indicated. The dotted lines and right axis show the same in velocity units. The dashed line shows \(2\) for the case of an alternative dark energy model with a different equation of state parameter \(w_{\text{de}}\) (and \(\Omega_m, \Omega_{\Lambda} = 0.3, 0.7\)).
We can see that, although challenging, a reasonable measurement of the redshift drift appears to be possible with a 42-m telescope. The best object gives $\sigma_z = 1.8$ cm/s and there exist 18 QSOs that are bright enough and/or lie at a high enough redshift to put them at $\sigma_z < 4$ cm/s.

Figure 2 tells us which QSO delivers the best accuracy and is hence the most suitable for a redshift drift experiment. However, for many practical reasons it will be desirable to include more than just the best object in the experiment. Doing so comes at a penalty though: the more objects that are included into the experiment the worse the final result will be because some of the fixed amount of observing time will have to be redistributed from the ‘best’ object to the less suited ones.

The dependence of the full experiment’s final, overall $\sigma_z$ on the telescope diameter, system throughput, total integration time and number of QSOs is shown in Figure 3. We can see that an overall accuracy of 2–3 cm/s is well within reach of the E-ELT, even when 20 or so objects are targeted for the experiment. However, the figure also shows that for a 30-m telescope it would be very time consuming indeed to achieve an accuracy better than 3 cm/s.

To further illustrate what can be achieved we show in Figure 4 three different simulations of the redshift drift experiment. The blue dots show the results that can be expected from monitoring the 20 best QSOs over a 20-year period, investing a total of 4 000 h of observing time. By construction these points represent the most precise measurement of $z$ that is possible with a set of 20 QSOs and the given set-up. However, since many of the selected QSOs lie near the redshift where $z = 0$ this experiment does not actually result in a positive detection of the effect. If we want to detect the effect with the highest possible significance we need to choose a different set of QSOs. The yellow squares show the result of selecting the 10 best QSOs according to this criterion.

However, neither of these datasets is particularly well suited to proving the existence of accelerated expansion, i.e. of a region where $z > 0$. Ideally, this would be achieved by obtaining a $z$ measurement at $z \approx 0.7$ — were it not for the atmosphere that restricts observations of the Lyman-$\alpha$ forest to $z > 1.7$. The best thing to do is to combine a measurement at the lowest possible redshift with a second measurement at the highest possible redshift, thereby gaining the best possible constraint on the slope of $z(z)$. The brown triangles in Figure 4 show the result of a simulation using appropriately selected QSOs: clearly, given these data one could confidently conclude that $z$ must turn positive at $z = 2$ for any reasonably well-behaved functional form of $z(z)$, i.e. regardless of the cosmological model. Thus we find that a redshift drift experiment on
the E-ELT will indeed be capable of providing unequivocal proof of the existence of past acceleration.

So will these constraints be able to compete with those coming from SN Ia, weak lensing or BAO measurements in 2037? No, they will not. However, the statistical significance of its constraints is not the only criterion by which to judge the value of a cosmological experiment. As we have seen, measuring the expansion history using $z$ is not only entirely independent of all other cosmological observations, it is fundamentally different from them – indeed unique. Moreover, it does not assume spatial flatness or require any other cosmological or astrophysical assumptions whatsoever. In fact, it does not even rely on any specific theory of gravity being correct. Instead, it can provide us with the most direct and inescapable evidence of acceleration possible. In that sense the redshift drift will not only be highly competitive, it will have the edge. Even in 2037.

References

The VLTI PRIMA instrument is shown during testing in Garching. The two combined beams of the PRIMA fringe sensor unit (FSU) B are seen, in red metrology laserlight, joining the FSU’s beam combiner in the background to the fibre injection optics. The second FSU is seen to the right. The PRIMA hardware was shipped to Paranal in July 2008 and underwent assembly, integration and verification in August. On-sky commissioning will begin in Period 82, and when complete the facility is expected to provide improvements in VLTI sensitivity, along with astrometry to better than 100 microarcseconds.