CHARACTERISATION OF THE OPTICAL PROPERTIES OF ATMOSPHERIC TURBULENCE

Methods for the Evaluation of an Astronomical Site

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Chapter 1

THEORETICAL BACKGROUND

1.1 Terminology

Turbulent flow Turbulent flows have some particular features in opposition to laminar flows, among which [?] an infinitely large number of exited degrees of freedom, high diffusion capabilities, a high level of vorticity and the property to be always dissipative.

Dissipation In a turbulent flow, transformation of kinetic energy into heat due to molecular friction (viscosity).

Diffusion In a turbulent flow, mixing and transport of quantities such as heat, momentum, water vapour, etc...

Turbulent eddies Fluid parcels in chaotic motion responsible for turbulent diffusion.

Conservative Passive additive A quantity characterizing an air parcel is conservative if not affected by its position in space. It is passive if it does not affect the statistics of the turbulence. It has been shown by [Corrsin 51] that the fluctuations of conservative passive additives inserted into a turbulent medium follow the same statistics as the velocity fluctuations.

Inner scale of turbulence (l_0): Spacing at which the inertial range form of the structure function equals its dissipation range form [Obukhov 49]. This definition produces slightly different results for temperature and humidity because the diffusion coefficient of water molecules in air is about 14% larger than that of heat in air [Hill 86].

Outer scale of turbulence (L_0): Largest scale size for which the eddies may be considered to be isotropic. [Clifford 78].

Taylor's frozen flow hypothesis According to which the entire spatial pattern of a random field is transported with the mean wind velocity. This is verified in the atmosphere for velocity and temperature inhomogeneities of characteristic size smaller thanL_0 [?].

Atmospheric surface layer The lowest few tens of meters of the atmospheric boundary layer where air-surface exchange processes dominate [Hill 86]. The wind direction is generally considered as constant and the driving parameter for the wind velocity profile is the surface roughness and the surrounding orography.

1.2 Thermal characteristics of the turbulent atmosphere

In a well developped turbulence - such as usually exists a few meters above ground - the thermal structure of the air is characterized by its isotropic temperature structure parameter [Clifford 78]:

$$C_T^2(\mathbf{r}) = \frac{<(T(\mathbf{r}) - T(\mathbf{r}+\Delta\mathbf{r}))^2>}{\Delta\mathbf{r}^{2/3}}\,; \mathrm{in}\,^{\circ}\mathrm{C}^2\mathrm{m}^{-2/3} \tag{1.1}$$

It is in fact an expression of the time variation of the potential temperature difference between two points in space: $<(\Theta(\mathbf{r})-\Theta(\mathbf{r}+\Delta\mathbf{r}))^2>$, which is usually written as a function of absolute temperature T assuming a constant height z above ground:

$$\Theta = T - \gamma_a z\,; \gamma_a \simeq 9.8\mathrm{K/km} \tag{1.2}$$

1.3 Index of refraction structure parameter

The atmosphere interacts with light through the spatial and temporal variations of air temperature, humidity and pressure which locally affect both real (refraction) and imaginary (absorption) parts of the instantaneous index of refraction. In the infrared, one distinguishes the contribution of the continuum (dry air and water vapour) from the anomalous dispersion due to infrared molecular resonances [Hill 80]. Moreover, apart for wavelengths very near an absorbtion line, the fluctuations of the imaginary part of the refraction index are at least two orders of magnitude below those of the real part.

The index of refraction n of the atmosphere, restricted to the contributions of the continuum, for a wavelength λ, pressure P, absolute temperature T, and water vapor pressure e is, at visible wavelengths:

$$n - 1 \;=\; 77.6\,10^{-6}(1 + 7.52\,10^{-3}\lambda^{-2})(P + 4810\,e/T)/T \tag{1.3}$$

The fluctuations of the refractivity may be expressed as a function of the fluctuations in temperature, pressure and humidity along:

$$dn(T,P,e) = \left[\frac{\partial n}{\partial T}\right]_{(P,e)} dT + \left[\frac{\partial n}{\partial P}\right]_{(T,e)} dP + \left[\frac{\partial n}{\partial e}\right]_{(T,P)} de \tag{1.4}$$

