The Optimal Speed for a Rechargeable Vehicle

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Abstract

This paper considers the optimal speed for a rechargeable vehicle, i.e. given a charging speed and a speed-dependent energy consumption, what is the driving speed that minimizes the sum of the driving and charging time, when a zero net change in the energy state of the vehicle is required.

1 Optimal, Constant Speed

A vehicle travels the distance s at a constant, positive speed v while constantly using a driving power $P_d(v)$ taking $t = s/v$ to cover the distance. Stop(s) need to be scheduled to charge the vehicle with the energy $E_s = tP_d(v)$ spent while driving. The charging stops are assumed to be sufficiently frequent to prevent the vehicle from running out of energy and each of a duration to maintain a constant charging power of P_c. While charging, any power consumed by the vehicle that reduces the power actually used for charging is a priori subtracted from P_c, which then is the effective charging power.

The total time spent driving and charging is thus

$$T(v) = t + \frac{E_s}{P_c} = \frac{s}{v}(1 + \frac{P_d(v)}{P_c}),$$

and the effective driving speed is

$$v_e = \frac{s}{T(v)} = v\frac{P_c}{P_c + P_d(v)}.$$

Of interest is the speed v_c that minimizes T. As such the derivative of T with respect to v is determined,

$$\begin{aligned} \frac{\partial T(v)}{\partial v} &= -\frac{s}{v^2}(1 + \frac{P_d(v)}{P_c}) + \frac{s}{v}(\frac{\partial P_d(v)}{\partial v}\frac{1}{P_c}) \\ &= \frac{s}{v}(\frac{1}{P_c}\frac{\partial P_d(v)}{\partial v} - \frac{1}{v}(1 + \frac{P_d(v)}{P_c})). \end{aligned}$$

This derivative thus has a zero when

$$\frac{1}{P_c}\frac{\partial P_d(v)}{\partial v} = \frac{1}{v}(1 + \frac{P_d(v)}{P_c}),$$

i.e. when

$$v\frac{\partial P_d(v)}{\partial v} = P_c + P_d(v). \tag{1}$$

2 Drag Equation

We now assume that the force required to keep the vehicle moving at the constant speed v is given by a very slightly generalized drag equation,

$$F = \frac{1}{2}\rho v^2 C_d A + F_c,$$

where ρ is the air density, C_d is the vehicle drag coefficient, A is the vehicle's reference area and where F_c is an additional, unspecified force independent of v.

It thus follows that the power required to keep the vehicle moving at v is the drag power $P_f(v)$, i.e.

$$P_f(v) = vF = \frac{1}{2}\rho v^3 C_d A + vF_c.$$

While consuming $P_f(v)$ for driving, the actual power consumption by the vehicle while driving may be increased by a power P_1 which is independent of the speed, for e.g. vehicle lights and infotainment system. As such, the driving power is

$$P_d(v) = P_f(v) + P_1 = \frac{1}{2}\rho v^3 C_d A + vF_c + P_1.$$

The derivative of this power with respect to v is thus

$$\frac{\partial P_d(v)}{\partial v} = \frac{3}{2}\rho v^2 C_d A + F_c.$$

The optimal speed v_c must therefore satisfy

$$v(\frac{3}{2}\rho v^2 C_d A + F_c) = P_c + \frac{1}{2}\rho v^3 C_d A + vF_c + P_1,$$

which has the single, real solution

$$v_c = \sqrt[3]{\frac{P_c + P_1}{\rho C_d A}},$$

where P_c is the effective charging power, P_1 is the speed-independent power consumption while driving, ρ is the air density, C_d is the vehicle drag coefficient and A is the vehicle reference area.

3 Limitations

The drag equation is useful at a range of speeds for very simple geometries like a sphere, for which the reference area equals the cross section. For an actual vehicle at relevant speeds, the reference area may differ significantly from the cross-section. This is especially true for irregular geometries like a vehicle with roof mounted bicycles.

In the above model the required driving power $P_d(v)$ is a third degree polynomial in v, where the quadratic term is zero.

Since any component of the driving power proportional to v is seen to not influence the optimal driving speed, the remaining limitations stem from driving power proportional to the square of v and higher order effects not described by the drag equation.

This may include power losses in the battery and drivetrain (except when proportional to the speed). The driving conditions are assumed to not change during the drive and ignore wind and precipitation, except effects that manifest themselves as F_c which could include a side wind.

Depending on e.g. ambient temperature, the power used for the cabin and battery temperature control system may consist of one component which is speed-independent and thus covered by P_1. It may also consist of a component that is proportional to the speed v, which will then not influence the optimal driving speed. If the power used for the vehicle's temperature control systems has a component which depends on non-linear powers of v, then this is not accounted for.

4 Conclusion and Examples

With the above limitations the optimal driving speed depends only on the effective charging power, the speed-independent power consumed while driving, the vehicle geometry and the air density. Any power consumption which is proportional to the speed does not influence the optimal driving speed (since driving e.g. half as fast would double the time during which this power was consumed, leading to the same energy consumption). We also note that when the speed-independent power consumption is negligible compared to the effective charging speed, then it has a negligible effect on the optimal driving speed, and when the speed-independent power consumption is significant, then this increases the optimal driving speed (since a higher speed reduces the time during which this power is spent).

With no change to the vehicle and driving conditions and a negligible speed-independent power consumption, a doubling of the optimal speed is seen to require an increase in the charging power by a factor 8. So if a charger able to deliver $P_c = 15kW$ is replaced with one able to deliver $P_c = 120kW$, then that will allow for a doubling of the optimal driving speed. Similarly, if a vehicle is able to charge with $P_c = 125kW$, it follows that a doubling of its optimal driving speed would require the charging power to be increased to $1MW$.