The local variations of the index n are responsible for the perturbations induced on the optical wavefront. Most authors agree to neglect humidity fluctuations for optical wavelength [Andreas 88] and pressure fluctuations at a fixed height over land. It appears that index of refraction fluctuations will be a function of potential temperature fluctuations alone. The potential temperature is a conservative passive additive, ie: it does not change the statistical laws of the turbulent medium. In this case, the same statistical law as for temperature also applies and one may define an atmospheric structure parameter for the index of refraction at $\lambda = 0.5\mu$m:

$$C_n^2 = \left(80 \cdot 10^{-6} \frac{P}{T^2}\right)^2 \; C_T^2 \, ; \mathrm{m}^{-2/3} \tag{1.5}$$

1.4 Full Width Half Maximum(FWHM)

In the presence of atmospheric turbulence, one notices that when a telescope has a diameter D larger than a value r_0, it is not anymore limited by diffraction and the standard relation in the first Airy disk of the diffraction pattern : FWHM = $1.22\lambda/D$ is not valid.

The resolution θ of a large telescope limited by the atmospheric turbulence may be defined according to the Strehl criterion:

$$\theta = (4/\pi)\lambda/r_0 \tag{1.6}$$

Where r_0, also called the Fried parameter, it is then convenient to express the effect of the atmosphere on images using the long exposure optical transfer function [Fried 66a]:

$$T(f) = \exp -3.44(\lambda f/r_0)^{5/3} \tag{1.7}$$

which corresponds to a nearly gaussian profile. The atmospheric index structure function is then related to the Fried parameter r_0 along [**?**]:

$$\mathrm{r}_0 = \left[0.423k^2(\cos\gamma)^{-1}\int C_n^2(H)\,dH\right]^{-3/5} \; ; \mathrm{m} \tag{1.8}$$

Where $k = 2\pi/\lambda$ and γ is the zenith angle .

An equivalent long exposure image size FWHM (Full Width Half Maximum) may then be defined, since it is the parameter directly accessible to the astronomer. A numerical computation gives /cite88DIA:

$$\mathrm{FWHM} = 0.976\lambda/r_0 = 5.345\lambda^{-1/5}\left[(\cos\gamma)^{-1}\int C_n^2(z)\,dz\right]^{3/5} \; ; \mathrm{radians} \tag{1.9}$$

Contrarily to r_0, the image size is not much dependent on wavelength (FWHM $\propto \lambda^{-1/5}$) and is the expression of the well known **Seeing**.

A commonly used parameter to characterize turbulence strength is the FWHM in arcsec and at $\lambda = 0.5\mu m$ for turbulence occurring between H_0 and H_1:

$$\mathrm{FWHM} = 2.0\,10^7 \left[(\cos\gamma)^{-1} \int_{H_0}^{H_1} C_n^2(z)\,dz \right]^{3/5} \text{; arcsec.} \tag{1.10}$$

1.5 Altitude and motion of turbulent layers

With the developments of new techniques to correct the optical wavefront disturbances created by the atmosphere, it appeared that site quality was not only determined by the path integral of the turbulence in the troposphere but also by the altitude at which the turbulence occurs. Scintillation and non-isoplanicity are two main parameters which prevent full compensation of wavefront errors with adaptive optics. Both are only produced by turbulence at high altitude. In addition to the standard Fried parameter r_0, scale height parameters were introduced for interferometry , speckle interferometry, and adaptive optics. All refer to the wavefront temporal (speckle lifetime) and spatial (isoplanatism) coherence.

Describing the effect of anisoplanatism on the performances of adaptive optics systems, [Fried 82] used the parameter:

$$h_F = \left[\frac{\int_0^\infty C_n^2(z) z^{5/3}\,dz}{\int_0^\infty C_n^2(z)\,dz} \right]^{3/5} \text{; m} \tag{1.11}$$

A slightly different form was given by [?] for stellar speckle interferometry with large apertures:

$$\overline{\mathrm{h}} = \left(\left[\frac{\int z^2 C_n^2(z)\,dz}{\int C_n^2(z)\,dz} \right] - \left[\frac{\int z C_n^2(z)\,dz}{\int C_n^2(z)\,dz} \right]^2 \right)^{1/2} \text{; m} \tag{1.12}$$

it is possible to determine an approximate isoplanetic patch size:

$$\theta = 0.36 \frac{r_0}{\overline{\mathrm{h}}} \tag{1.13}$$

The same formulae apply also to speckle life time, replacing altitude by wind speed as a weighing parameter [?]. For the speckle boiling time, [Aime 86] found the relation:

$$\theta = 0.47 \frac{r_0}{\Delta \mathrm{v}} \text{; s} \tag{1.14}$$

Where Δv is the dispersion of the horizontal wind speed as a function of altitude.

Finally, the index of scintillation defined as the variance of relative irradiance fluctuations as seen through a finite aperture was defined by [?] as:

$$\sigma_I^2/I^2 = 19.12\,\lambda^{-7/6}(\cos\gamma)^{-11/6}\int_0^\infty C_n^2(z)z^{5/6}\,dz \tag{1.15}$$

which holds for small apertures only (a few centimeters). For a large telescope of diameter D, one uses a slightly different relation:

$$\sigma_I^2/I^2 \propto D^{-7/3}(\cos\gamma)^{-3}\int_0^\infty C_n^2(z)z^2\,dz \tag{1.16}$$

Both relations are not valid close to the horizon where saturation occurs.

1.6 Image motion

1.6.1 Stellar image motion

Short exposures taken at the focus of a large telescope limited by the atmospheric turbulence have a speckle structure. The speckles are moving in time and one calls image motion the motion of the center of gravity of the speckles. It is a representation of the average phase fluctuations over the whole pupil of the telescope.

Its frequency spectrum extends to more than 100Hz according to wind velocities in the atmosphere. Exposure time lower than 10ms are thus required to be insured to freeze the motion.

The Fried parameter r_0 or the resulting image size FWHM are defined for long exposures where the integration time is long enough to include all motions of the individual speckles in the image. It was shown during the Lassca campaign [?] that a long exposure image could be obtained at the 2.2m telescope by adding at least 300 consecutive short exposure(speckle) images. This corresponds to a minimum equivalent exposure time of 10s.

On such a time base, the fluctuations of the angle of arrival of the wavefront obey Gaussian statistics and their variance is isotropic. It is related to the Fried parameter by [?]:

$$\sigma_x^2 = \sigma_y^2 = \sigma_{xy}^2/2 \simeq (3.44/2\pi^2)\lambda^2 D^{-1/3} r_0^{-5/3} \simeq 0.18\lambda^2 D^{-1/3} r_0^{-5/3} \tag{1.17}$$

The standard deviation of image motion is independent of the wavelength and decreases when the averaging aperture increases. It is thus possible to evaluate r_0 and consequently the image quality of a large telescope by measuring image motion through any aperture. The smaller aperture leads to the highest accuracy at equivalent signal to noise ratios.

This method is the most commonly used for site studies worldwide. It is more reliable with fixed telescopes(star trails) since mechanical spurious motions cannot be distinguished from atmospheric effects.

1.6.2 Differential image motion

This method has several advantages since the telescope spurious motions are subtracted out and because it does not need a good optical quality. The only constraint on image is the circularity of the spots for the centroiding algorithm. Contrarily to direct image size measurements which require an aperture at least two times larger than the seeing Fried parameter r_0 ($r_0 = 20\,cm$ for a 0.5 arcsec seeing at $\lambda = 0.5\,\mu m$), the differential motion monitor is not limited by the size of the telescope.

Achieving a large accuracy to be able to measure site quality even during excellent seeing conditions ($FWHM \leq 0.5 arcsec$) requires a method insensitive to mechanical vibrations or to the telescope optical quality. The problem of measuring differential angle of arrival through two relatively small aperture of variable separation has been considered by [Fried 75] and analytical solution given by [**?**].

The lateral and longitudinal covariance of angle of arrival fluctuations are computed for aperture spacing d belonging to the inertial range (from 5 to 30 cm or even a few meters):

$$< \alpha(x,y)\alpha(x+d,y) > \;=\; 0.097\lambda^2 d^{-1/3} r_0^{-5/3} \quad (1.18)$$

$$< \alpha(x,y)\alpha(x,y+d) > \;=\; 0.145\lambda^2 d^{-1/3} r_0^{-5/3} \quad (1.19)$$

The covariance of phase fluctuations is a measure of the spatial homogeneity of the phase of the wavefront. According to the inertial model of the turbulence, the lateral covariance is larger than the longitudinal covariance and thus the differential motion is smaller in the direction perpendicular to the pupil separation.

The variance of the differential motion between two pupils of diameter D at the distance d of each other is equal to the sum of the two motions (uncorrelated statistics) minus the sum of the common motions (given by the covariance function):

$$\sigma_{\|}^2 \simeq 2(0.18D^{-1/3} - 0.097d^{-1/3})\lambda^2 r_0^{-5/3} \quad (1.20)$$

$$\sigma_{\perp}^2 \simeq 2(0.18D^{-1/3} - 0.145d^{-1/3})\lambda^2 r_0^{-5/3} \quad (1.21)$$

The sensitivity of image motions to changes in seeing quality is proportional to the coefficients:

$$\begin{aligned} K_{\|} &= 2(0.18D^{-1/3} - 0.097d^{-1/3}) \\ K_{\perp} &= 2(0.18D^{-1/3} - 0.145d^{-1/3}) \\ K_{xy} &= 2(0.18D^{-1/3}) = 2K_x = 2K_y \end{aligned}$$

Computing the variation of K with pupil size for various pupil separations we see that a separation larger than 20 cm offers very little gain in sensitivity. On the other hand, using pupil diameter smaller then 3 cm decreases the accuracy of the measurement since K becomes very sensitive to pupil characteristics.

1.6.3 Relation of image motion to image size

We recall that, in general, if * represents any type of above mentioned motions:

$$\sigma_*^2 = K_* \lambda^2 r_0^{-5/3} \tag{1.22}$$

In other words, for measurements at zenith angle γ, the integrated turbulence along the zenital path is the following function of image motion:

$$\int C_n^2(H)\, dH = 0.0599\, \frac{\sigma_*^2}{K_*}\, m^{-2/3} \tag{1.23}$$

It is important to note again that the wavelength is not present in this equation, ie: for a given amount of fluctuations of the atmospheric index of refraction, the apparent motion of the image does not depend on the detector wavelength sensitivity.

Finally, site quality defined as the long exposure FWHM, is related to the variance of image motion in square radian by 1.9:

$$\begin{aligned} \mathrm{FWHM} &= 0.976\lambda/r_0 = 5.345\lambda^{-1/5} \left[\int C_n^2(z)\, dz\right]^{3/5} \quad ; \mathrm{radian} \\ &= 0.976\lambda^{-1/5} \left[\frac{\sigma_*^2 \cos\gamma}{K_*}\right]^{3/5} \quad ; \mathrm{radian} \end{aligned}$$

Expressing the FWHM in arcsec, and at $\lambda = 0.5\mu$m, one obtains the following conversion formula:

$$\mathrm{FWHM} \simeq 3.70710^6 \left[\frac{\sigma_*^2 \cos\gamma}{K_*}\right]^{3/5} \quad ; \mathrm{arcsec} \tag{1.24}$$

$$\simeq 1.55 \left[\frac{\sigma_*^{"2} \cos\gamma}{K_*}\right]^{3/5} \quad ; \mathrm{arcsec} \tag{1.25}$$

where $\sigma_*^{"}$ is the standard deviation of the image motion in units of arc seconds.

1.7 The atmospheric lightpath

1.7.1 A partition

For the purpose of site studies, the environment of an astronomical candidate site is divided into three parts: the ground layer, the boundary layer and the free atmosphere. Though no clear physical limit exist between them, such a partition is nevertheless representative of distinct regimes and distance scales. This partition implies to use several instruments probing the different atmospheric layers described below and offering some redundancy for inter-comparison [?].

The consistency of these options has been tested during a full scale campaign in February 1986, named LASSCA (LA Silla Seeing CAmpaign) and described in [?] and [?].

1.7.2 The ground layer

The ground layer turbulence is located between ground level and the first tens of meter above the summit. Because it is often produced by wind shear, the strength of the turbulence at ground level depends on summit shape, ground temperature, colour and rugosity. On large summits, it may vary from point to point if the ground constitution is not uniform. It may even be negligible at nighttime in the case of narrow ridges perpendicular to the wind direction where the thermal non-homogeneities develop on the lee side rather than on the summit. The proximity of thermal sources upwind such as buildings or large stone accretions is of course a major danger. In any case, a study of the ground layer is necessary if one wants to determine the optimal height of the telescope above ground.

While the turbulence in the stably stratified ground layer at night is often negligible and is weakly coupled to what may happen above in the boundary layer, it behaves quite in a different manner during daytime and at sunset. During daytime convective regime, the ground layer is the normal way for heat exchanges between ground and air, the turbulence has the shape of needles (plumes) pointing upwards and extends to a height of several hundred meters over the ground. Ground layer and boundary layer are then mixed together.

1.7.3 The boundary layer

Meteorologist use to name Planetary Boundary Layer the part of the atmosphere which lies under the first temperature inversion often noticeable in plains by a cloud layer. By similarity, we may define in mountainous area the boundary layer which extends to one half kilometer or more above the summit.

It is the most difficult part of the study since the turbulence at this level is not only related to summit shape and ground rugosity but also to the topography in the surroundings ten kilometers and to the wind rose. Large differences may be found from one peak to the other because of different patterns of orographic disturbances. The boundary layer carries the thermal history of its motion inside pressure fields in the shape of a few meter thick turbulent areas stacked above each other. That is why boundary layer turbulent activity is often poorly related to standard meteorological parameters, it is thus hardly predictable, and if models were to be made, they would be valid only on a given summit.

1.7.4 The high atmosphere

The high atmosphere, above one kilometer, is also called free atmosphere because it represents large scale circulation regardless of local orography. It may nevertheless present local particularities such as inversion layers in relation with vertical wind shear and high mountain ranges, or temporary large scale perturbations in the presence of the Jet Stream. Recent attempts to model refractive-index structure profile turbulence out of standard meteorological data from radiosonde seem to produce encouraging results ([**?**, Coulman 88]).

1.8 The instruments

1.8.1 Seeing monitoring

The image quality at the focus of a large telescope limited by the atmosphere is the result of an accumulation of wavefront corrugations each time the light from the star travels through one turbulent layer. This effect, also called atmospheric seeing because it impedes the ground telescopes from distinguishing fine structures and faint objects in the sky, can be studied either as a whole, or as a function of the altitude distribution of turbulence.

Only one system allows to remotely determine the altitude of the turbulent layers, it is the Scidar (Scintillation Detection and Ranging) [Azouit 86]. It is based on the spatio-temporal analysis of shadow patterns produced by double stars in the pupil plane of a large telescope. Though a transportable version with a plastic refractor has been successfully tested, it remains a rather heavy instruments due to the large aperture (80cm) required.

Speckle analysis is the only optical method which can provide information on temporal parameters [**?**] but only with large apertures (2m) and at the cost of much computing time.

In the ESO-VLT campaign, the differential image motion monitor (DIMM) was the main tool for the measurements of seeing quality. This technique, described in Chap. **??**, consists in measuring the decorrelation of the wavefront slope perturbations between two areas on the entrance pupil plane of a small telescope. From the statistics of the differential tilt, the image size of a large telescope limited by the atmosphere can be deduced. It was possible for the first time, with this method, to measure seeing accurately and in real time with a transportable telescope.

A new powerful instrument recently invented [**?**] measuring curvature rather than tilt has a shorter time response and thus permits the temporal frequency analysis of the atmospheric motion.

1.8.2 Ground layer monitoring

The ground turbulence is monitored by fast temperature sensors made out of thin platinium wire. A microthermal sensor, associated to a fast response anemometer, provides an estimate of the local temperature structure parameter every minute (§**??**). On the candidate site the actual altitude distribution of turbulence is measured from several sensors on a 30m mast as described in §**??**.

1.8.3 Boundary layer monitoring

The acoustic sounder (Sodar) described in Chap. **??** delivers a measure of the acoustic energy back-scattered by thermal turbulence as a function of altitude from 30m to 800m in 40m slices. With the help of a calibration coefficient determined experimentally ([**?**]) , one may estimate the altitude distribution of the atmospheric temperature structure parameter.

Because of absorbsion of sound by air, the sodar altitude range varies with the air relative humidity and temperature and with the acoustic frequency chosen. Though maximum range decreases in very dry sites, the Sodar was very helpful to understand the general behaviour of the boundary layer. It becomes even more useful when doppler shift is measured to monitor wind velocity. The knowledge of the wind flow and turbulence distribution up to 1000m permits to map a whole area an improves the possibilities of prediction.

1.8.4 Free atmosphere monitoring

Anybody experienced visually the scintillation of stars which is due to the combination of light having passed through different parts of the atmosphere. As described in Chap. **??**, when monitored through a small aperture, the index of scintillation at the zenith, is related to the temperature structure parameter, with a weighting function such that the scintillation is much stronger if the same turbulence occurrs at a higher altitude.

We use a scintillometer developed by [Azouit 86], which points the same star as the seeing monitor. Scintillation presents a determinate seasonal trend, being more important in winter when the wind velocity at the level of the tropopause is at the highest.